Aufgabe 3. geg: $z^4 + 4z^2 + 16 = 0$

Substitution: $u = z^2$

$\Rightarrow u^2 + 4u + 16 = 0$

$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow u = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 16}}{2}$ //

$\frac{-4 \pm \sqrt{-48}}{2}$

$u = \frac{-4 \pm i\sqrt{48}}{2}$

$u = -2 \pm i\sqrt{12}$

$u = -2 \pm 2i\sqrt{3}$

Rücksubstitution: $u = z^2$

$z^2 = -2 \pm 2i\sqrt{3}$

$\Rightarrow z_1 = \sqrt{-2 - 2i\sqrt{3}} = -1 + i\sqrt{3}$

$z_2 = -\sqrt{-2 - 2i\sqrt{3}} = 1 + i\sqrt{3}$

$z_3 = \sqrt{-2 + 2i\sqrt{3}} = -1 - i\sqrt{3}$

$z_4 = -\sqrt{-2 + 2i\sqrt{3}} = 1 - i\sqrt{3}$

Gaussschen Zahlenebene: