

$$1) \quad a) \quad \|A\|_{\infty} = 3$$

$$A^{-1} = \begin{pmatrix} -1 & 0 & 20'000 \\ 0 & 1 & 0 \\ 1 & 0 & -10'000 \end{pmatrix} \quad \|A^{-1}\|_{\infty} = 20'001$$

$$\text{cond}(A) = \|A\|_{\infty} \cdot \|A^{-1}\|_{\infty} = 60'003$$

b) Relativ:

$$\frac{\|x - \tilde{x}\|_{\infty}}{\|x\|_{\infty}} \leq \text{cond}(A) \cdot \frac{\|b - \tilde{b}\|_{\infty}}{\|b\|_{\infty}} \leq 0.01$$

$$\|b\|_{\infty} = 1$$

$$60'003 \cdot \frac{\|b - \tilde{b}\|_{\infty}}{1} \leq 0.01 \quad \Rightarrow \quad 60'003 \cdot \|b - \tilde{b}\|_{\infty} \leq 0.01$$

$$\|b - \tilde{b}\|_{\infty} = 1.66583 \cdot 10^{-7}$$

$$b - \tilde{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ \epsilon \end{pmatrix} \Rightarrow \epsilon \leq 1.66583 \cdot 10^{-7}$$

$$d) \quad x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\tilde{x} = \begin{pmatrix} -0.99666683 \\ 0.99833342 \end{pmatrix}$$

$$\delta x = \tilde{x} - x = \begin{pmatrix} 0.00333317 \\ 0 \\ -0.00166658 \end{pmatrix}$$

$$\|\delta x\|_\infty = 0.00333317$$

$$\frac{\|\delta x\|_\infty}{\|x\|_\infty} = 0.00333317$$

$$d) \quad \|A - \tilde{A}\|_\infty = 3 \cdot 10^{-7}$$

$$f = \text{cond}(A) \cdot \frac{\|A - \tilde{A}\|_\infty}{\|A\|_\infty} = 60003 \cdot \frac{3 \cdot 10^{-7}}{3} = 0.0060003$$

$$\frac{\|x - \tilde{x}\|_\infty}{\|x\|_\infty} \leq \frac{\text{cond}(A)}{1-f} \left( \frac{\|A - \tilde{A}\|}{\|A\|} + \frac{\|b - \tilde{b}\|}{\|b\|} \right)$$

$$\tilde{x} = \begin{pmatrix} -0.99866806 \\ 0.99999998 \\ 0.99933398 \end{pmatrix}$$

$$\delta x = x - \tilde{x} = \begin{pmatrix} -0.00133194 \\ 0.00000002 \\ 0.00066602 \end{pmatrix}$$

$$\frac{\|\delta x\|_\infty}{\|x\|_\infty} = 0.00133194$$

$$\frac{\text{cond}(A)}{1-f} \left( \frac{\|A - \tilde{A}\|}{\|A\|} + \frac{\|b - \tilde{b}\|}{\|b\|} \right) = \frac{60'003}{1-0.002} \cdot \left( \frac{3 \cdot 10^7}{3} \right)$$

$$+ \epsilon)$$

$$\hookrightarrow \epsilon \leq 6.56583 \cdot 10^{-8}$$