Physics 2
Homework 7
Chapters 37-40

1. (10 pts) Muons created by collisions of high energy cosmic rays with Earth's upper atmosphere pass through the atmosphere and arrive at Earth's surface. The average lifetime of a muon at rest is $\tau = 2.2\ \mu$s. The muons are created a distance $d \approx 10$ km above the surface. The mass of a muon is 105.7 MeV/c^2.

   a) About how far can a muon travel in $\Delta t = 2.2\ \mu$s, if it moves at nearly the speed of light? Can a muon travel the distance from the upper atmosphere to Earth's surface in this time?

   b) How far, on average, can a muon travel before decaying, if it moves at $v = 0.999c$? Can the muon travel from the upper atmosphere to Earth's surface before decaying? Why or why not?

   c) Explain how the trip is possible, as observed in the rest frame of the muon. Justify your answer with explicit calculation.

2. (10 pts) Newton's Second Law, $\boldsymbol{F} = d\boldsymbol{p}/dt = d(\gamma m\boldsymbol{v})/dt$, is equivalent to $\boldsymbol{F}_\parallel = \gamma^3 m\boldsymbol{a}_\parallel$ and $\boldsymbol{F}_\perp = \gamma m\boldsymbol{a}_\perp$, where the subscripts indicate vector components parallel and perpendicular to the velocity $\boldsymbol{v}$.

   a) Consider a particle of mass m moving in one dimension under the influence of a net force of constant magnitude F pointing in the direction of motion. Suppose the particle starts from rest at time $t = 0$. Calculate the speed v of the particle at time t. Show that the speed $v < c$ for all times t, and that $v \to c$ as $t \to \infty$.

   b) Consider a particle of mass m and charge q in a uniform magnetic field of magnitude B. The velocity of the particle is perpendicular to the magnetic field. Find the radius r of the path in terms of m, q, B, and v. Find the period T of the motion, in terms of the same quantities. Show that the results reduce correctly for $v \ll c$. You may assume energy losses due to radiation are negligible.

3. (10 pts) High energy particle accelerators convert part of the energy of colliding particles into the masses of particles produced in the collisions. Consider a collision of two protons that produces two charged kaons. The mass of the proton is $m_p = 938.3$ MeV/c^2, and the mass of each kaon is $m_K = 493.7$ MeV/c^2. The reaction is

$$p + p \to p + p + K^+ + K^-$$

a) The total energy (kinetic energy and rest energy) and total momentum is conserved. Suppose one of the protons is at rest in the laboratory frame. Calculate the minimum kinetic energy that the other proton must have (in the laboratory frame) just before the collision if the reaction is to occur. You may use a Lorentz transformation to relate the velocities in the laboratory frame to the reference frame in which the total momentum of the two protons is zero.

b) Find the ratio of the minimum kinetic energy of part (a) to the rest energy of the two kaons.

c) Suppose the two protons move in the laboratory frame at the same speed just before the collision. How much (kinetic) energy must be imparted to the protons for the reaction to proceed? Repeat part (b) for this scenario.

d) Energy frontier particle accelerators are designed to collide beams of particles moving in opposite directions, not beams of particles with fixed targets. Why?

4. (10 pts) The coordinates of two events are (x_1, t_1) and (x_2, t_2) in the reference frame S, and (x'_1, t'_1) and (x'_2, t'_2) in S'. The events occur on the x-axis of S, S' moves at velocity $\boldsymbol{u} = u\hat{\mathbf{\imath}}$ relative to S, where $u > 0$, and the axes of S and S' coincide at $t = t' = 0$. Let $\Delta x = x_2 - x_1$, $\Delta t = t_2 - t_1$, $\Delta x' = x'_2 - x'_1$, and $\Delta t' = t'_2 - t'_1$. Event 2 follows Event 1 in S; *i.e.*, $\Delta t > 0$.

   a) Suppose $\Delta x > c\Delta t$. Show that the order of the events depends on the frame of reference. Find the velocities u of S' relative to S for which (i) Event 1 precedes Event 2, (ii) Event 1 follows Event 2, and (iii) Events 1 and 2 are simultaneous. Can either event cause the other? Why or why not?

   b) Suppose $\Delta x < c\Delta t$. Show that (i) the order of the events is the same in all reference frames S', regardless of the velocity u, and (ii) there exists a reference frame in which the two events occur at the same location. Find the velocity u for this frame. Can either event cause the other, in principle? Explain.

5. (10 pts) An electron at rest is struck by a photon.

   a) If the photon has wavelength $\lambda = 0.0900$ nm and recoils backward after the collision, find the magnitude of the momentum of the electron immediately after the collision. What are the initial and final energies of the photon? the final kinetic energy of the electron?

   b) If the photon has wavelength $\lambda = 0.1050$ nm and scatters at an angle of $\pi/3$ radians, find the momentum (magnitude and direction) of the electron immediately after the collision. What are the initial and final energies of the photon? the final kinetic energy of the electron?

6. (10 pts) A block of mass m rests on a horizontal, frictionless surface. The block is attached to a Hooke's Law spring of constant k, oriented horizontally. Let x be the displacement of the block from its equilibrium position, and let p be the x-component of its momentum. Assume the motion is one dimensional, and the

speed $v \ll c$.

a) Write down the energy E of the block as a function of its momentum p and displacement from equilibrium x. Define the potential energy $U(x)$ to be zero for $x = 0$.

b) In quantum systems the uncertainty in observables is often (1) comparable to their values and (2) close to the theoretical minimum allowed by the Heisenberg Uncertainty Principle, $\Delta x\, \Delta p \geq \hbar/2$. Estimate the order of magnitude of the minimum possible energy E of the mass-spring oscillator. Show that the minimum energy approaches zero as $\hbar \to 0$.

c) What value of x gives the value of E found in part (b)? Find the ratio of the kinetic energy K to the potential energy U for this x.

7. (10 pts) The Sun radiates energy at the rate $P = 3.86 \times 10^{26}$ W. Blue supergiants have surface temperatures around $T = 30,000$ K and radiate energy in the visual part of the electromagnetic spectrum at rates $100,000$ times greater than the Sun.

a) Stars can be (well) approximated as blackbodies. At what wavelength does the intensity of a blue supergiant reach its maximum value? Why are blue supergiants blue?

b) Suppose the total power radiated by a blue supergiant is $100,000$ times greater than the total power radiated by the Sun. The radius of the Sun is $R = 6.96 \times 10^5$ km. Calculate the radius of a blue supergiant, in units of the solar radius R.

c) Consider the assumption of part (b). Is the power radiated in the visual part of the spectrum proportional to the total power radiated? Explain.

8. (10 pts) Consider a particle of mass m moving in one dimension under the influence of a potential $U(x)$ that is an even function of position x, $U(-x) = U(x)$.

a) Show that if $\psi(x)$ satisfies the time-independent Schrödinger wave equation for some energy E, then $\psi(-x)$ also satisfies the wave equation, for the same value of E.

b) Recall the wave functions and allowed energies for the infinite square well. The energies are nondegenerate; there exists only a single (linearly independent) wave function for each allowed energy. In one spatial dimension, this feature persists for all potentials. Therefore, if $\psi(x)$ and $\psi(-x)$ satisfy the wave equation for the same energy E, then $\psi(-x) = C\psi(x)$, for some constant C. This relation holds for all x, $-\infty < x < \infty$. Show that only two values for C are possible, namely, $C = \pm 1$. Verify that the wave functions for the infinite square well are even or odd about the center of the well.

c) Consider the potential for the simple harmonic oscillator. The wave function of the ground state is even, with no nodes (points where $\psi(x) = 0$). The first excited state is odd, with one node. The parity (even or odd) of the wave function for each energy level alternates as the energy increases from one level to the next. Sketch the first four wave functions. Include the correct number of nodes and exponential decay into the classically forbidden regions.

9. (10 pts) Consider a particle of mass m moving in an infinite square well in one spatial dimension. The particle is free to move between $x = 0$ and $x = L$, and the wave function is

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar}\,,$$

where the constants c_n obey the condition

$$\sum_{n=1}^{\infty} |c_n|^2 = 1\,,$$

and the functions $\psi_n(x)$, $n = 1, 2, 3 \ldots$ are the stationary states corresponding to the allowed energies E_n,

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}\,,$$

$$E_n = \frac{n^2 h^2}{8mL^2}\,.$$

The wave function $\Psi(x,t)$ satisfies the time-dependent Schrödinger wave equation, for any choice of the constants c_n.

a) Note that the wave functions for the stationary states are not only normalized, but also *orthogonal*; that is,

$$\int_{-\infty}^{\infty} dx\, \psi_n^*(x)\psi_m(x) = 0 \quad \text{for } m \neq n\,.$$

Show that the wave function $\Psi(x,t)$ is normalized; that is,

$$\int_{-\infty}^{\infty} dx\, |\Psi(x,t)|^2 = 1\,.$$

b) Consider the case $c_1 = c_3 = 1/\sqrt{2}$, $c_n = 0$ for $n \neq 1, 3$. Find the probability distribution function for this case; does $\Psi(x,t)$ for this case represent a stationary state? Why or why not?

c) For the case of part (b), find the (angular) frequency of oscillation of the probability distribution function. What is the physical interpretation of this oscillation frequency?

10. (10 pts) Consider the beryllium atom. Write down the electron configuration of the ground state ($1s^2$, $2s^2$, …); identify the next two atoms, in order of increasing atomic number Z, with chemical properties similar to beryllium; and write down their ground-state electron configurations.