Physics 1
Homework 6
Chapters 18-20

1. (2 pts) The star Rigel A, a blue-white supergiant in the constellation Orion, radiates energy at about 4.6×10^{31} W. If the surface temperature is $12,000$ K, and the emissivity is approximately one (a good approximation), calculate the radius of Rigel. Express your answer in astronomical units.

2. (6 pts) A brass ring has diameter $d_B = 10.00$ cm at a temperature of 20°C. The ring is heated until it is large enough to slip over an aluminum rod that has diameter $d_A = 10.01$ cm at 20°C. Assume the coefficients of linear expansion are constant with respect to temperature.

   a) To separate the ring and rod, the system is cooled. At what temperature will they separate? Is this physically possible?

   b) Repeat part (a) for an aluminum rod of diameter $d_A = 10.02$ cm at 20°C.

3. (4 pts) Two metal bars with the same spatial dimensions are placed in contact side by side. The first bar is gold, and the second is silver. On one side of the two bars is a heat reservoir at temperature 80.0°C. On the other side is a heat reservoir at temperature 30.0°C. Heat can flow between the reservoirs and the bars, but insulation is wrapped around the bars to prevent heat from otherwise flowing into or out of the system. Find the temperature between the two bars when the flow of heat has reached a steady state.

4. (3 pts) An aluminum container of mass $M = 300$ g holds water with mass $m_1 = 200$ g. Initially, the container and water are at 10°C. Then water with mass $m_2 = 100$ g at 100°C is added to the container. What is the final temperature of the system?

5. (5 pts) Ice with mass $m_I = 250$ g and temperature $T_I = 0$°C is added to water with mass $m_W = 600$ g and temperature $T_W = 18$°C. The system reaches thermal equilibrium within an insulated container.

   a) What is the equilibrium temperature?

   b) What mass of ice remains?

6. (8 pts) Consider a closed rectangular path in a pV diagram. The path begins at point A, at the upper left corner of the path, with minimum volume and maximum pressure. An isobar connects point A to point B, at the upper right of the path, with maximum volume and pressure. An isochor connects point B to point C, at

the lower right of the path, with maximum volume and minimum pressure. An isobar connects point C to point D, at the lower left of the path, with minimum volume and pressure. An isochor connects point D to point A. A curved path within the rectangle begins at point A and ends at point C; along this path the pressure decreases with increasing volume. When a gas is taken from A to C along the curved path, its internal energy changes by $\Delta E_{AC} = 800$ J. When the gas is taken along the path ABC, the work it does is $W_{ABC} = 500$ J.

a) How much heat is added to the system along the path ABC?

b) Suppose the pressure at A is five times greater than the pressure at C. How much work does the gas do as it is taken from C to D?

c) The gas is taken along path CDA. How much heat is exchanged with the environment?

d) The change in the internal energy of the gas along the isochor from D to A is $\Delta E_{DA} = 500$ J. How much heat is added to the system as it goes from C to D?

7. (5 pts) An ideal gas is taken through a cycle that is rectangular in a pV diagram. The system begins at pressure p_0, volume V_0, and temperature T_0. The pressure then increases along an isochor until the pressure is $3p_0$. The volume then increases along an isobar until it reaches $3V_0$. The pressure then decreases along an isochor until it reaches the initial pressure. Finally, the volume decreases along an isobar until the system reaches its initial state.

   a) What is the net work per cycle done by the gas?

   b) Find the net heat absorbed by the gas per cycle.

   c) Suppose $T_0 = 0°$C and the number of moles of gas is $n = 1$ mol. What is the numerical value of the net work per cycle done by the gas?

8. (6 pts) The temperature of one mole of a monatomic, ideal gas is initially $T_0 = 300$ K. The gas absorbs heat $Q = 500$ J in an isochoric process. The gas then loses the same amount of heat in an isobaric process.

   a) What is the final temperature of the gas?

   b) How much work is done by the gas? How much work is done on the gas?

9. (6 pts) Assume air can be modeled as an ideal, diatomic gas. Given n moles of air at temperature T_0 and pressure p_0, how much energy must be expended as work to compress it to one tenth of its original volume, if

   a) the compression occurs along an isotherm?

   b) the compression occurs along a reversible adiabatic path?

   c) Evaluate your results to parts (a) and (b) for $n = 5.00$ mol, $T_0 = 20.0°$C, and $p_0 = 1.00$ atm. Express your answers in kilojoules (kJ).

10. (4 pts) Determine the heat capacity of two moles of an ideal, diatomic gas at constant volume if

    a) the rotational degrees of freedom are active;

    b) the rotational and vibrational degrees of freedom are active.

    c) For parts (a) and (b), find the heat capacity at constant pressure.

11. (6 pts) A Carnot engine running in reverse is an ideal refrigerator. In a reversed Carnot cycle, heat Q_C is extracted from a cold reservoir and heat Q_H is released into a hot reservoir.

    a) Prove that the work needed to operate the refrigerator is

    $$W = \frac{T_H - T_C}{T_C} Q_C \,.$$

    b) Show that the coefficient of performance is

    $$\mathrm{COP} = \frac{T_C}{T_H - T_C} \,.$$

12. (5 pts) 273 g of pure liquid water at 0°C is placed in contact with a thermal reservoir at −1°C. The water and reservoir are separated after all the water has become ice at 0°C.

    a) Find the change in the entropy of the water. What is the physical significance of the sign of your result?

    b) Show that the answer to part (a) is consistent with the second law of thermodynamics.

13. (5 pts) A mole of diatomic, ideal gas expands, doubling its volume. During the expansion, the pressure is directly proportional to the volume. Find the change in entropy of the gas.

14. (8 pts) A mole of monatomic, ideal gas has initial pressure $p_0 = 1.0$ atm and initial volume $V_0 = 10$ L. The gas undergoes an isochoric process, and the pressure increases by a factor of five. The gas then expands along a reversible isotherm until the pressure returns to its original value. The gas then contracts along an isobar until it reaches its original state. For this cycle,

    a) What is the net work done by the gas?

    b) How much heat is absorbed by the gas?

    c) How much heat is released by the gas?

    d) What is the efficiency?

15. (5 pts) Consider n moles of monatomic, ideal gas. The gas begins at temperature T_0. The gas then follows an isochor to a higher temperature $3T_0$. The gas then doubles

its volume by expanding along an isotherm. The temperature of the gas decreases to its original value along an isochor. Finally, the gas returns to its original volume along an isotherm.

a) What is the net heat absorbed by the gas during this cycle?

b) What is the efficiency of an engine in this cycle?

16. (8 pts) A mole of ideal, monatomic gas is taken through a cycle that is rectangular in a pV diagram. The system begins at pressure p_0, volume V_0, and temperature T_0. The pressure then increases along an isochor until the pressure is $3p_0$. The volume then increases along an isobar until it reaches $2V_0$. The pressure then decreases along an isochor until it reaches the initial pressure. Finally, the volume decreases along an isobar until the system reaches its initial state. In terms of T_0, find

    a) the heat that enters the gas per cycle,

    b) the heat released by the gas per cycle,

    c) the efficiency of an engine in this cycle,

    d) the efficiency of an engine in a Carnot cycle with the same minimum and maximum temperatures.

17. (2 pts) A mass $m = 250$ g of water is heated from 20°C to 80°C. Find the change in the entropy of the water.

18. (3 pts) In an isobaric process, n moles of an ideal gas triples in volume. What is the change in entropy of the gas?

19. (4 pts) Suppose the pressure increases from 1.00 atm to 20.0 atm during the compression stroke of a gasoline engine, that the process is a reversible, adiabatic one, and that the gas may be treated as ideal, with $\gamma = 7/5$.

    a) What is the ratio of the final volume to the initial volume?

    b) What is the ratio of the final temperature to the initial temperature?

20. (5 pts) The air-standard Diesel cycle is an idealization of the cycle in a Diesel engine. At the point of maximum compression (minimum volume), fuel is injected into the cylinder. The combustion step is approximated by expansion of the gas in the cylinder along an isobar. The combustion is followed by an adiabatic expansion to maximum volume, during which the cylinder supplies power. The power step is followed by the exhaust step, which can be represented by an isochoric process to minimum pressure. Intake begins at this minimum pressure and is followed by compression along an adiabatic path to the point of maximum compression. Let A be the point of minimum pressure, B be the point of minimum volume, C be the point reached after isobaric expansion, and D be the point of maximum volume reached during the adiabatic expansion. Show that the efficiency of an engine in the air-standard Diesel cycle is

$$e = 1 - \frac{1}{\gamma}\left(\frac{T_D - T_A}{T_C - T_B}\right) .$$