1. Show

(a) $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{k}\times\hat{i}=\hat{j}$ (2pt),

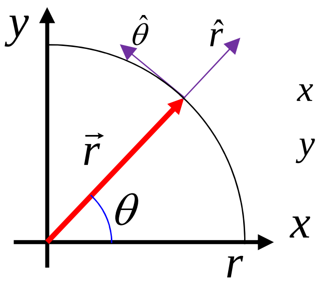
(b) $\vec{a}\cdot(\vec{b}\times\vec{c})=\vec{b}\cdot(\vec{c}\times\vec{a})=\vec{c}\cdot(\vec{a}\times\vec{b})$ (2pt), (c) $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}(\vec{a}\cdot\vec{c})-\vec{c}(\vec{a}\cdot\vec{b})$ (2pt),

(d) $\frac{d}{dt}(\vec{a}\cdot\vec{b})=\frac{d\vec{a}}{dt}\cdot\vec{b}+\vec{a}\cdot\frac{d\vec{b}}{dt}$ (2pt) (e) $\frac{d}{dt}(\vec{a}\times\vec{b})=\frac{d\vec{a}}{dt}\times\vec{b}+\vec{a}\times\frac{d\vec{b}}{dt}$ (2pt).

2. The following figures show two-dimensional “polar coordinates” (left) and three-dimensional “spherical coordinates” (right).

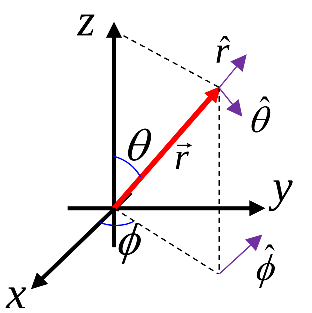
Show that the velocity vectors can be written as

polar : $\mathrm{v}=\hat{r}\frac{dr}{dt}+\hat{\theta}(r\frac{d\theta}{dt})$, spherical : $\mathrm{v}=\hat{r}\frac{dr}{dt}+\hat{\phi}(r\frac{d\phi}{dt}\sin\theta)+\hat{\theta}(r\frac{d\theta}{dt})$ (6pt).



$x=r\cos\theta$

$y=r\sin\theta$

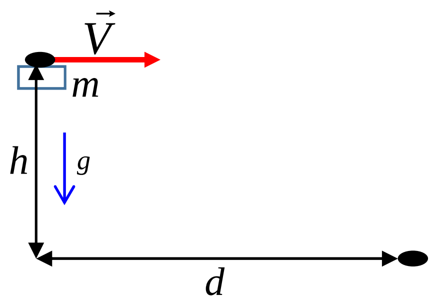


$x=r\sin\theta\cos\phi$

$y=r\sin\theta\sin\phi$

$z=r\cos\theta$

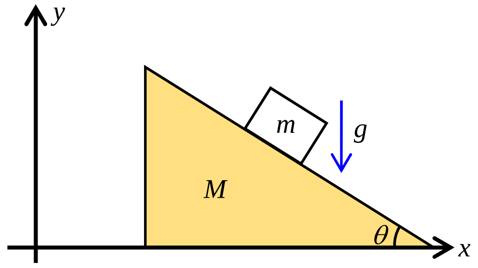
3. An African swallow releases an object (mass m) during the flight at a constant height of h to hit a target on the ground at the horizontal distance d. What should be the speed V of the loaded swallow? (6pt) What is the angle between the moving direction of the dropped object and the horizon at the impact? (4pt)



4. Consider a counterclockwise circular motion of which radius is a and the speed is $\mathrm{v}_0+\mathrm{v}_0\sin\omega t$, where v_0 and ω are constants. Find the acceleration of the motion. If the period of circulation is $8\pi/\omega$, find the relation between v_0 and ω (10pt).

5. Consider a moving particle. (a) Show that the position relative to the center of the circle is perpendicular to the velocity when the motion is circular (4pt). (b) Show that the acceleration and the velocity are perpendicular for a motion with a constant speed (6pt).

6. A block of mass m is sliding on a frictionless wedge of mass M which is movable along a horizontal plane as shown in the following figure. What is the accelerations of the wedge and the block? Assume that the motions are confined in a plane and the size of the block is negligible (15pt).

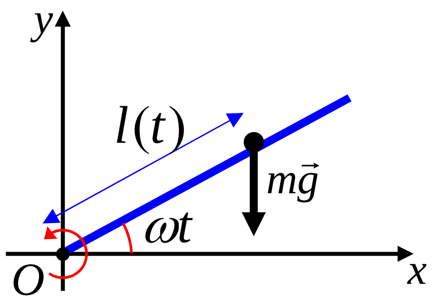


7. A stone of mass m is suspended on a vertically standing rod by a string of constant length l . The stone is in a uniform circular motion (with period T) of which trajectory is parallel to the ground. Find the angle between the rod and the string. Neglect friction (10pt).

8. Consider a small bead of mass m moving on a frictionless inclined plane that makes an angle θ with the ground. Let the initial velocity $\vec{v}_0$ of the bead is parallel to the ground. Find the trajectory of the motion. You can use any coordinate system you like (10pt).

9. A small bead of mass m under gravity is sliding along a frictionless rod, which is rotating around a fixed point at one end with a constant angular velocity ω as the following figure. $l\,(t)$ is the distance from the bead to the center of rotation. Show that

$$\frac{d^2 l(t)}{dt^2} - \omega^2 l(t) + g \sin \omega t = 0$$



and verify that $l(t) = A\cosh \omega t + B\sinh \omega t + (g \sin \omega t)/(2\omega^2)$ satisfies the above condition. Neglect the size of the bead (15pt).

※ $\cosh x = \dfrac{e^x + e^{-x}}{2}$, $\sinh x = \dfrac{e^x - e^{-x}}{2}$