Ch. 13-2 Hash Tables

- Array or linked list
  – Overall $O(n)$ time
- Binary search trees
  – Expected $O(\log n)$-time search, insertion, and deletion
  – But, $O(n)$ in the worst case
- Balanced binary search trees
  – Guarantees $O(\log n)$-time search, insertion, and deletion
  – Red-black tree, AVL tree
- Balanced k-ary trees
  – Guarantees $O(\log n)$-time search, insertion, and deletion w/ smaller constant factor
  – 2-3 tree, 2-3-4 tree, B-trees
- Hash table
  – Expected $O(1)$-time search, insertion, and deletion

- Stack, queue, priority queue
  - do not support *search* operation
  - i.e., do not support *dictionary*
- But, hash table does not support finding the minimum (or maximum) element
- Applications that need radically fast operations
  - 119 emergent calls and locating caller's address
  - Air flight information system
  - 주민등록 시스템

Figure 1
Address calculator

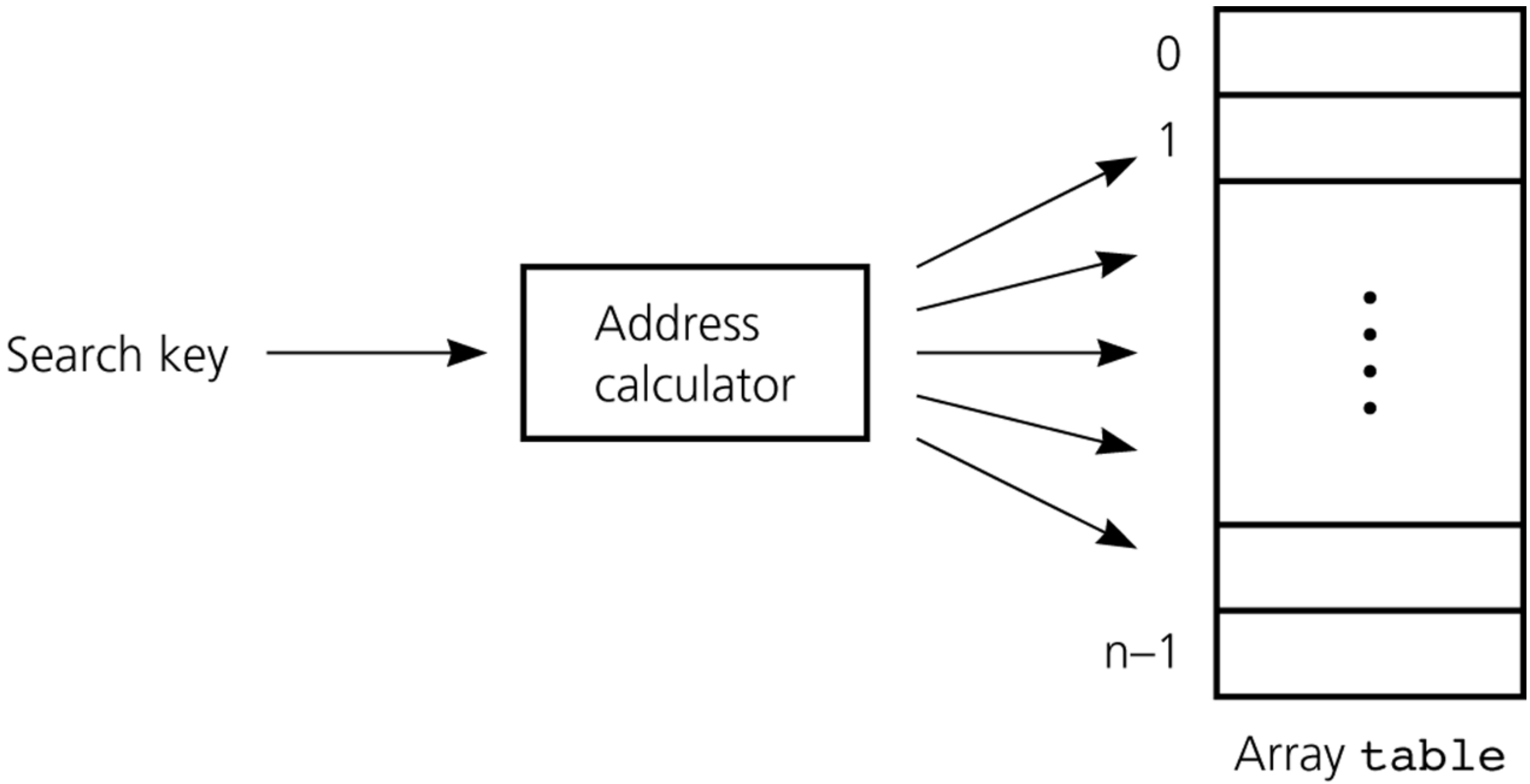
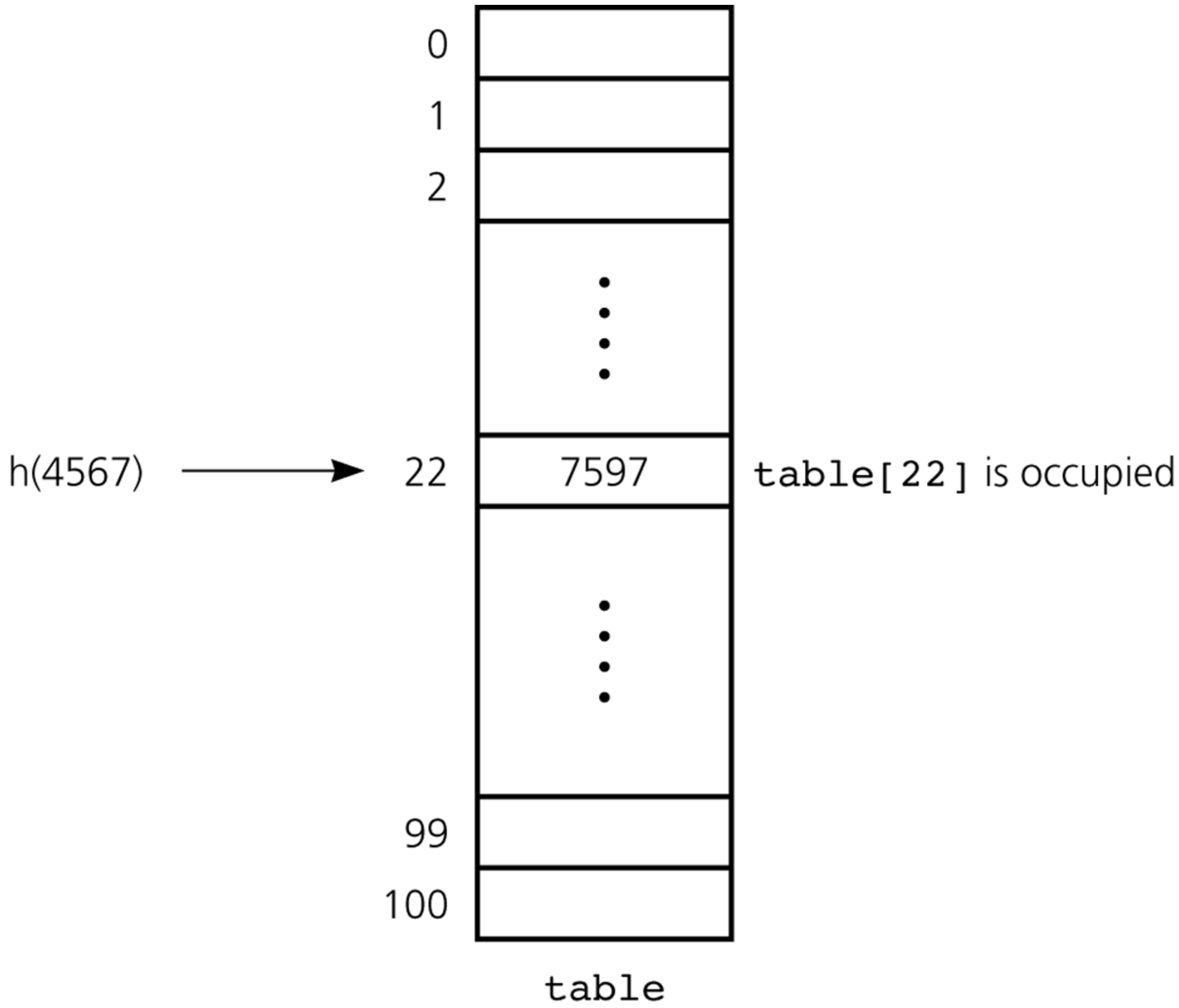


Figure 2

A collision



Insert

```
tableInsert(x)
{  // A[ ]: hash table, x: new key to insert
     if (A[h(x)] is not occupied) {
           A[h(x)] = x;
     } else {
           Find an appropriate index i by a collision-resolution method;
           A[i] =x ;
     }
}
```

Hash Functions

- Toy functions
  - Selection digits
    - $h(001364825) = 35$
  - Folding
    - $h(001364825) = 1190$
- Modulo arithmetic
  - $h(x) = x \bmod tableSize$
  - *tableSize* is recommended to be prime
- Multiplication method
  - $h(x) = (xA \bmod 1) * tableSize$
  - A: constant in (0, 1)
  - *tableSize* is not critical, usually 2^p for an integer p

Collision Resolution

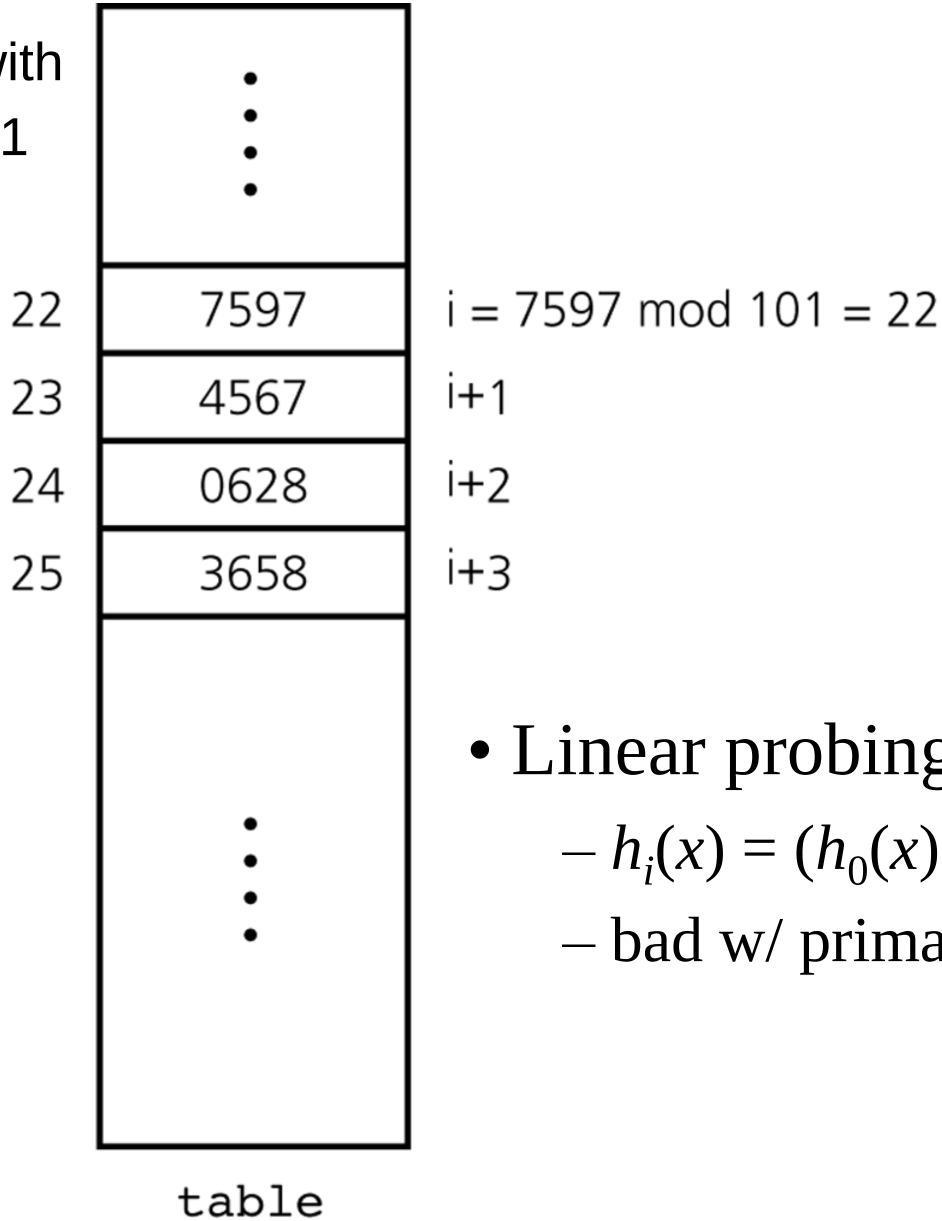
- Collision
  - The situation that two keys are mapped into the same location in the hash table
- Collision resolution
  - resolves collision by a seq. of hash values
  - $h_0(x)$, $h_1(x)$, $h_2(x)$, $h_3(x)$, …

Collision-Resolution Methods

- Open addressing (resolves in the array)
  - Linear probing
    - $h_i(x) = (h_0(x) + i) \bmod tableSize$
  - Quadratic probing
    - $h_i(x) = (h_0(x) + i^2) \bmod tableSize$
  - Double hashing
    - $h_i(x) = (\alpha(x) + i \bullet \beta(x)) \bmod tableSize$
    - $\alpha(x)$, $\beta(x)$: hash functions
- Separate chaining
  - Each *table*[*i*] is maintained by a linked list

Figure 3

Linear probing with
$h(x) = x \bmod 101$

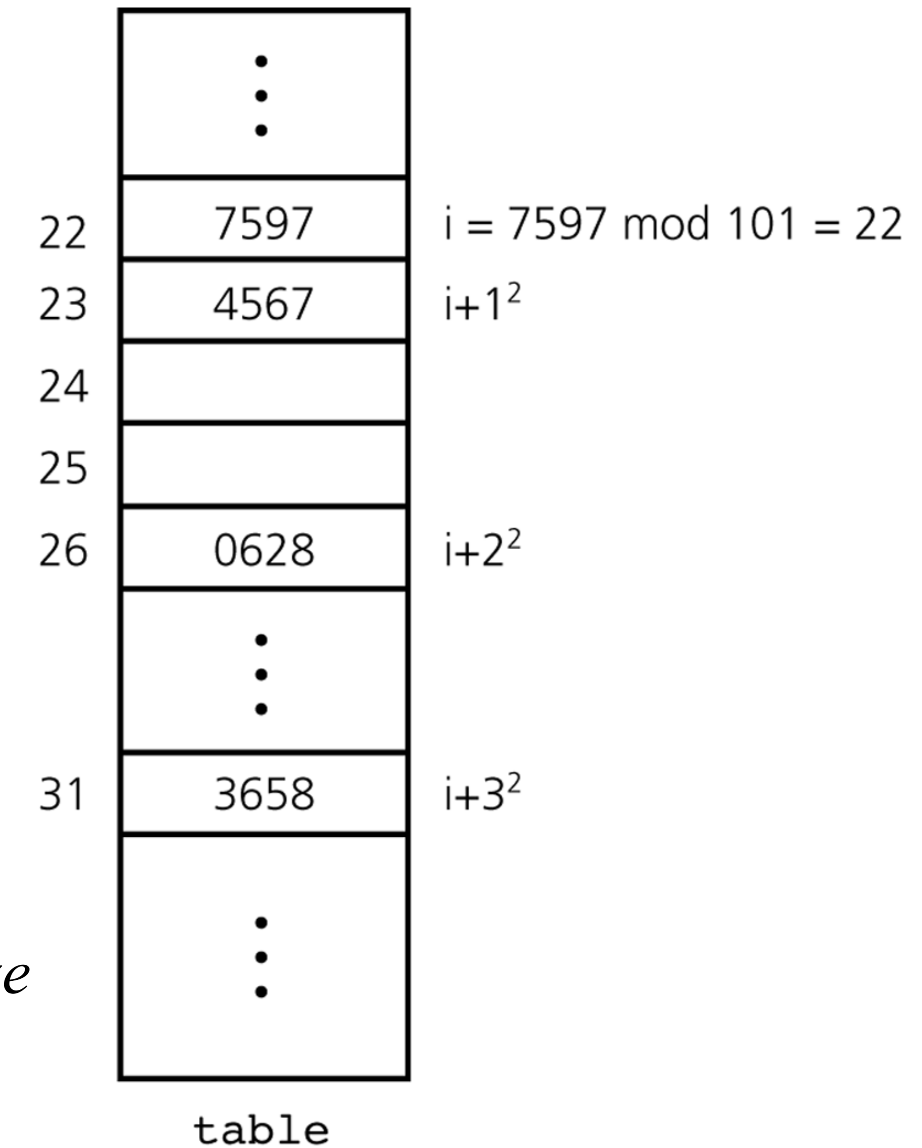


- Linear probing
  - $h_i(x) = (h_0(x) + i) \bmod tableSize$
  - bad w/ primary clustering

Figure 4

Quadratic probing with

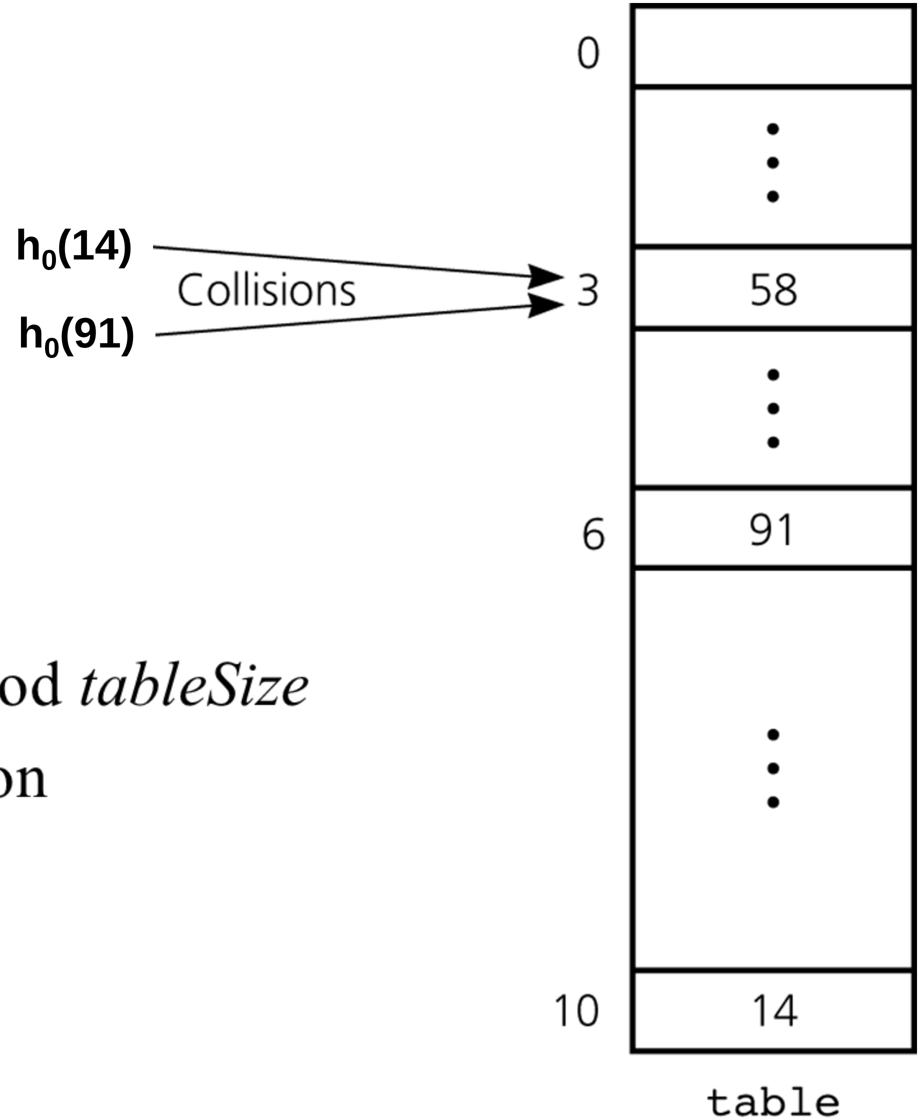
$h(x) = x \bmod 101$



- Quadratic probing
  - $h_i(x) = (h_0(x) + i^2) \bmod tableSize$
  - bad w/ secondary clustering

Figure 5

Double hashing during the insertion of 58, 14, and 91

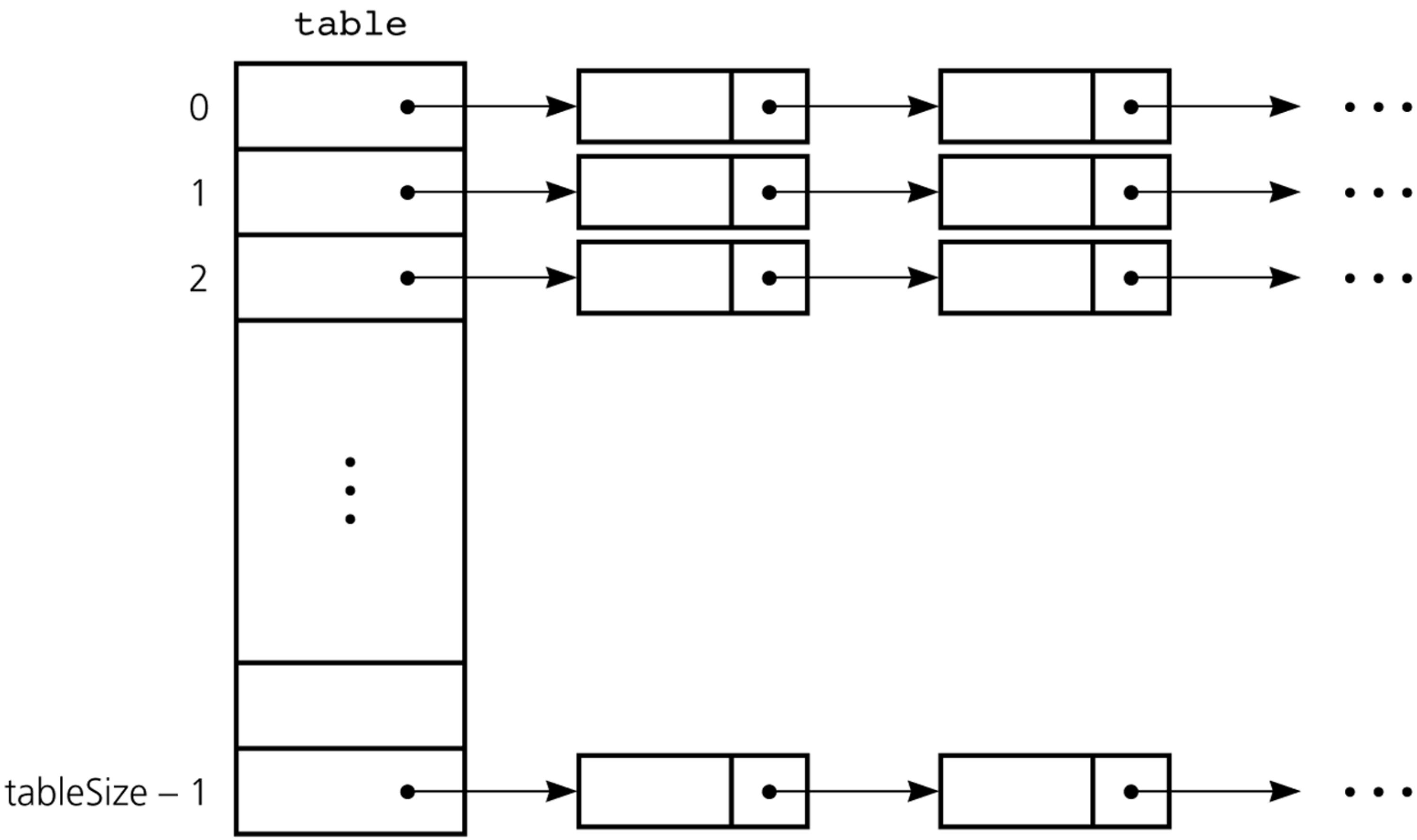


- Double hashing
  - $h_i(x) = (h_0(x) + i \cdot \beta(x)) \bmod tableSize$
  - $\beta(x)$: another hash function

- Increasing the size of hash table
  - Load factor α
    - The rate of occupied slots in the table
    - A high load factor harms performance
      - We need to increase the size of hash table
  - Increasing the hash table
    - Roughly double the table size
    - Rehash all the items on the new table

Figure 6

Separate chaining

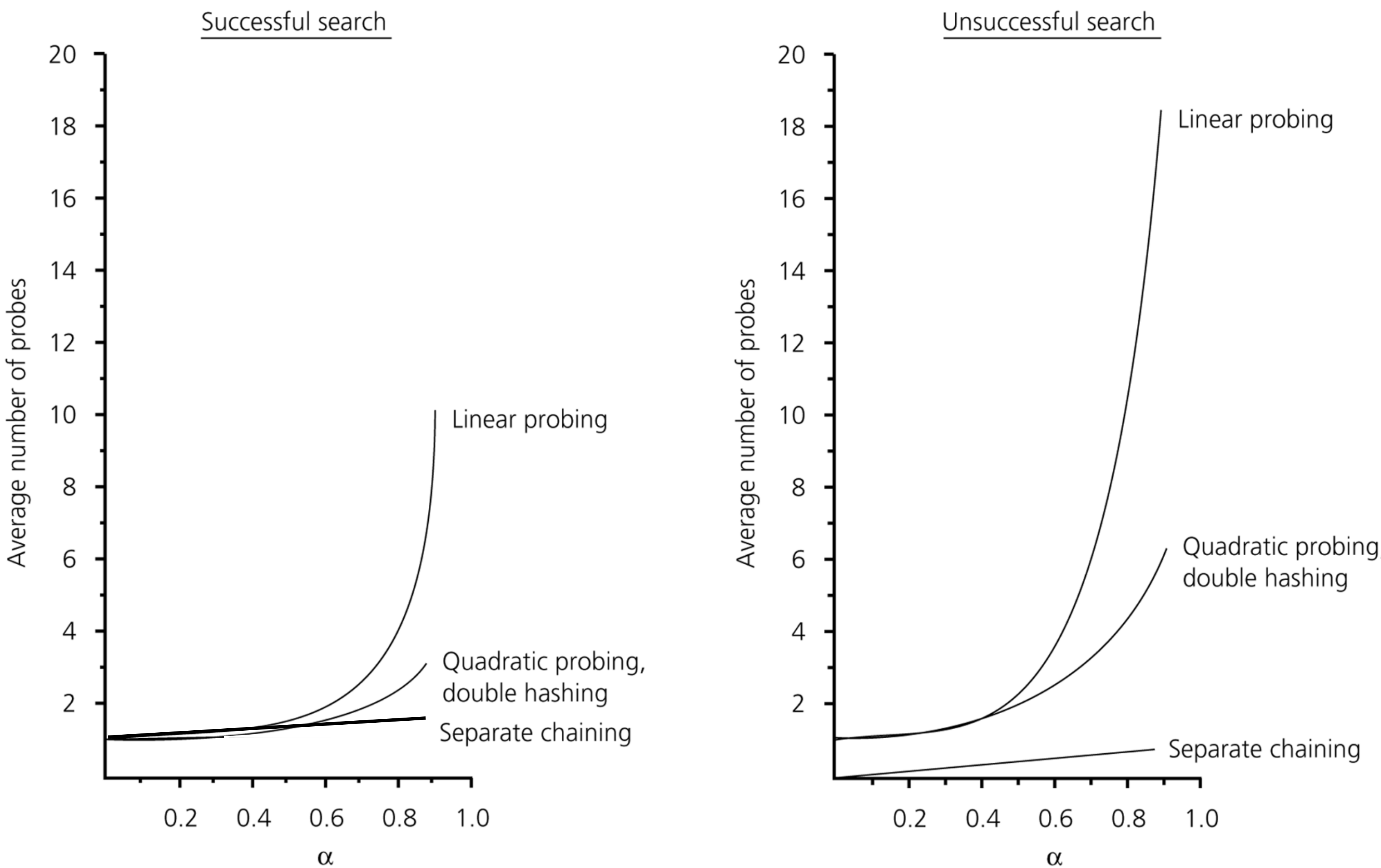


Each location of the hash table contains a reference to a linked list

The Efficiency of Hashing

- Approximate average # of comparisons for a search
  - Linear probing
    - $\frac{1}{2}(1 + \frac{1}{1-\alpha})$ for a successful search
    - $\frac{1}{2}(1 + \frac{1}{(1-\alpha)^2})$ for an unsuccessful search
  - Quadratic probing and double hashing
    - $-\ln(1-\alpha)/\alpha$ for a successful search
    - $\frac{1}{1-\alpha}$ for an unsuccessful search
  - Separate chaining (except the access for the indexing array)
    - $1 + \alpha/2$ for a successful search
    - α for an unsuccessful search

The Relative Efficiency of Collision-Resolution Methods



Good Hash Functions

- should be easy and fast to compute
- should scatter the data evenly on the hash table

Observation

- Load factor가 낮을 때는 probing 방법들은 대체로 큰 차이가 없다.
- Successful search는 insertion할 당시의 궤적을 그대로 밟는다.