Chapter 37

Relativity

Lecture 29, 30

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37 Summary

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The Postulates

- Einstein's special theory of relativity is based on two postulates:

1. The laws of physics are the same for observers in all inertial reference frames. No one frame is preferred over any other.
2. The speed of light in vacuum has the same value *c* in all directions and in all inertial reference frames.

Time Dilation

- For an observer moving with relative speed *v*, the measured time interval is

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} = \frac{\Delta t_0}{\sqrt{1-\beta^2}} = \gamma\,\Delta t_0 \quad \text{(time dilation).}$$

Eq. 37-7 to 9

Length Contraction

- For an observer moving with relative speed v, the measured length is

$$L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma}$$

Eq. 37-13

The Lorentz Transformation

- The Lorentz transformation equations relate the space time coordinates of a single event as seen by observers in two inertial frames and are given by

$$\begin{aligned} x' &= \gamma(x - vt), \\ y' &= y, \\ z' &= z, \\ t' &= \gamma(t - vx/c^2). \end{aligned}$$

Eq. 37-21

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37-1 Simultaneity and Time Dilation

Learning Objectives

37.01 Identify the two postulates of (special) relativity and the type of frames to which they apply.

37.02 Identify the speed of light as the ultimate speed and give its approximate value.

37.03 Explain how the space and time coordinates of an event can be measured with a three-dimensional array of clocks and measuring rods and how that eliminates the need of a signal's travel time to an observer.

37.04 Identify that the relativity of space and time has to do with transferring measurements between two inertial frames in relative motion but we still can use classical kinematics and Newtonian mechanics within a frame.

37.05 Identify that for reference frames with relative motion, simultaneous events in one of the frames will generally not be simultaneous in the other frame.

37.06 Explain what is meant by the entanglement of the spatial and temporal separations between two events.

37-1 Simultaneity and Time Dilation

Learning Objectives

37.07 Identify the conditions in which a temporal separation of two events is a proper time.

37.08 Identify that if the temporal separation of two events is a proper time as measured in one frame, that separation is greater (dilated) as measured in another frame.

37.09 Apply the relationship between proper time Δt_0, dilated time Δt, and the relative speed v between two frames.

37.10 Apply the relationships between the relative speed v, the speed parameter β, and the Lorentz factor γ.

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37-1 Simultaneity and Time Dilation

Einstein's special theory of relativity is based on two postulates:

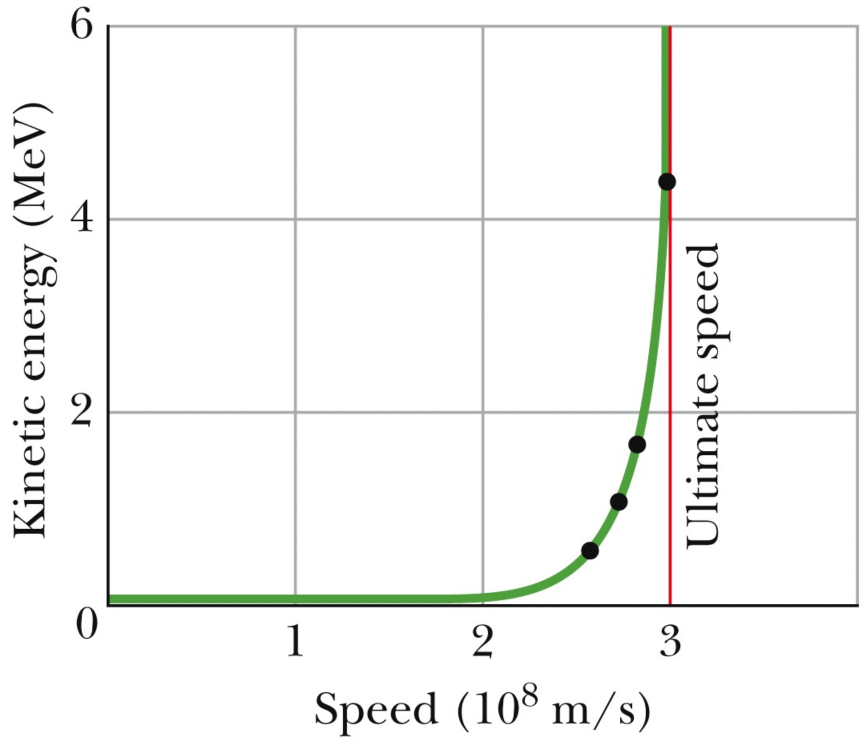
1. **The Relativity Postulate:** The laws of physics are the same for observers in all inertial reference frames. No one frame is preferred over any other.

2. **The Speed of Light Postulate:** The speed of light in vacuum has the same value c in all directions and in all inertial reference frames.

We can also phrase this postulate to say that there is in nature an ultimate speed *c*, the same in all directions and in all inertial reference frames. Light happens to travel at this ultimate speed. However, no entity that carries energy or information can exceed this limit. Moreover, no particle that has mass can actually reach speed *c*, no matter how much or for how long that particle is accelerated.

Both postulates have been exhaustively tested, and no exceptions have ever been found.

37-1 Simultaneity and Time Dilation



The dots show measured values of the kinetic energy of an electron plotted against its measured speed. No matter how much energy is given to an electron (or to any other particle having mass), its speed can never equal or exceed the ultimate limiting speed c. (The plotted curve through the dots shows the predictions of Einstein's special theory of relativity.)

37-1 Simultaneity and Time Dilation

An **event** is something that happens, and every event can be assigned three space coordinates and one time coordinate. Among many possible events are (1) the turning on or off of a tiny light bulb, (2) the collision of two particles, and (3) the sweeping of the hand of a clock past a marker on the rim of the clock.

If two observers are in relative motion, they will not, in general, agree as to whether two events are simultaneous. If one observer finds them to be simultaneous, the other generally will not.

Simultaneity is not an absolute concept but rather a relative one, depending on the motion of the observer.

37-1 Simultaneity and Time Dilation

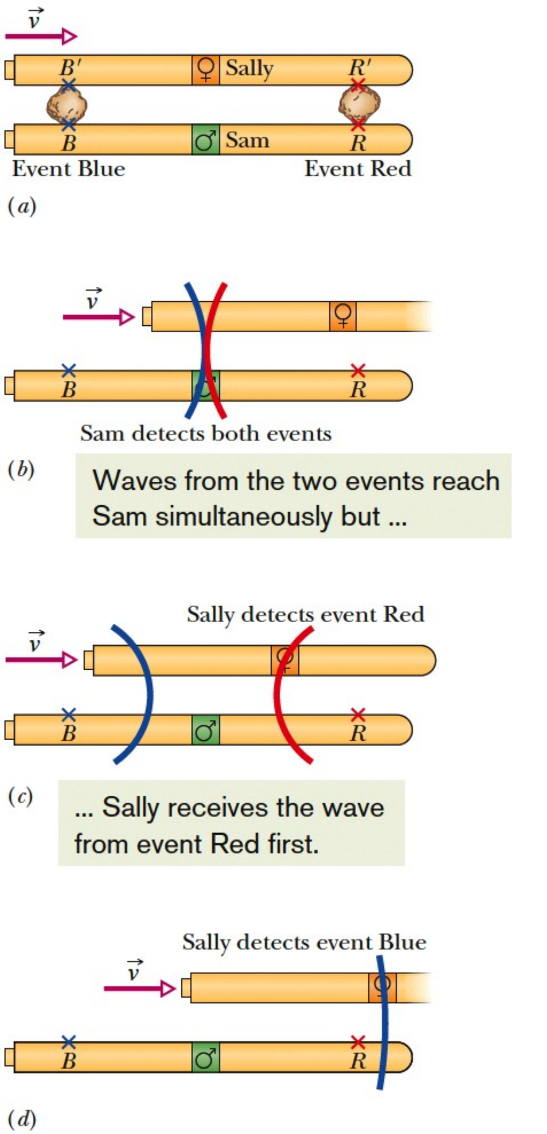


Figure: The spaceships of Sally and Sam and the occurrences of events from Sam's view. Sally's ship moves rightward with velocity ***v***.

(a) Event Red occurs at positions RR' and event Blue occurs at positions BB'; each event sends out a wave of light.
(b) Sam simultaneously detects the waves from event Red and event Blue.
(c) Sally detects the wave from event Red.
(d) Sally later detects the wave from event Blue.

If the relative speed of the observers is very much less than the speed of light, then measured departures from simultaneity are so small that they are not noticeable. Such is the case for all our experiences of daily living; that is why the relativity of simultaneity is unfamiliar.

37-1 Simultaneity and Time Dilation

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The time interval between two events depends on how far apart they occur in both space and time; that is, their spatial and temporal separations are entangled.

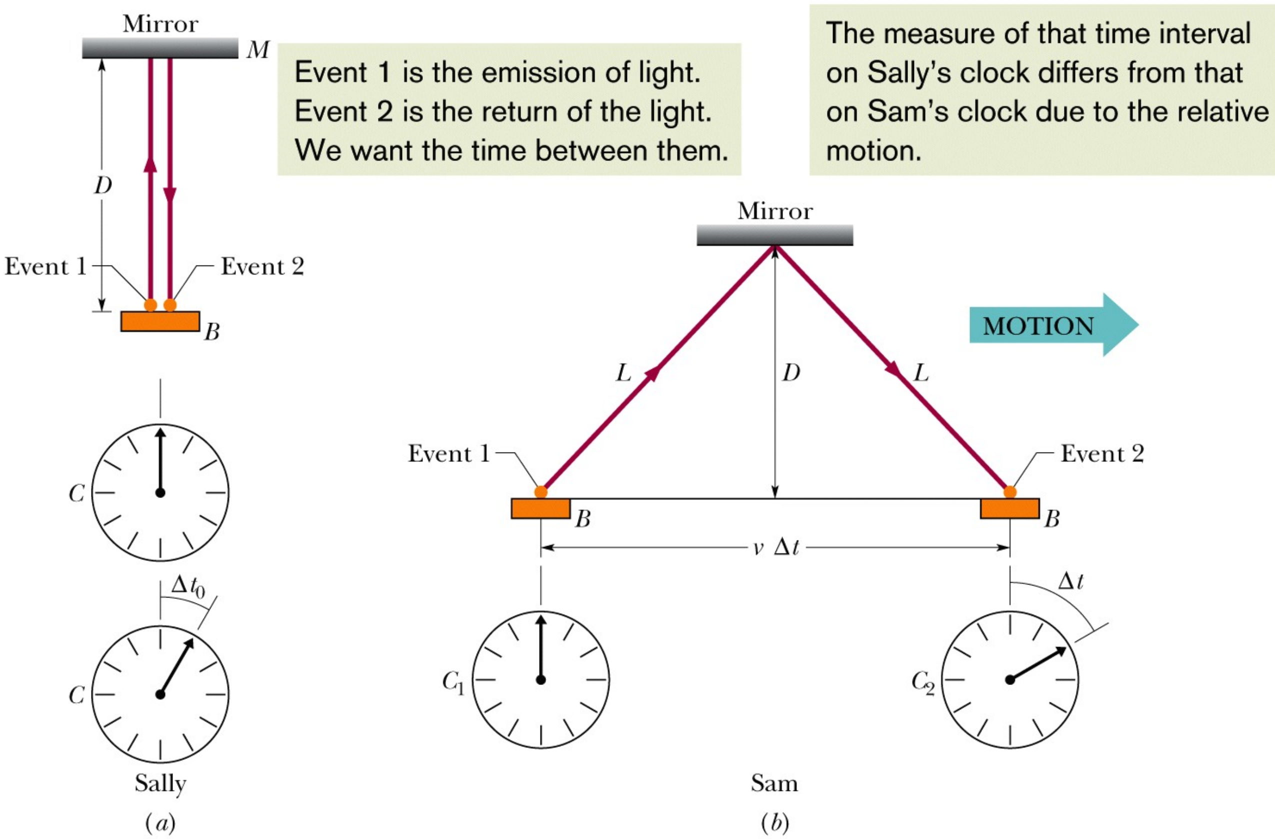

Figure (a) shows the basics of an experiment Sally conducts while she and her equipment—a light source, a mirror, and a clock—ride in a train moving with constant velocity ***v*** relative to a station. A pulse of light leaves the light source B (event 1), travels vertically upward, is reflected vertically downward by the mirror, and then is detected back at the source (event 2).

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37-1 Simultaneity and Time Dilation

As in the example, if two successive events occur *at the same place* in an inertial reference frame, the time interval Δt_0 between them, *measured on a single clock* where they occur, is called the **proper time Δt_0**. Observers in frames moving relative to that frame such as observers on the track watching Sally and her equipment move past, will always measure a larger value Δt for the time interval on their clocks, an effect known as **time dilation**.

When two events occur at the same location in an inertial reference frame, the time interval between them, measured in that frame, is called the **proper time interval** or the **proper time.** Measurements of the same time interval from any other inertial reference frame are always greater.

If the relative speed between the two frames is v, then

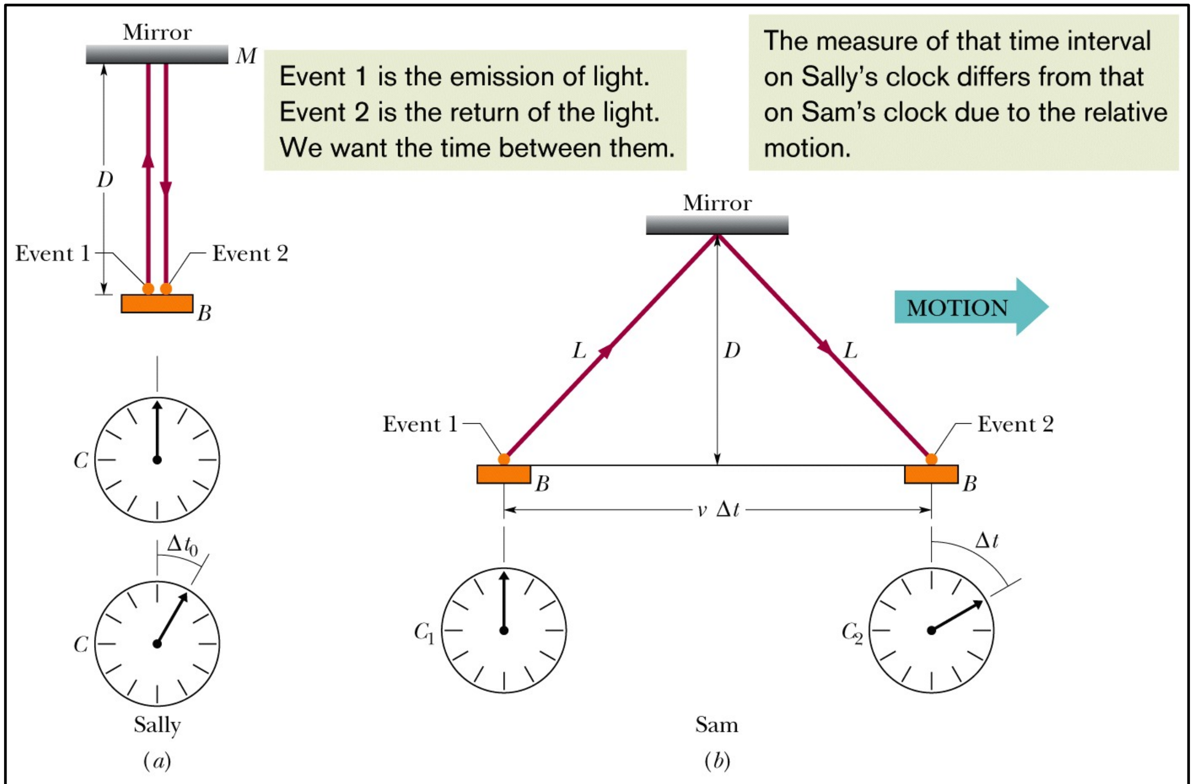
$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} = \frac{\Delta t_0}{\sqrt{1-\beta^2}} = \gamma\,\Delta t_0,}$$

$$\boxed{0 \le \beta \le 1}$$

$$\boxed{1 \le \gamma}$$

where $\boxed{\beta = v/c}$ is the speed parameter and $\boxed{\gamma = 1/\sqrt{1-\beta^2}}$ is the Lorentz factor.

37-1 Simultaneity and Time Dilation



$$\Delta t_0 = \frac{2D}{c} \quad \text{(Sally)}.$$

$$\Delta t = \frac{2L}{c} \quad \text{(Sam)},$$

$$L = \sqrt{(\tfrac{1}{2}v\,\Delta t)^2 + D^2}.$$

$$L = \sqrt{(\tfrac{1}{2}v\,\Delta t)^2 + (\tfrac{1}{2}c\,\Delta t_0)^2}.$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}.$$

37-2 The Relativity of Length

Learning Objectives

37.11 Identify that because spatial and temporal separations are entangled, measurements of the lengths of objects may be different in two frames with relative motion.

37.12 Identify the condition in which a measured length is a proper length.

37.13 Identify that if a length is a proper length as measured in one frame, the length is less (contracted) as measured in another frame that is in relative motion parallel to the length.

37.14 Apply the relationship between contracted length L, proper length L_0, and the relative speed v between two frames.

37-2 The Relativity of Length

The length L_0 of an object measured in the rest frame of the object is its **proper length** or **rest length.** Measurements of the length from any reference frame that is in relative motion *parallel* to that length are always less than the proper length.

If the relative speed between frames is *v*, the contracted length *L* and the proper length L_0 are related by

$$\boxed{L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma}}$$

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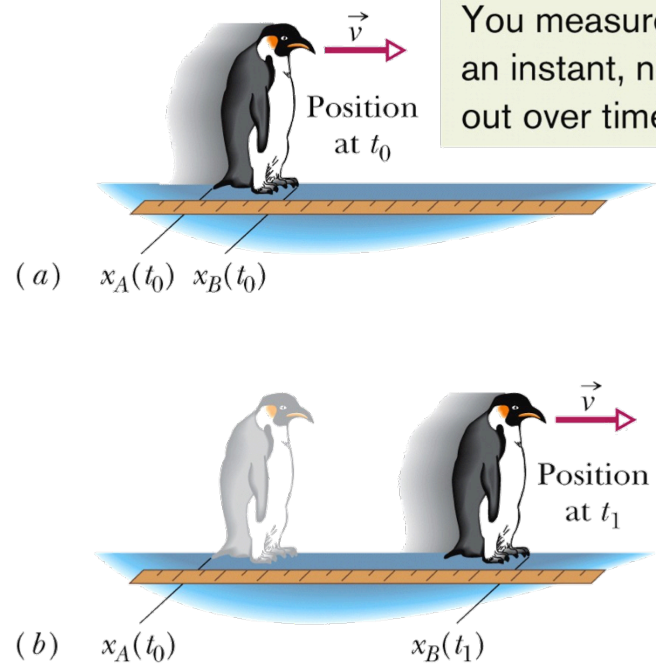
37-2 The Relativity of Length

Does a moving object really shrink?

Reality is based on observations and measurements; if the results are always consistent and if no error can be determined, then what is observed and measured is real. In that sense, the object really does shrink. However, a more precise statement is that the object is really measured to shrink — motion affects that measurement and thus reality.

You measure a width at an instant, not spread out over time.



If you want to measure the front-to-back length of a penguin while it is moving, you must mark the positions of its front and back simultaneously (in your reference frame), as in (a), rather than at different times, as in (b). How to do that is not trivial.

37-2 The Relativity of Length

Sally is in the train moving with velocity v.
Sam is in the platform and try to measure the platform length.

$$L_0 = v\,\Delta t \quad \text{(Sam)}.$$

$$L = v\,\Delta t_0 \qquad \text{(Sally)}.$$

$$\frac{L}{L_0} = \frac{v\,\Delta t_0}{v\,\Delta t} = \frac{1}{\gamma},$$

$$L = \frac{L_0}{\gamma},$$

37-2 The Relativity of Length

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Sample Problem 37.03 Time dilation and length contraction as seen from each frame

In Fig. 37-8, Sally (at point A) and Sam's spaceship (of proper length $L_0 = 230$ m) pass each other with constant relative speed v. Sally measures a time interval of 3.57 μs for the ship to pass her (from the passage of point B in Fig. 37-8*a* to the passage of point C in Fig. 37-8*b*). In terms of c, what is the relative speed v between Sally and the ship?

KEY IDEAS

Let's assume that speed v is near c. Then:

1. This problem involves measurements made from two (inertial) reference frames, one attached to Sally and the other attached to Sam and his spaceship.
2. This problem also involves two events: the first is the passage of point B past Sally (Fig. 37-8*a*) and the second is the passage of point C past her (Fig. 37-8*b*).
3. From either reference frame, the other reference frame passes at speed v and moves a certain distance in the time interval between the two events:

$$v = \frac{\text{distance}}{\text{time interval}}. \tag{37-17}$$

Because speed v is assumed to be near the speed of light, we must be careful that the distance and the time interval in Eq. 37-17 are measured in the *same* reference frame.

Calculations: We are free to use either frame for the measurements. Because we know that the time interval Δt between the two events measured from Sally's frame is 3.57 μs, let us also use the distance L between the two events measured from her frame. Equation 37-17 then becomes

$$v = \frac{L}{\Delta t}. \tag{37-18}$$

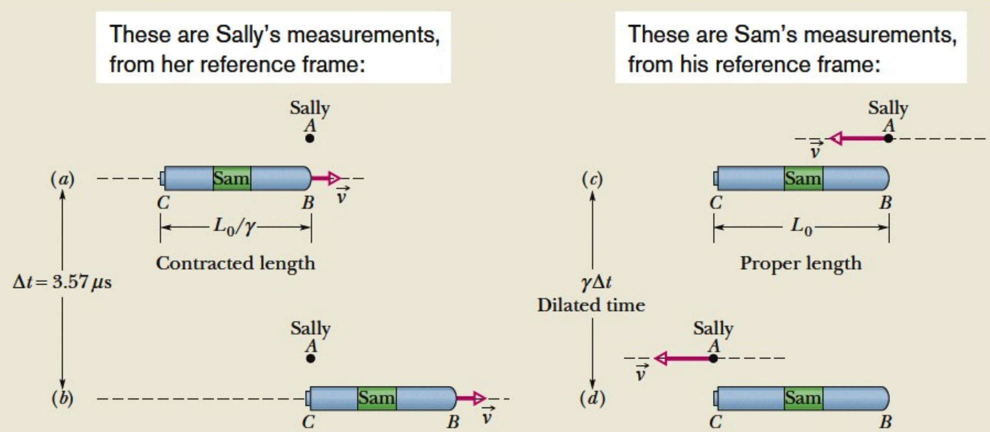


Figure 37-8 (*a*)–(*b*) Event 1 occurs when point B passes Sally (at point A) and event 2 occurs when point C passes her. (*c*)–(*d*) Event 1 occurs when Sally passes point B and event 2 occurs when she passes point C.

37-2 The Relativity of Length

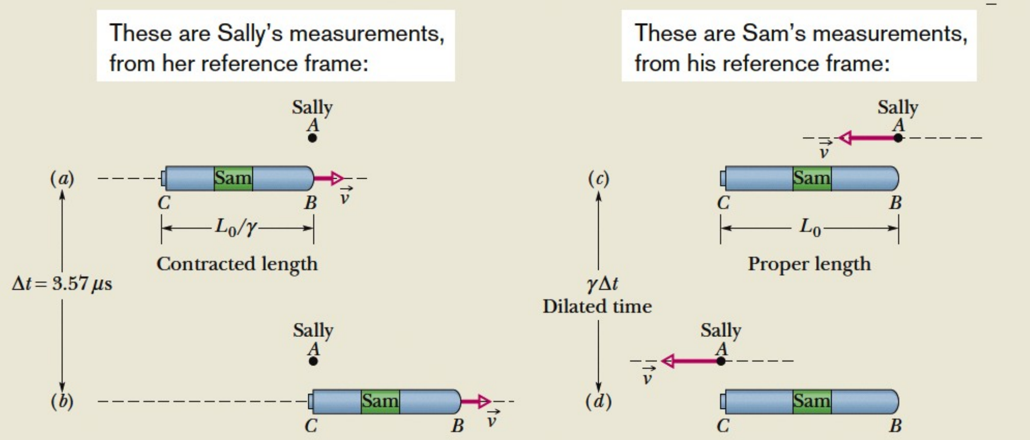


Figure 37-8 (*a*)–(*b*) Event 1 occurs when point *B* passes Sally (at point *A*) and event 2 occurs when point *C* passes her. (*c*)–(*d*) Event 1 occurs when Sally passes point *B* and event 2 occurs when she passes point *C*.

We do not know L, but we can relate it to the given L_0: The distance between the two events as measured from Sam's frame is the ship's proper length L_0. Thus, the distance L measured from Sally's frame must be less than L_0, as given by Eq. 37-13 ($L = L_0/\gamma$) for length contraction. Substituting L_0/γ for L in Eq. 37-18 and then substituting Eq. 37-8 for γ, we find

$$v = \frac{L_0/\gamma}{\Delta t} = \frac{L_0\sqrt{(1-(v/c)^2}}{\Delta t}.$$

Solving this equation for v (notice that it is on the left and also buried in the Lorentz factor) leads us to

$$v = \frac{L_0 c}{\sqrt{(c\,\Delta t)^2 + L_0^2}}$$

$$= \frac{(230\text{ m})c}{\sqrt{(299\,792\,458\text{ m/s})^2(3.57\times10^{-6}\text{ s})^2 + (230\text{ m})^2}}$$

$$= 0.210c. \qquad \text{(Answer)}$$

Note that only the relative motion of Sally and Sam matters here; whether either is stationary relative to, say, a space station is irrelevant. In Figs. 37-8*a* and *b* we took Sally to be stationary, but we could instead have taken the ship to be stationary, with Sally moving to the left past it. Event 1 is again when Sally and point *B* are aligned (Fig. 37-8*c*), and event 2 is again when Sally and point *C* are aligned (Fig. 37-8*d*). However, we are now using Sam's measurements. So the length between the two events in *his* frame is the proper length L_0 of the ship and the time interval between them is not Sally's measurement Δt but a dilated time interval $\gamma\,\Delta t$.

Substituting Sam's measurements into Eq. 37-17, we have

$$v = \frac{L_0}{\gamma\Delta t},$$

which is exactly what we found using Sally's measurements. Thus, we get the same result of $v = 0.210c$ with either set of measurements, *but we must be careful not to mix the measurements from the two frames.*

37-3 The Lorentz Transformation

Learning Objectives

37.15 For frames with relative motion, apply the Galilean transformation to transform an event's position from one frame to the other.

37.16 Identify that a Galilean transformation is approximately correct for slow relative speeds but the Lorentz transformations are the correct transformations for any physically possible speed.

37.17 Apply the Lorentz transformations for the spatial and temporal separations of two events as measured in two frames with a relative speed v.

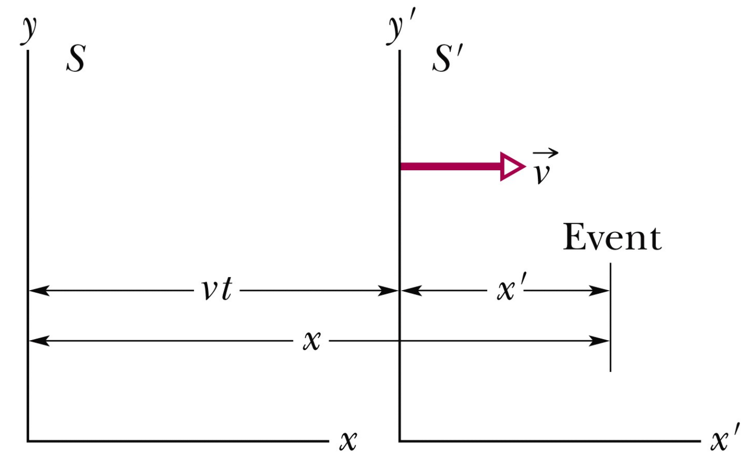
37.18 From the Lorentz transformations, derive the equations for time dilation and length contraction.

37.19 From the Lorentz transformations show that if two events are simultaneous but spatially separated in one frame, they cannot be simultaneous in another frame in relative motion.

37-3 The Lorentz Transformation

The **Lorentz transformation** equations relate the *spacetime* coordinates of a single event as seen by observers in two inertial frames, *S* and *S'*, where *S'* is moving relative to *S* with velocity *v* in the positive *x* and *x'* direction. The four coordinates are related by

$$\boxed{\begin{aligned} x' &= \gamma(x - vt), \\ y' &= y, \\ z' &= z, \\ t' &= \gamma(t - vx/c^2) \end{aligned}}$$



Two inertial reference frames: frame *S'* has velocity ***v*** relative to frame *S*.

Note that the spatial values x and the temporal values t are bound together in the first and last equations. This entanglement of space and time was a prime message of Einstein's theory, a message that was long rejected by many of his contemporaries.

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37-3 The Lorentz Transformation

The Lorentz transformations in terms of any pair of events 1 and 2, with spatial and temporal separations is given in Table 37-2.

Table 37-2 The Lorentz Transformation Equations for Pairs of Events

1. $\Delta x = \gamma(\Delta x' + v\,\Delta t')$	1′. $\Delta x' = \gamma(\Delta x - v\,\Delta t)$
2. $\Delta t = \gamma(\Delta t' + v\,\Delta x'/c^2)$	2′. $\Delta t' = \gamma(\Delta t - v\,\Delta x/c^2)$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

Frame S' moves at velocity v relative to frame S.

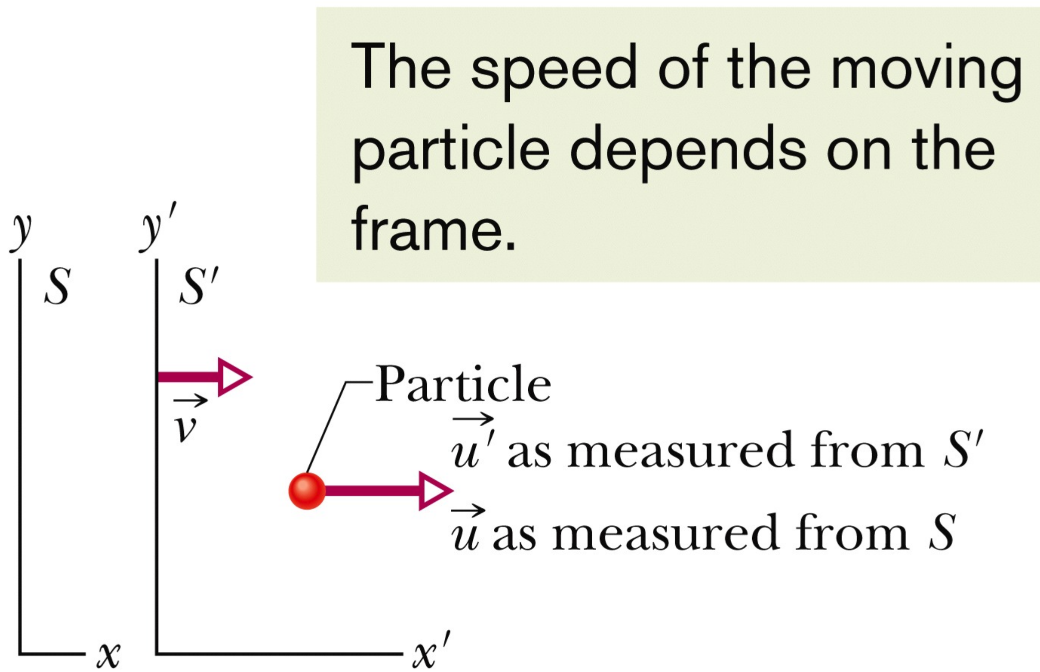
37-4 The Relativity of Velocities

Learning Objectives

37.20 With a sketch, explain the arrangement in which a particle's velocity is to be measured relative to two frames that have relative motion.

37.21 Apply the relationship for a relativistic velocity transformation between two frames with relative motion.

37-4 The Relativity of Velocities



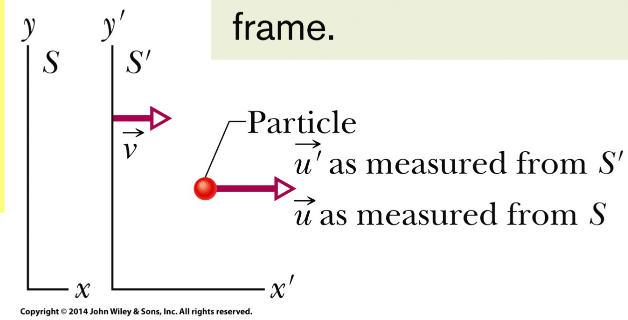
Reference frame *S'* moves with velocity ***v*** relative to frame *S*. A particle has velocity ***u'*** relative to reference frame *S'* and velocity ***u*** relative to reference frame *S*.

37-4 The Relativity of Velocities

When a particle is moving with speed *u'* in the positive *x'* direction in an inertial reference frame *S'* that itself is moving with speed *v* parallel to the *x* direction of a second inertial frame *S*, the speed *u* of the particle as measured in *S* is

$$\boxed{u = \frac{u' + v}{1 + u'v/c^2}}$$

The speed of the moving particle depends on the frame.



Be careful to substitute the correct signs for the velocities. Above Equation reduces to the classical, or **Galilean**, velocity transformation equation,

$$\boxed{u = u' + v}$$

Reference frame *S'* moves with velocity ***v*** relative to frame *S*. A particle has velocity ***u'*** relative to reference frame *S'* and velocity ***u*** relative to reference frame *S*.

when we apply the formal test of letting *c*-> ∞. In other words, relativistic equation is correct for all physically possible speeds, but classical equation is approximately correct for speeds much less than *c*.

37-4 The Relativity of Velocities

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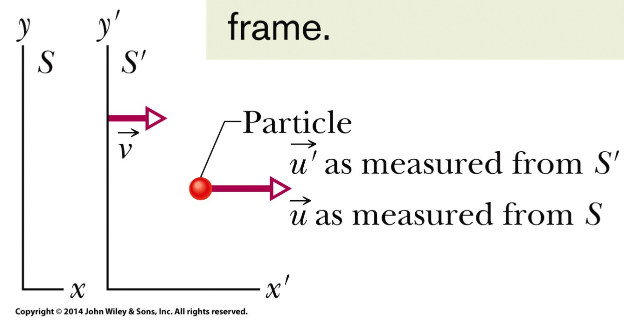
When a particle is moving with speed *u'* in the positive *x'* direction in an inertial reference frame *S'* that itself is moving with speed *v* parallel to the *x* direction of a second inertial frame *S*, the speed *u* of the particle as measured in *S* is

$$\boxed{u = \frac{u' + v}{1 + u'v/c^2}}$$

The speed of the moving particle depends on the frame.



$$\Delta x = \gamma(\Delta x' + v\,\Delta t')$$

$$\Delta t = \gamma\left(\Delta t' + \frac{v\,\Delta x'}{c^2}\right).$$

$$\frac{\Delta x}{\Delta t} = \frac{\Delta x' + v\,\Delta t'}{\Delta t' + v\,\Delta x'/c^2}.$$

$$\frac{\Delta x}{\Delta t} = \frac{\Delta x'/\Delta t' + v}{1 + v(\Delta x'/\Delta t')/c^2}.$$

$$\Rightarrow \quad \boxed{u = \frac{u' + v}{1 + u'v/c^2}}$$

Reference frame *S'* moves with velocity ***v*** relative to frame *S*. A particle has velocity ***u'*** relative to reference frame *S'* and velocity ***u*** relative to reference frame *S*.

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37-5 Doppler Effect for Light

Learning Objectives

37.22 Identify that the frequency of light as measured in a frame attached to the light source (the rest frame) is the proper frequency.

37.23 For source–detector separations increasing and decreasing, identify whether the detected frequency is shifted up or down from the proper frequency, identify that the shift increases with an increase in relative speed, and apply the terms blue shift and red shift.

37.24 Identify radial speed.

37.25 For source–detector separations increasing and decreasing, apply the relationships between proper frequency f_0, detected frequency f, and radial speed v.

37.26 Convert between equations for frequency shift and wavelength shift.

37.27 When a radial speed is much less than light speed, apply the approximation relating wavelength shift $\Delta\lambda$, proper wavelength λ_0, and radial speed v.

37-5 Doppler Effect for Light

Learning Objectives

37.28 Identify that for light (not sound) there is a shift in the frequency even when the velocity of the source is perpendicular to the line between the source and the detector, an effect due to time dilation.

37.29 Apply the relationship for the transverse Doppler effect by relating detected frequency f, proper frequency f_0, and relative speed v.

37-5 Doppler Effect for Light

When a light source and a light detector move relative to each other, the wavelength of the light as measured in the rest frame of the source is the proper wavelength λ_0. The detected wavelength λ is either longer (a red shift) or shorter (a blue shift) depending on whether the source–detector separation is increasing or decreasing.

When the separation is increasing, the wavelengths & frequencies are related by

$$\lambda = \lambda_0 \sqrt{\frac{1+\beta}{1-\beta}}$$

$$f = f_0 \sqrt{\frac{1-\beta}{1+\beta}}$$

where $\beta = v/c$ and v is the relative radial speed (along a line through the source and detector). If the separation is decreasing, the signs in front of the β symbols are reversed.

37-5 Doppler Effect for Light

For speeds much less than *c*, the magnitude of the Doppler wavelength shift $\Delta\lambda = \lambda - \lambda_0$ is approximately related to v by

$$v = \frac{|\Delta\lambda|}{\lambda_0} c \quad \text{(radial speed of light source, } v \ll c\text{).}$$

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}} \Rightarrow f = f_0(1 - \beta + \tfrac{1}{2}\beta^2) \quad \text{(source and detector separating, } \beta \ll 1\text{).}$$

Suppose a star (or any other light source) moves away from us with a radial speed v that is low enough (β is small enough) for us to neglect the β^2 term in Eq. 37-33. Then we have

$$f = f_0(1 - \beta). \tag{37-34}$$

Because astronomical measurements involving light are usually done in wavelengths rather than frequencies, let's rewrite Eq. 37-34 as

$$\frac{c}{\lambda} = \frac{c}{\lambda_0}(1 - \beta),$$

or

$$\lambda = \lambda_0(1 - \beta)^{-1}.$$

Because we assume β is small, we can expand $(1 - \beta)^{-1}$ in a power series. Doing so and retaining only the first power of β, we have

$$\lambda = \lambda_0(1 + \beta),$$

or

$$\beta = \frac{\lambda - \lambda_0}{\lambda_0}. \tag{37-35}$$

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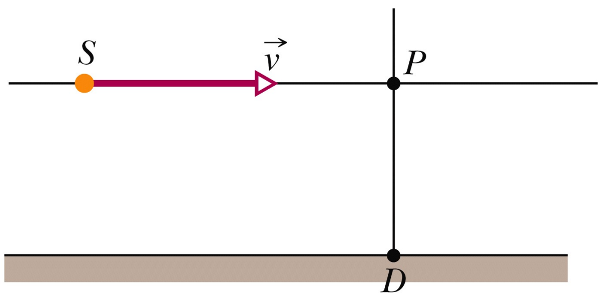
37-5 Doppler Effect for Light

Transverse Doppler Effect: In the figure a source S moves past a detector D. When S reaches point P, the velocity of S is perpendicular to the line joining P and D, and at that instant S is moving neither toward nor away from D. If the source is emitting *sound* waves of frequency f_0, D detects that frequency (with no Doppler effect) when it intercepts the waves that were emitted at point P. However, if the source is emitting *light* waves, there is still a Doppler effect, called the *transverse Doppler effect*. In this situation, the detected frequency of the light emitted when the source is at point P is

$$f = f_0\sqrt{1-\beta^2}$$



For low speeds (β<<1), this equation can be expanded in a power series in β and approximated as

$$f = f_0(1 - \tfrac{1}{2}\beta^2) \quad \text{(low speeds).}$$

Here the first term is what we would expect for sound waves, and again the relativistic effect for low-speed light sources and detectors appears with the β^2 term.

37-6 Momentum and Energy

Learning Objectives

37.30 Identify that the classical expressions for momentum and kinetic energy are approximately correct for slow speeds whereas the relativistic expressions are correct for any physically possible speed.

37.31 Apply the relationship between momentum, mass, and relative speed.

37.32 Identify that an object has a mass energy (or rest energy) associated with its mass.

37.33 Apply the relationships between total energy, rest energy, kinetic energy, momentum, mass, speed, the speed parameter, and the Lorentz factor.

37.34 Sketch a graph of kinetic energy versus the ratio v/c (of speed to light speed) for both classical and relativistic expressions of kinetic energy.

37.35 Apply the work–kinetic energy theorem to relate work by an applied force and the resulting change in kinetic energy.

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37-6 Momentum and Energy

Learning Objectives

37.36 For a reaction, apply the relationship between the *Q* value and the change in the mass energy.

37.37 For a reaction, identify the correlation between the algebraic sign of *Q* and whether energy is released or absorbed by the reaction.

37-6 Momentum and Energy

In relativistic mechanics the definition of linear momentum is,

$$\boxed{\vec{p} = \gamma m \vec{v} \quad \text{(momentum).}}$$

This equation gives the correct definition of momentum for all physically possible speeds. For a speed much less than c, it reduces to the classical definition of momentum ($\mathbf{p} = m\mathbf{v}$) .

An object's mass m and the equivalent energy E_0 are related by

$$\boxed{E_0 = mc^2,}$$

which, without the subscript *0*, is the best-known science equation of all time. This energy E_0 that is associated with the mass of an object is called **mass energy** or **rest energy**. The second name suggests that E_0 is an energy that the object has even when it is at rest, simply because it has mass.
And if we assume that the object's potential energy is zero, then its total energy E is the sum of its mass energy and its kinetic energy:

$$\boxed{E = E_0 + K = mc^2 + K.}$$

Another expression for total energy E is $E = \gamma mc^2$

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37-6 Momentum and Energy

An expression for kinetic energy that is correct for all physically possible speeds, including speeds close to c is given by

$$K = E - mc^2 = \gamma mc^2 - mc^2$$
$$= mc^2(\gamma - 1) \quad \text{(kinetic energy)},$$

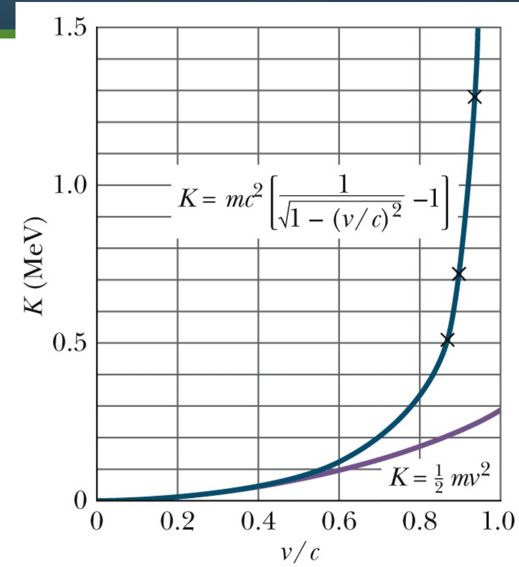
where $\gamma (= 1/\sqrt{1 - (v/c)^2})$ is the Lorentz factor for the object's motion.

The connection between the relativistic momentum and kinetic energy is thus given by

$$(pc)^2 = K^2 + 2Kmc^2.$$

and

$$E^2 = (pc)^2 + (mc^2)^2.$$



The relativistic and classical equations for the kinetic energy of an electron, plotted as a function of v/c. Note that the two curves blend together at low speeds and diverge widely at high speeds. Experimental data (at the × marks) show that at high speeds the relativistic curve agrees with experiment but the classical curve does not.

37 Summary

The Postulates

- Einstein's special theory of relativity is based on two postulates:

1. The laws of physics are the same for observers in all inertial reference frames. No one frame is preferred over any other.
2. The speed of light in vacuum has the same value *c* in all directions and in all inertial reference frames.

Time Dilation

- For an observer moving with relative speed *v*, the measured time interval is

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} = \frac{\Delta t_0}{\sqrt{1-\beta^2}} = \gamma\,\Delta t_0 \quad \text{(time dilation).}$$

Eq. 37-7 to 9

Length Contraction

- For an observer moving with relative speed v, the measured length is

$$L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma}$$

Eq. 37-13

The Lorentz Transformation

- The Lorentz transformation equations relate the space time coordinates of a single event as seen by observers in two inertial frames and are given by

$$\begin{aligned} x' &= \gamma(x - vt), \\ y' &= y, \\ z' &= z, \\ t' &= \gamma(t - vx/c^2). \end{aligned}$$

Eq. 37-21

37 Summary

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Relativity of Velocities

- Relativistic addition of velocities is given by

$$u = \frac{u' + v}{1 + u'v/c^2}$$ **Eq. 37-29**

Relativistic Doppler Effect

- When the separation between the detector and the light source is increasing, the wavelengths are related by

$$\lambda = \lambda_0\sqrt{\frac{1+\beta}{1-\beta}}$$ **Eq. 37-32**

- For speeds much less than c, the magnitude of the Doppler wavelength shift is approximately related to v by

$$v = \frac{|\Delta\lambda|}{\lambda_0}c \qquad (v \ll c).$$ **Eq. 37-36**

Transverse Doppler Effect

- If the relative motion of the light source is perpendicular to a line joining the source and detector, then

$$f = f_0\sqrt{1-\beta^2}.$$ **Eq. 37-37**

Momentum and Energy

- The following definitions of linear momentum ***p*** , kinetic energy *K*, and total energy *E* for a particle of mass *m* are valid at any physically possible speed:

$$\vec{p} = \gamma m\vec{v}$$ **Eq. 37-42**

$$E = mc^2 + K = \gamma mc^2$$ **Eq. 37-47&48**

$$K = mc^2(\gamma - 1)$$ **Eq. 37-52**

- These equations lead to the relationships

$$(pc)^2 = K^2 + 2Kmc^2$$ **Eq. 37-54**

$$E^2 = (pc)^2 + (mc^2)^2.$$ **Eq. 37-55**