

Chapter 21-27

Summary

Seon-Hee Seo

21 Summary

Electric Charge

- The strength of a particle's electrical interaction with objects around it depends on its electric charge, which can be either positive or negative.

Conductors and Insulators

- Conductors are materials in which a significant number of electrons are free to move. The charged particles in nonconductors (insulators) are not free to move.

Conservation of Charge

- The net electric charge of any isolated system is always conserved.

Coulomb's Law

- The magnitude of the electrical force between two charged particles is proportional to the product of their charges and inversely proportional to the square of their separation distance.

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1| |q_2|}{r^2}$$

Eq. 21-4

The Elementary Charge

- Electric charge is quantized (restricted to certain values).
- e is the elementary charge

$$e = 1.602 \times 10^{-19} \text{ C.}$$

Eq. 21-12

22 Summary

Definition of Electric Field

- The electric field at any point

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Eq. 22-1

Electric Field Lines

- provide a means for visualizing the directions and the magnitudes of electric fields

Field due to a Point Charge

- The magnitude of the electric field E set up by a point charge q at a distance r from the charge is

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$$

Eq. 22-3

Field due to an Electric Dipole

- The magnitude of the electric field set up by the dipole at a distant point on the dipole axis is

$$E = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3}$$

Eq. 22-9

Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

Eq. 22-26

22 Summary

Force on a Point Charge in an Electric Field

- When a point charge q is placed in an external electric field \vec{E}

$$\vec{F} = q\vec{E}.$$

Eq. 22-28

Dipole in an Electric Field

- The electric field exerts a torque on a dipole

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

Eq. 22-34

- The dipole has a potential energy U associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}.$$

Eq. 22-38

23 Summary

Gauss' Law

- Gauss' law is

$$\epsilon_0 \Phi = q_{\text{enc}}$$

Eq. 23-6

- the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

Eq. 23-6

Applications of Gauss' Law

- surface of a charged conductor

$$E = \frac{\sigma}{\epsilon_0}$$

Eq. 23-11

- Within the surface $E=0$.
- line of charge

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Eq. 23-12

- Infinite non-conducting sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

Eq. 23-13

- Outside a spherical shell of charge

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Eq. 23-15

- Inside a uniform spherical shell

$$E = 0$$

Eq. 23-16

- Inside a uniform sphere of charge

$$E = \left(\frac{q}{4\pi\epsilon_0 R^3} \right) r.$$

Eq. 23-20

24 Summary (I)

Electric Potential

- The electric potential V at point P in the electric field of a charged object:

$$V = \frac{-W_{\infty}}{q_0} = \frac{U}{q_0}, \quad \text{Eq. 24-2}$$

Electric Potential Energy

- Electric potential energy U of the particle-object system:

$$U = qV. \quad \text{Eq. 24-3}$$

- If the particle moves through potential ΔV :

$$\Delta U = q \Delta V = q(V_f - V_i). \quad \text{Eq. 24-4}$$

Mechanical Energy

- Applying the conservation of mechanical energy gives the change in kinetic energy:

$$\Delta K = -q \Delta V. \quad \text{Eq. 24-9}$$

- In case of an applied force in a particle

$$\Delta K = -q \Delta V + W_{\text{app}}. \quad \text{Eq. 24-11}$$

- In a special case when $\Delta K=0$:

$$W_{\text{app}} = q \Delta V \quad (\text{for } K_i = K_f). \quad \text{Eq. 24-12}$$

Finding V from E

- The electric potential difference between two point i and f is:

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}. \quad \text{Eq. 24-18}$$

24 Summary (II)

Potential due to a Charged Particle

- due to a single charged particle at a distance r from that particle :

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Eq. 24-26

- due to a collection of charged particles

$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

Eq. 24-27

Potential due to an Electric Dipole

- The electric potential of the dipole is

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

Eq. 24-30

Potential due to a Continuous Charge Distribution

- For a continuous distribution of charge:

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

Eq. 24-32

Calculating E from V

- The component of E in any direction is:

$$E_s = -\frac{\partial V}{\partial s}$$

Eq. 24-40

Electric Potential Energy of a System of Charged Particle

- For two particles at separation r :

$$U = W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

Eq. 24-46

25 Summary

Capacitor and Capacitance

- The capacitance of a capacitor is defined as:

$$q = CV$$

Eq. 25-1

Determining Capacitance

- Parallel-plate capacitor:

$$C = \frac{\epsilon_0 A}{d}$$

Eq. 25-9

- Cylindrical Capacitor:

$$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$$

Eq. 25-14

- Spherical Capacitor:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}$$

Eq. 25-17

- Isolated sphere:

$$C = 4\pi\epsilon_0 R$$

Eq. 25-18

Capacitor in parallel and series

- In parallel:

$$C_{eq} = \sum_{j=1}^n C_j$$

Eq. 25-19

- In series

$$\frac{1}{C_{eq}} = \sum_{j=1}^n \frac{1}{C_j}$$

Eq. 25-20

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2$$

Eq. 25-21&22

- Energy density (u)

$$u = \frac{1}{2}\epsilon_0 E^2$$

Eq. 25-25

25 Summary

Capacitance with a Dielectric

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ , called the dielectric constant, which is characteristic of the material.

Gauss' Law with a Dielectric

- When a dielectric is present, Gauss' law may be generalized to

$$\epsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q.$$

Eq. 25-36

26 Summary

Current

- The electric current i in a conductor is defined by

$$i = \frac{dq}{dt}. \quad \text{Eq. 26-1}$$

Current Density

- Current is related to current density by

$$i = \int \vec{J} \cdot d\vec{A}, \quad \text{Eq. 26-4}$$

Drift Speed of the Charge Carriers

- Drift speed of the charge carriers in an applied electric field is related to current density by

$$\vec{J} = (ne)\vec{v}_d, \quad \text{Eq. 26-7}$$

Resistance of a Conductor

- Resistance R of a conductor is defined by

$$R = \frac{V}{i} \quad \text{Eq. 26-8}$$

- Similarly the resistivity and conductivity of a material is defined by

$$\rho = \frac{1}{\sigma} = \frac{E}{J} \quad \text{Eq. 26-10\&12}$$

- Resistance of a conducting wire of length L and uniform cross section is

$$R = \rho \frac{L}{A} \quad \text{Eq. 26-16}$$

Change of ρ with Temperature

- The resistivity of most material changes with temperature and is given as

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0). \quad \text{Eq. 26-17}$$

26 Summary

Ohm's Law

- A given device (conductor, resistor, or any other electrical device) obeys Ohm's law if its resistance R (defined by **Eq. 26-8** as V/i) is independent of the applied potential difference V .

Resistivity of a Metal

- By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}.$$

Power

- The power P , or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV$$

Eq. 26-26

- If the device is a resistor, we can write

$$P = i^2 R = \frac{V^2}{R}$$

Eq. 26-27&28

27 Summary

Emf

- The **emf** (work per unit charge) of the device is

$$\mathcal{E} = \frac{dW}{dq} \quad (\text{definition of } \mathcal{E}). \quad \text{Eq. 27-1}$$

Single-Loop Circuits

- Current in a single-loop circuit:

$$i = \frac{\mathcal{E}}{R + r}, \quad \text{Eq. 27-4}$$

Power

- The rate P of energy transfer to the charge carriers is $P = iV$ Eq. 27-14

- The rate P_r at which energy is dissipated as thermal energy in the battery is $P_r = i^2 r$. Eq. 27-16

- The rate P_{emf} at which the chemical energy in the battery changes is $P_{\text{emf}} = i\mathcal{E}$. Eq. 27-17

Series Resistance

- When resistances are in series

$$R_{\text{eq}} = \sum_{j=1}^n R_j \quad \text{Eq. 27-7}$$

Parallel Resistance

- When resistances are in parallel

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^n \frac{1}{R_j} \quad \text{Eq. 27-24}$$

RC Circuits

- The charge on the capacitor increases according to $q = C\mathcal{E}(1 - e^{-t/RC})$ Eq. 27-33

- During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R} \right) e^{-t/RC} \quad \text{Eq. 27-34}$$

- During the discharging, the current is

$$i = \frac{dq}{dt} = - \left(\frac{q_0}{RC} \right) e^{-t/RC} \quad \text{Eq. 27-40}$$