Chapter 22

Electric Fields

Lecture 3

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22-4 The Electric Field Due to a Line of Charge

Learning Objectives

22.16 For a uniform distribution of charge, find the linear charge density λ for charge along a line, the surface charge density σ for charge on a surface, and the volume charge density ρ for charge in a volume.

22.17 For charge that is distributed uniformly along a line, find the net electric field at a given point near the line by splitting the distribution up into charge elements dq and then summing (by integration) the electric field vectors $d\mathbf{E}$ set up at the point by each element.

22.18 Explain how symmetry can be used to simplify the calculation of the electric field at a point near a line of uniformly distributed charge.

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22-4 The Electric Field Due to a Line of Charge

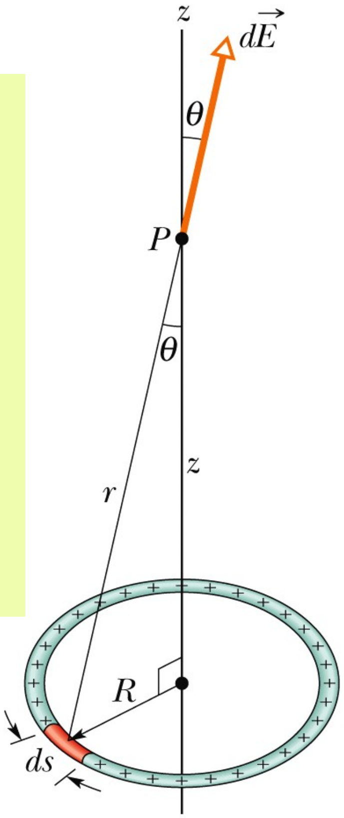
Key Concepts

- The equation for the electric field set up by a particle does not apply to an extended object with charge (said to have a **continuous charge distribution**).
- **To find the electric field of an extended object at a point**, we first consider the electric field set up by a charge element dq in the object, where the element is small enough for us to apply the equation for a particle. Then we sum, via integration, components of the electric fields *d**E*** from all the charge elements.
- Because the individual electric fields *d**E*** have different magnitudes and point in different directions, we first see if symmetry allows us to cancel out any of the components of the fields, to simplify the integration.

22-4 The Electric Field Due to a Line of Charge

Charged Ring

Canceling Components - Point P is on the axis: In the Figure (right), consider the charge element on the opposite side of the ring. It too contributes the field magnitude *d**E*** but the field vector leans at angle θ in the opposite direction from the vector from our first charge element, as indicated in the side view of Figure (bottom). Thus the two perpendicular components cancel. All around the ring, this cancelation occurs for every charge element and its symmetric partner on the opposite side of the ring. So we can neglect all the perpendicular components.



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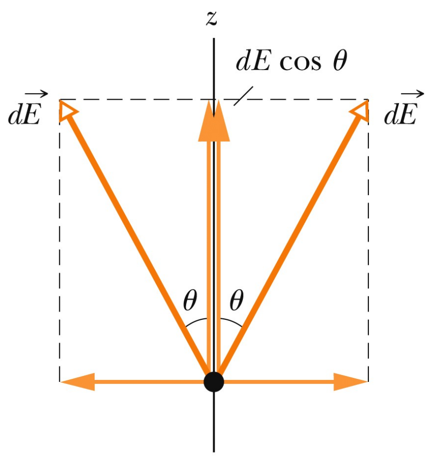
A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field *d**E*** at point *P*.



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The components perpendicular to the z axis cancel; the parallel components add.

22-4 The Electric Field Due to a Line of Charge

Charged Ring

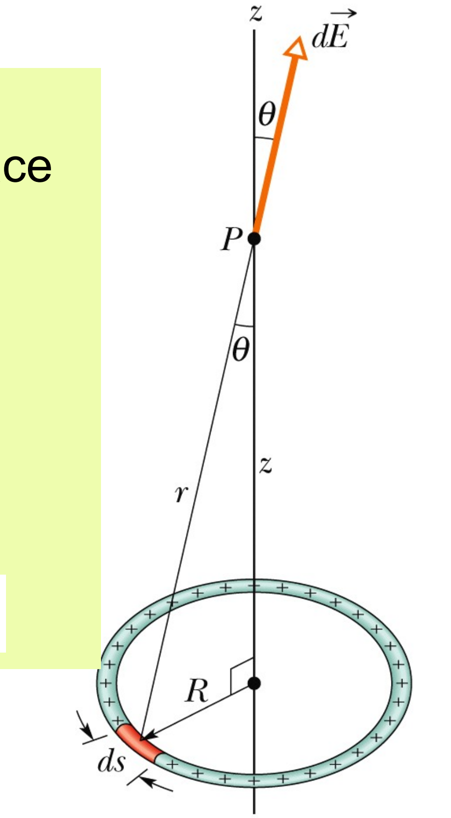
Adding Components. From the figure (bottom), we see that the parallel components each have magnitude $d\boldsymbol{E}\cos\theta$. We can replace $\cos\theta$ by using the right triangle in the Figure (right) to write

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}$$

And, $$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{(z^2 + R^2)^{3/2}}\,ds$$ gives us the parallel field component from each charge element

charge density:

$$\lambda = \frac{q}{2\pi R}$$



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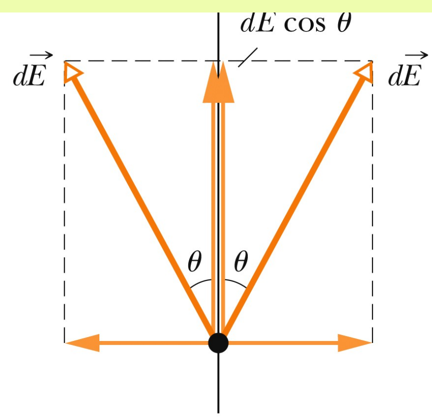
A ring of uniform positive charge. A differential element of charge occupies a length ds (greatly exaggerated for clarity). This element sets up an electric field $d\boldsymbol{E}$ at point P.



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The components perpendicular to the z axis cancel; the parallel components add.

22-4 The Electric Field Due to a Line of Charge

Charged Ring

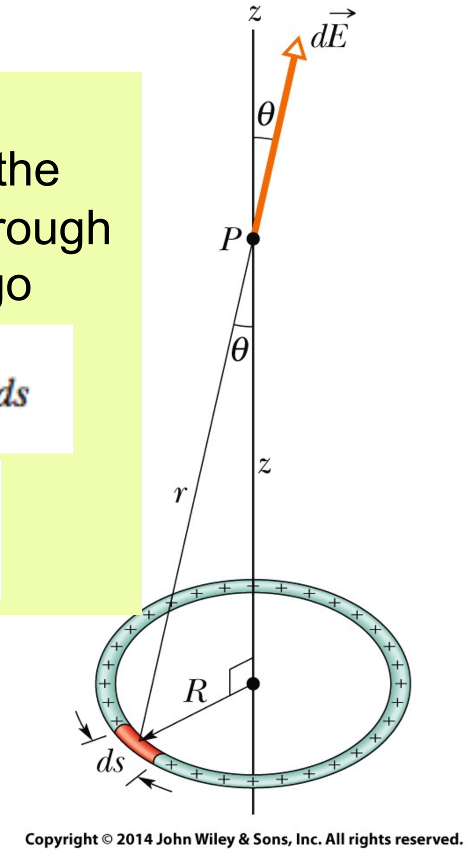
Integrating. Because we must sum a huge number of these components, each small, we set up an integral that moves along the ring, from element to element, from a starting point (call it *s=0*) through the full circumference (*s=2πR*). Only the quantity *s* varies as we go through the elements. We find

$$E = \int dE \cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \int_0^{2\pi R} ds$$

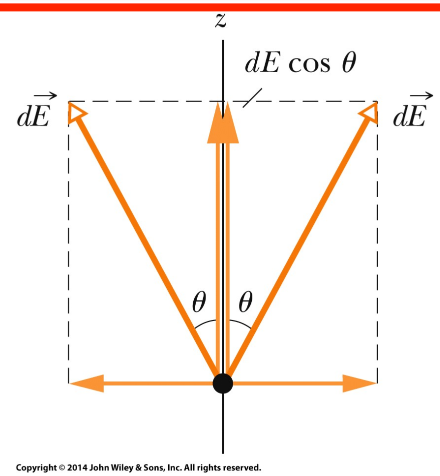
$$\lambda = \frac{q}{2\pi R}$$

Finally,

$$\boxed{E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad \text{(charged ring)}}$$



A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field *d**E*** at point *P*.



The components perpendicular to the z axis cancel; the parallel components add.

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22-5 The Electric Field Due to a Charged Disk

Learning Objectives

22.19 Sketch a disk with uniform charge and indicate the direction of the electric field at a point on the central axis if the charge is positive and if it is negative.

22.20 Explain how the equation for the electric field on the central axis of a uniformly charged ring can be used to find the equation for the electric field on the central axis of a uniformly charged disk.

22.21 For a point on the central axis of a uniformly charged disk, apply the relationship between the surface charge density σ, the disk radius R, and the distance z to that point.

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22-5 The Electric Field Due to a Charged Disk

We superimpose a ring on the disk as shown in the Figure, at an arbitrary radius $r \le R$. The ring is so thin that we can treat the charge on it as a charge element dq. To find its small contribution dE to the electric field at point P, on the axis, in terms of the ring's charge dq and radius r we can write

$$dE = \frac{dq\, z}{4\pi\varepsilon_0 (z^2 + r^2)^{3/2}}.$$

$$\sigma = \frac{q}{\pi r^2}$$

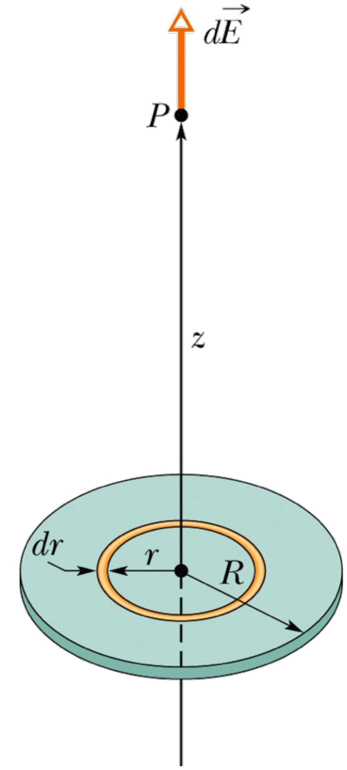
Then, we can sum all the $d\mathbf{E}$ contributions with

$$dq = \pi\sigma(2r)dr$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr,$$

We find

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \quad \text{(charged disk)}}$$



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A disk of radius R and uniform positive charge. The ring shown has radius r and radial width dr. It sets up a differential electric field $d\mathbf{E}$ at point P on its central axis.

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22-6 A Point Charge in an Electric Field

Learning Objectives

22.22 For a charged particle placed in an external electric field (a field due to other charged objects), apply the relationship between the electric field ***E*** at that point, the particle's charge *q*, and the electrostatic force ***F*** that acts on the particle, and identify the relative directions of the force and the field when the particle is positively charged and negatively charged.

22.23 Explain Millikan's procedure of measuring the elementary charge.

22.24 Explain the general mechanism of ink-jet printing.

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22-6 A Point Charge in an Electric Field

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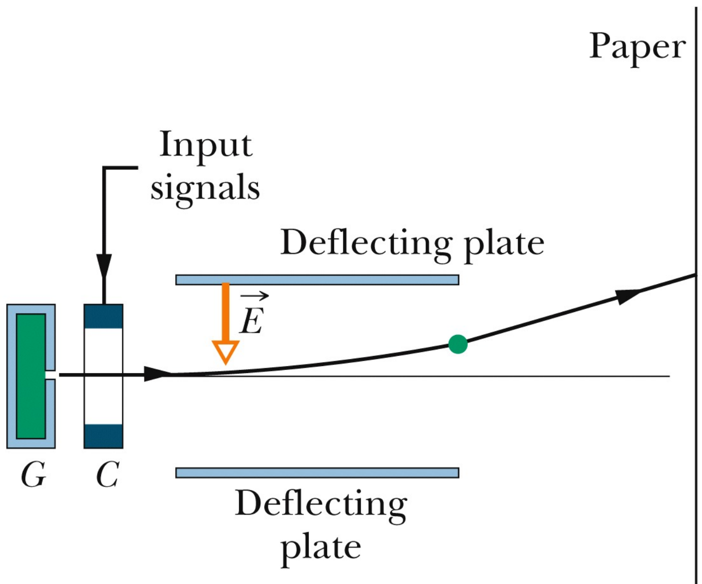
If a particle with charge q is placed in an external electric field $\boldsymbol{E}$, an electrostatic force $\boldsymbol{F}$ acts on the particle:

$$\boxed{\vec{F} = q\vec{E}.}$$

The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.



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Ink-jet printer. Drops shot from generator *G* receive a charge in charging unit *C*. An input signal from a computer controls the charge and thus the effect of field $\boldsymbol{E}$ on where the drop lands on the paper.

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22-7 A Dipole in an Electric Field

Learning Objectives

22.25 On a sketch of an electric dipole in an external electric field, indicate the direction of the field, the direction of the dipole moment, the direction of the electrostatic forces on the two ends of the dipole, and the direction in which those forces tend to rotate the dipole, and identify the value of the net force on the dipole.

22.26 Calculate the torque on an electric dipole in an external electric field by evaluating a cross product of the dipole moment vector and the electric field vector, in magnitude-angle notation and unit-vector notation.

22.27 For an electric dipole in an external electric field, relate the potential energy of the dipole to the work done by a torque as the dipole rotates in the electric field.

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22-7 A Dipole in an Electric Field

Learning Objectives (Contd.)

22.28 For an electric dipole in an external electric field, calculate the potential energy by taking a dot product of the dipole moment vector and the electric field vector, in magnitude-angle notation and unit-vector notation.

22.29 For an electric dipole in an external electric field, identify the angles for the minimum and maximum potential energies and the angles for the minimum and maximum torque magnitudes.

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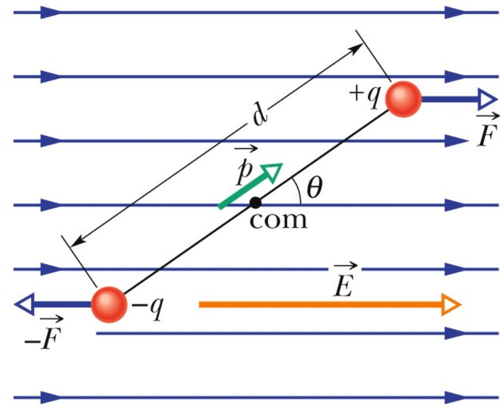
22-7 A Dipole in an Electric Field

The **torque** on an electric dipole of dipole moment ***p*** when placed in an external electric field ***E*** is given by a cross product:

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}}$$

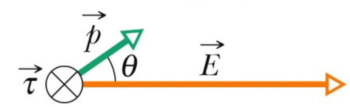
A **potential energy *U*** is associated with the orientation of the dipole moment in the field, as given by a dot product:

$$\boxed{U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}}$$



(a)

The dipole is being torqued into alignment.



(b)

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(a) An electric dipole in a uniform external electric field ***E***. Two centers of equal but opposite charge are separated by distance *d*. The line between them represents their rigid connection.
(b) Field E: causes a torque ***τ*** on the dipole. The direction of ***τ*** is into the page, as represented by the symbol (x-in a circle) .

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22 Summary

Definition of Electric Field

- The electric field at any point

$$\vec{E} = \frac{\vec{F}}{q_0}.$$

Eq. 22-1

Electric Field Lines

- provide a means for visualizing the directions and the magnitudes of electric fields

Field due to a Point Charge

- The magnitude of the electric field ***E*** set up by a point charge *q* at a distance *r* from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$

Eq. 22-3

Field due to an Electric Dipole

- The magnitude of the electric field set up by the dipole at a distant point on the dipole axis is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

Eq. 22-9

Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Eq. 22-26

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22 Summary

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Force on a Point Charge in an Electric Field

- When a point charge q is placed in an external electric field ***E***

$$\boxed{\vec{F} = q\vec{E}.}$$ Eq. 22-28

Dipole in an Electric Field

- The electric field exerts a torque on a dipole

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}.}$$ Eq. 22-34

- The dipole has a potential energy *U* associated with its orientation in the field

$$\boxed{U = -\vec{p} \cdot \vec{E}.}$$ Eq. 22-38

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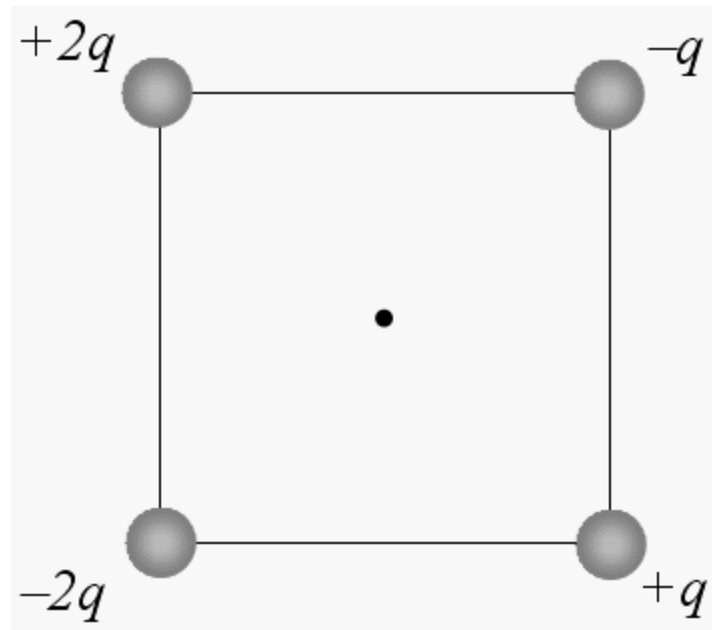
22.4.7. Four charges are located on the corners of a square as shown in the drawing. What is the direction of the net electric field at the point labeled P?

a) toward the upper left corner of the square

b) toward the middle of the right side of the square

c) toward the middle of the bottom side of the square

d) toward the lower right corner of the square



e) There is no direction. The electric field at P is zero N/C.

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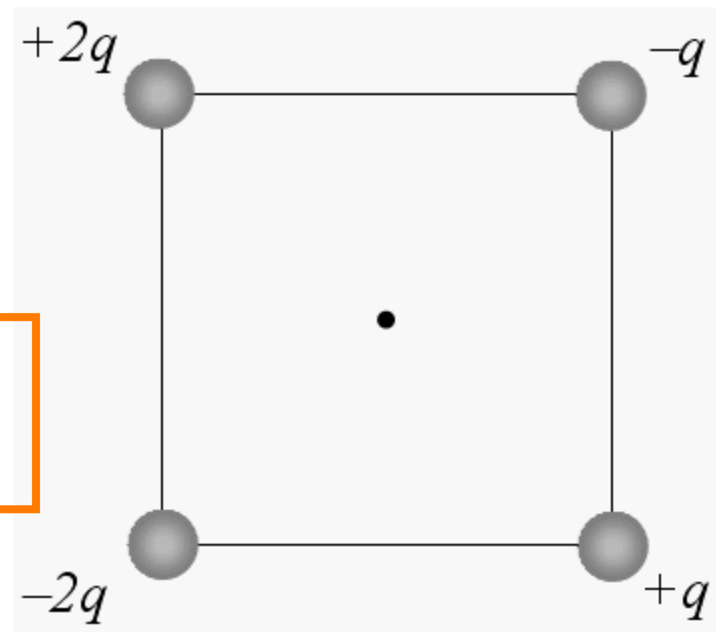

b) toward the middle of the right side of the square

c) toward the middle of the bottom side of the square

d) toward the lower right corner of the square

e) There is no direction. The electric field at P is zero N/C.

22.6.1. Consider a line of charge of length L that has a linear charge density λ that is located on the x axis beginning at $x = d$. Which one of the following expressions allows one to calculate the electric field at the origin?

a) $E = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_d^L \frac{dx}{x^2}$

b) $E = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_0^L x^2 dx$

c) $E = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_d^{d+L} \frac{dx}{x^2}$

d) $E = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_0^L \frac{dx}{x}$

e) $E = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_0^{d+L} \frac{dx}{x}$

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c) $\boxed{E = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_d^{d+L} \frac{dx}{x^2}}$

d) $E = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_0^L \frac{dx}{x}$

e) $E = \dfrac{\lambda}{4\pi\varepsilon_0}\displaystyle\int_0^{d+L} \frac{dx}{x}$

22.6.2. A uniformly charged rod with a linear charge density λ is located along the *y* axis as shown. The rod extends to infinity in both directions. Which of the following expressions gives the magnitude of the electric field at a point P located a distance *d* from the rod on the *x* axis?

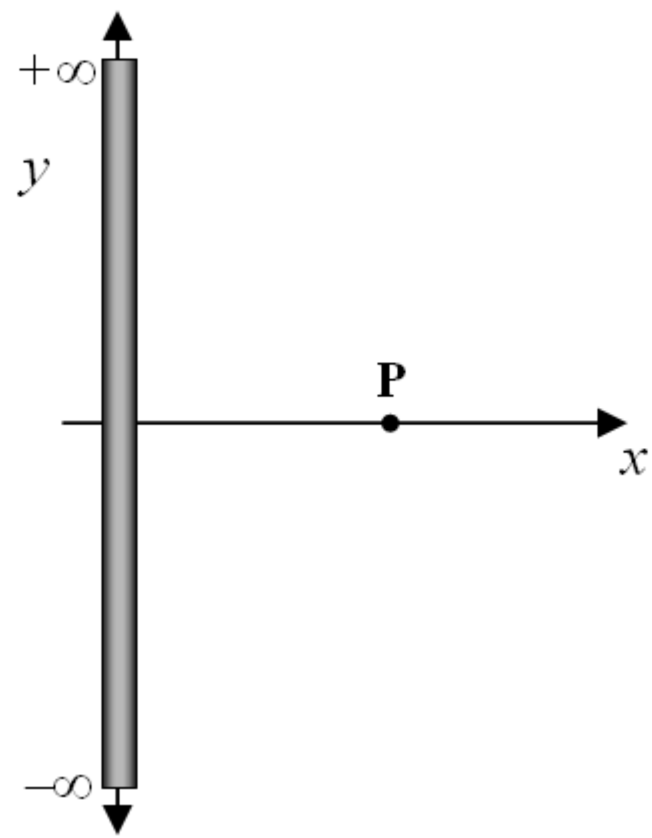
a) $E = \dfrac{\lambda}{4\pi\varepsilon_0 x}$

b) $E = \dfrac{\lambda}{4\pi\varepsilon_0 x^2}$

c) $E = \dfrac{\lambda x}{4\pi\varepsilon_0}$

d) $E = \dfrac{\lambda x^2}{4\pi\varepsilon_0}$

e) $E = 0$ N/C



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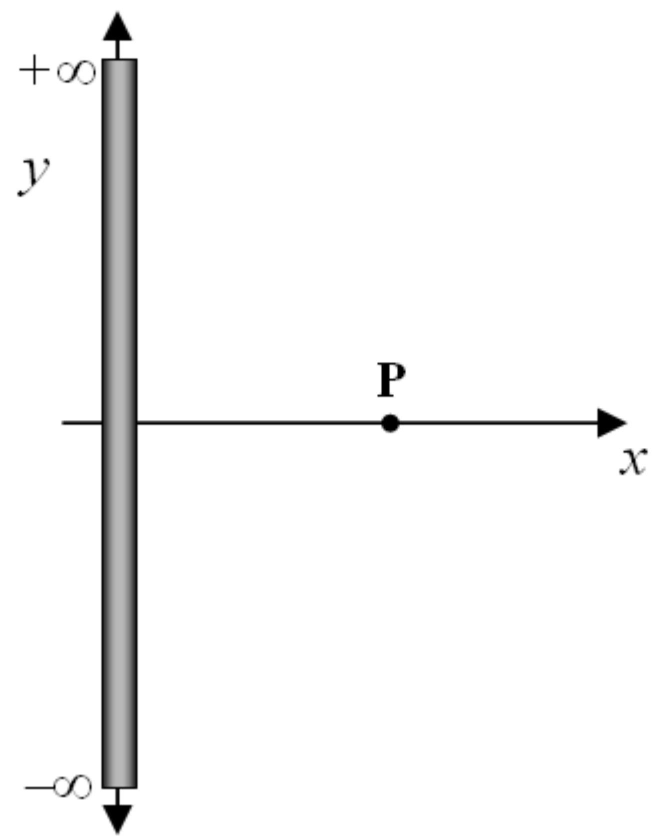
d) $E = \dfrac{\lambda x^2}{4\pi\varepsilon_0}$

e) $E = 0$ N/C



22.7.1. Two parallel infinite sheets of charge carry equal charge distributions σ of opposite sign. Which of the following expressions gives the electric field in the region between the infinite sheets?

a) $E = \sigma\varepsilon_0$

b) $E = \dfrac{\sigma}{\varepsilon_0}$

c) $E = \dfrac{\sigma}{2\varepsilon_0}$

d) $E = \dfrac{\sigma}{4\varepsilon_0}$

e) This cannot be answered without knowing the distance between the sheets.

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