

36 pts

1. The fig.1 shows an object whose profile in x - y plane is given by $f(x) = 3/4 - \cos(\pi x)/4$. The object is a filled solid which is rotationally symmetric around the x axis with uniform density of ρ .

(a) Find the Center of Mass of the object in a coordinate (x, y, z) .

(b) What's the rotational inertia about x axis of this object? (hint : make use of disk's I)

(c) Is this statement true? "A rigid body moves with all its parts in the same velocity and without any change in its shape." Explain.

(d) The scale of the drawing is 1: 1m. Also this object is made of Aluminum (density : 2.7 g/cm^3). If this object is rolling down on the rail which is inclined at an angle of 30 degrees using its **groove** from 10 m high building as shown in fig.2, what's the speed of the center of mass at the bottom? No sliding is involved. Also find the frictional force f_s .

(hint : $\sin 2x = (1 - \cos^2 x)/2$, $d(AB) = A dB + B dA$, $\int B dA = AB - \int A dB$, $\sin 2\pi = \sin(360 \text{ degrees})$

$\int dx \sin(\pi x)$, $\int dx \cos(\pi x)$ in a range $(-1, 1) = 0$.)

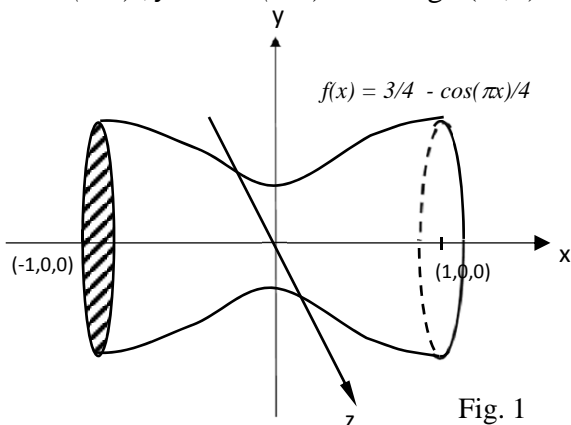


Fig. 1

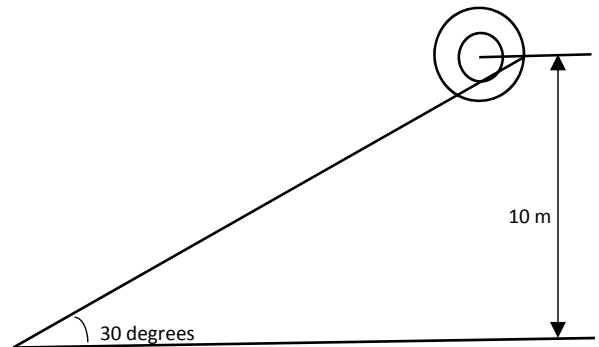


Fig. 2

36 pts

2. Assume that the fig.1 is a space ship design. The unit of the drawing is in 100 m. ((1,0,0) corresponds to (100m, 0, 0)). Namely, the length of the ship will be 200m, and the largest radius is 100 m. The space ship will be a hollow shell (not a filled solid) with a shell thickness of 1 m.

(a) Assuming this space ship will be built with carbon fiber (density(ρ) = 1.6 g/cm^3), what's the rotational inertia about x axis in kg m^2 ? (hint: make use of rotational inertia formula for rings and disks)

(b) For many years of long journey, it is desirable for astronauts to feel the same gravitational acceleration inside the space ship as on the earth. It can be realized if the spaceship is rotating around x axis during its voyage. How much angular velocity is needed to achieve the centrifugal (centripetal) acceleration of g ? Astronauts will reside around the area of the largest radius (refer to the picture below) on both sides of the ship.

(c) How much energy is required to achieve this rotational motion? Also find the torque needed to stop this rotation in 12 hours.

(d) If this spaceship (while rotating around x axis in fig.1 with the angular velocity you found in (b)) is orbiting at 1 revolution/day around the earth at altitude of 300 km, find the total angular momentum of the spaceship. (earth radius : 6400 km, use the earth as a rest frame, assume axis for two rotations are parallel, angular momentums are in the same direction. The orbit is circular.)

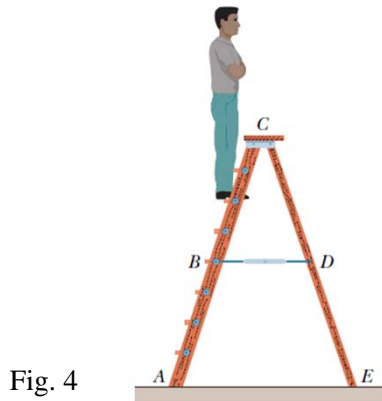


Fig. 3

From 2001 : A Space Odyssey

18 pts

3. For the stepladder shown in fig. 4, sides AC and CE are each 2.5 m long and hinged at C. Bar BD is a tie-rod 0.5 m long. Half way up. A man weighing 900 N climbs 2 m along the ladder. Assuming that the floor is frictionless and neglecting the mass of the ladder, find (a) the tension in the tie-rod and the magnitudes of the forces on the ladder from the floor at (b) A and (c) E.



10 pts

4. In fig. 5, a uniform beam with a weight of 80 N and a length of 3.2 m is hinged at its lower end, and a horizontal force of magnitude 50 N acts at its upper end. The beam is held vertical by a cable that makes angle $\theta = 25^\circ$ with the ground and is attached to the beam at height $h = 1.6$ m. What are the tension in the cable and the force on the beam from the hinge?

