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Definition : binary operation

S : set, $*$: binary operation
$* : S \times S \to S$
$*(a, b) = a * b$
$< S, * >$ (* : 적절한 조건 $\to$ Group(군), Ring(환), Field(체))

1.

Z = set of integers
$(Z, +)$

2.

$Z_n = \{0, 1, ..., n-1\}$ (when n : 양의정수)
$(Z_n, +_n)$
$+_n$: modulo n

3.

$< M_n(R), + > < M_n(R), \cdot >$

4.

$R_{2\pi} = [0, 2\pi)$, $+_{2\pi}$
$< R_{2\pi}, +_{2\pi} >$

5.

$U_n = \{z \in C | z^n = 1\}$ (n-th root of unity)
$< U_n, \cdot >$ ($\because (ab)^n = a^n b^n = 1$)
when $z = 1(cos\frac{2\pi}{n} + isin\frac{2\pi}{n}), z^n = 1$
$U_n = \left\{1, z, z^2, ..., z^{n-1}\right\}$

6.

$u = z \in C||z| = 1$ (circle)
$< u, \cdot >$

not binary operation

1.

$< Z, / >$

2.

$< M(R), + >$ ($M(R)$은 모든 크기에 해당하는 행렬)

Definition

$< S, * >$

commutative

$a * b = b * a$

associative

$(a * b) * c = a * (b * c)$

Commot(?)

$|S| < \infty$
$S = \{a_1, a_2, ..., a_n\}$
for all i, j, $a_i \cdot a_j = a_k$ for some k

Definition : issomorphism

$< S, * >, < S', *' >$
$\phi : S- > S'$
1) ϕ : one to one, onto.
2) $\phi(a * b) = \phi(a) *' \phi(b)$ (homomorphic property)
$\Leftrightarrow$
ϕ is issomorphism
S, S' 사이에 ϕ가 존재한다면 $S\ = S'$ (isomorphism)

1.

$< R(¸), + >, < R + (X¸), \cdot >$
$x- > a^x$ (some $a > 0$)
one to one

2.

$U_n = \left\{1, z, z^2, ..., z^{n-1}\right\} < U_n, \cdot > \simeq < Z_n, +_n >$
$z^i \to i$
$\phi(z^i \cdot z^j) = \phi(z^{i+j\%n})) = i + j\%n$

3.

$< Z, + >, < 2Z, + >$
$Z \to 2Z \ n \to 2n$
one to one
$\phi(n+m) = \phi(n) + \phi(m)$

How to proof not issomrophism

$(S, *)! \simeq (S', *')$
assume $< S, * > \simeq < S', *' >$
then " ... ..." holds
structure prop.
$< Q, + >, < R, + >$
$|Q| = |Z| = \aleph_0$
$|R| > \aleph_0$

1.

$< Z, \cdot >! \simeq < Z+, \cdot >$
if) ϕ exists
$x = 0 or 1 \Leftrightarrow x \cdot x = x \Leftrightarrow \phi(x) \cdot \phi(x) = \phi(x) \Leftrightarrow \phi(x) = 1$
$\phi(0) = 1, \phi(1) = 1$
not one to one
contradiction. so, $< Z, \cdot >! \simeq < Z+, \cdot >$

2.

$< Z, + >! \simeq < Q, + >$
$|Z| = |Q|$
if) ϕ exists $x is None \Leftrightarrow x + x = 3 \Leftrightarrow \phi(x) + \phi(x) = \phi(3) = c in Q$
$\phi(v) = \frac{c}{2}$
v is None
contradiction. so, $< Z, + >! \simeq < Q, + >$

3.

$< R, \cdot > \simeq < C, \cdot >$
$C = \{a + bi | a, b in R\}$
$|C| = |R|$
$x^2 = -1$
?????

I don't know

????
$(G, \cdot)$: Group $G \simeq G'$
n = dim V ¡ inf $V = F^n (Fis Ror C, i think?)$
$|G| = n$
when n=4
$Z_4, Z_2 x Z_2$

Group

$< G, * >$: Group
$\Leftrightarrow$
0) * : binary operation (it might be) (closure)
1) * is associative
2) exists e in G s.t a * e = a (= e * a) (some a in G)
e : identity
3) for all a in G, exists a' s.t. a*a' = e (= a'*a)
a' : inverse of a
()로 약화해도 됨 * 기준으로 방향 중요.

uniqueness of e

if exists e, e'
$e = e * e' = e'$
contradiction

uniqueness of a'

if exists a', a''
$a' = a' * e = a' * (a * a'') = (a' * a) * a'' = e * a'' = a''$
contradiction