고급수학 및 연습 1 중간고사

(2014년 4월 19일 오후 1:00-3:00)

학번:	이름:

모든 문제의 답에 풀이과정을 명시하시오. (총점 200점)

Problem 1. [40 pts] Determine whether each of the following series is convergent or divergent.

(a) (10 pts) $\displaystyle\sum_{n=1}^{\infty}\left(\sin^2\frac{1}{n}-\frac{1}{2^n}\right)$

(b) (10 pts) $\displaystyle\sum_{n=2}^{\infty}\left(\frac{n}{n-1}\right)^{n^2}$

(c) (10 pts) $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{n!-3^n+2014}$

(d) (10 pts) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\arctan\frac{1}{n}\right)$

Problem 2. [20 pts] Find the values of s for which the series $\displaystyle\sum_{n=1}^{\infty}\frac{n-\sqrt{n-1}}{2n^s+1}$ is convergent.

Problem 3. [20 pts] Find the values of x for which each of the following series converges.

(a) (10 pts) $\displaystyle\sum_{n=0}^{\infty}(-1)^n(\sqrt{n+1}-\sqrt{n})x^n$

(b) (10 pts) $\displaystyle\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{n}\right)x^n$

Problem 4. [15 pts] Find the values of x for which each of the following series converges.

(a) (5 pts) $\displaystyle\sum_{n=0}^{\infty}\sqrt{n}x^n$

(b) (10 pts) $\displaystyle\sum_{n=0}^{\infty}nx^{n^2}$

Problem 5. [15 pts] Let $x=\tanh^{-1}y$ be the inverse function of $y=\tanh x$. Show that

$$\tanh^{-1}y=\sum_{n=0}^{\infty}\frac{y^{2n+1}}{2n+1}\qquad\text{for}\quad |y|<1$$

Problem 6. [20 pts] For the function $f(x)=e^x+e^{2x}$, show that $y=f(x)$ has the well-defined inverse function $x=g(y)$ for $y>0$, and find the 2nd-degree Taylor polynomial $T_2^2g(y)$ at $y=2$.

Problem 7. [15 pts] Calculate the following integral

$$\int_0^{\frac{1}{10}}\frac{dx}{1+x^4}$$

within an error of less than 10^{-10}.

Problem 8. [15 pts] Find all real numbers x which satisfy the following equation.

$$1+\frac{x}{2!}+\frac{x^2}{4!}+\frac{x^3}{6!}+\cdots=0$$

Problem 9. [20 pts]

(a) (10 pts) Sketch the curve defined by the polar equation $r^2=2a^2\cos 2\theta$ $(a>0)$ in polar coordinates. For the point A on the curve with $\theta=\dfrac{5\pi}{6}$, and for the points $B(-a,0)$ and $C(a,0)$ in rectangular coordinates, find $\angle BAC$.

(b) (10 pts) For the curve given by $r=1+\cos\theta$ in polar coordinates and the line given by $y=x$ in rectangular coordinates, express all their intersection points in terms of rectangular coordinates.

Problem 10. [20 pts] Sketch the surface defined by

$$\rho=2\cos\varphi+2\sin\varphi\sin\theta,\quad \rho>0$$

in spherical coordinates. Find the ranges of ρ, φ and θ respectively on this surface.