Honor Calculus II Midterm Exam

(October 18, 13:00-15:00)

학번:	이름:

모든 문제의 답에 풀이과정을 명시하시오. (총점 200점)

1. (25 pts) Let S be the surface in $\mathbb{R}^3$ defined by

$$(x-1)^2+2(y-2)^2+3(z-3)^2=1.$$

For $P \in S$, let T_P denote the tangent plane of S at P. Prove that the curve

$$\{\, P \in S \mid \mathrm{T}_P \ni (0,0,0)\,\}$$

is contained in a plane in $\mathbb{R}^3$.

2. (25 pts) Show that if a C^2 function f satisfies

$$f(tx,ty)=t^2 f(x,y)$$

for any (x,y) and any real number $t \in \mathbb{R}$, then

$$f(x,y)=\frac{1}{2}\left[x^2\frac{\partial^2 f}{\partial x^2}(0,0)+2xy\frac{\partial^2 f}{\partial x \partial y}(0,0)+y^2\frac{\partial^2 f}{\partial y^2}(0,0)\right].$$

3. (30 pts) For the function

$$f(x,y)=\sin(x\cos y)$$

(a) find the local maximum points, local minimum points, and saddle points of f ;
(b) find the third-degree Taylor polynomial of f at $(0,0)$.

4. (25 pts) Determine the minimum of $xy+yz+zx$ on the surface $x^2+y^2-z^2=1$.

5. (30 pts) Let $F:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear map with $\det F'(0,0)=2$.
(a) Find $\det F'(1,0)$.
(b) For the function $G(x,y)=(x^2, x^2-y^2)$, find $\det(F\circ G)'(1,1)$.

6. (25 pts) Find the line integral $\int_X \mathbf{F}\cdot d\mathbf{s}$ of the vector field

$$\mathbf{F}(x,y,z)=(2xe^{x^2+y^3}+z\cos y,\ 3y^2e^{x^2+y^3}-xz\sin y,\ x\cos y)$$

along the curve $X(t)=(\cos t, \sin t, t),\ \ 0\le t\le 2\pi$.

7. (20 pts) For the function $f(x,y,z)=x^3+y^3+z^3+3xyz$,
(a) show that there exists a differentiable function $g(x,y)$ defined on an neighborhood of $(1,1)$ such that $g(1,1)=1$ and $f(x,y,g(x,y))=6$.
(b) Find $\operatorname{grad} g(1,1)$.

8. (20 pts) Consider the function

$$f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0).\end{cases}$$

(a) Show that f is continuous at $(0,0)$.
(b) Determine whether f is differentiable at $(0,0)$.