Chapter 25

Capacitance

Lecture 7 & 8

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 → Modified by S.H. Seo

25 Summary

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Capacitor and Capacitance

- The capacitance of a capacitor is defined as:

$$\boxed{q = CV}$$ **Eq. 25-1**

Determining Capacitance

- Parallel-plate capacitor:

$$\boxed{C = \frac{\varepsilon_0 A}{d}.}$$ **Eq. 25-9**

- Cylindrical Capacitor:

$$\boxed{C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}.}$$ **Eq. 25-14**

- Spherical Capacitor:

$$\boxed{C = 4\pi\varepsilon_0 \frac{ab}{b-a}.}$$ **Eq. 25-17**

- Isolated sphere:

$$\boxed{C = 4\pi\varepsilon_0 R.}$$ **Eq. 25-18**

Capacitor in parallel and series

- In parallel:

$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j}$$ **Eq. 25-19**

- In series

$$\boxed{\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}}$$ **Eq. 25-20**

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2.$$ **Eq. 25-21&22**

- Energy density (*u*)

$$u = \tfrac{1}{2}\varepsilon_0 E^2.$$ **Eq. 25-25**

25-1 Capacitance

Learning Objectives

25.01 Sketch a schematic diagram of a circuit with a parallel-plate capacitor, a battery, and an open or closed switch.

25.02 In a circuit with a battery, an open switch, and an uncharged capacitor, explain what happens to the conduction electrons when the switch is closed.

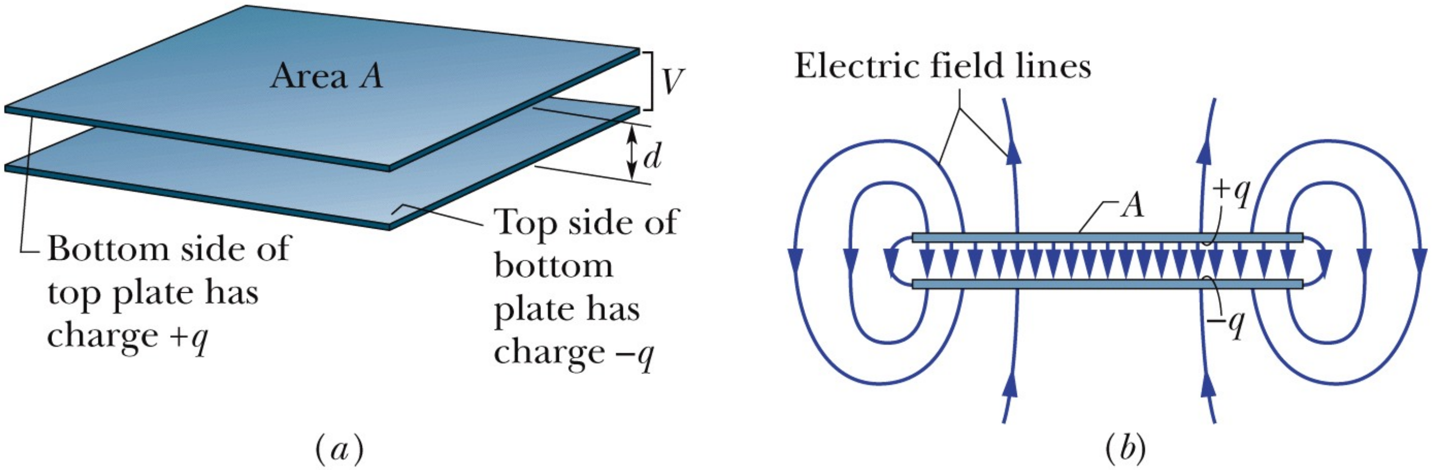
25.03 For a capacitor, apply the relationship between the magnitude of charge *q* on either plate (“the charge on the capacitor”), the potential difference V between the plates (“the potential across the capacitor”), and the capacitance C of the capacitor.

25-1 Capacitance

A capacitor consists of two isolated conductors (the plates) with charges $+q$ and $-q$. Its **capacitance** C is defined from

$$\boxed{q = CV.}$$

where V is the potential difference between the plates.



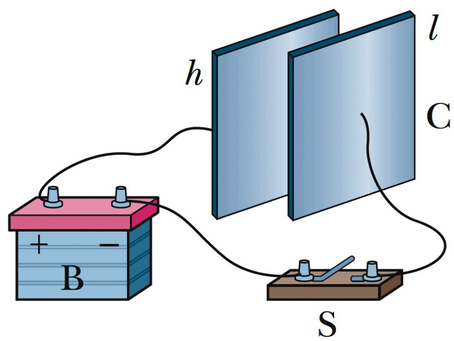
A parallel-plate capacitor, made up of two plates of area A separated by a distance d. The charges on the facing plate surfaces have the same magnitude q but opposite signs

As the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the “fringing” of the field lines there.

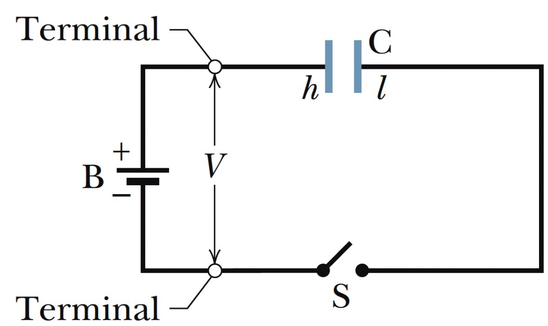
25-1 Capacitance

Charging Capacitor

When a circuit with a battery, an open switch, and an uncharged capacitor is completed by closing the switch, conduction electrons shift, leaving the capacitor plates with opposite charges.



(a)



(b)

In Fig. a, a battery B, a switch S, an uncharged capacitor C, and interconnecting wires form a circuit. The same circuit is shown in the schematic diagram of Fig. b, in which the symbols for a battery, a switch, and a capacitor represent those devices. The battery maintains potential difference V between its terminals. The terminal of higher potential is labeled + and is often called the positive terminal; the terminal of lower potential is labeled - and is often called the negative terminal.

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25-2 Calculating the Capacitance

Learning Objectives

25.04 Explain how Gauss' law is used to find the capacitance of a parallel-plate capacitor.

25.05 For a parallel-plate capacitor, a cylindrical capacitor, a spherical capacitor, and an isolated sphere, calculate the capacitance.

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25-2 Calculating the Capacitance

Calculating electric field and potential difference

To relate the electric field ***E*** between the plates of a capacitor to the charge *q* on either plate, we shall use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \quad \Rightarrow \quad q = \varepsilon_0 EA$$

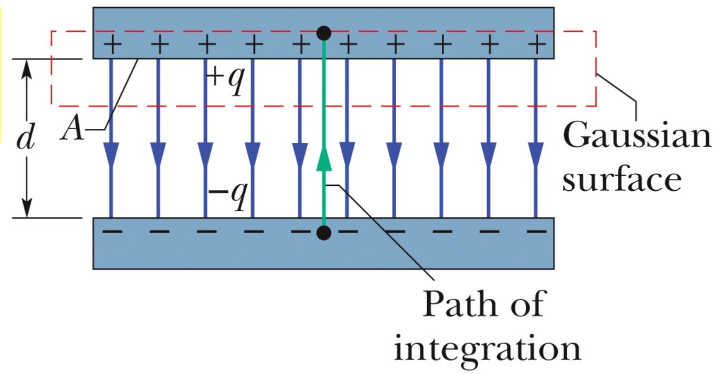
We use Gauss' law to relate *q* and *E*. Then we integrate the *E* to get the potential difference.

the potential difference between the plates of a capacitor is related to the field ***E*** by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

Letting V represent the difference $V_f - V_i$, we can then recast the above equation as:

$$V = \int_-^+ E\, ds = E \int_0^d ds = Ed.$$



A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration is taken along a path extending directly from the negative plate to the positive plate.

25-2 Calculating the Capacitance

Parallel-Plate Capacitor

We assume, as Figure suggests, that the plates of our parallel-plate capacitor are so large and so close together that we can neglect the fringing of the electric field at the edges of the plates, taking ***E*** to be constant throughout the region between the plates.

We use Gauss' law to relate *q* and *E*. Then we integrate the *E* to get the potential difference.

We draw a Gaussian surface that encloses just the charge *q* on the positive plate

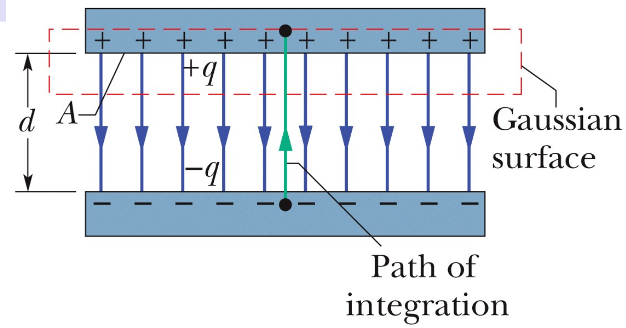
$$q = \varepsilon_0 EA$$

where A is the area of the plate. And therefore,

$$V = \int_{-}^{+} E\,ds = E\int_0^d ds = Ed.$$

Now if we substitute *q* in the above relations to *q=CV*, we get,

$$\boxed{C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor).}}$$



A charged parallel-plate capacitor. A Gaussian surface encloses the charge on the positive plate. The integration is taken along a path extending directly from the negative plate to the positive plate.

25-2 Calculating the Capacitance

Cylindrical Capacitor

Figure shows, in cross section, a cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b. We assume that $L >> b$ so that we can neglect the fringing of the electric field that occurs at the ends of the cylinders. Each plate contains a charge of magnitude q. Here, charge and the field magnitude E is related as follows,

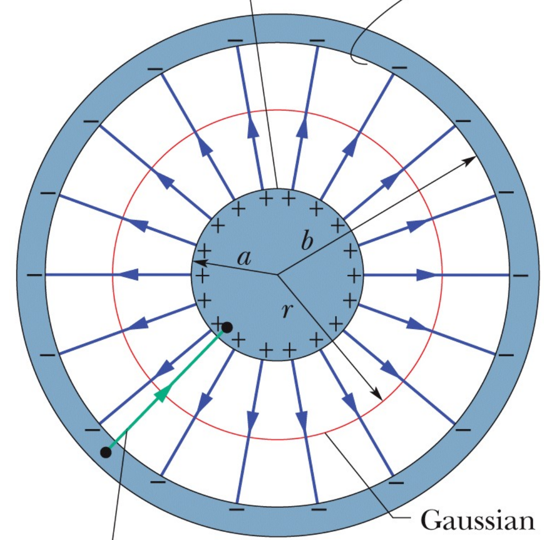
$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$

Solving for E field:

$$V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right),$$

From the relation $C = q/V$, we then have

$$\boxed{C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).}}$$



A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

25-2 Calculating the Capacitance

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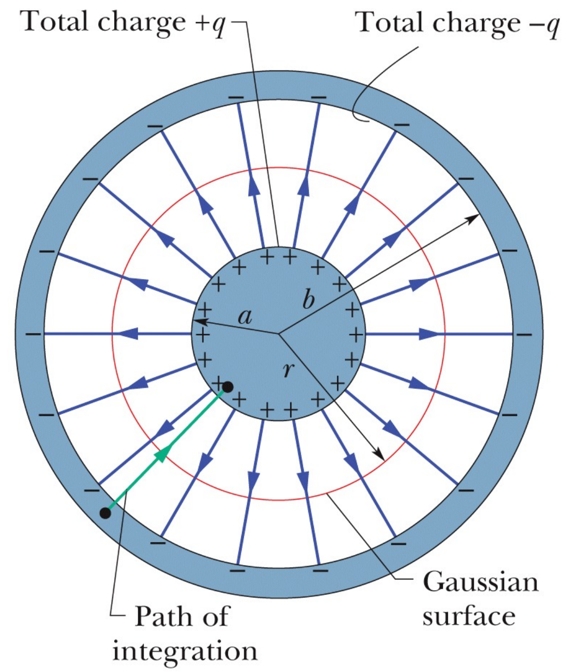
Others…

For **spherical capacitor** the capacitance is:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{(spherical capacitor).}$$

Capacitance of an **isolated sphere (b→ inf.)**:

$$C = 4\pi\varepsilon_0 R \quad \text{(isolated sphere).}$$



A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate) and the radial path of integration. This figure also serves to illustrate a spherical capacitor in a cross section through its center.

Checkpoint 2

For capacitors charged by the same battery, does the charge stored by the capacitor increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased. (c) The radius of the outer spherical shell of a spherical capacitor is increased.

Answer: (a) decreases (b) increases
(c) increases

25-3 Capacitors in Parallel and in Series

Learning Objectives

25.06 Sketch schematic diagrams for a battery and (a) three capacitors in parallel and (b) three capacitors in series.

25.07 Identify that capacitors in parallel have the same potential difference, which is the same value that their equivalent capacitor has.

25.08 Calculate the equivalent of parallel capacitors.

25.09 Identify that the total charge stored on parallel capacitors the sum of the charges stored on the individual capacitors.

25.10 Identify that capacitors in series have the same charge, which is the same value that their equivalent capacitor has.

25.11 Calculate the equivalent of series capacitors.

25.12 Identify that the potential applied to capacitors in series is equal to the sum of the potentials across the individual capacitors.

25-3 Capacitors in Parallel and in Series

Learning Objectives (Cont'd.)

25.13 For a circuit with a battery and some capacitors in parallel and some in series, simplify the circuit in steps by finding equivalent capacitors, until the charge and potential on the final equivalent capacitor can be determined, and then reverse the steps to find the charge and potential on the individual capacitors.

25.14 For a circuit with a battery, an open switch, and one or more uncharged capacitors, determine the amount of charge that moves through a point in the circuit when the switch is closed.

25.15 When a charged capacitor is connected in parallel to one or more uncharged capacitors, determine the charge and potential difference on each capacitor when equilibrium is reached.

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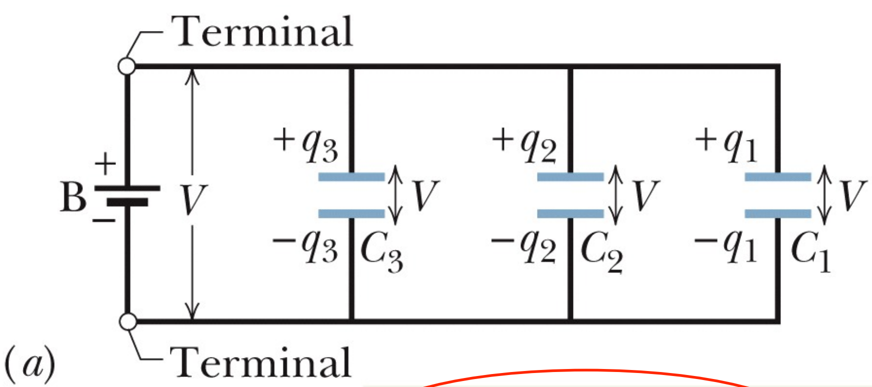
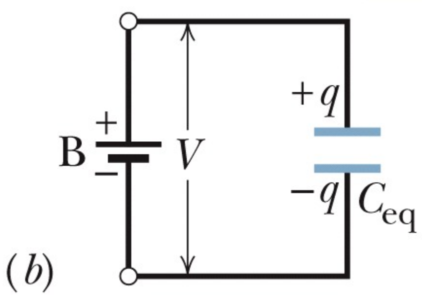
25-3 Capacitors in Parallel and in Series

In parallel

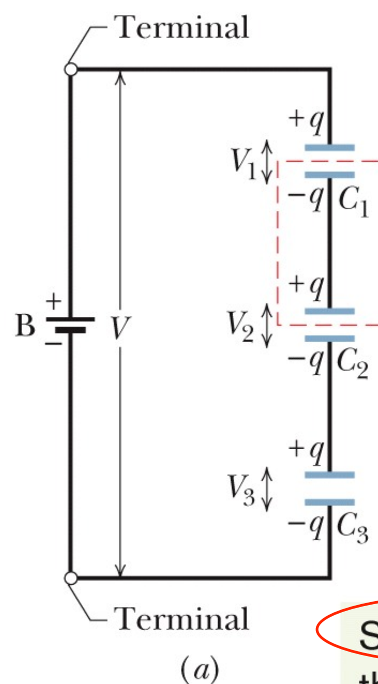


Parallel capacitors and their equivalent have the same V ("par-V").

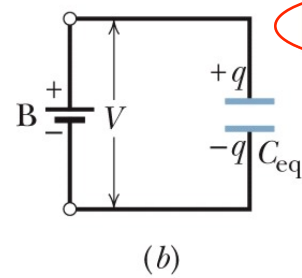


$$q = cV$$

In series



Series capacitors and their equivalent have the same q ("seri-q").



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25-3 Capacitors in Parallel

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad \text{and} \quad q_3 = C_3 V.$$

The total charge on the parallel combination of Fig. 25-8*a* is then

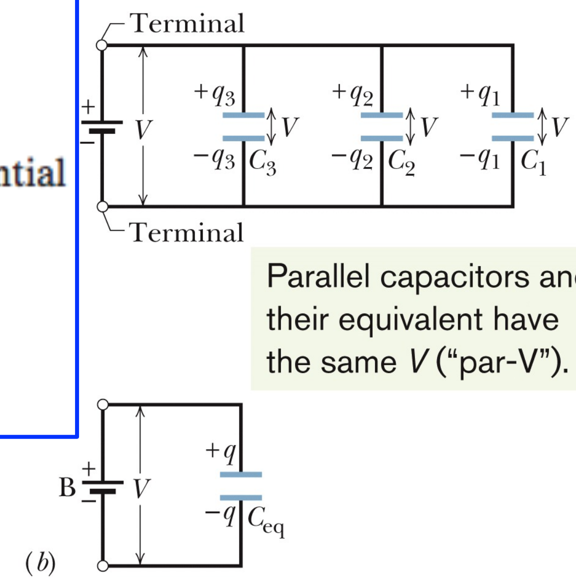
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

The equivalent capacitance, with the same total charge q and applied potential difference V as the combination, is then

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

a result that we can easily extend to any number n of capacitors, as

$$C_{eq} = \sum_{j=1}^{n} C_j \quad (n \text{ capacitors in parallel}).$$



Parallel capacitors and their equivalent have the same V (“par-V”).

Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.

25-3 Capacitors in Series

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When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

The total potential difference V due to the battery is the sum

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

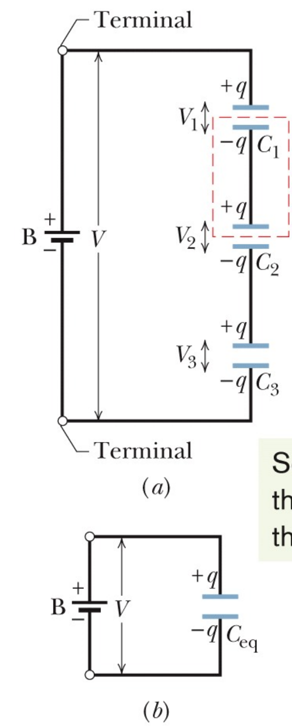
The equivalent capacitance is then

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$



Series capacitors and their equivalent have the same q ("seri-q").

Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same *total* potential difference V as the actual series capacitors.

25 Summary

Capacitor and Capacitance

- The capacitance of a capacitor is defined as:

$$\boxed{q = CV}$$ **Eq. 25-1**

Determining Capacitance

- Parallel-plate capacitor:

$$\boxed{C = \frac{\varepsilon_0 A}{d}.}$$ **Eq. 25-9**

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Capacitor in parallel and series

- In parallel:

$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j}$$ **Eq. 25-19**

- In series

$$\boxed{\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}}$$ **Eq. 25-20**

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$\boxed{U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2}$$ **Eq. 25-21&22**

- Energy density (*u*)

$$\boxed{u = \tfrac{1}{2}\varepsilon_0 E^2.}$$ **Eq. 25-25**

25 Summary

Capacitance with a Dielectric

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ, called the dielectric constant, which is characteristic of the material.

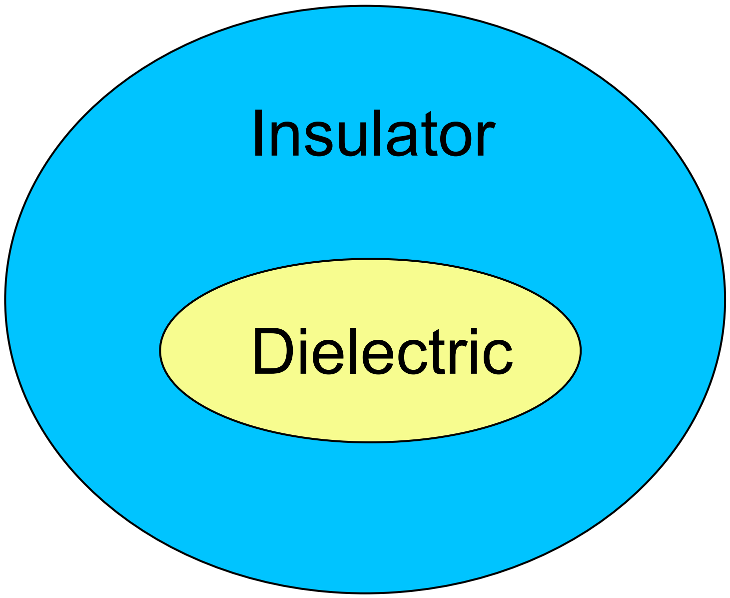
Gauss' Law with a Dielectric

- When a dielectric is present, Gauss' law may be generalized to

$$\boxed{\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q.}$$

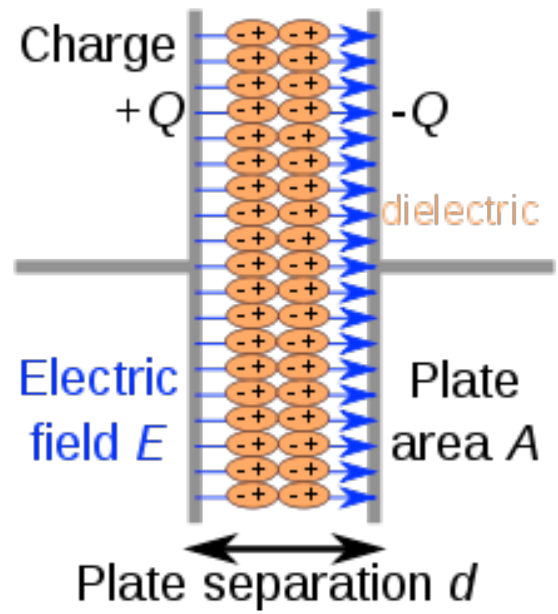
Eq. 25-36

Insulator vs. Dielectric (material)



Insulator:
internal electrons do not move freely

Dielectric:
Insulator that can be polarized by an applied E field.



25-4 Energy Stored in an Electric Field

Learning Objectives

25.16 Explain how the work required to charge a capacitor results in the potential energy of the capacitor.

25.17 For a capacitor, apply the relationship between the potential energy U, the capacitance C, and the potential difference V.

25.18 For a capacitor, apply the relationship between the potential energy, the internal volume, and the internal energy density.

25.19 For any electric field, apply the relationship between the potential energy density u in the field and the field's magnitude E.

25.20 Explain the danger of sparks in airborne dust.

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25-4 Energy Stored in an Electric Field

The **electric potential energy** *U* of a charged capacitor,

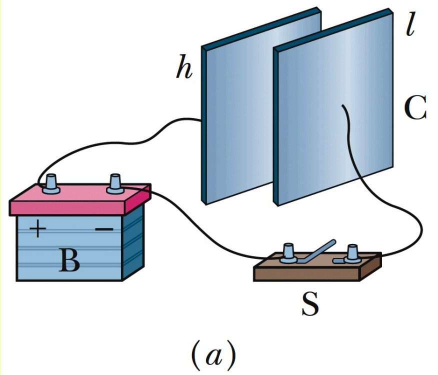
$$\boxed{U = \frac{q^2}{2C} \quad \text{(potential energy).}}$$

and,

$$\boxed{U = \tfrac{1}{2}CV^2 \quad \text{(potential energy).}}$$

$$\boxed{q = CV}$$

is equal to the work required to charge the capacitor. This energy can be associated with the capacitor's electric field ***E***.



$$\boxed{U = qV.}$$

$$dW = Vdq' = \frac{q'}{C}dq' \Rightarrow W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}$$

25-4 Energy Stored in an Electric Field

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Every electric field, in a capacitor or from any other source, has an associated stored energy. In vacuum, the **energy density** *u* (potential energy per unit volume) in a field of magnitude *E* is

$$\boxed{u = \tfrac{1}{2}\varepsilon_0 E^2 \quad \text{(energy density).}}$$

Parallel capacitor

$$u = \frac{U}{volume} = \frac{U}{Ad} = \frac{CV^2}{2Ad}$$

$$C = \varepsilon_0 \frac{A}{d}$$

$$u = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

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25-5 Capacitor with a Dielectric

Learning Objectives

25.21 Identify that capacitance is increased if the space between the plates is filled with a dielectric material.

25.22 For a capacitor, calculate the capacitance with and without a dielectric.

25.23 For a region filled with a dielectric material with a given dielectric constant *k*, identify that all electrostatic equations containing the permittivity constant ε_0 are modified by multiplying that constant by the dielectric constant to get k ε_0.

25.24 Name some of the common dielectrics.

25.25 In adding a dielectric to a charged capacitor, distinguish the results for a capacitor (a) connected to a battery and (b) not connected to a battery.

25.26 Distinguish polar dielectrics from non-polar dielectrics.

25.27 In adding a dielectric to a charged capacitor, explain what happens to the electric field between the plates in terms of what happens to the atoms in the dielectric.

25-5 Capacitor with a Dielectric

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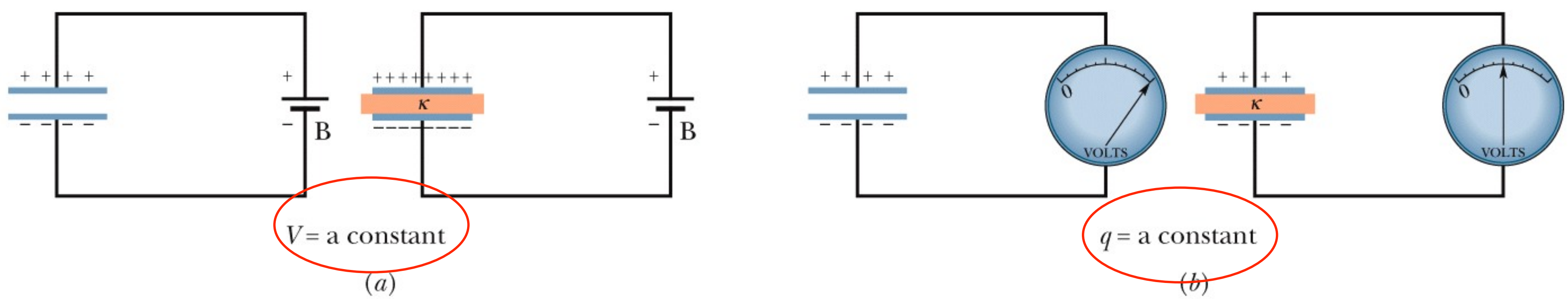
If the space between the plates of a capacitor is completely filled with a **dielectric material**, the capacitance C in vacuum (or, effectively, in air) is multiplied by the material's **dielectric constant** κ, (Greek kappa) which is a number greater than 1.

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.



(a)If the potential difference between the plates of a capacitor is maintained, as by the presence of battery B, the effect of a dielectric is to increase the charge on the plates.

(b) If the charge on the capacitor plates is maintained, as in this case by isolating the capacitor, the effect of a dielectric is to reduce the potential difference between the plates. The scale shown is that of a potentiometer, a device used to measure potential difference (here, between the plates). A capacitor cannot discharge through a potentiometer.

Dielectric constant

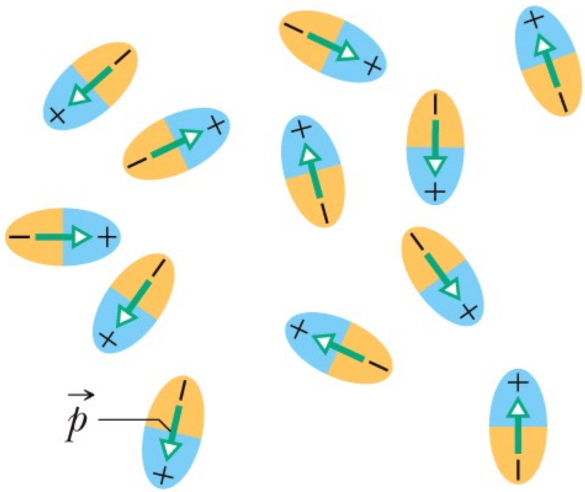
TABLE 17–3 Dielectric constants (at 20°C)

Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Vinyl (plastic)	2–4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Rubber, neoprene	6.7	12×10^6
Porcelain	6–8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

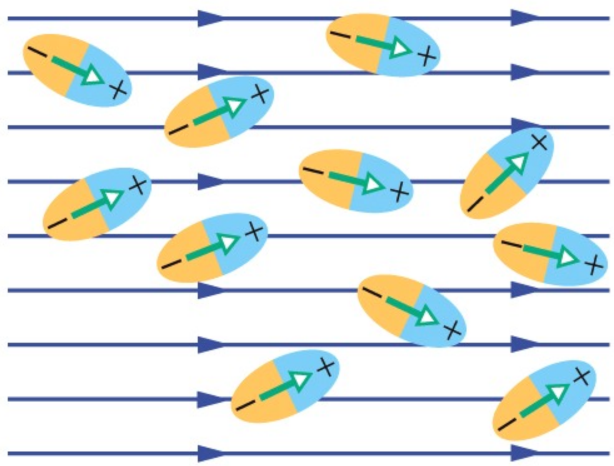
25-5 Capacitor with a Dielectric

An Atomic View

Polar Dielectrics



(a)



(b)

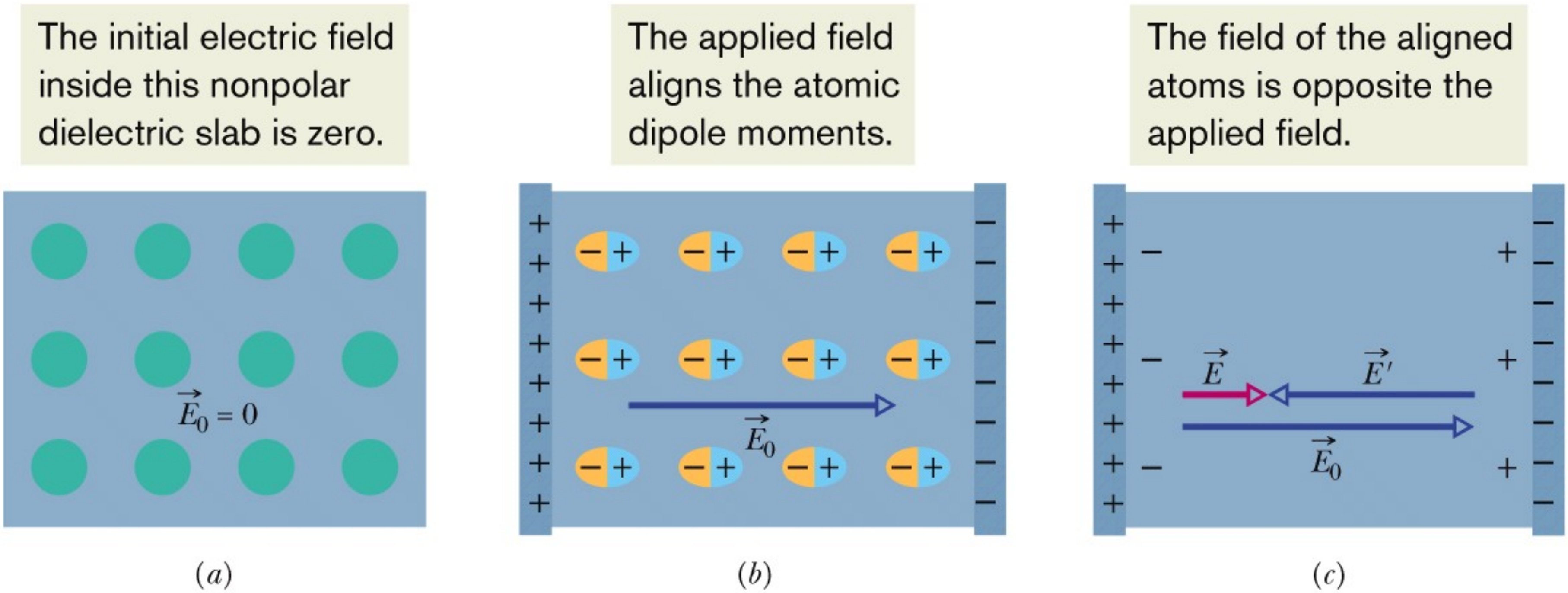
(a) Molecules with a permanent electric dipole moment, showing their random orientation in the absence of an external electric field.

(b) An electric field is applied, producing partial alignment of the dipoles. Thermal motion prevents complete alignment.

25-5 Capacitor with a Dielectric

An Atomic View

Nonpolar Dielectrics



25-6 Dielectrics and Gauss' Law

Learning Objectives

25.28 In a capacitor with a dielectric, distinguish free charge from induced charge.

25.29 When a dielectric partially or fully fills the space in a capacitor, find the free charge, the induced charge, the electric field between the plates (if there is a gap, there is more than one field value), and the potential between the plates.

25-6 Dielectrics and Gauss' Law

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- Inserting a dielectric into a capacitor causes induced charge to appear on the faces of the dielectric and weakens the electric field between the plates.
- The induced charge is less than the free charge on the plates.

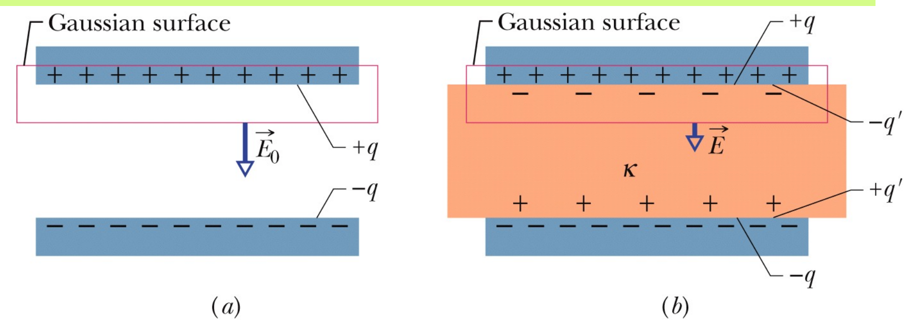
When a dielectric is present, Gauss' law may be generalized to

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss' law with dielectric).}$$

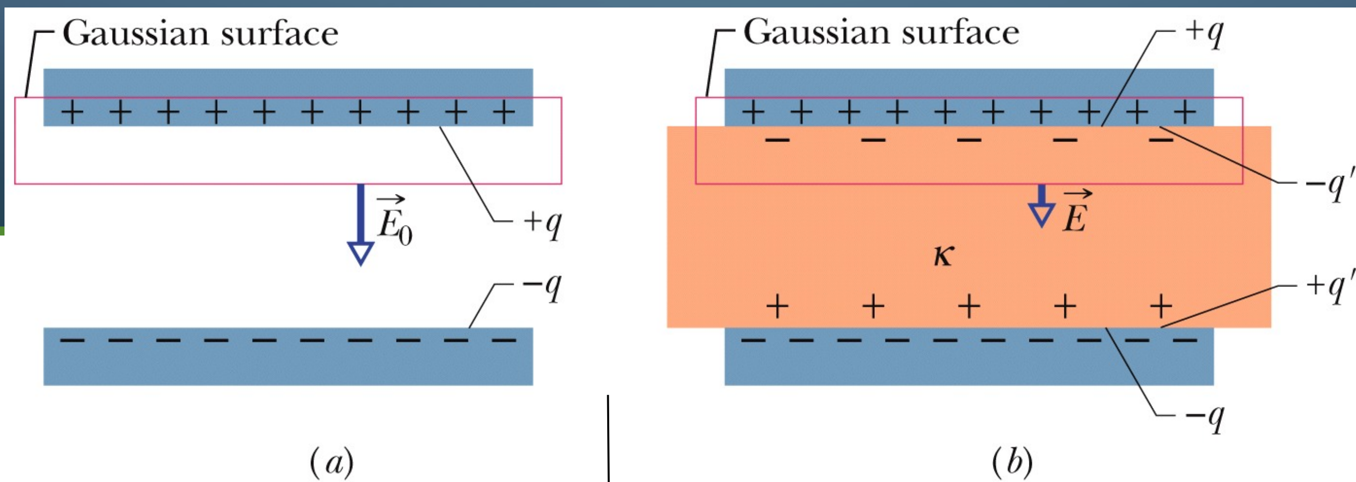
where q is the free charge. Any induced surface charge is accounted for by including the dielectric constant k inside the integral.

Note:
The flux integral now involves $\kappa\boldsymbol{E}$, not just $\boldsymbol{E}$. The vector $\varepsilon_0\kappa\boldsymbol{E}$ is sometimes called the electric displacement $\boldsymbol{D}$, so that the above equation can be written in the form

$$\oint \vec{D} \cdot d\vec{A} = q.$$



A parallel-plate capacitor (a) without and (b) with a dielectric slab inserted. The charge q on the plates is assumed to be the same in both cases.



Gauss's Law

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q$$

$$\Rightarrow \quad E_0 = \frac{q}{\varepsilon_0 A}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q - q'$$

$$E = \frac{q - q'}{\varepsilon_0 A} \qquad E = \frac{E_0}{\kappa} = \frac{q}{\kappa \varepsilon_0 A}$$

$$\Rightarrow \quad q - q' = \frac{q}{\kappa}$$

$$E = \frac{q}{\varepsilon_0 \kappa A} \qquad \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = \frac{q}{\kappa}$$

Electric displacement

$$\boxed{\vec{D} = \varepsilon_0 \kappa \vec{E}} \qquad \boxed{\oint \vec{D} \cdot d\vec{A} = q} \Leftarrow \boxed{\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q}$$

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25 Summary

Capacitor and Capacitance

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$$q = CV$$ **Eq. 25-1**

Determining Capacitance

- Parallel-plate capacitor:

$$C = \frac{\varepsilon_0 A}{d}.$$ **Eq. 25-9**

- Cylindrical Capacitor:

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- Spherical Capacitor:

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- Isolated sphere:

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Capacitor in parallel and series

- In parallel:

$$C_{eq} = \sum_{j=1}^{n} C_j$$ **Eq. 25-19**

- In series

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$ **Eq. 25-20**

Potential Energy and Energy Density

- Electric Potential Energy (U):

$$U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2$$ **Eq. 25-21&22**

- Energy density (*u*)

$$u = \tfrac{1}{2}\varepsilon_0 E^2.$$ **Eq. 25-25**

25 Summary

Capacitance with a Dielectric

- If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C is increased by a factor κ, called the dielectric constant, which is characteristic of the material.

Gauss' Law with a Dielectric

- When a dielectric is present, Gauss' law may be generalized to

$$\boxed{\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q.}$$

Eq. 25-36