Quiz #3 (CSE 4190.313)

Wednesday, May 11, 2016

Name: _____ **ID No:** _____

1. (10 points) If A is a Markov matrix, show that the sum of the components of $A\mathbf{x}$ equals the sum of the components of $\mathbf{x}$. Deduce that if $A\mathbf{x} = \lambda\mathbf{x}$ with $\lambda \neq 1$, the components of the eigenvector add to zero.

2. (10 points) Find the rank and all four eigenvalues for each of the following matrices. Which eigenvectors correspond to nonzero eigenvalues?

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

3. (8 points) The identity transformation takes every vector to itself: $T\mathbf{x} = \mathbf{x}$. Find the corresponding matix, if the first basis is $\mathbf{v}_1 = (1, 0)$, $\mathbf{v}_2 = (0, 1)$, and the second basis is $\mathbf{w}_1 = (1, 2)$, $\mathbf{w}_2 = (3, 4)$.

4. (12 points) True or false, with a good reason or a counterexample.

   (a) (3 points) An invertible matrix cannot be similar to a singular matrix.

   (b) (3 points) A symmetric matrix cannot be similar to a nonsymmetric matrix.

   (c) (3 points) A cannot be similar to $-A$ unless $A = 0$.

   (d) (3 points) $A - I$ cannot be similar to $A + I$.