Chapter 30

Induction and Inductance

Lecture 15,16

Seon-Hee Seo

2016.10.12-10.17

Copyright © 2014 John Wiley & Sons, Inc. All rights reserved. → Modified by S.H. Seo

30 Summary

WILEY

Magnetic Flux

- The magnetic flux through an area *A* in a magnetic field ***B*** is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$ **Eq. 30-1**

- If B: is perpendicular to the area and uniform over it, Eq. 30-1 becomes

$$\Phi_B = BA \quad (\vec{B} \perp A, \vec{B} \text{ uniform}).$$ **Eq. 30-2**

Faraday's Law of Induction

- The induced *emf* is,

$$\mathscr{E} = -\frac{d\Phi_B}{dt}$$ **Eq. 30-4**

- If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\mathscr{E} = -N\frac{d\Phi_B}{dt}.$$ **Eq. 30-5**

Lenz's Law

- An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current.

Emf and the Induced Magnetic Field

- The induced emf is related to ***E*** by

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s},$$ **Eq. 30-19**

- Faraday's law in its most general form,

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$ **Eq. 30-20**

30-1 Faraday's Law and Lenz's Law

Learning Objectives

30.01 Identify that the amount of magnetic field piercing a surface (not skimming along the surface) is the magnetic flux Φ through the surface.

30.02 Identify that an area vector for a flat surface is a vector that is perpendicular to the surface and that has a magnitude equal to the area of the surface.

30.03 Identify that any surface can be divided into area elements (patch elements) that are each small enough and flat enough for an area vector *d**A*** to be assigned to it, with the vector perpendicular to the element and having a magnitude equal to the area of the element.

30.04 Calculate the magnetic flux Φ through a surface by integrating the dot product of the magnetic field vector ***B*** and the area vector *d**A*** (for patch elements) over the surface, in magnitude-angle notation and unit-vector notation.

30.05 Identify that a current is induced in a conducting loop while the number of magnetic field lines intercepted by the loop is changing.

WILEY

30-1 Faraday's Law and Lenz's Law

Learning Objectives (Contd.)

30.06 Identify that an induced current in a conducting loop is driven by an induced emf.

30.07 Apply Faraday's law, which is the relationship between an induced emf in a conducting loop and the rate at which magnetic flux through the loop changes.

30.08 Extend Faraday's law from a loop to a coil with multiple loops.

30.09 Identify the three general ways in which the magnetic flux through a coil can change.

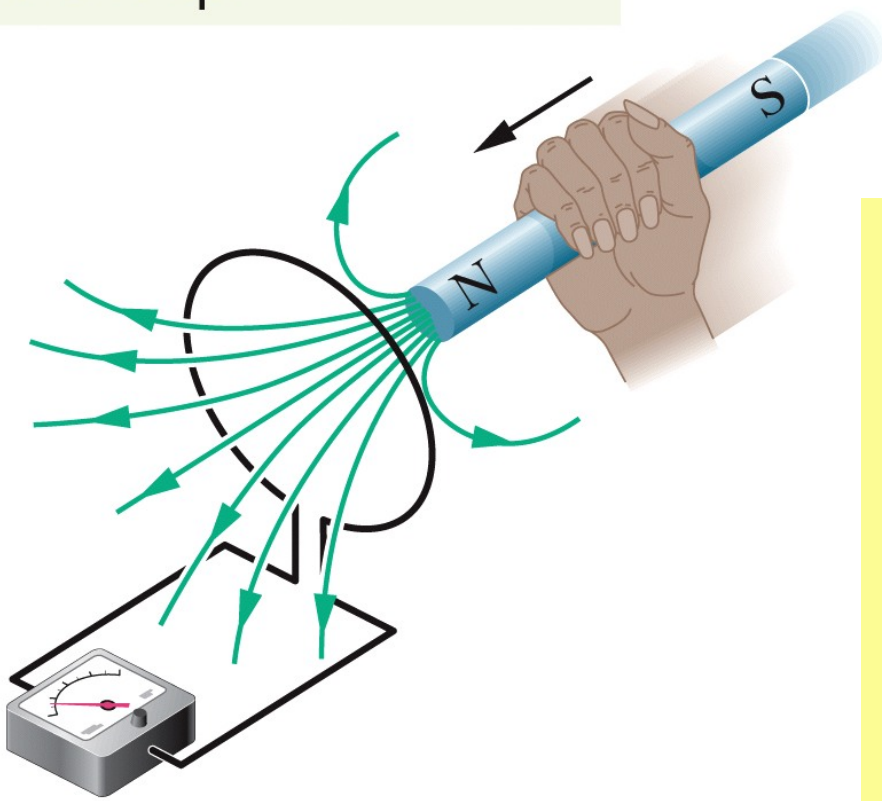
30.10 Use a right-hand rule for Lenz's law to determine the direction of induced emf and induced current in a conducting loop.

30.11 Identify that when a magnetic flux through a loop changes, the induced current in the loop sets up a magnetic field to oppose that change.

30.12 If an emf is induced in a conducting loop containing a battery, determine the net emf and calculate the corresponding current in the loop.

30-1 Faraday's Law and Lenz's Law

The magnet's motion creates a current in the loop.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

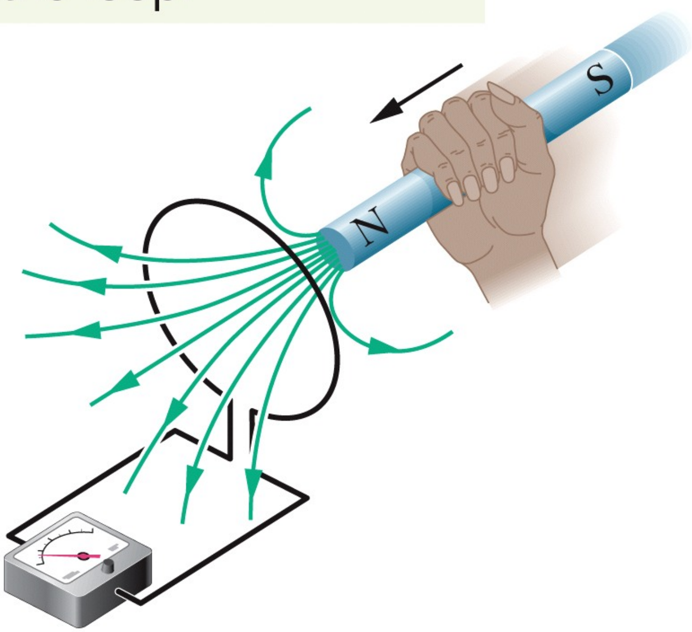
1st Experiment

- If we move a bar magnet toward the loop, a current suddenly appears in the circuit.
- The current disappears when the magnet stops moving.
- If we then move the magnet away, a current again suddenly appears, but now in the opposite direction.

30-1 Faraday's Law and Lenz's Law

1st Experiment

The magnet's motion creates a current in the loop.

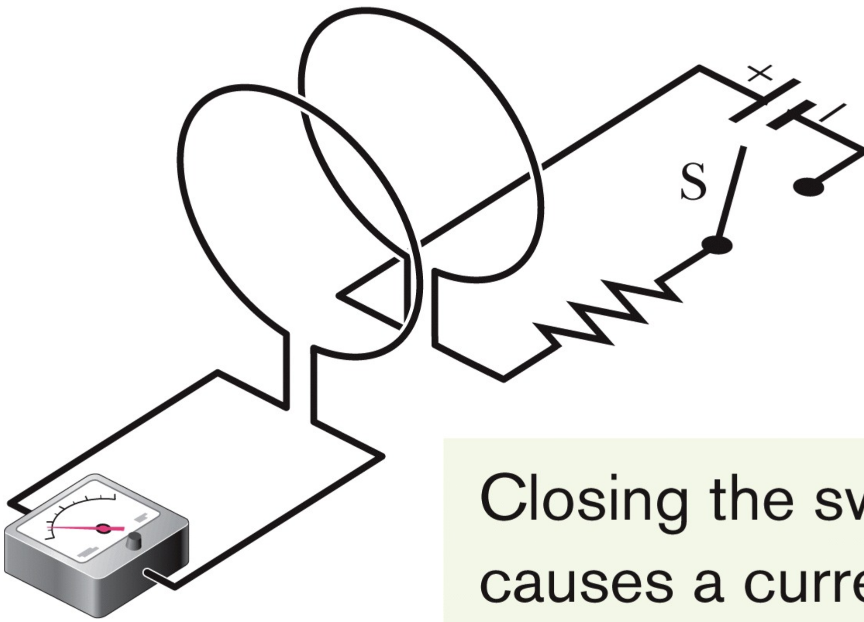


Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion of the magnet produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions from the north pole effects.

30-1 Faraday's Law and Lenz's Law

2nd Experiment



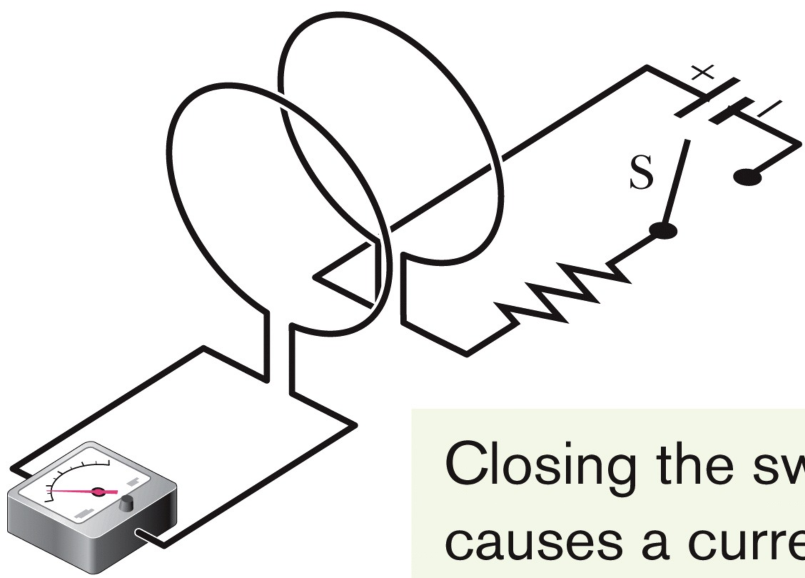
Closing the switch causes a current in the left-hand loop.

Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

- If we close switch S to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop.
- If the switch remains closed, no further current is observed.
- If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction.

30-1 Faraday's Law and Lenz's Law

2nd Experiment



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

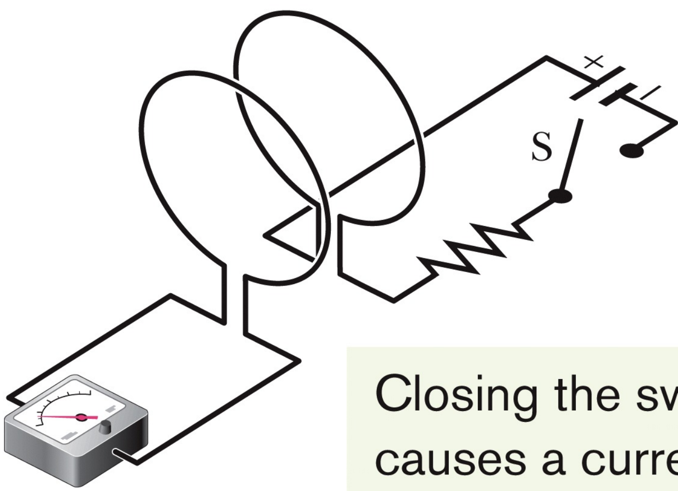
Closing the switch causes a current in the left-hand loop.

- We get an induced current (from an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large).

- The induced emf and induced current in these experiments are apparently caused when something changes — but what is that "something"? Faraday knew.

30-1 Faraday's Law and Lenz's Law

Faraday's Law of Induction



Closing the switch causes a current in the left-hand loop.

Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

The magnetic flux Φ_B through an area *A* in a magnetic field ***B*** is defined as

$$\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A}}$$

where the integral is taken over the area. The SI unit of magnetic flux is the weber, where *1 Wb = 1 T • m²*.

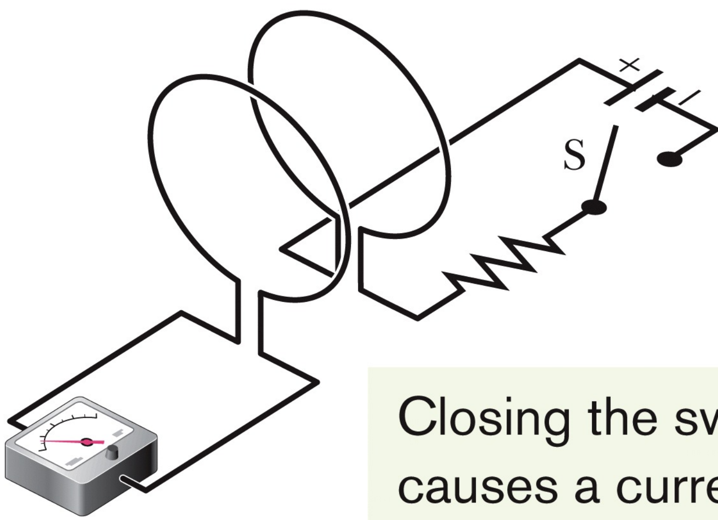
If ***B*** is perpendicular to the area and uniform over it, the flux is

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

30-1 Faraday's Law and Lenz's Law

Faraday's Law of Induction

The magnitude of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Closing the switch causes a current in the left-hand loop.

Faraday's Law.
With the notion of magnetic flux, we can state Faraday's law in a more quantitative and useful way:

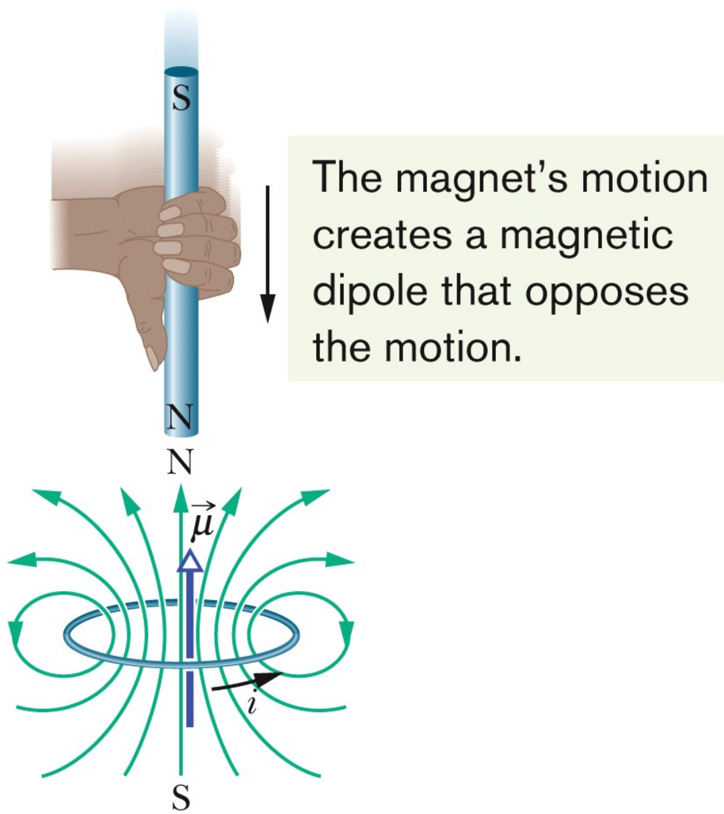
$$\boxed{\mathscr{E} = -\frac{d\Phi_B}{dt}}$$

the induced emf tends to oppose the flux change and the minus sign indicates this opposition.
This minus sign is referred to as Lenz's Law.

30-1 Faraday's Law and Lenz's Law

Lenz's Law

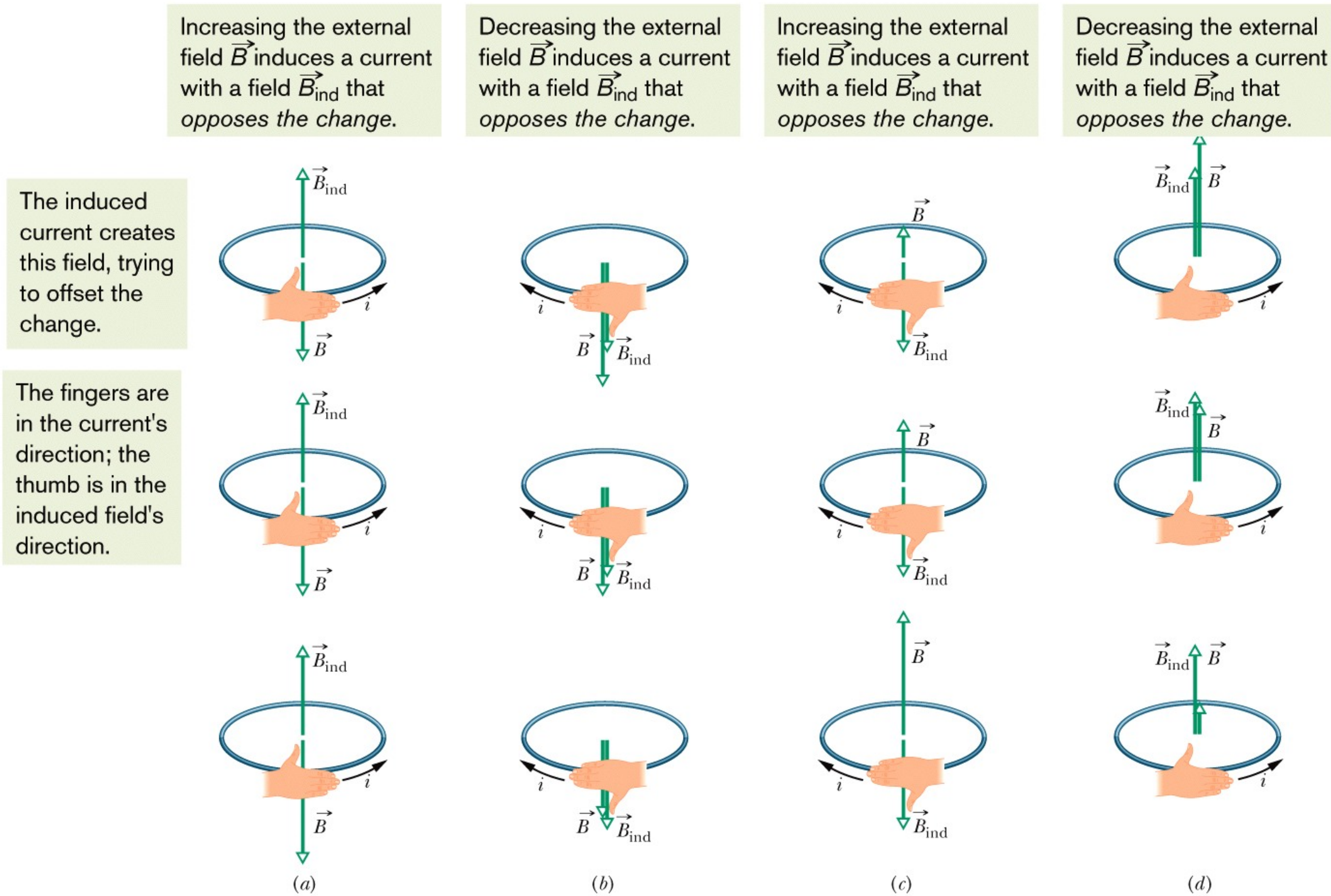
An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current. The induced emf has the same direction as the induced current.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment **μ** oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

30-1 Faraday's Law and Lenz's Law



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

30-2 Induction and Energy Transfer

Learning Objectives

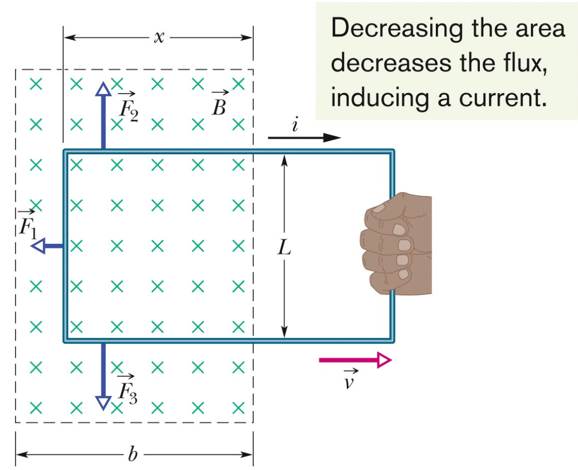
30.13 For a conducting loop pulled into or out of a magnetic field, calculate the rate at which energy is transferred to thermal energy.

30.14 Apply the relationship between an induced current and the rate at which it produces thermal energy.

30.15 Describe eddy currents.

30-2 Induction and Energy Transfer

In the figure, a rectangular loop of wire of width *L* has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in the figure show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity ***v***.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

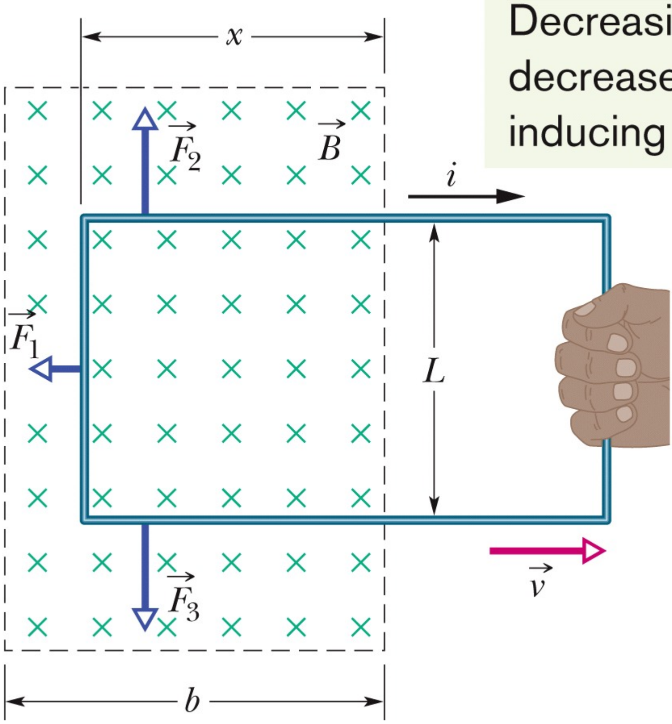
Flux change: Therefore, in the figure a magnetic field and a conducting loop are in relative motion at speed *v* and the flux of the field through the loop is changing with time (here the flux is changing as the area of the loop still in the magnetic field is changing).

Rate of Work: To pull the loop at a constant velocity ***v***, you must apply a constant force ***F*** to the loop because a magnetic force of equal magnitude but opposite direction acts on the loop to oppose you. The rate at which you do work — that is, the power — is then

$$P = Fv,$$

where F is the magnitude of the force you apply to the loop.

30-2 Induction and Energy Transfer



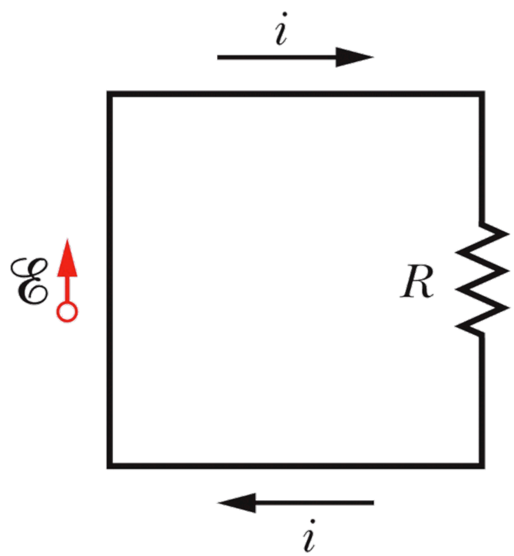
Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Decreasing the area decreases the flux, inducing a current.

$$\Phi_B = BA = BLx.$$

the magnitude of induced emf

$$\boxed{\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL\frac{dx}{dt} = BLv,}$$



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

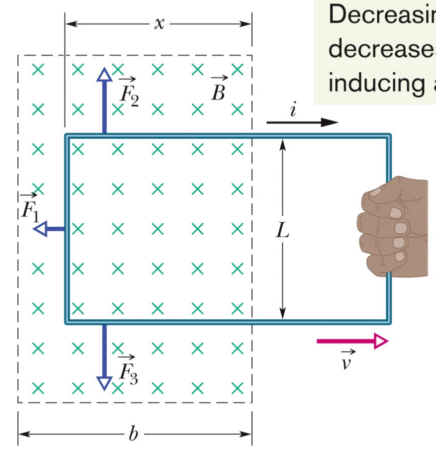
A circuit diagram for the loop of above figure while the loop is moving.

WILEY

30-2 Induction and Energy Transfer

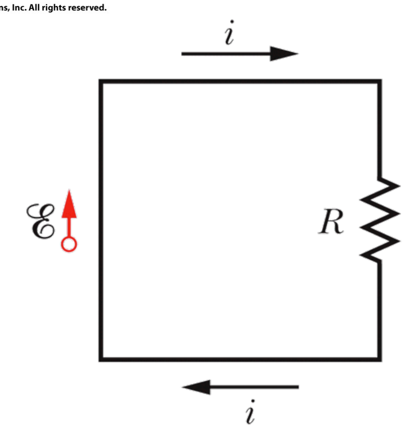
Induced Current: Figure (bottom) shows the loop as a circuit: induced *emf* is represented on the left, and the collective resistance R of the loop is represented on the right. To find the magnitude of the induced current, we can apply the equation $i = \mathscr{E}/R$, which gives

$$\boxed{i = \frac{BLv}{R}.}$$



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

In the Fig. (top), the deflecting forces acting on the three segments of the loop are marked $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, and $\boldsymbol{F_3}$. Note, however, that from the symmetry, forces $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ are equal in magnitude and cancel. This leaves only force $\boldsymbol{F_1}$, which is directed opposite your force $\boldsymbol{F}$ on the loop and thus is the force opposing you.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

A circuit diagram for the loop of above figure while the loop is moving.

WILEY

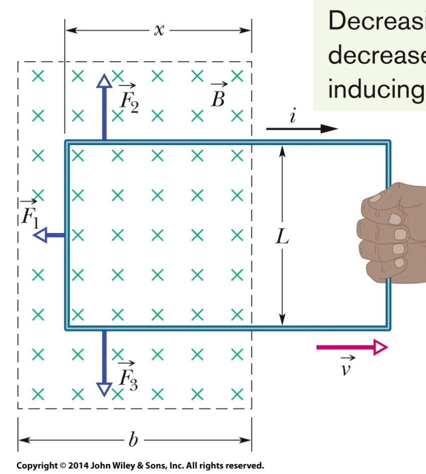
30-2 Induction and Energy Transfer

So, $\boldsymbol{F} = -\boldsymbol{F_1}$. the magnitude of $\boldsymbol{F_1}$ thus

$$F = F_1 = iLB \sin 90° = iLB.$$ (from $$\vec{F}_d = i\vec{L} \times \vec{B}.$$)

where the angle between **B** and the length vector **L** for the left segment is 90°. This gives us

$$\boxed{F = \frac{B^2L^2v}{R}.}$$

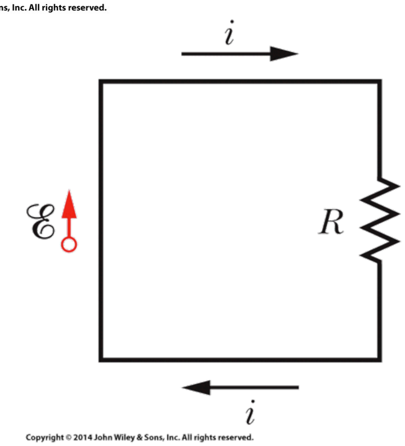


Because B, L, and R are constants, the speed v at which you move the loop is constant if the magnitude F of the force you apply to the loop is also constant.

Rate of Work: We find the rate at which you do work on the loop as you pull it from the magnetic field:

$$\boxed{P = Fv = \frac{B^2L^2v^2}{R}}$$

NOTE: The work that you do in pulling the loop through the magnetic field appears as thermal energy in the loop.



A circuit diagram for the loop of above figure while the loop is moving.

30 Summary

Magnetic Flux

- The magnetic flux through an area *A* in a magnetic field ***B*** is defined as

$$\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A}}$$ **Eq. 30-1**

- If B: is perpendicular to the area and uniform over it, Eq. 30-1 becomes

$$\boxed{\Phi_B = BA \quad (\vec{B} \perp A, \vec{B} \text{ uniform}).}$$ **Eq. 30-2**

Faraday's Law of Induction

- The induced *emf* is,

$$\boxed{\mathscr{E} = -\frac{d\Phi_B}{dt}}$$ **Eq. 30-4**

- If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\boxed{\mathscr{E} = -N\frac{d\Phi_B}{dt}.}$$ **Eq. 30-5**

Lenz's Law

- An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current.

Emf and the Induced Magnetic Field

- The induced emf is related to ***E*** by

$$\boxed{\mathscr{E} = \oint \vec{E} \cdot d\vec{s},}$$ **Eq. 30-19**

- Faraday's law in its most general form,

$$\boxed{\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$ **Eq. 30-20**

WILEY

30-3 Induced Electric Field

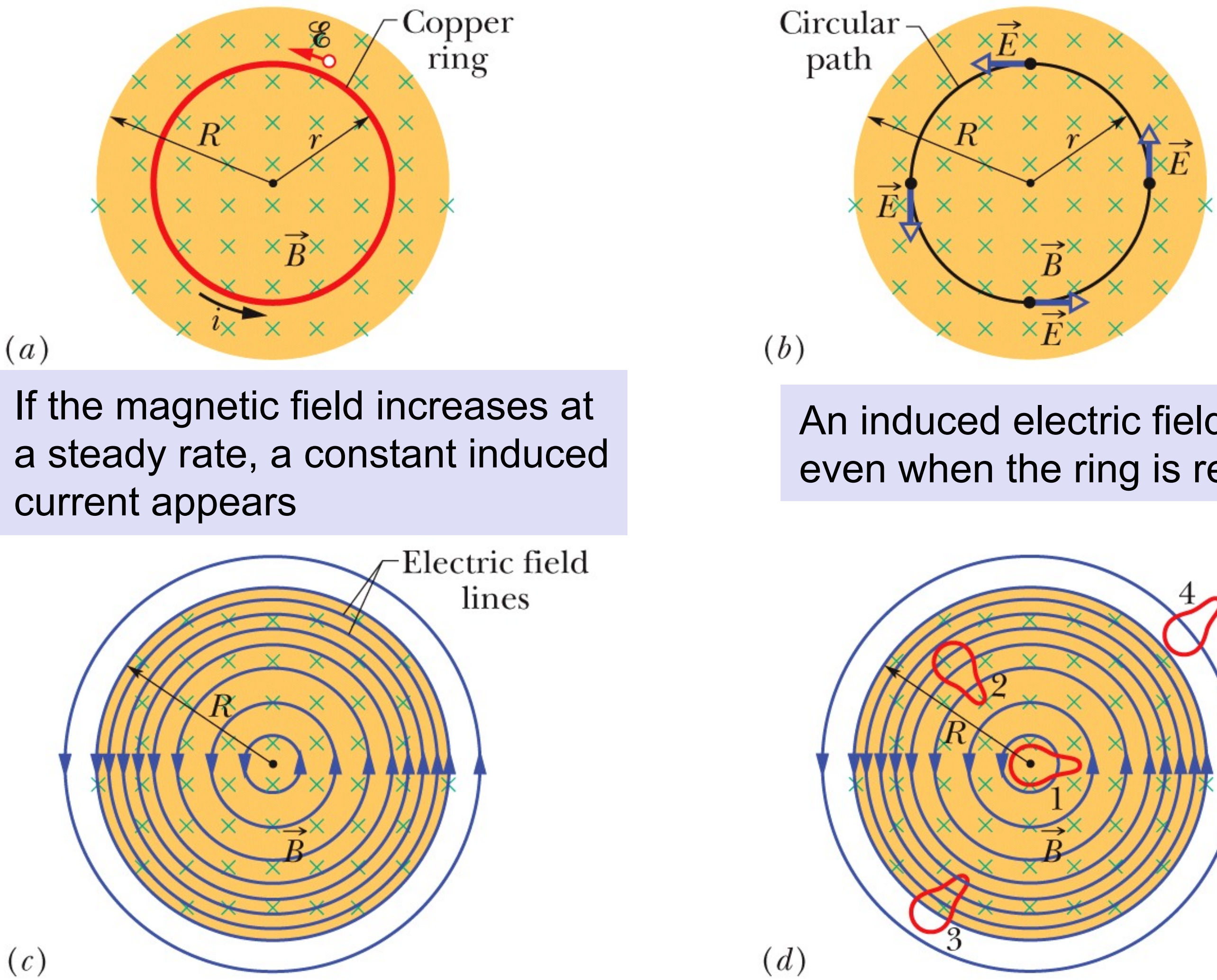
Learning Objectives

30.16 Identify that a changing magnetic field induces an electric field, regardless of whether there is a conducting loop.

30.17 Apply Faraday's law to relate the electric field ***E*** induced along a closed path (whether it has conducting material or not) to the rate of change $d\Phi/dt$ of the magnetic flux encircled by the path.

30.18 Identify that an electric potential cannot be associated with an induced electric field.

30-3 Induced Electric Field



If the magnetic field increases at a steady rate, a constant induced current appears

An induced electric field exists even when the ring is removed

The complete picture of the induced electric field

Four similar closed paths that enclose identical areas

30-3 Induced Electric Field

An emf is induced by a changing magnetic flux even if the loop is an imaginary line.

The changing magnetic field induces an electric field ***E*** at every point of such a loop; the induced *emf* is related to ***E*** by

$$\boxed{\mathscr{E} = \oint \vec{E} \cdot d\vec{s}.}$$

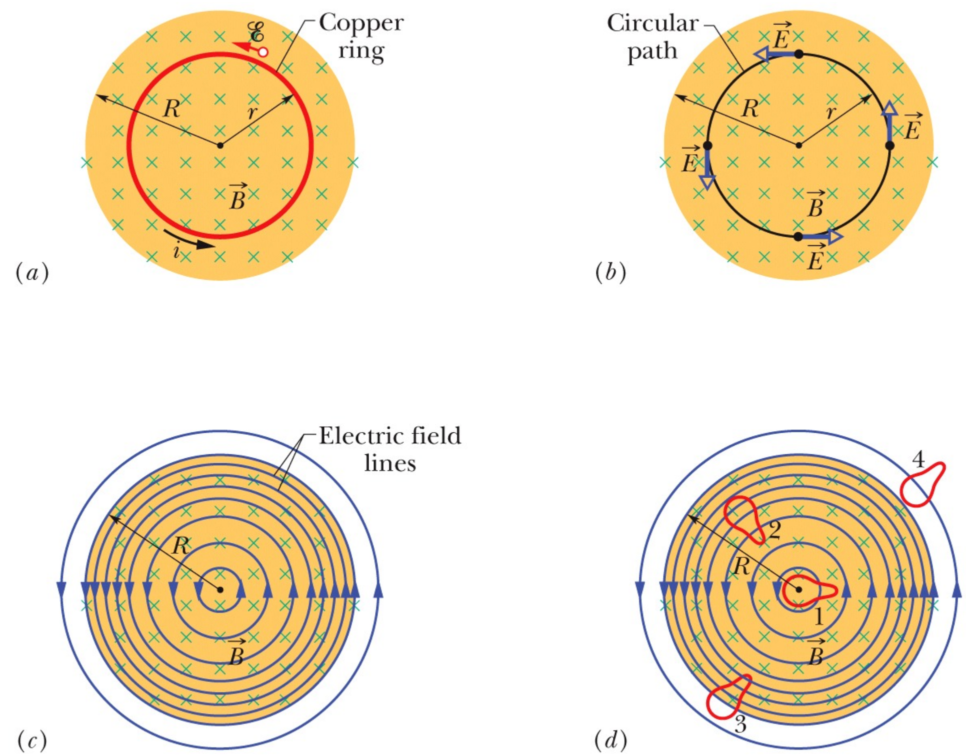
Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}. \qquad \oint \vec{E} \cdot d\vec{s} = 0.$$

30-3 Induced Electric Field

Using the induced electric field, we can write Faraday's law in its most general form as

$$\boxed{\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

A changing magnetic field produces an electric field.

$$\varepsilon_1 = \varepsilon_2 > \varepsilon_3 > \varepsilon_4 = 0$$

30-4 Inductors and Inductance

Learning Objectives

30.19 Identify an inductor.

30.20 For an inductor, apply the relationship between inductance L, total flux $N\Phi$, and current i.

30.21 For a solenoid, apply the relationship between the inductance per unit length L/l, the area A of each turn, and the number of turns per unit length n.

WILEY

30-4 Inductors and Inductance

An inductor is a device that can be used to produce a known magnetic field in a specified region. If a current *i* is established through each of the *N* windings of an inductor, a magnetic flux Φ_B links those windings. The inductance *L* of the inductor is

$$\boxed{L = \frac{N\Phi_B}{i}}$$

The SI unit of inductance is the *henry (H)*, where *1 henry = 1H=1T•m²/A*.

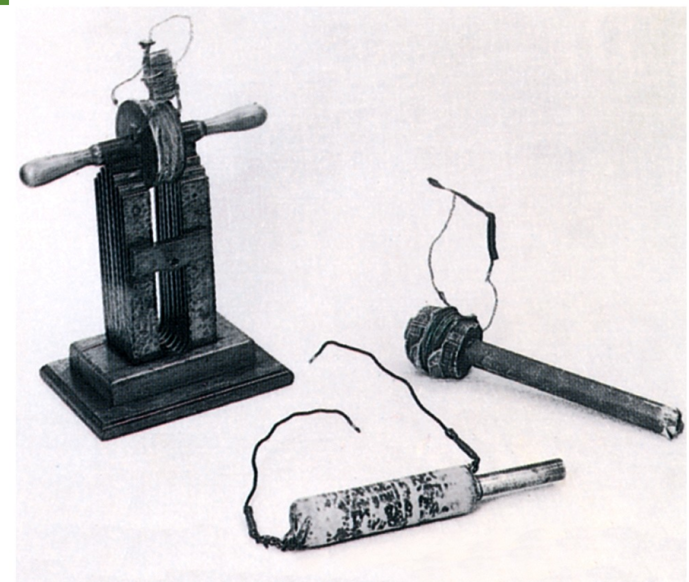
The inductance per unit length near the middle of a long solenoid of cross-sectional area *A* and *n* turns per unit length is

$$N\Phi_B = (nl)(BA).$$

$$B = \mu_0 in$$

$$\boxed{\frac{L}{l} = \mu_0 n^2 A}$$

→ L depends only on geometry of inductor



The Royal Institution/Bridgeman Art Library/NY

The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife's petticoats.

30-5 Self-Induction

Learning Objectives

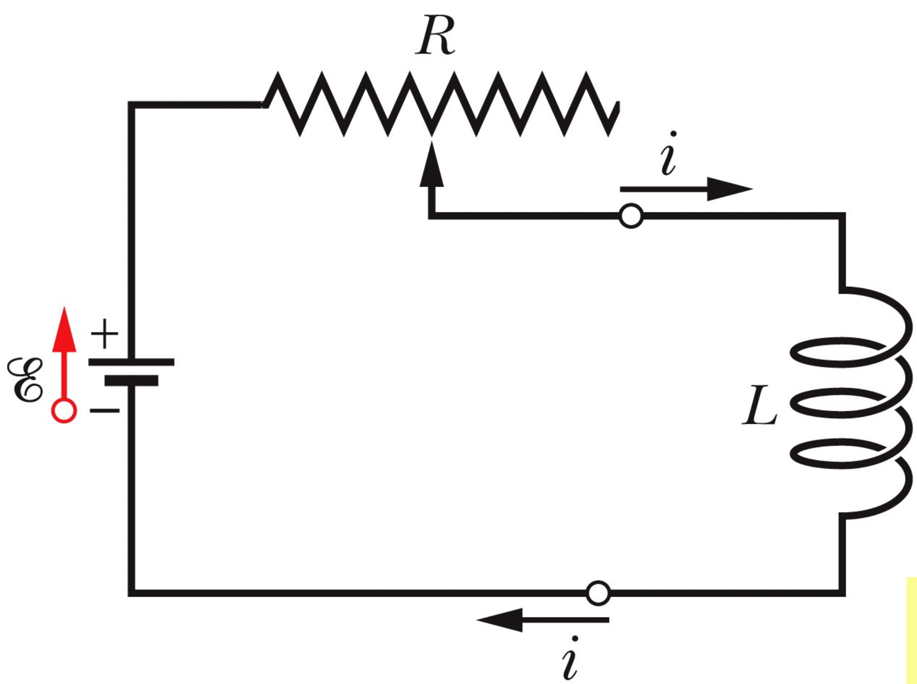
30.22 Identify that an induced emf appears in a coil when the current through the coil is changing.

30.23 Apply the relationship between the induced *emf* in a coil, the coil's inductance L, and the rate di/dt at which the current is changing.

30.24 When an *emf* is induced in a coil because the current in the coil is changing, determine the direction of the *emf* by using Lenz's law to show that the *emf* always opposes the change in the current, attempting to maintain the initial current.

30-5 Self-Induction

WILEY



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

$$N\Phi_B = Li.$$

$$\mathscr{E}_L = -\frac{d(N\Phi_B)}{dt}.$$

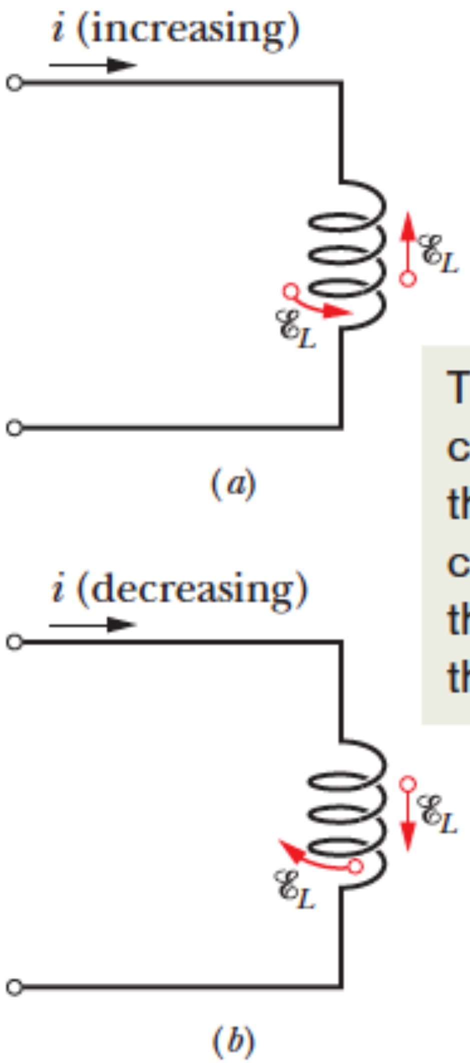
$$\boxed{\mathscr{E}_L = -L\frac{di}{dt} \quad \text{(self-induced emf).}}$$

Note: Thus, in any inductor (such as a coil, a solenoid, or a toroid) a self-induced *emf* appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced *emf*; only the rate of change of the current counts.

An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.

30-5 Self-Induction

WILEY



The changing current changes the flux, which creates an emf that opposes the change.

WILEY

30-6 RL Circuits

Learning Objectives

30.25 Sketch a schematic diagram of an RL circuit in which the current is rising.

30.26 Write a loop equation (a differential equation) for an RL circuit in which the current is rising.

30.27 For an RL circuit in which the current is rising, apply the equation *i(t)* for the current as a function of time.

30.28 For an RL circuit in which the current is rising, find equations for the potential difference *V* across the resistor, the rate *di/dt* at which the current changes, and the *emf* of the inductor, as functions of time.

30.29 Calculate an inductive time constant τ_L.

30.30 Sketch a schematic diagram of an RL circuit in which the current is decaying.

30-6 RL Circuits

Learning Objectives

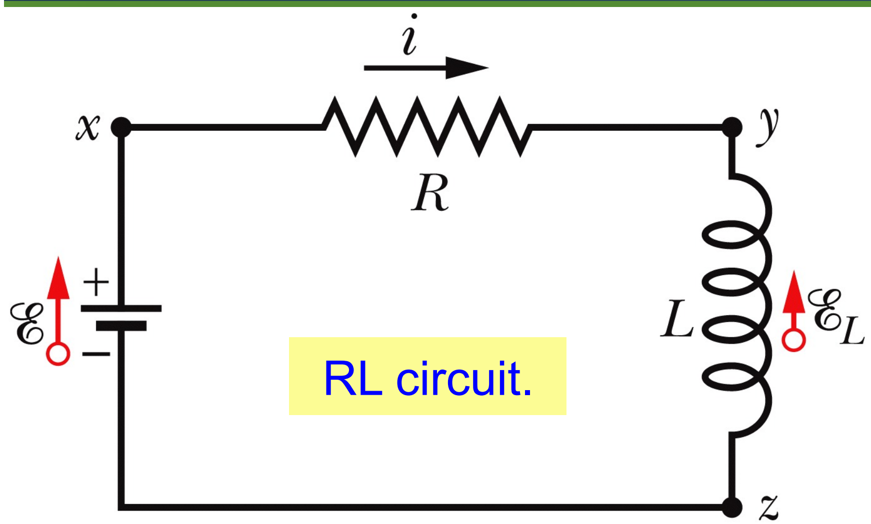
30.31 Write a loop equation (a differential equation) for an RL circuit in which the current is decaying.

30.32 For an RL circuit in which the current is decaying, apply the equation *i(t)* for the current as a function of time.

30.33 From an equation for decaying current in an RL circuit, find equations for the potential difference *V* across the resistor, the rate *di/dt* at which current is changing, and the *emf* of the inductor, as functions of time.

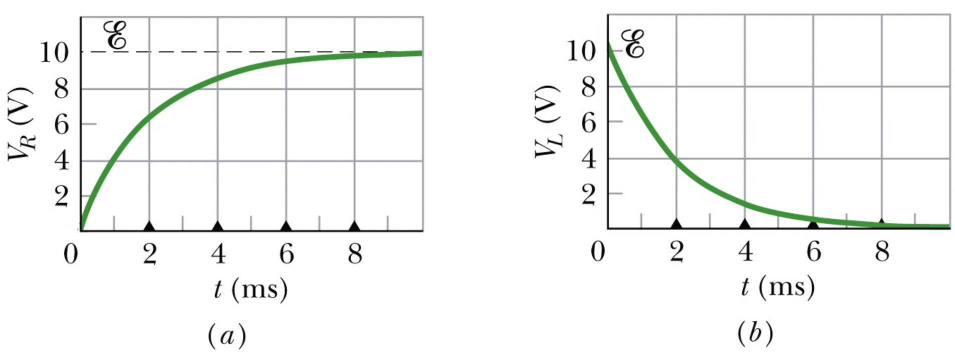
30.34 For an RL circuit, identify the current through the inductor and the *emf* across it just as current in the circuit begins to change (the initial condition) and a long time later when equilibrium is reached (the final condition).

30-6 RL Circuits



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

The resistor's potential difference turns on.
The inductor's potential difference turns off.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Loop rule

$$-iR - L\frac{di}{dt} + \mathscr{E} = 0$$

$$L\frac{di}{dt} + Ri = \mathscr{E} \quad (RL \text{ circuit}).$$

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L})$$

$$\tau_L = \frac{L}{R}$$

inductive time constant

$$i = \frac{\mathscr{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

RC circuit

$$R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E}$$

$$q = C\mathscr{E}(1 - e^{-t/RC})$$

$$\tau = RC$$

30-7 Energy Stored in a Magnetic Field

Learning Objectives

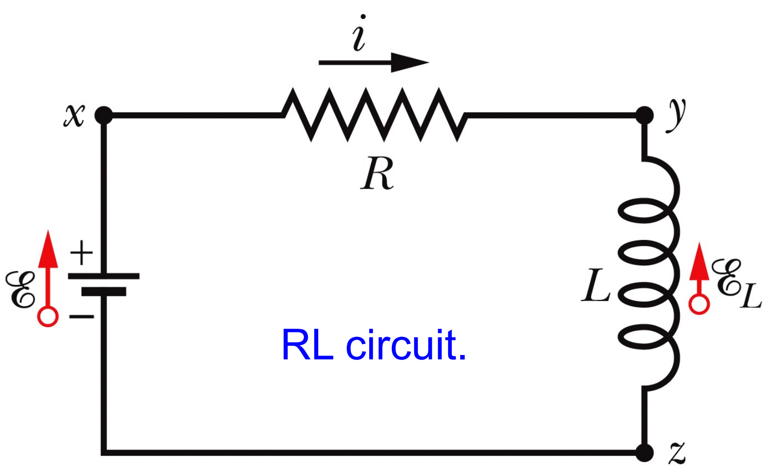
30.35 Describe the derivation of the equation for the magnetic field energy of an inductor in an *RL* circuit with a constant *emf* source.

30.36 For an inductor in an *RL* circuit, apply the relationship between the magnetic field energy U, the inductance L, and the current i.

30-7 Energy Stored in a Magnetic Field

If an inductor L carries a current i, the inductor's magnetic field stores an energy given by

$$\boxed{U_B = \tfrac{1}{2}Li^2}$$



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

$$L\frac{di}{dt} + Ri = \mathscr{E} \quad (RL \text{ circuit}).$$

$$\mathscr{E}i = Li\frac{di}{dt} + i^2R,$$

$$P = i^2R = \frac{V^2}{R}$$

$$P = P_L + P_R$$

$$P_L = \frac{dU_B}{dt} = Li\frac{di}{dt}. \Rightarrow U_B = \tfrac{1}{2}Li^2$$

30-7 Energy Stored in a Magnetic Field

If an inductor L carries a current i, the inductor's magnetic field stores an energy given by

$$U_B = \tfrac{1}{2}Li^2$$

A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

KEY IDEA

The energy stored in the magnetic field of a coil at any time depends on the current through the coil at that time, according to Eq. 30-49 ($U_B = \tfrac{1}{2}Li^2$).

Calculations: Thus, to find the energy $U_{B\infty}$ stored at equilibrium, we must first find the equilibrium current. From Eq. 30-41, the equilibrium current is

$$i_\infty = \frac{\mathscr{E}}{R} = \frac{12\ \text{V}}{0.35\ \Omega} = 34.3\ \text{A}. \qquad (30\text{-}51)$$

Then substitution yields

$$U_{B\infty} = \tfrac{1}{2}Li_\infty^2 = (\tfrac{1}{2})(53 \times 10^{-3}\ \text{H})(34.3\ \text{A})^2$$
$$= 31\ \text{J}. \qquad \text{(Answer)}$$

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

Calculations: Now we are being asked: At what time t will the relation

$$U_B = \tfrac{1}{2}U_{B\infty}$$

be satisfied? Using Eq. 30-49 twice allows us to rewrite this energy condition as

$$\tfrac{1}{2}Li^2 = (\tfrac{1}{2})\tfrac{1}{2}Li_\infty^2$$

or

$$i = \left(\frac{1}{\sqrt{2}}\right)i_\infty. \qquad (30\text{-}52)$$

This equation tells us that, as the current increases from its initial value of 0 to its final value of i_∞, the magnetic field will have half its final stored energy when the current has increased to this value. In general, we know that i is given by Eq. 30-41, and here i_∞ (see Eq. 30-51) is $\mathscr{E}/R$; so Eq. 30-52 becomes

$$\frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) = \frac{\mathscr{E}}{\sqrt{2}R}.$$

By canceling $\mathscr{E}/R$ and rearranging, we can write this as

$$e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293,$$

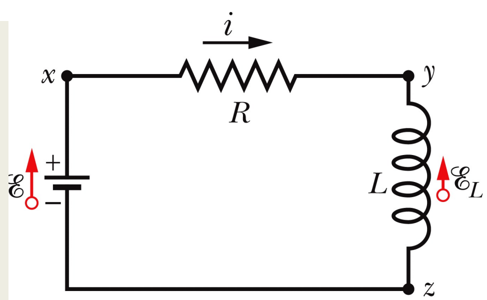
which yields

$$\frac{t}{\tau_L} = -\ln 0.293 = 1.23$$

or

$$t \approx 1.2\tau_L. \qquad \text{(Answer)}$$

Thus, the energy stored in the magnetic field of the coil by the current will reach half its equilibrium value 1.2 time constants after the emf is applied.



opyright © 2014 John Wiley & Sons, Inc. All rights reserved.

An RL circuit.

30-8 Energy Density of a Magnetic Field

Learning Objectives

30.37 Identify that energy is associated with any magnetic field.

30.38 Apply the relationship between energy density u_B of a magnetic field and the magnetic field magnitude B.

30-8 Energy Density of a Magnetic Field

WILEY

Consider a length *l* near the middle of **a long solenoid** of cross-sectional area *A* carrying current i; the volume associated with this length is *Al*. The energy UB stored by the length *l* of the solenoid must lie entirely within this volume because the magnetic field outside such a solenoid is approximately zero. Moreover, the stored energy must be uniformly distributed within the solenoid because the magnetic field is (approximately) uniform everywhere inside. Thus, the energy stored per unit volume of the field is

$$u_B = \frac{U_B}{Al}$$

$$\boxed{U_B = \tfrac{1}{2}Li^2}$$

We have,

$$\boxed{u_B = \frac{Li^2}{2Al} = \frac{L}{l}\frac{i^2}{2A}.}$$

here *L* is the inductance of length *l* of the solenoid

Substituting for *L/l* we get

$$\frac{L}{l} = \mu_0 n^2 A$$

$$u_B = \tfrac{1}{2}\mu_0 n^2 i^2$$

And we can write the **energy density** as

$$B = \mu_0 i n$$

$$\boxed{u_B = \frac{B^2}{2\mu_0}}$$

WILEY

30-8 Mutual Induction

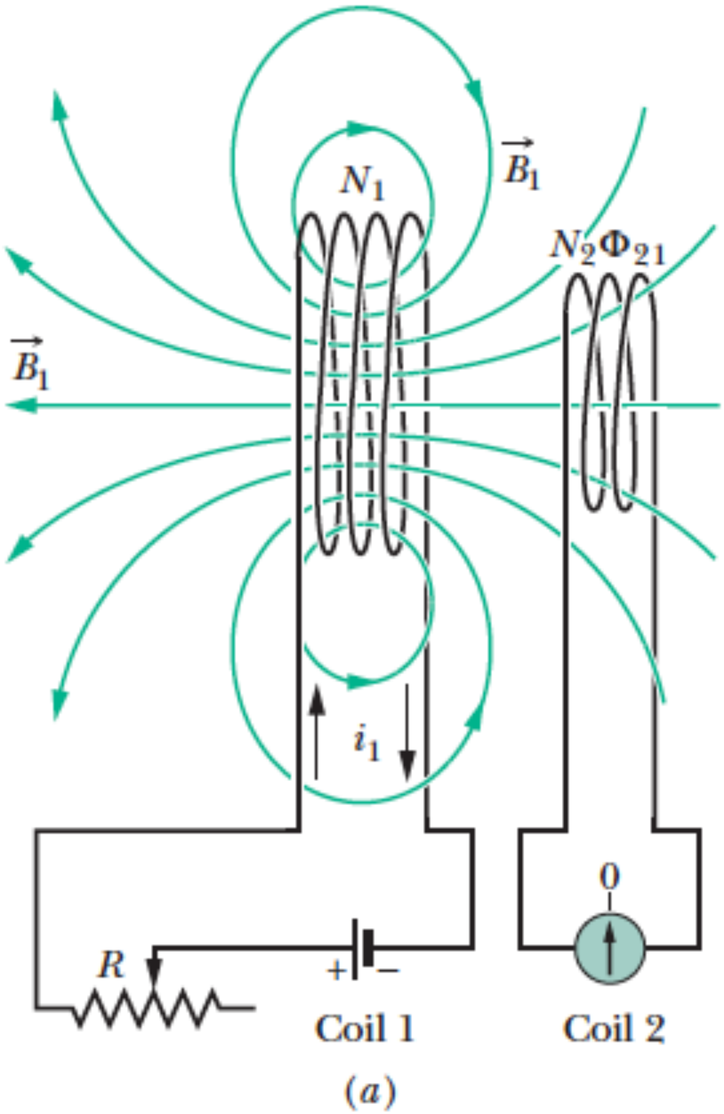
Learning Objectives

30.39 Describe the mutual induction of two coils and sketch the arrangement.

30.40 Calculate the mutual inductance of one coil with respect to a second coil (or some second current that is changing).

30.41 Calculate the *emf* induced in one coil by a second coil in terms of the mutual inductance and the rate of change of the current in the second coil.

30-8 Mutual Induction



Mutual inductance of coil2 with respect to coil1

$$M_{21} = \frac{N_2\Phi_{21}}{i_1},$$

$$M_{21}i_1 = N_2\Phi_{21}.$$

$$M_{21}\frac{di_1}{dt} = N_2\frac{d\Phi_{21}}{dt}.$$

$$\boxed{\mathscr{E}_2 = -M_{21}\frac{di_1}{dt},}$$

$$\mathscr{E} = -N\frac{d\Phi_B}{dt}.$$

Mutual inductance of coil1 with respect to coil2

$$\boxed{\mathscr{E}_1 = -M_{12}\frac{di_2}{dt}.}$$

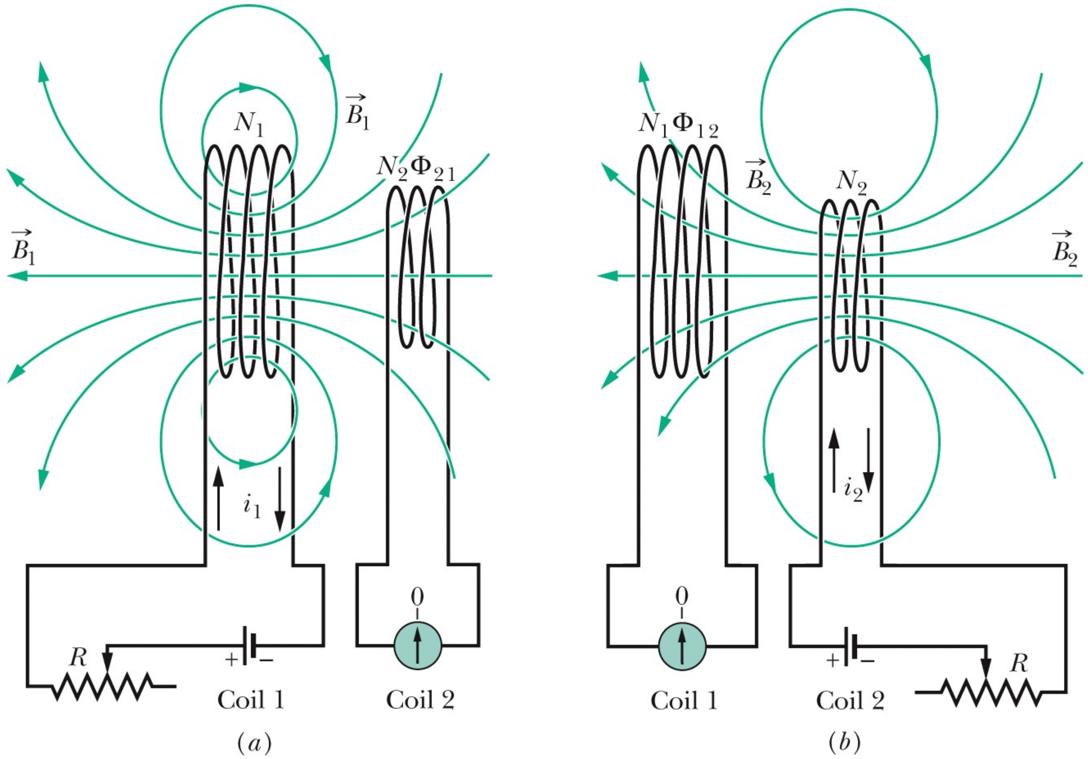
$$M_{21} = M_{12} = M,$$

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$

$$\mathscr{E}_1 = -M\frac{di_2}{dt}.$$

30-8 Mutual Induction

WILEY



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Mutual induction.
(a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an *emf* is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt}.$$

30 Summary

Magnetic Flux

- The magnetic flux through an area *A* in a magnetic field ***B*** is defined as

$$\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A}}$$ **Eq. 30-1**

- If B: is perpendicular to the area and uniform over it, Eq. 30-1 becomes

$$\boxed{\Phi_B = BA \quad (\vec{B} \perp A, \vec{B}\text{ uniform}).}$$ **Eq. 30-2**

Faraday's Law of Induction

- The induced *emf* is,

$$\boxed{\mathscr{E} = -\frac{d\Phi_B}{dt}}$$ **Eq. 30-4**

- If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\boxed{\mathscr{E} = -N\frac{d\Phi_B}{dt}.}$$ **Eq. 30-5**

Lenz's Law

- An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current.

Emf and the Induced Magnetic Field

- The induced emf is related to ***E*** by

$$\boxed{\mathscr{E} = \oint \vec{E} \cdot d\vec{s},}$$ **Eq. 30-19**

- Faraday's law in its most general form,

$$\boxed{\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$ **Eq. 30-20**

WILEY

30 Summary

Inductor

- The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i}$$ Eq. 30-28

- The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$ Eq. 30-31

Self-Induction

- This self-induced *emf* is,

$$\mathscr{E}_L = -L\frac{di}{dt}.$$ Eq. 30-35

Mutual Induction

- The mutual induction is described by,

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$ Eq. 30-64

$$\mathscr{E}_1 = -M\frac{di_2}{dt}$$ Eq. 30-65

Series *RL* Circuit

- Rise of current,

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L})$$ Eq. 30-41

- Decay of current

$$i = i_0 e^{-t/\tau_L}$$ Eq. 30-45

Magnetic Energy

- the inductor's magnetic field stores an energy given by

$$U_B = \tfrac{1}{2}Li^2$$ Eq. 30-49

- The density of stored magnetic energy,

$$u_B = \frac{B^2}{2\mu_0}$$ Eq. 30-55