1 Earth : $g_E = \frac{GM}{R^2}$

Planet : $g_P = \frac{Gm}{r^2}$

$\begin{cases} m = 0.05M \\ r = 0.4R \end{cases}$

$\therefore g_P = \frac{G(0.05)M}{(0.4R)^2} = 0.05 \times \frac{1}{0.4^2} \times \frac{GM}{R^2}$

$= \frac{0.05}{0.4^2} \times g_E = \frac{0.05}{0.4^2} \times 9.8 m/s^2 = 3.06 m/s^2$

2 $P_1 = \frac{f_1}{\pi r^2}$, $P_2 = \frac{f_2}{\pi (2r)^2}$

$P_1 = P_2$

$\frac{f_1}{\pi r^2} = \frac{f_2}{\pi (2r)^2}$

$f_2 = \frac{f_1}{r^2} \cdot (2r)^2 = 4f_1 = 4 \times 16N$

$\therefore 64N$

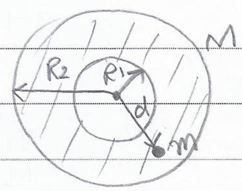
3 $V(\text{before}) = 0$

$V(\text{after}) = -G\frac{m(2m)}{(d/2)} - G\frac{m(2m)}{(d/2)} = -4G\frac{m^2}{d} - 4G\frac{m^2}{d} = -8G\frac{m^2}{d}$

4 object volume $= \frac{30g}{2.0 g/cm^3} = 15 cm^3$

$\therefore$ object density $= \frac{210g}{15cm^3} = 14 g/cm^3$

5



$|F| = \frac{GM'm}{d^2}$

The volume of spherical shell : $\frac{4}{3}\pi(R_2^3 - R_1^3)$

Mass : M

$\rightarrow$ density : $\frac{M}{\frac{4}{3}\pi(R_2^3 - R_1^3)}$

The mass located between the spheres of radius d and radius R_1 $\rightarrow$ Force

$\rightarrow \frac{4}{3}\pi d^3 - \frac{4}{3}\pi R_1^3 = \frac{4}{3}\pi(d^3 - R_1^3)$

$\therefore M' = \frac{M}{\frac{4}{3}\pi(R_2^3 - R_1^3)} \times \frac{4}{3}\pi(d^3 - R_1^3) = \frac{(d^3 - R_1^3)M}{R_2^3 - R_1^3}$

$$\therefore F = \frac{GM'm}{d^2} = G \cdot \frac{(d^3 - R_1^3)M}{R_2^3 - R_1^3} \times \frac{m}{d^2}$$

$$= \frac{GMm(d^3 - R_1^3)}{d^2(R_2^3 - R_1^3)}$$

$\boxed{6}$ v: speed of the water

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.8 \times 30} = 24\ m/s$$

$\boxed{7}$ $$-G\frac{M\cancel{m}}{r_1} + \frac{1}{2}\cancel{m}v_1^2 = -G\frac{M\cancel{m}}{r_2} + \frac{1}{2}\cancel{m}v_2^2$$

$$-GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{2}v_1^2 = \frac{1}{2}v_2^2$$

$$-2GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + v_1^2 = v_2^2$$

$$\therefore v_2 = \sqrt{v_1^2 - 2GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$= \left[(5.0\times10^4)^2 - 2(6.67\times10^{-11})(2\times10^{30})\left(\frac{1}{2\times10^{11}} - \frac{1}{2.5\times10^{10}}\right)\right]^{1/2}$$

$$= \left[(5.0\times10^4)^2 - 2(6.67\times10^{-11})(2\times10^{30})(-3.5\times10^{-11})\right]^{1/2}$$

$$= \left[(5.0\times10^4)^2 + (93.38\times10^8)\right]^{1/2}$$

$$= \sqrt{118.38\times10^8}$$

$$= 1.1\times10^5\ m/s$$

$\boxed{8}$ $$\int_{\theta=0}^{\frac{\pi}{2}}\int_{\phi=0}^{2\pi}(P_0 - P)\cdot\cos\theta\, R^2\sin\theta\, d\theta\, d\phi = F_{hemi}$$

$$= R^2(P_0 - P)\cdot 2\pi \cdot \underbrace{\int_0^{\pi/2}\sin\theta\cos\theta\, d\theta}$$

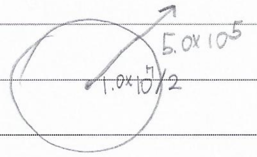
$$\int_0^{\pi/2}\frac{1}{2}\sin 2\theta\, d\theta = -\frac{1}{2}\cos 2\theta\Big|_0^{\pi/2} \times \frac{1}{2} = -\frac{1}{4}(-1-1) = \frac{1}{2}$$

$$= 2\pi R^2 \cdot \frac{1}{2}\cdot(P_0 - P)$$

$$= \pi R^2(P_0 - P)$$

$$\therefore F_{hemi} \times 2 = 2\pi R^2(P_0 - P)$$

9 $M = 10000\,kg$



Orbit radius $= (5.0\times10^5) + (0.5\times10^7) = 5.5\times10^6\,m$

speed $v = \dfrac{2\pi R}{T} = \dfrac{2\pi\times5.5\times10^6}{5000} = 6912\,m/s$

$\dfrac{GM_pM}{R^2} = \dfrac{Mv^2}{R} \longrightarrow M_p = \dfrac{v^2R}{G} = \dfrac{6912^2\times(5.5\times10^6)}{6.67\times10^{-11}} = 3.94\times10^{24}\,kg$

on surface, $g = \dfrac{GM_p}{R_p^2} = \dfrac{(6.67\times10^{-11})\times(3.94\times10^{24})}{(0.5\times10^7)^2} = 10.5\,m/s^2$

$\therefore$ The weight of an 80kg astronaut

$= (80kg)\times(10.5m/s^2) = 840\,N$

10 $\rho_{air} = 1.0\,kg/m^3$

$\Sigma F = 0$

$-3000N - 500N - 2000N - W_{Gas} + F_B = 0$

$-5500N - M_{Gas}g + \rho_{air}Vg = 0$

$\therefore M_{Gas} = \dfrac{(1.0kg/m^3)\times(2000m^3)\times(9.8m/s^2) - 5500N}{9.8m/s^2} = 1439\,kg$

$\therefore \rho_{Gas} = \dfrac{1439kg}{2000m^3} = 0.72\,kg/m^3$