Chapter 36

Diffraction

Lecture 27, 28

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 → Modified by S.H. Seo

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36 Summary

Diffraction

- When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference.

Single-Slit Diffraction

- A single-slit diffraction patterns satisfy

$$\boxed{a \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \ldots}$$

Eq. 36-3

- The intensity of the diffraction pattern at any given angle θ is

$$\boxed{I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2,}$$

Eq. 36-5

where

$$\boxed{\alpha = \frac{\pi a}{\lambda} \sin \theta}$$

Eq. 36-6

Circular Aperture Diffraction

- Diffraction by a circular aperture or a lens with diameter *d* produces a central maximum and concentric maxima and minima, with the first minimum at an angle θ given by

$$\boxed{\sin \theta = 1.22 \frac{\lambda}{d}}$$

Eq. 36-12

Rayleigh's Criterion

- Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\boxed{\theta_R = 1.22 \frac{\lambda}{d}}$$

Eq. 35-14

36-1 Single-Slit Diffraction

Learning Objectives

36.01 Describe the diffraction of light waves by a narrow opening and an edge, and also describe the resulting interference pattern.

36.02 Describe an experiment that demonstrates the Fresnel bright spot.

36.03 With a sketch, describe the arrangement for a single-slit diffraction experiment.

36.04 With a sketch, explain how splitting a slit width into equal zones leads to the equations giving the angles to the minima in the diffraction pattern.

36.05 Apply the relationships between width a of a thin, rectangular slit or object, the wavelength λ, the angle θ to any of the minima in the diffraction pattern, the distance to a viewing screen, and the distance between a minimum and the center of the pattern.

36.06 Sketch the diffraction pattern for monochromatic light, identifying what lies at the center and what the various bright and dark fringes are called (such as "first minimum").

36-1 Single-Slit Diffraction

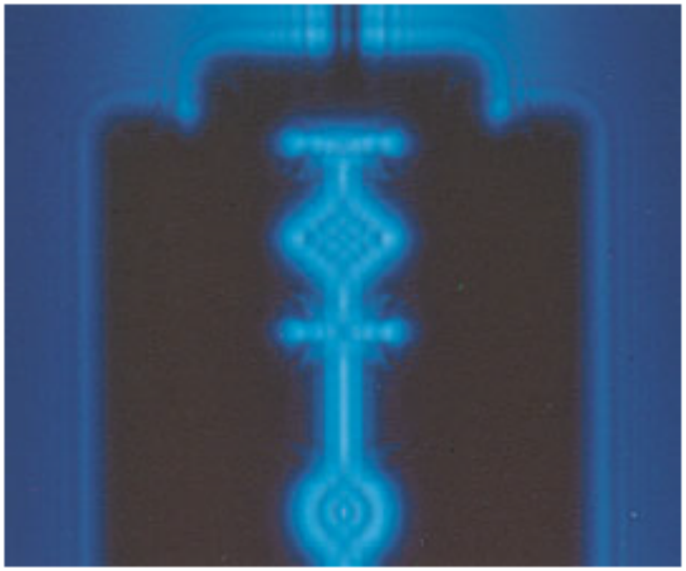
Learning Objectives

36.07 Identify what happens to a diffraction pattern when the wavelength of the light or the width of the diffracting aperture or object is varied.

36-1 Single-Slit Diffraction

When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference.

This type of interference is called diffraction.



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36-1 Single-Slit Diffraction

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This pair of rays cancel each other at P_1. So do all such pairings.

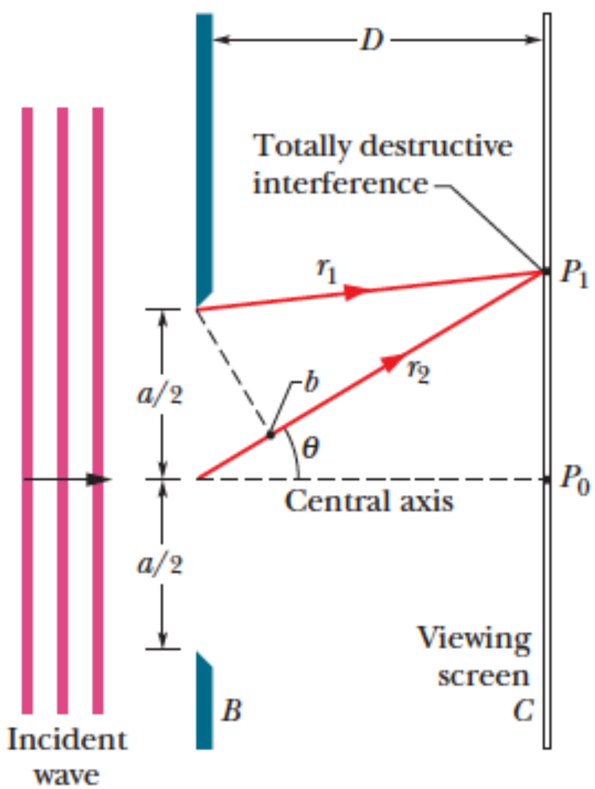


Figure 36-4 Waves from the top points of two zones of width $a/2$ undergo fully destructive interference at point P_1 on viewing screen C.

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2},$$ @ 1st minimum

$$a\sin\theta = \lambda \qquad \text{(first minimum).}$$

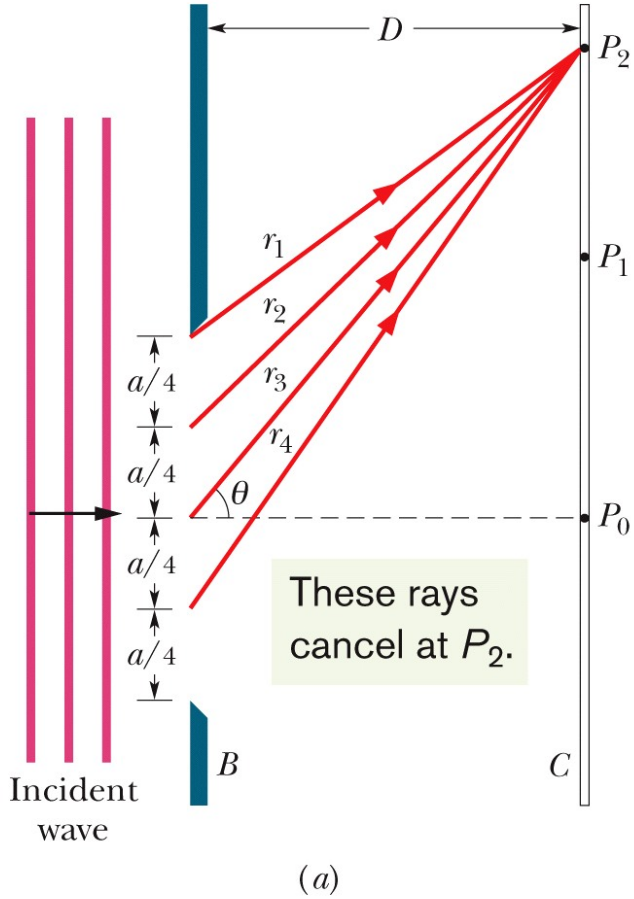
$$a\sin\theta = 2\lambda \qquad \text{(second minimum).}$$

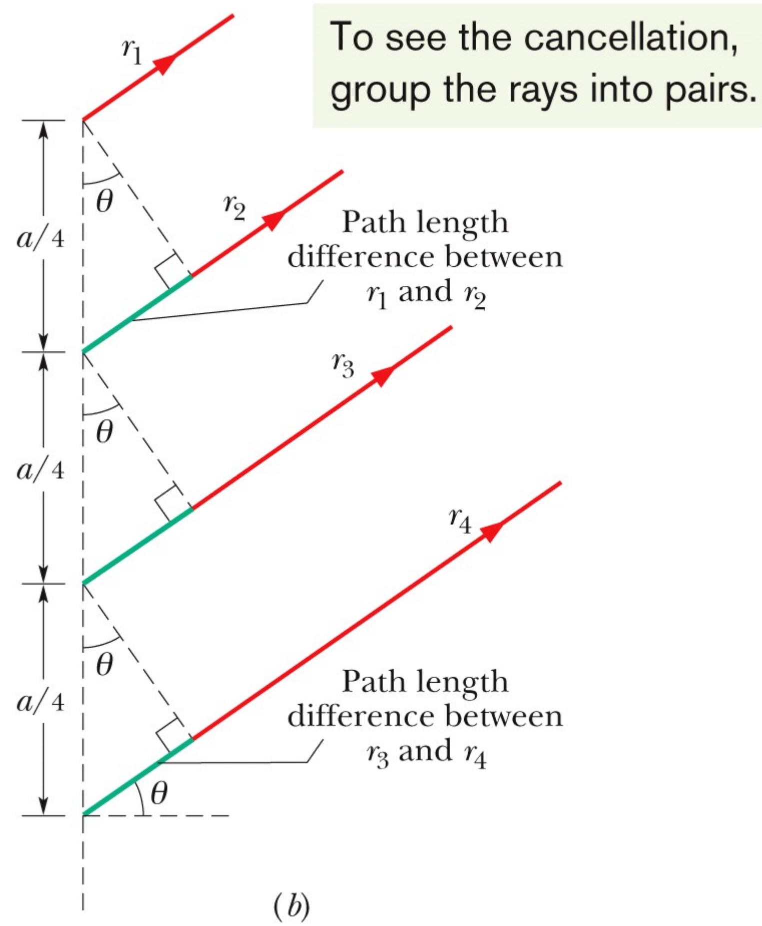
$$\boxed{a\sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \quad \text{(minima—dark fringes).}}$$

36-1 Single-Slit Diffraction

$\frac{a}{2}\sin\theta = \frac{\lambda}{2},$	$a\sin\theta = \lambda$	@ 1st minimum
$\frac{a}{4}\sin\theta = \frac{\lambda}{2}$	$a\sin\theta = 2\lambda$	@ 2nd minimum
$\frac{a}{6}\sin\theta = \frac{\lambda}{2}$	$a\sin\theta = 3\lambda$	@ 3rd minimum
$\downarrow$	$\downarrow$	$\downarrow$
$\frac{a}{2m}\sin\theta = \frac{\lambda}{2}$	$a\sin\theta = m\lambda$	@ mth minimum

36-1 Single-Slit Diffraction





(a) Waves from the top points of four zones of width a/4 undergo fully destructive interference at point P_2. (b) For $D >> a$, we can approximate rays r_1, r_2, r_3, and r_4 as being parallel, at angle θ to the central axis.

36-2 Intensity in Single-Slit Diffraction

Learning Objectives

36.08 Divide a thin slit into multiple zones of equal width and write an expression for the phase difference of the wavelets from adjacent zones in terms of the angle θ to a point on the viewing screen.

36.09 For single-slit diffraction, draw phasor diagrams for the central maximum and several of the minima and maxima off to one side, indicating the phase difference between adjacent phasors, explaining how the net electric field is calculated, and identifying the corresponding part of the diffraction pattern.

36.10 Describe a diffraction pattern in terms of the net electric field at points in the pattern.

36.11 Evaluate α, the convenient connection between angle θ to a point in a diffraction pattern and the intensity I at that point.

36.12 For a given point in a diffraction pattern, at a given angle, calculate the intensity I in terms of the intensity I_m at the center of the pattern.

Division of single slit into 18 zones

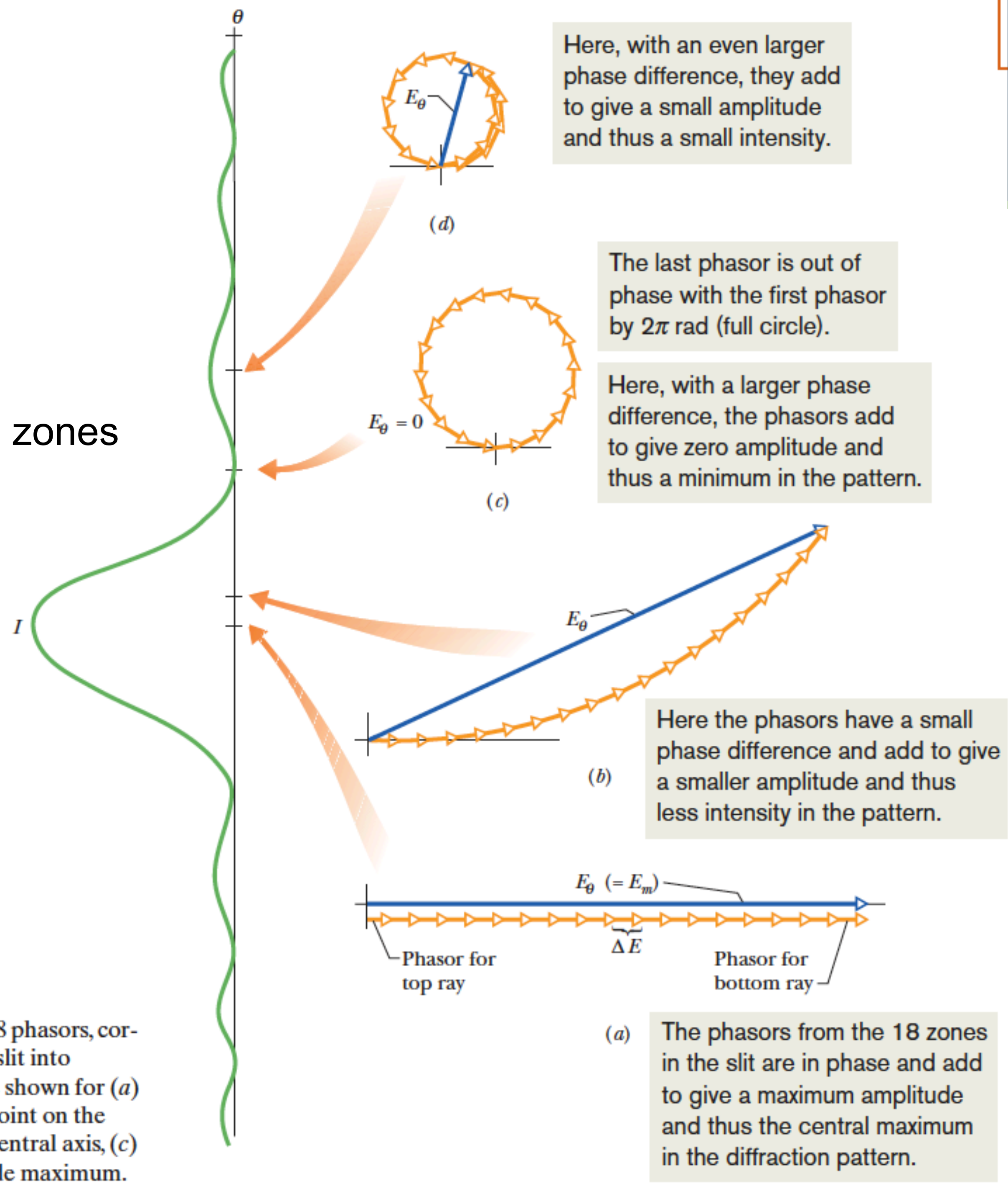


Figure 36-7 Phasor diagrams for $N = 18$ phasors, corresponding to the division of a single slit into 18 zones. Resultant amplitudes E_θ are shown for (*a*) the central maximum at $\theta = 0$, (*b*) a point on the screen lying at a small angle θ to the central axis, (*c*) the first minimum, and (*d*) the first side maximum.

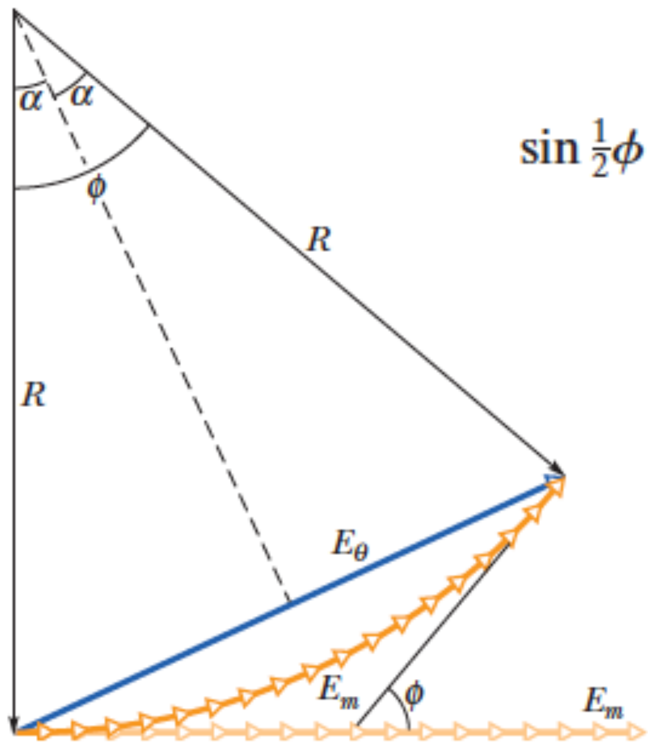
36-2 Intensity in Single-Slit Diffraction

The intensity of the diffraction pattern at any given angle θ is

$$\boxed{I(\theta) = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2,}$$

where, I_m is the intensity at the center of the pattern and

$$\boxed{\alpha = \tfrac{1}{2}\phi = \frac{\pi a}{\lambda}\sin\theta.}$$



$$\sin\tfrac{1}{2}\phi = \frac{E_\theta}{2R}. \qquad \phi = \frac{E_m}{R}. \longrightarrow E_\theta = \frac{E_m}{\frac{1}{2}\phi}\sin\tfrac{1}{2}\phi.$$

$$\frac{I(\theta)}{I_m} = \frac{E_\theta^2}{E_m^2}. \longrightarrow I(\theta) = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2.$$

$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \frac{2\pi}{\lambda}\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}. \qquad \phi = \left(\frac{2\pi}{\lambda}\right)(a\sin\theta),$$

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36-2 Intensity in Single-Slit Diffraction

The plots show the relative intensity in single-slit diffraction for three values of the ratio *a/λ*. The wider the slit is, the narrower is the central diffraction maximum.

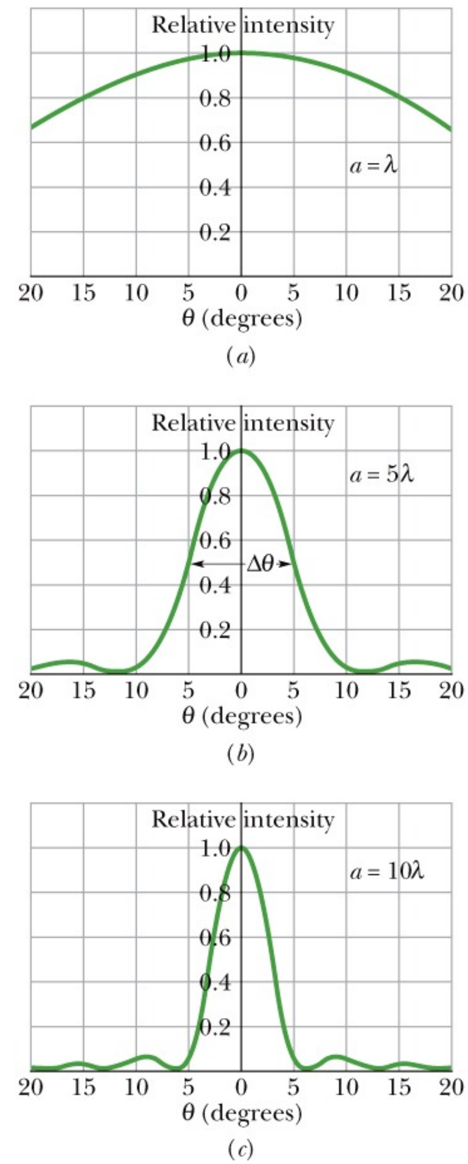
$$a \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \quad \text{(minima—dark fringes).}$$

For m = 1

$$a = \lambda \longrightarrow \sin\theta = 1$$

$$a = 10\lambda \longrightarrow \sin\theta = \frac{1}{10}$$

The larger "a" (wider slit distance), the smaller θ



36-3 Diffraction by a Circular Aperture

Learning Objectives

36.13 Describe and sketch the diffraction pattern from a small circular aperture or obstacle.

36.14 For diffraction by a small circular aperture or obstacle, apply the relationships between the angle θ to the first minimum, the wavelength λ of the light, the diameter d of the aperture, the distance D to a viewing screen, and the distance y between the minimum and the center of the diffraction pattern.

36.15 By discussing the diffraction patterns of point objects, explain how diffraction limits visual resolution of objects.

36.16 Identify that Rayleigh's criterion for resolvability gives the (approximate) angle at which two point objects are just barely resolvable.

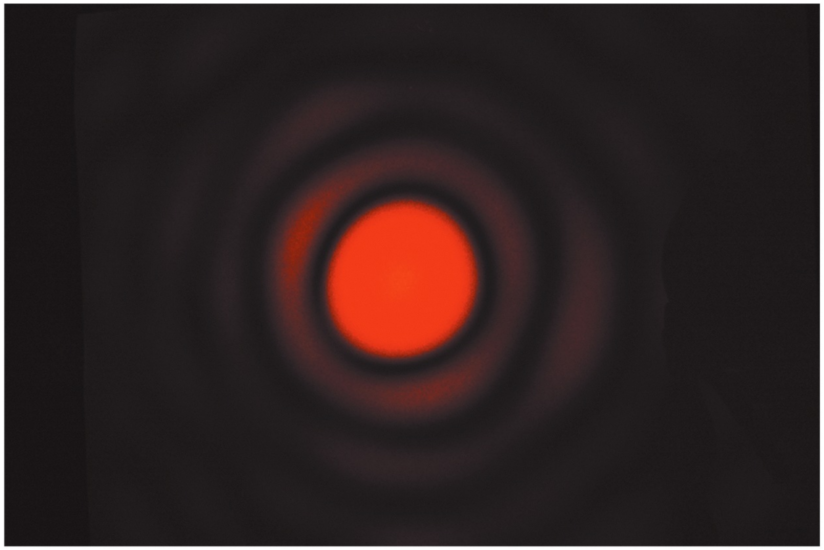
36.17 Apply the relationships between the angle θ_R in Rayleigh's criterion, the wavelength λ of the light, the diameter d of the aperture (for example, the diameter of the pupil of an eye), the angle θ subtended by two distant point objects, and the distance L to those objects.

36-3 Diffraction by a Circular Aperture

Diffraction by a circular aperture or a lens with diameter *d* produces a central maximum and concentric maxima and minima, given by

$$\sin\theta = 1.22\frac{\lambda}{d} \quad \text{(first minimum—circular aperture).}$$

The angle θ here is the angle from the central axis to any point on that (circular) minimum.



Courtesy Jearl Walker

$$\sin\theta = \frac{\lambda}{a} \quad \text{(first minimum—single slit),}$$

which locates the first minimum for a long narrow slit of width a. The main difference is the factor 1.22, which enters because of the circular shape of the aperture.

The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

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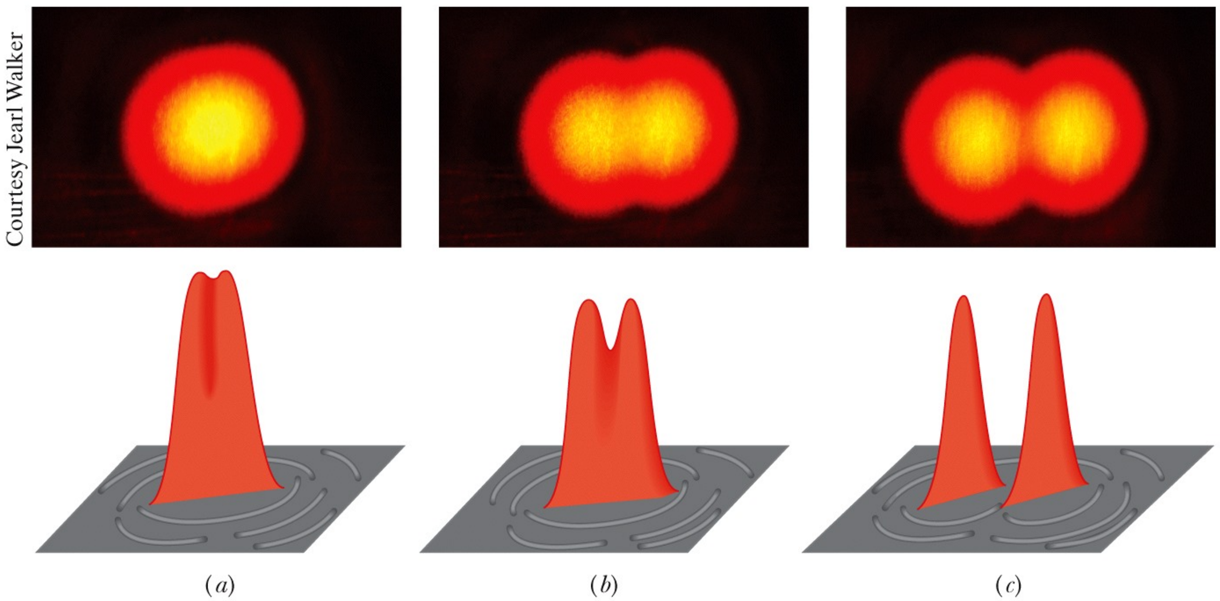
36-3 Diffraction by a Circular Aperture

Resolvability



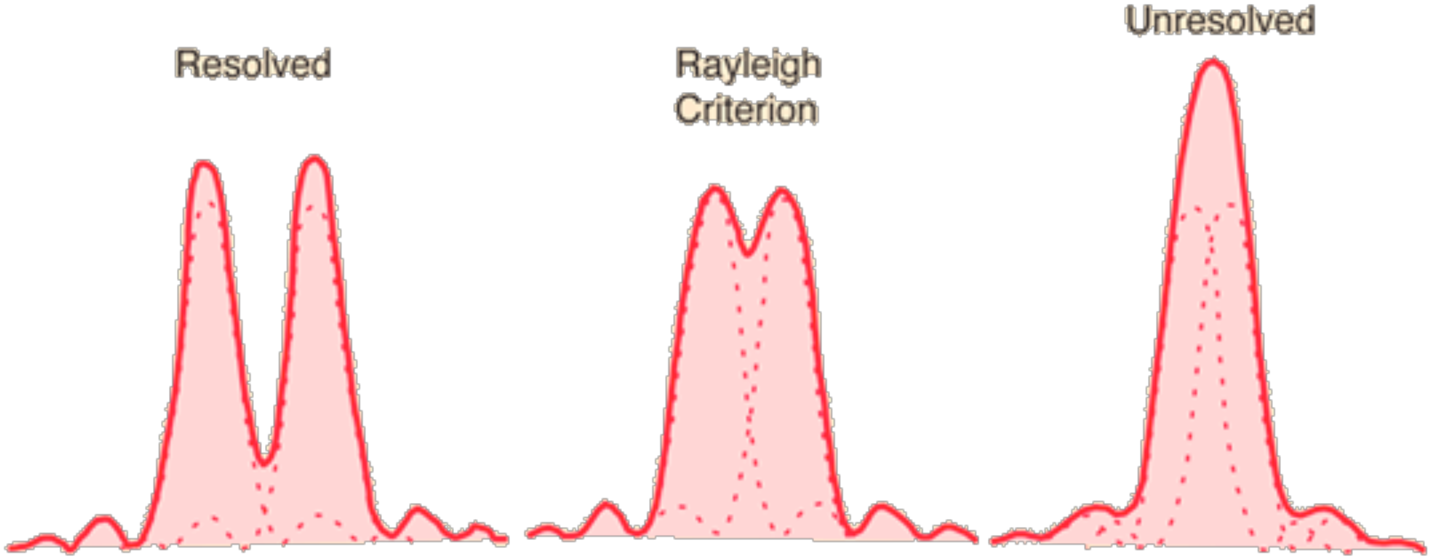
The images of two point sources (stars) formed by a converging lens. At the bottom, representations of the image intensities. In (a) the angular separation of the sources is too small for them to be distinguished, in (b) they can be marginally distinguished, and in (c) they are clearly distinguished. Rayleigh's criterion is satisfied in (b), with the central maximum of one diffraction pattern coinciding with the first minimum of the other.

Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_R = 1.22\frac{\lambda}{d} \quad \text{(Rayleigh's criterion).}$$

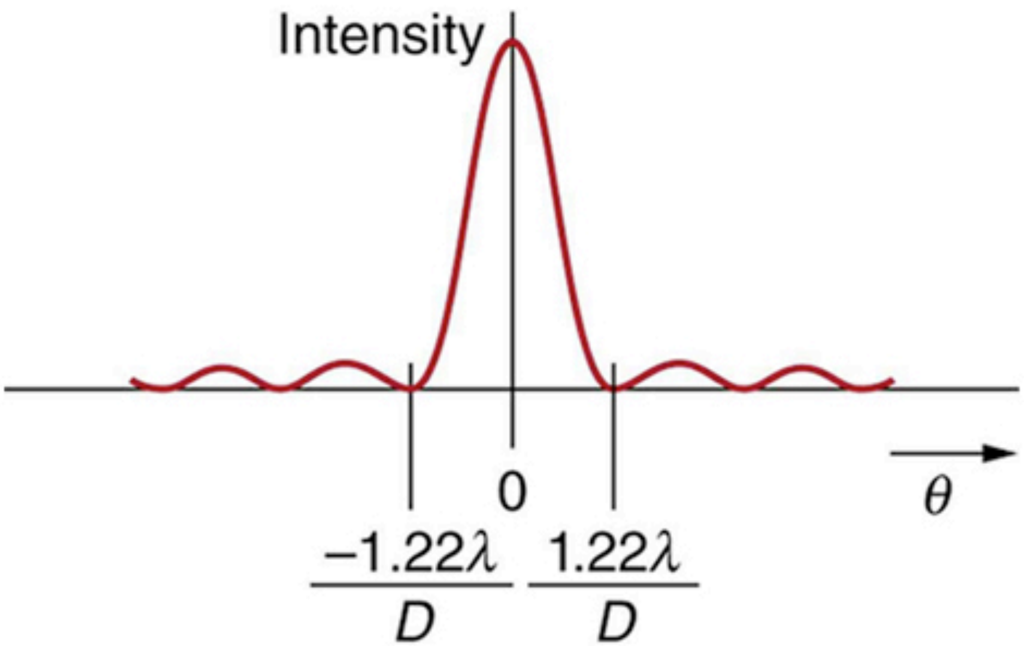
in which *d* is the diameter of the aperture through which the light passes.

Rayleigh's criterion

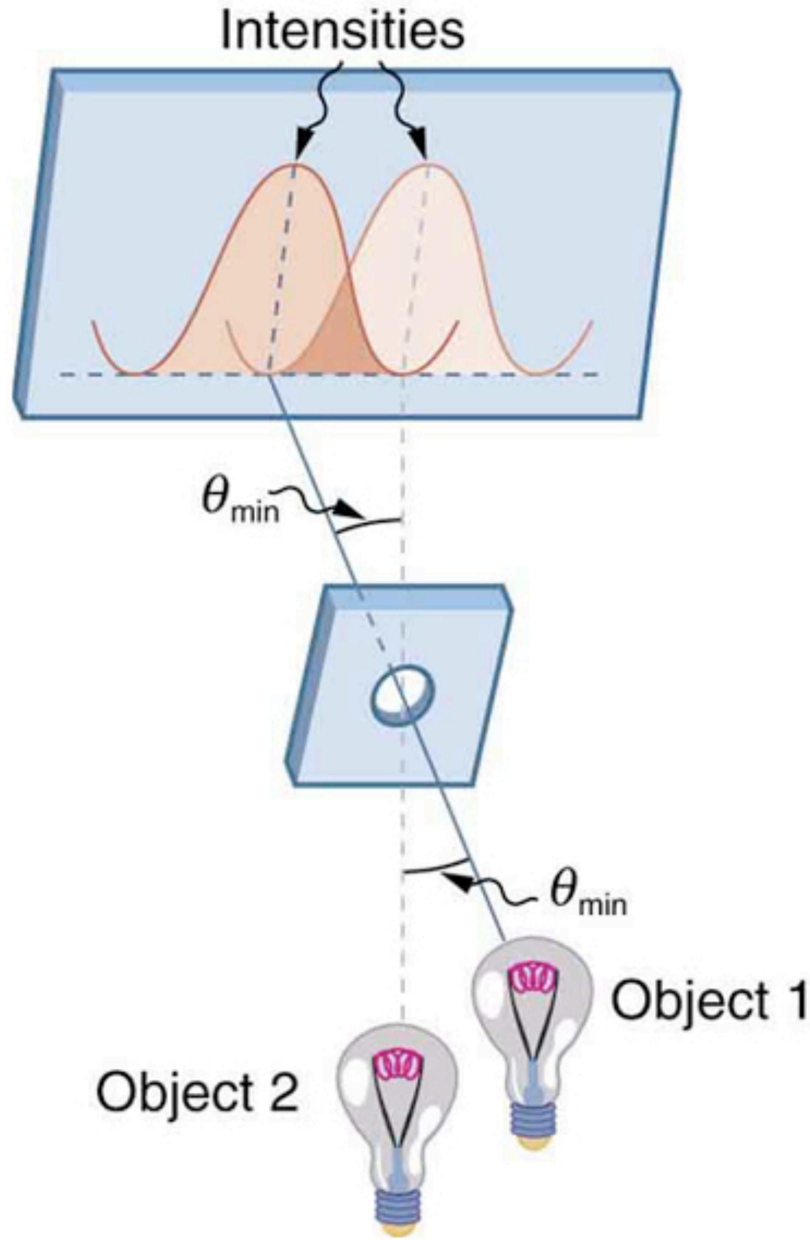


$$\sin\theta = 1.22\frac{\lambda}{d} \quad \text{(first minimum—circular aperture).}$$

If θ is small $\theta = 1.22\dfrac{\lambda}{d}$

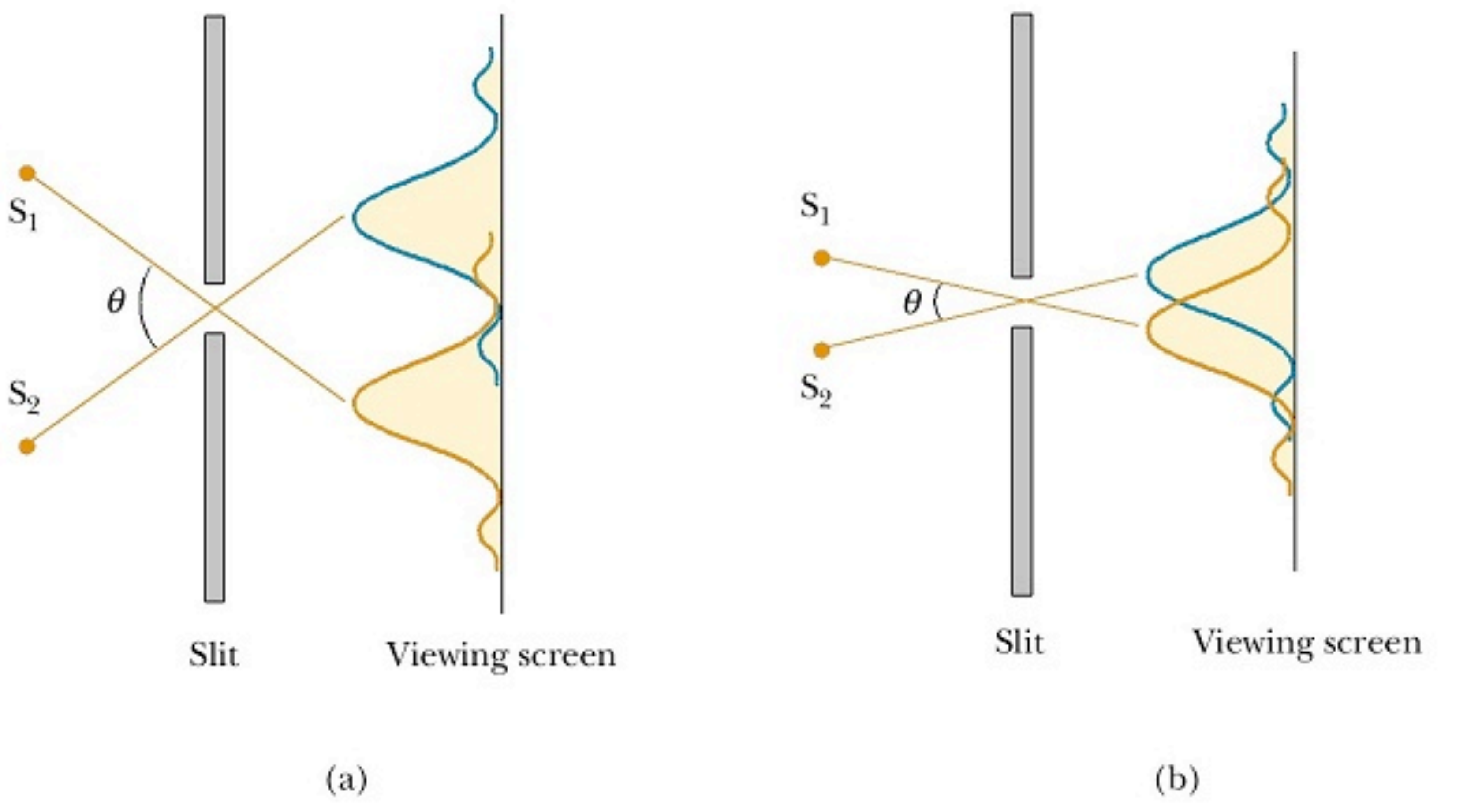


(a)



(b)

More resolvable for larger angle θ



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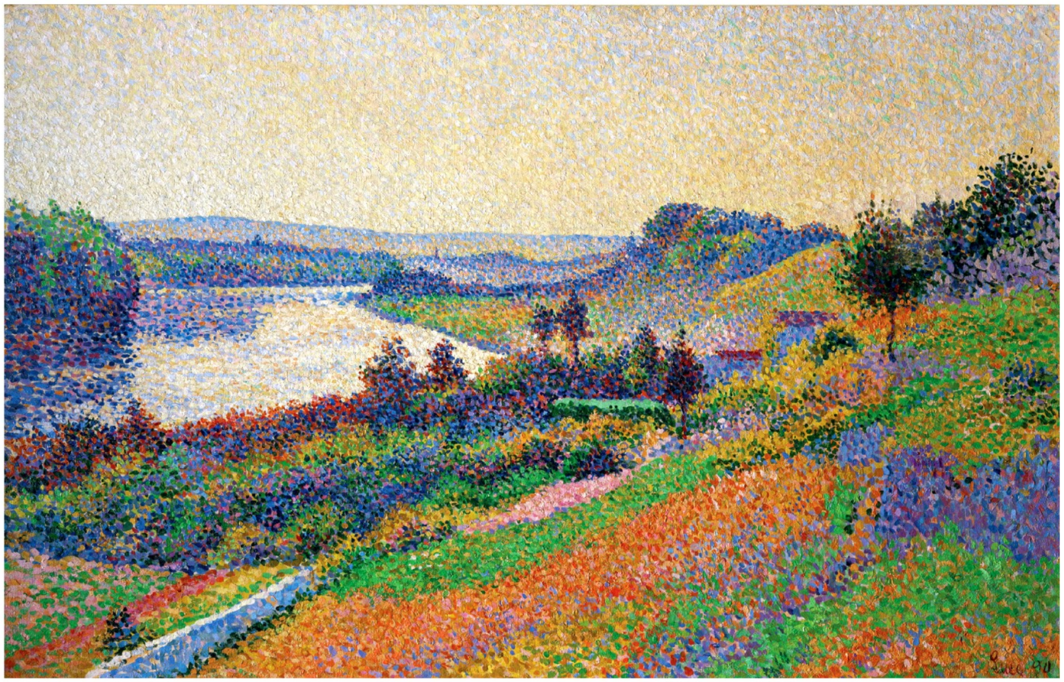
36-3 Diffraction by a Circular Aperture

Pointillism



Maximilien Luce, *The Seine at Herblay*, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

Rayleigh's criterion can explain the arresting illusions of color in the style of painting known as pointillism. In this style, a painting is made not with brush strokes in the usual sense but rather with a myriad of small colored dots. One fascinating aspect of a pointillistic painting is that when you change your distance from it, the colors shift in subtle, almost subconscious ways. This color shifting has to do with whether you can resolve the colored dots.

Checkpoint 4

Suppose that you can barely resolve two red dots because of diffraction by the pupil of your eye. If we increase the general illumination around you so that the pupil decreases in diameter, does the resolvability of the dots improve or diminish? Consider only diffraction. (You might experiment to check your answer.)

Answer:

Resolvability improves.

36-4 Diffraction by a Double Slit

Learning Objectives

36.18 In a sketch of a double-slit experiment, explain how the diffraction through each slit modifies the two-slit interference pattern, and identify the diffraction envelope, the central peak, and the side peaks of that envelope.

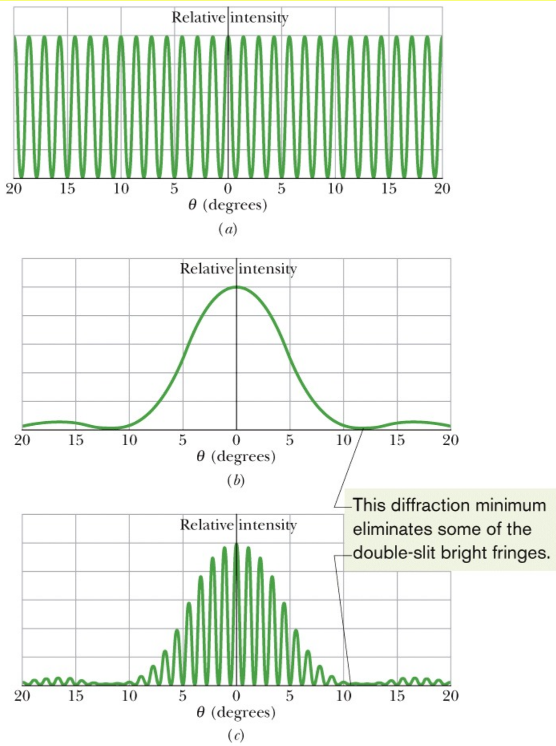
36.19 For a given point in a double-slit diffraction pattern, calculate the intensity I in terms of the intensity I_m at the center of the pattern.

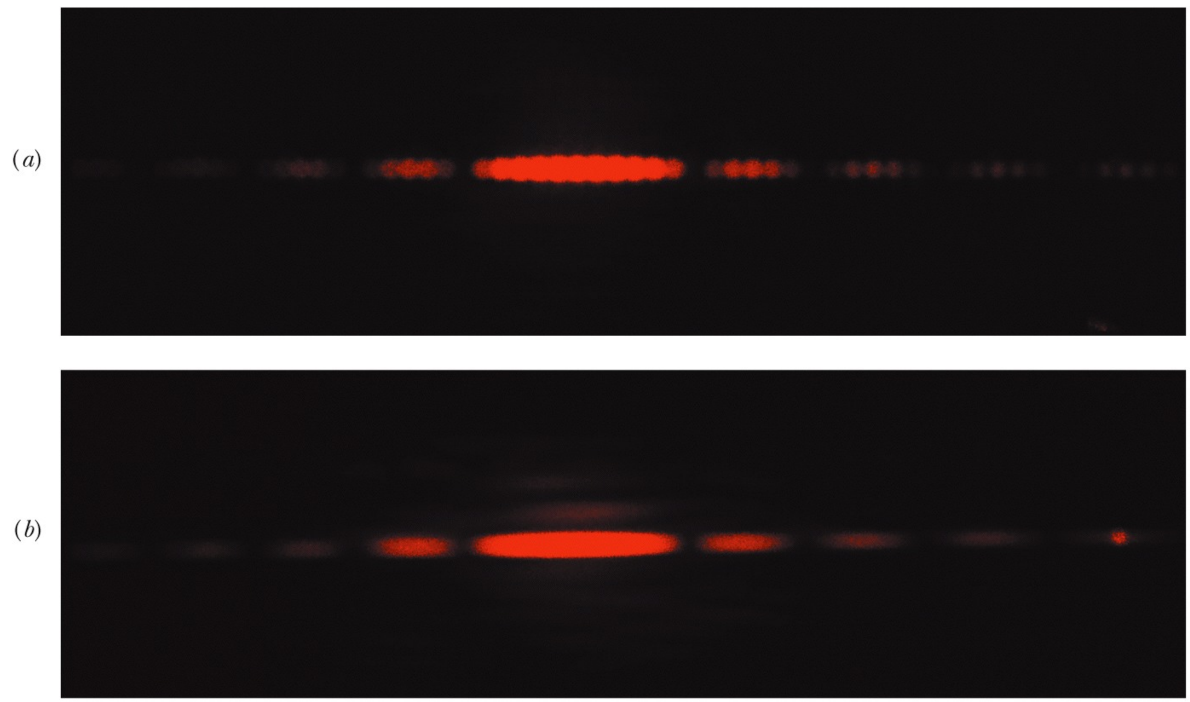
36.20 In the intensity equation for a double-slit diffraction pattern, identify what part corresponds to the interference between the two slits and what part corresponds to the diffraction by each slit.

36.21 For double-slit diffraction, apply the relationship between the ratio d/a and the locations of the diffraction minima in the single-slit diffraction pattern, and then count the number of two-slit maxima that are contained in the central peak and in the side peaks of the diffraction envelope.

36-4 Diffraction by a Double Slit

Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.



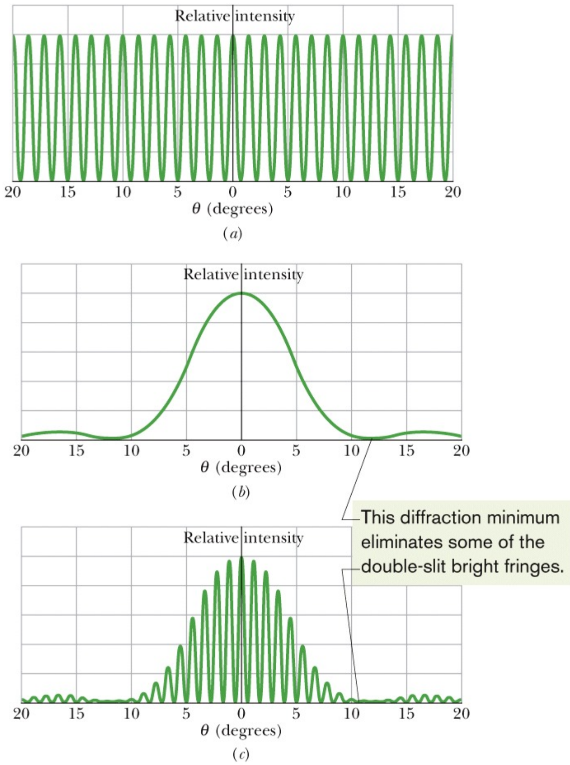


Courtesy Jearl Walker

(a) The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits. (b) The intensity plot for diffraction by a typical slit of width a (not vanishingly narrow). (c) The intensity plot to be expected for two slits of width a. The curve of (b) acts as an envelope, limiting the intensity of the double-slit fringes in (a). Note that the first minima of the diffraction pattern of (b) eliminate the double-slit fringes that would occur near 12° in (c).

36-4 Diffraction by a Double Slit

Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.



For identical slits with width a and center-to-center separation *d*, the intensity in the pattern varies with the angle θ from the central axis as

$$\boxed{I(\theta) = I_m(\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{(double slit)},}$$

in which

$$\boxed{\beta = \frac{\pi d}{\lambda}\sin\theta}$$

And

$$\boxed{\alpha = \frac{\pi a}{\lambda}\sin\theta.}$$

Note carefully that the right side of double slit equation is the product of I_m and two factors. (1) The interference factor $\cos^2\beta$ is due to the interference between two slits with slit separation *d*. (2) The diffraction factor $[(\sin a)/a]^2$ is due to diffraction by a single slit of width *a*.

36-5 Diffraction Gratings

Learning Objectives

36.22 Describe a diffraction grating and sketch the interference pattern it produces in monochromatic light.

36.23 Distinguish the interference patterns of a diffraction grating and a double-slit arrangement.

36.24 Identify the terms line and order number.

36.25 For a diffraction grating, relate order number m to the path length difference of rays that give a bright fringe.

36.26 For a diffraction grating, relate the slit separation d, the angle θ to a bright fringe in the pattern, the order number m of that fringe, and the wavelength λ of the light.

36.27 Identify the reason why there is a maximum order number for a given diffraction grating.

36.28 Explain the derivation of the equation for a line's half-width in a diffraction-grating pattern.

36-5 Diffraction Gratings

Learning Objectives

36.29 Calculate the half-width of a line at a given angle in a diffraction-grating pattern.

36.30 Explain the advantage of increasing the number of slits in a diffraction grating.

36.31 Explain how a grating spectroscope works.

36-5 Diffraction Gratings

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A diffraction grating is a series of "slits" used to separate an incident wave into its component wavelengths by separating and displaying their diffraction maxima. Diffraction by *N* (multiple) slits results in maxima (lines) at angles θ such that

$$d \sin\theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—lines)},$$

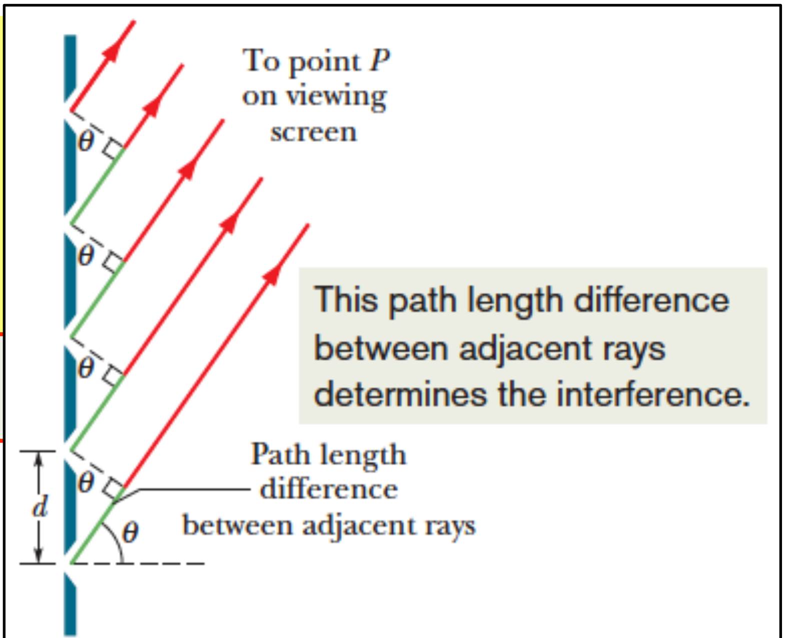
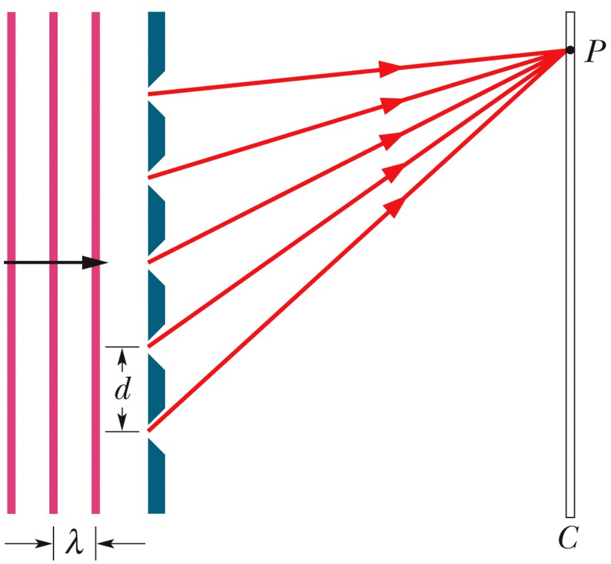


Figure 36-20 The rays from the rulings in a diffraction grating to a distant point *P* are approximately parallel. The path length difference between each two adjacent rays is $d \sin\theta$, where θ is measured as shown. (The rulings extend into and out of the page.)



An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a distant viewing screen *C*.

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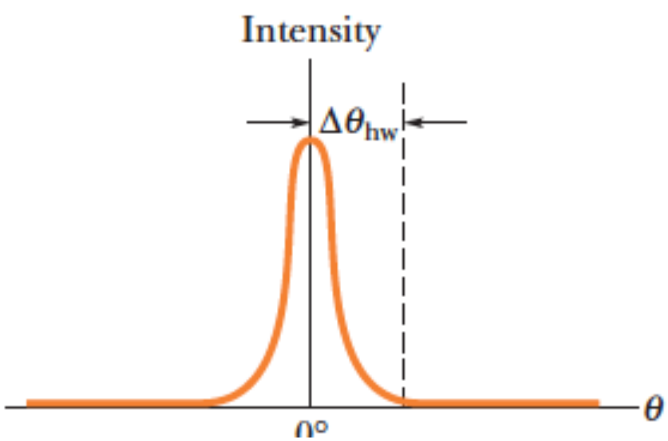
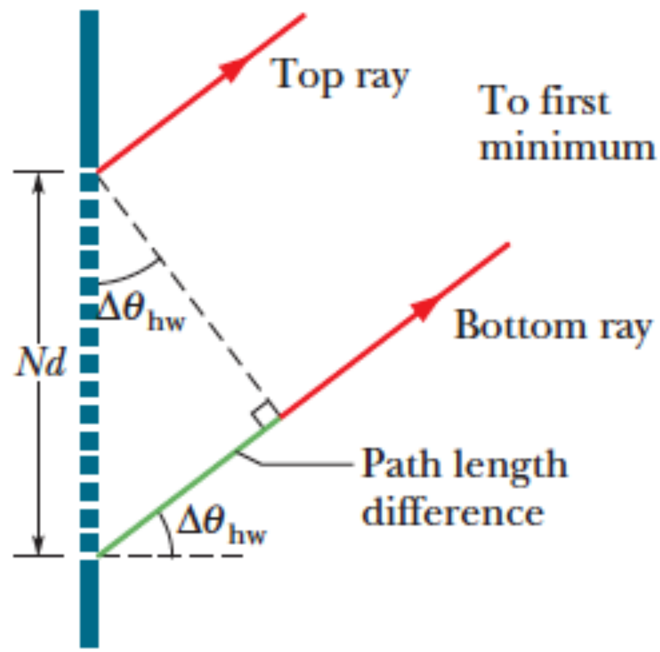
36-5 Diffraction Gratings

A line's **half-width** is the angle from its center to the point where it disappears into the darkness and is given by

$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta} \quad \text{(half-width of line at } \theta\text{).}$$





Note that for light of a given wavelength λ and a given ruling separation d, the widths of the lines decrease with an increase in the number N of rulings. Thus, of two diffraction gratings, the grating with the larger value of N is better able to distinguish between wavelengths because its diffraction lines are narrower and so produce less overlap.

36-6 Gratings: Dispersion and Resolving Power

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Learning Objectives

36.32 Identify dispersion as the spreading apart of the diffraction lines associated with different wavelengths.

36.33 Apply the relationships between dispersion D, wavelength difference $\Delta\lambda$, angular separation $\Delta\theta$, slit separation d, order number m, and the angle θ corresponding to the order number.

36.34 Identify the effect on the dispersion of a diffraction grating if the slit separation is varied.

36.35 Identify that for us to resolve lines, a diffraction grating must make them distinguishable.

36.36 Apply the relationship between resolving power R, wavelength difference $\Delta\lambda$, average wavelength l_{avg}, number of rulings N, and order number m.

36.37 Identify the effect on the resolving power R if the number of slits N is increased.

36-6 Gratings: Dispersion and Resolving Power

The dispersion D of a diffraction grating is a measure of the angular separation $\Delta\theta$ of the lines it produces for two wavelengths differing by $\Delta\lambda$. For order number m, at angle θ, the dispersion is given by

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta} \quad \text{(dispersion).}$$

Thus, to achieve higher dispersion we must use a grating of smaller grating spacing d and work in a higher-order m. Note that the dispersion does not depend on the number of rulings N in the grating. The SI unit for D is the degree per meter or the radian per meter.

Kristen Brochmann/Fundamental Photographs

The fine rulings, each 0.5 μm wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms colored "lanes" that are the composite of the diffraction patterns from the rulings.

36-6 Gratings: Dispersion and Resolving Power

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$$d \sin\theta = m\lambda.$$

$$d(\cos\theta)\, d\theta = m\, d\lambda.$$

$$d(\cos\theta)\, \Delta\theta = m\, \Delta\lambda$$

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}.$$

36-6 Gratings: Dispersion and Resolving Power

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The resolving power *R* of a diffraction grating is a measure of its ability to make the emission lines of two close wavelengths distinguishable. For two wavelengths differing by $\Delta\lambda$ and with an average value of λ_{avg}, the resolving power is given by

$$\boxed{R = \frac{\lambda_{\text{avg}}}{\Delta\lambda} = Nm}$$

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{Nd\cos\theta}.$$

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta} \quad \text{(dispersion)}.$$

$$\longrightarrow \quad \frac{\lambda}{N} = m\,\Delta\lambda, \quad \longrightarrow \quad R = \frac{\lambda}{\Delta\lambda} = Nm.$$

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36-7 X-Ray Diffraction

Learning Objectives

36.38 Identify approximately where x rays are located in the electromagnetic spectrum.

36.39 Define a unit cell.

36.40 Define reflecting planes (or crystal planes) and interplanar spacing.

36.41 Sketch two rays that scatter from adjacent planes.

36.42 For the intensity maxima in x-ray scattering by a crystal, apply the relationship between the interplanar spacing *d*, the angle θ of scattering, the order number m, and the wavelength l of the x rays.

36.43 Given a drawing of a unit cell, demonstrate how an interplanar spacing can be determined.

36-7 X-Ray Diffraction

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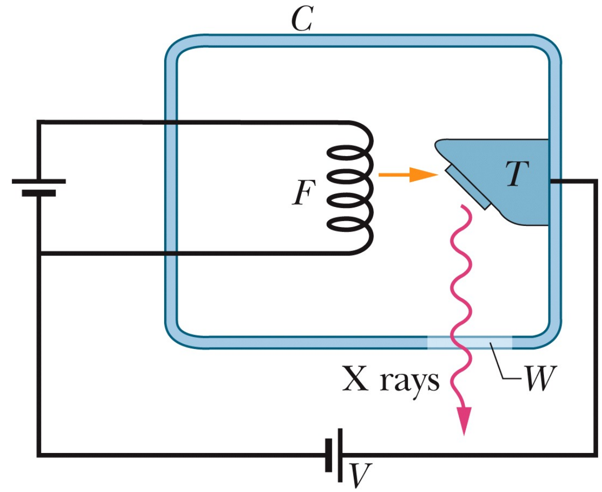
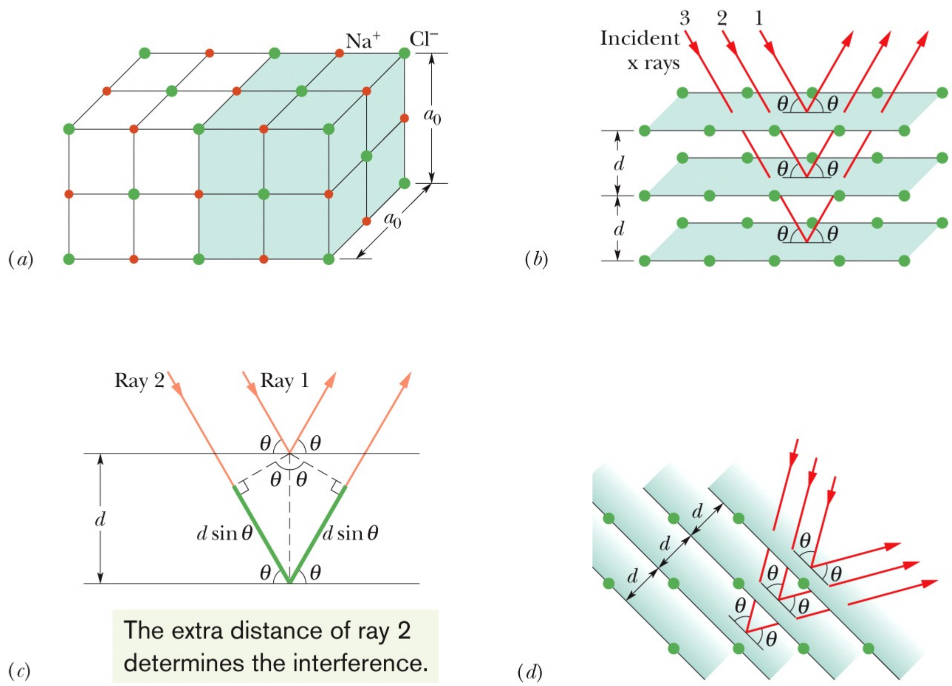
X rays are electromagnetic radiation whose wavelengths are of the order of 1 Å (=10^{-10} *m*). Figure (right) shows that x rays are produced when electrons escaping from a heated filament *F* are accelerated by a potential difference *V* and strike a metal target *T*.





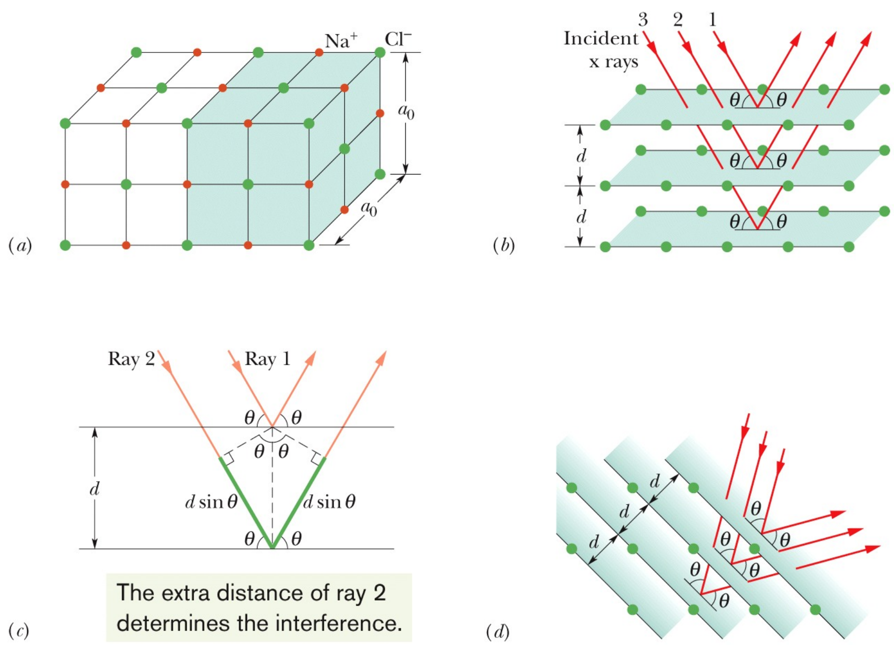
(a) The cubic structure of NaCl, showing the sodium and chlorine ions and a unit cell (shaded). (b) Incident x rays undergo diffraction by the structure of (a). The x rays are diffracted as if they were reflected by a family of parallel planes, with angles measured relative to the planes (not relative to a normal as in optics). (c) The path length difference between waves effectively reflected by two adjacent planes is $2d\sin\theta$. (d) A different orientation of the incident x rays relative to the structure. A different family of parallel planes now effectively reflects the x rays.

36-7 X-Ray Diffraction

As shown in figure below if x rays are directed toward a crystal structure, they undergo Bragg scattering, which is easiest to visualize if the crystal atoms are considered to be in parallel planes.

For x rays of wavelength λ scattering from crystal planes with separation d, the angles u at which the scattered intensity is maximum are given by

$$\boxed{2d \sin \theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots}$$



(a) The cubic structure of NaCl, showing the sodium and chlorine ions and a unit cell (shaded). (b) Incident x rays undergo diffraction by the structure of (a). The x rays are diffracted as if they were reflected by a family of parallel planes, with angles measured relative to the planes (not relative to a normal as in optics). (c) The path length difference between waves effectively reflected by two adjacent planes is $2d\sin\theta$. (d) A different orientation of the incident x rays relative to the structure. A different family of parallel planes now effectively reflects the x rays.

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36 Summary

Diffraction

- When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference.

Single-Slit Diffraction

- A single-slit diffraction patterns satisfy

$$\boxed{a \sin\theta = m\lambda, \quad \text{for } m = 1, 2, 3, \ldots}$$ **Eq. 36-3**

- The intensity of the diffraction pattern at any given angle θ is

$$\boxed{I(\theta) = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2,}$$ **Eq. 36-5**

where

$$\boxed{\alpha = \frac{\pi a}{\lambda} \sin\theta}$$ **Eq. 36-6**

Circular Aperture Diffraction

- Diffraction by a circular aperture or a lens with diameter *d* produces a central maximum and concentric maxima and minima, with the first minimum at an angle θ given by

$$\boxed{\sin\theta = 1.22 \frac{\lambda}{d}}$$ **Eq. 36-12**

Rayleigh's Criterion

- Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\boxed{\theta_R = 1.22 \frac{\lambda}{d}}$$ **Eq. 35-14**

36 Summary

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Double-Slit Diffraction

- Waves passing through two slits, each of width a, whose centers are a distance *d* apart, display diffraction patterns whose intensity *I* at angle *θ* is

$$I(\theta) = I_m(\cos^2 \beta)\left(\frac{\sin \alpha}{\alpha}\right)^2$$

Eq. 36-19

X-Ray Diffraction

- Diffraction maxima (due to constructive interference) occur if the incident direction of the wave, measured from the surfaces of these planes, and the wavelength l of the radiation satisfy Bragg's law:

$$2d \sin \theta = m\lambda, \quad \text{for } m = 1, 2, 3, \ldots$$

Eq. 36-12

Diffraction Gratings

- Diffraction by *N* (multiple) slits results
- in maxima (lines) at angles *θ* such that

$$d \sin \theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$

Eq. 36-25

with the half-widths of the lines given by

$$\Delta\theta_{hw} = \frac{\lambda}{Nd \cos \theta}$$

Eq. 36-28

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d \cos \theta}$$

Eq. 36-29&30

$$R = \frac{\lambda_{avg}}{\Delta\lambda} = Nm.$$

Eq. 36-31&32