

Handwritten derivation of the typing rule for the `isnil` function, showing the application of various typing rules (Form, Var, App, TApp) to derive the final typing judgment $\Gamma' \vdash \text{isnil } X \ell : \text{Bool}$.

$$\begin{array}{c}
 \text{Form} \\
 \hline
 \Gamma' \vdash Y : * \\
 \hline
 \text{Var} \quad \text{Var} \quad \text{Form} \quad \text{Var} \\
 \hline
 \Gamma' \vdash \text{cons} : \forall Y. Y \rightarrow \text{List } Y \rightarrow \text{List } Y \quad \Gamma' \vdash f : X \rightarrow Y \quad \Gamma' \vdash \ell : \text{List } X \quad \Gamma' \vdash \text{head} : \forall X. \text{List } X \rightarrow X \\
 \hline
 \Gamma' \vdash \text{cons } Y : Y \rightarrow \text{List } Y \rightarrow \text{List } Y \quad \Gamma' \vdash f(\text{head } X \ell) : Y \\
 \hline
 \Gamma' \vdash \text{cons } Y \text{ 1} : \text{List } Y \rightarrow \text{List } Y \\
 \hline
 \text{Form} \quad \text{Var} \\
 \hline
 \Gamma' \vdash \text{isnil} : \forall X. \text{List } X \rightarrow \text{Bool} \quad \Gamma' \vdash X : * \quad \Gamma' \vdash \text{tail} : \forall X. \text{List } X \rightarrow \text{List } X \\
 \hline
 \Gamma' \vdash \text{isnil } X : \text{List } X \rightarrow \text{Bool} \quad \Gamma' \vdash \ell : \text{List } X \quad \Gamma' \vdash \text{tail } X \ell : \text{List } X \\
 \hline
 \text{Form} \quad \text{Var} \quad \text{Form} \quad \text{Var} \\
 \hline
 \Gamma' \vdash \text{isnil } X \ell : \text{Bool} \quad \Gamma' \vdash Y : * \quad \Gamma' \vdash \text{nil} : \forall X. \text{List } X \rightarrow \text{Bool} \quad \Gamma' \vdash m : \text{List } X \rightarrow \text{List } Y \\
 \hline
 \Gamma' \vdash \text{isnil } X \ell : \text{Bool} \quad \Gamma' \vdash \text{nil } Y : \text{List } Y \quad \Gamma' \vdash \text{cons } Y \text{ 1 2} : \text{List } Y
 \end{array}$$
$$\Gamma, m: (List\ X) \rightarrow (List\ Y). \vdash \lambda \ell: List\ X. \text{map } \ell : List\ X \rightarrow List\ Y$$
$$\frac{\Gamma \vdash \text{fix} : (List\ X \rightarrow List\ Y) \rightarrow List\ X \rightarrow List\ Y}{\Gamma \vdash \text{fix} : (List\ X \rightarrow List\ Y) \rightarrow List\ X \rightarrow List\ Y} \text{App}$$
$$X:*, Y:* \vdash \lambda f: X \rightarrow Y. \quad \sim : (X \rightarrow Y) \rightarrow \text{List } X \rightarrow \text{List } Y$$

TAB₃

$$\vdash \lambda X. \lambda Y. \sim : \forall X. \forall Y. (X \rightarrow Y) \rightarrow \text{List } X \rightarrow \text{List } Y$$

$$2). \text{ pair} = \lambda X. \lambda f: X. \lambda s: X. \lambda b: X \rightarrow X \rightarrow X. b f s$$

$$\text{fst} = \lambda X. \lambda p: (X \rightarrow X \rightarrow X) \rightarrow X. p (\lambda f: X. \lambda s: Y. f)$$

$$\text{snd} = \lambda X. \lambda p: (X \rightarrow X \rightarrow X) \rightarrow X. p (\lambda f: X. \lambda s: Y. s)$$

$$\text{rev}' = \lambda X. \lambda p: (X \rightarrow X \rightarrow X) \rightarrow X. \text{pair } X (\text{cons } X (\text{head } X (\text{snd } X p)) (\text{fst } X p)) (\text{tail } X (\text{snd } X p))$$

$$\text{rev}'' = \lambda X. \text{fix} (\lambda r: (X \rightarrow X \rightarrow X) \rightarrow X. \lambda p: (X \rightarrow X \rightarrow X) \rightarrow X. \text{if } \text{isnil } X (\text{snd } X p) \text{ then } p \text{ else } r (\text{rev}' X p))$$

$$\text{reverse} = \lambda X. \lambda l: \text{List } X. \text{fst } X (\text{rev}'' X (\text{pair } X (\text{nil } X) l))$$

$$3). \text{ pair} = \lambda X. \lambda Y. \lambda f: X. \lambda s: Y. \lambda R. \lambda b: X \rightarrow Y \rightarrow R. b f s$$

$$\text{fst} = \lambda X. \lambda Y. \lambda p: \lambda R (X \rightarrow Y \rightarrow R) \rightarrow R. p X (\lambda f: X. \lambda s: Y. f)$$

$$\text{snd} = \lambda X. \lambda Y. \lambda p: \lambda R (X \rightarrow Y \rightarrow R) \rightarrow R. p Y (\lambda f: X. \lambda s: Y. s)$$

$$\text{biggest} = \lambda X. \lambda p: (X \rightarrow \text{List } X \rightarrow R) \rightarrow R. \lambda c: (X \rightarrow X \rightarrow \text{Bool}). \text{if } c (\text{fst } X (\text{List } X) p) (\text{head } X (\text{snd } X (\text{List } X) p))$$

$$\text{then } (\lambda p'. \text{pair } X (\text{List } X) (\text{fst } X (\text{List } X) p') (\text{cons } X (\text{head } X (\text{snd } X p))$$

$$\text{snd } X (\text{List } X) p')) (\text{biggest } X (\text{pair } (\text{fst } X p) (\text{tail } X (\text{snd } X p))))$$

$$\text{else } (\lambda p'. \text{pair } X (\text{fst } X p') (\text{cons } X (\text{fst } X p) (\text{snd } X p'))$$

$$(\text{biggest } X (\text{pair } (\text{head } X (\text{snd } X p)) (\text{tail } X (\text{snd } X p))))$$

$$\text{sort} = \lambda X. \lambda c: (X \rightarrow X \rightarrow \text{Bool}). \text{fix} (\lambda s: \text{List } X \rightarrow \text{List } X. \lambda l: \text{List } X.$$

$$(\lambda p'. (\text{cons } X (\text{fst } X (\text{List } X) p') (\text{snd } X (\text{List } X) p'))$$

$$(\text{biggest } X (\text{pair } X (\text{List } X) (\text{head } X l) (\text{tail } X l)) c))$$