Exam

Friday, 15. December 2023 11:34

1.

I two args. of type $(\gamma \to \varepsilon)$ & $(\gamma \to \varepsilon) \to \varepsilon$ (λx, λy)

II create term f:

$f = \lambda x \lambda y . y (\lambda z : \gamma . x z)$

III try to type f with $\alpha \to (\alpha \to \varepsilon) \to \varepsilon$

=> not possible as z has type γ => x has to have type γ

2.

a) data T a b c = A a | B b c | C b b a

b) data Unit = Unit a

c) data T a b c = T | F | B a (a times?)

d) ..

3.

a) $(\delta \xrightarrow{x} \delta \to \alpha) \to (\gamma \xrightarrow{y} \alpha) \to (\alpha \xrightarrow{z} \beta) \to \delta \xrightarrow{f} \gamma \xrightarrow{g} \beta$

$\lambda x . \lambda y . \lambda z . \lambda f . \lambda g . \; z (x f f)$

$\lambda x . \lambda y . \lambda z . \lambda f . \lambda g . \; z (y g)$

b) $(\alpha \xrightarrow{x} (\beta \to \gamma)) \to ((\alpha \to (\gamma \to \delta)) \to (\alpha \to (\beta \to \delta)))$

c) $((((\alpha \to \beta) \xrightarrow{x} \alpha) \to \alpha) \to \beta) \to \beta$

$\lambda x^{(((\alpha \to \beta) \to \alpha) \to \alpha) \to \beta} . x (\lambda y^{\alpha} z^{\beta} . z)$

d)

$\lambda x . x$ can be typed void -> void

4.

a) $\lambda x . x$

b) $(\lambda x . y)(\Omega)$

c) $\Lambda \alpha . \lambda x : \alpha . x$

5.

Let true: $\lambda x \lambda y . x$

false: $\lambda x \lambda y . y$

Ω: non-terminating

id: $\lambda x . x$

a)

$(\lambda b . \lambda t . \lambda f . b t f)(\lambda x y . y)(\Omega)(\lambda x . x)$

$\to (\lambda t . \lambda f . (\lambda x y . y) t f)(\Omega)(\lambda x . x)$

$\to \lambda f . (\lambda x y . y)(\Omega) f$

$\to \lambda f . \lambda y . y \to \lambda y . y$

b) infinite loop.

6.

a) g(5) => !f(4) => !f(3) => !f(2) => !f(1)

true, false, true, false, true

If n odd => true, otherwise false

g computes if a number is odd.

b)

$Y = \lambda f . (\lambda x . f(x x))(\lambda x . f(x x))$

$Y H = (\lambda f . (\lambda x . f(x x))(\lambda x . f(x x)))H$

$\to (\lambda x . H(x x))(\lambda x . H(x x))$

$\to H((\lambda x . H(x x))^{\lambda f} (\lambda x . H(x x))^{\lambda n})$

$\to$ if $(\lambda x . H(x x) = 1)$ then true else if $(\lambda x . H(x x)) = 0$ then false else $(\lambda x . H(x x))((\lambda x . H(x x) - 1))$

c)

$(Y H) 2 = ((\lambda f . (\lambda x . f(x x))(\lambda x . f(x x)))H) 2$

$\to (\lambda x . H(x x))(\lambda x . H(x x)) 2$

$\to (H(2 2))(H(2 2))$

$\to$