Easy tasks:

1. $\lambda f^{\gamma \to \varepsilon} . \lambda g^{(\gamma \to \varepsilon) \to \varepsilon} . g\,(\lambda x^{\gamma} . f\,x)$

2. a) `data T a b c = A a | B b c | C b b a`

   b) `data Unit = Unit a`

3. 1) $\lambda f^{\delta \to \delta \to \alpha} . \lambda g^{\gamma \to \alpha} . \lambda h^{\alpha \to \beta} . \lambda x^{\delta} . \lambda y^{\gamma} . h\,(f\,(x\,x))$

   $\lambda f^{\delta \to \delta \to \alpha} . \lambda g^{\gamma \to \alpha} . \lambda h^{\alpha \to \beta} . \lambda x^{\delta} . \lambda y^{\gamma} . h\,(g\,(y))$

   2) $\lambda f^{\alpha \to (\beta \to \gamma)} . (\lambda f . f)$

   3) $\lambda f^{(((\alpha \to \beta) \to \alpha) \to \alpha) \to \beta} . f\,(\lambda x^{\alpha}\, y^{\beta} . y)$ $\lambda x^{\alpha}\, y^{\beta} . x)))$

   4) $\lambda x . x$ -> Id

4. 1) $\lambda x . x$

   2) $\lambda f . (\lambda x . f\,(x\,x))\,(\lambda x . f\,(x\,x))$ -> Y-combinator

   3) $\Lambda \alpha . \lambda x : \alpha x$

5. Call-by-name:

(If False Ω λx.x)

$\Rightarrow (\lambda b.\lambda t.\lambda f.btf)(\lambda xy.y)\,\Omega\,(\lambda x.x)$

$\Rightarrow \lambda x.\lambda y.(\lambda xy.y)\,x\,y)\,\Omega\,(\lambda x.x)$

$\Rightarrow \lambda y.(\lambda xy.y)\,\Omega\,y$

$\Rightarrow \lambda y.y$

Call-by-value:

Reaches infinite loop.

6. 1) g computes if n is odd (true) or even (false)

2) $Y = \lambda f.(\lambda x.f(x\,x))(\lambda x.f(x\,x))$

$Y\,H = \lambda f.(\lambda x.f(x\,x))(\lambda x.f(x\,x))\,H$

$\Rightarrow (\lambda x.H(x\,x))(\lambda x.H(x\,x))$

$\Rightarrow H((\lambda x.H(x\,x))(\lambda x.H(x\,x)))$

* Now substitute $(\lambda x.H(x\,x))(\lambda x.H(x\,x))$ for f in H

$\rightarrow (\lambda n.$ if n=1 then true else if n=0 then false else not $(((\lambda x.H(x\,x))(\lambda x.H(x\,x)))(n-1))$

* Simplify inner expression

$\Rightarrow H((\lambda x.H(x\,x))(\lambda x.H(x\,x)))(n-1)$

* Substitute back into larger expression:

$\rightarrow (\lambda n.$ if n=1 then true else if n=0 then false else not$(H((\lambda x.H(x\,x))(\lambda x.H(x\,x)))(n-1))$

$\Rightarrow f(n-1)$ has been replaced by $H((\lambda x.H(x\,x))(\lambda x.H(x\,x)))(n-1)$

3) $(YH)2 = H((\lambda x.H(xx))(\lambda x.H(xx))2$

$\Rightarrow$ if $2=1$ then true else if $2=0$ then false else

not $(((\lambda x.H(xx))(\lambda x.H(xx)))(2-1)$

$\Rightarrow$ if false then true else if false then false else

not $(H((\lambda x.H(xx))(\lambda x.H(xx)))1)$

$\Rightarrow$ not(if $1=1$ then true else if $1=0$ then false

else not $(((\lambda x.H(xx))(\lambda x.H(xx)))(1-1))$

The computation might continue but we can see that this should evaluate to true, thus the final result should be not (true) $\Rightarrow$ false.