

Easy tasks:

$$1. \lambda f^{\gamma \rightarrow \varepsilon} . \lambda g^{(\gamma \rightarrow \varepsilon) \rightarrow \varepsilon} . g(\lambda x^{\gamma} . f x)$$

$$2. \text{data } T a b c = A a | B b c | C b b a$$

$$b) \text{data Unit} = \text{Unit } a$$

$$3. 1) \lambda f^{\delta \rightarrow \delta \rightarrow \alpha} . \lambda g^{\gamma \rightarrow \alpha} . \lambda h^{\alpha \rightarrow \beta} . \lambda x^{\delta} . \lambda y^{\gamma} . h(f(x x))$$

$$\lambda f^{\delta \rightarrow \delta \rightarrow \alpha} . \lambda g^{\gamma \rightarrow \alpha} . \lambda h^{\alpha \rightarrow \beta} . \lambda x^{\delta} . \lambda y^{\gamma} . h(g(y))$$

$$2) \lambda f^{\alpha \rightarrow (\beta \rightarrow \gamma)} . (\lambda f . f)$$

$$3) \lambda f^{(((\alpha \rightarrow \beta) \rightarrow \alpha) \rightarrow \alpha) \rightarrow \beta} . f(\lambda x^{\alpha} y^{\beta} . y)$$

$$4) \lambda x . x \rightarrow \text{Id}$$

$$4. 1) \lambda x . x$$

$$2) \lambda f . (\lambda x . f(x x))(\lambda x . f(x x)) \rightarrow Y\text{-combinator}$$

$$3) \lambda \alpha . \lambda x : \alpha . x$$

5. Call-by-name:

$$(\text{If False } \Omega \lambda x.x)$$

$$\Rightarrow (\lambda b. \lambda t. \lambda f. b \text{ if } (\lambda xy. y) \Omega (\lambda x.x))$$

$$\Rightarrow \lambda x. \lambda y. (\lambda xy. y) x y \Omega (\lambda x.x)$$

$$\Rightarrow \lambda y. (\lambda xy. y) \Omega y$$

$$\Rightarrow \lambda y. y$$

Call-by-value:

Reaches infinite loop.

6. 1) g computes if n is odd (true) or even (false)

$$2) \quad Y = \lambda f. (\lambda x. f(x x)) (\lambda x. f(x x))$$

$$Y H = \lambda f. (\lambda x. f(x x)) (\lambda x. f(x x)) H$$

$$\Rightarrow (\lambda x. H(x x)) (\lambda x. H(x x))$$

$$\Rightarrow H((\lambda x. H(x x)) (\lambda x. H(x x)))$$

* Now substitute $(\lambda x. H(x x)) (\lambda x. H(x x))$ for f in H

$\rightarrow (\lambda n. \text{if } n=1 \text{ then true else if } n=0 \text{ then false else}$

$$\text{not } (((\lambda x. H(x x)) (\lambda x. H(x x))) (n-1)))$$

* Simplify inner expression

$$\Rightarrow H((\lambda x. H(x x)) (\lambda x. H(x x))) (n-1)$$

* Substitute back into larger expression:

$\rightarrow (\lambda n. \text{if } n=1 \text{ then true else if } n=0 \text{ then false else}$

$$\text{not } (H((\lambda x. H(x x)) (\lambda x. H(x x))) (n-1)))$$

$$\Rightarrow f(n-1) \text{ has been replaced by } H((\lambda x. H(x x)) (\lambda x. H(x x))) (n-1))$$

$$3) (\gamma H)^2 = H((\lambda x. H(x x))(\lambda x. H(x x)))^2$$

\Rightarrow if $2=1$ then true else if $2=0$ then false else

$$\text{not } (((\lambda x. H(x x))(\lambda x. H(x x))))(2-1)$$

\Rightarrow if false then true else if false then false else

$$\text{not } (1 - (((\lambda x. H(x x))(\lambda x. H(x x)))) 1)$$

\Rightarrow not (if $1=1$ then true else if $1=0$ then false

else not $(((\lambda x. H(x x))(\lambda x. H(x x))))(1-1)$

The computation might continue but we can see that this should evaluate to true, thus the final result should be $\text{not (true)} \Rightarrow$ false.