Semi-Automated Direction-Driven Functional Conversion

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Relational Programming

One relation to solve many problems

Nondeterminism

Completeness of search

Relational Conversion: Easy

Given a function

```
let rec add x y =
  match x with
   | O → y
   | S x₁ → S (add x₁ y)
```

generate miniKanren relation

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁)) ]
```

Principal Directions of miniKanren Relations

Every argument of a relation can be either `in` or `out`

For addition relation $\texttt{add}^o$ `x y z` there are 8 directions:

- *Forward* direction: $\texttt{add}^o$ `in in out` — addition
- *Backward* direction: $\texttt{add}^o$ `out out in` — decomposition
- *Predicate*: $\texttt{add}^o$ `in in in`
- *Generator*: $\texttt{add}^o$ `out out out`
- $\texttt{add}^o$ `in out in` — subtraction
- $\texttt{add}^o$ `out in in` — subtraction
- $\texttt{add}^o$ `out in out`
- $\texttt{add}^o$ `in out out`

Each Direction is a Function

Each Direction is a Function (kind of)

Straightforward functions:

- *Forward* direction: `add`^{o} `in in out` — addition
- `add`^{o} `in out in` — subtraction
- `add`^{o} `out in in` — subtraction
- *Predicate*: `add`^{o} `in in in`

Relations:

- *Backward* direction: `add`^{o} `out out in` — decomposition
- *Generator*: `add`^{o} `out out out`
- `add`^{o} `out in out`
- `add`^{o} `in out out`

These relations are functions which return multiple answers (list monad)

miniKanren Comes with an Overhead

Unifications

Occurs-check

Scheduling complexity

Functional Conversion

Given a relation and a principal direction, construct a functional program which generates the same answers as miniKanren would

Preserve completeness of the search

Both inputs and outputs are expected to be ground

Example: Addition in the Forward Direction

```
let rec addᵒ x y z = conde [
  (x ≡ O  ∧  y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁  ∧
     addᵒ x₁ y z₁  ∧
     z ≡ S z₁)) ]
```

```
addIIO :: Nat → Nat → Nat
addIIO x y =
  case x of
    O → y
    S x₁ → S (addIIO x₁ y)
```

Addition in the Backward Direction: Nondeterminism

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁)) ]
```

```
addOOI :: Nat → Stream (Nat, Nat)
addOOI z =
  return (O, z) `mplus`
  case z of
    O → Empty
    S z₁ → do
      (x₁, y) ← addOOI z₁
      return (S x₁, y)
```

Free Variables in Answers: Generators

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧ z ≡ S z₁ ∧ addᵒ x₁ y z₁) ) ]
```

```
addIOO :: Nat → Stream (Nat, Nat)
addIOO x = case x of
  O → do
    z ← genNat
    return (z, z)
  S x₁ → do
    (y, z₁) ← addIOO x₁
    return (y, S z₁)

genNat :: Stream Nat
genNat = Mature O (S <$> genNat)
```

Predicates

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁)) ]
```

```
addIII :: Nat → Nat → Nat → Stream ()
addIII x y z =
  case x of
    O | y == z → return ()
      | otherwise → Empty
    S x₁ →
      case z of
        O → Empty
        S z₁ → addIII x₁ y z₁
```

Conversion Scheme

- Normalization
- Mode analysis
- Functional conversion

Normalization: Flat Term

Flat terms: a var or a constructor which takes *distinct* vars as arguments:

$$\mathcal{FT}_V = V \cup \{\mathcal{C}_i\,(x_1, \ldots, x_{k_i}) \mid x_j \in V, x_j - distinct\}$$

Examples:

$$\begin{aligned}
C\,(x_1, x_2) \equiv C\,(C\,(y_1, y_2)\,, y_3) &\iff x_1 \equiv C\,(y_1, y_2) \land x_2 \equiv y_3 \\
C\,(C\,(x_1, x_2)\,, x_3) \equiv C\,(C\,(y_1, y_2)\,, y_3) &\iff x_1 \equiv y_1 \land x_2 \equiv y_2 \land x_3 \equiv y_3 \\
x \equiv C\,(y, y) &\iff x \equiv C\,(y_1, y_2) \land y_1 \equiv y_2
\end{aligned}$$

Normalization: Goal

$$
\begin{array}{llll}
\mathcal{K}_V^N & = & \bigvee(c_1, \ldots, c_n), c_i \in \mathrm{Conj}_V & \text{normal form} \\
\mathrm{Conj}_V & = & \bigwedge(g_1, \ldots, g_n), g_i \in \mathrm{Base}_V & \text{normal conjunction} \\
\mathrm{Base}_V & = & V \equiv \mathcal{FT}_V & \text{flat unification} \\
 & \mid & R_i(x_1, \ldots, x_{k_i}), x_j \in V, x_j - \mathit{distinct} & \text{flat call}
\end{array}
$$

Mode of a Variable

Mode of a variable: mapping between its instantiations

Ground term contains no fresh variables
Free variable: a fresh variable, no info about its instantiation

Once we know that a variable is *ground*, it stays *ground* in subsequent conjuncts

Mode *in*: $\textit{ground} \to \textit{ground}$
Mode *out*: $\textit{free} \to \textit{ground}$

Mercury uses more complicated modes

Modded Goal

Assign mode to every variable, make sure they are consistent

Modded Unification Types

$$
\begin{aligned}
\textit{assignment} &: x^{\text{out}} \equiv \mathcal{T}^{\text{in}} \text{ and } x^{\text{in}} \equiv y^{\text{out}} \\
\textit{guard} &: x^{\text{in}} \equiv \mathcal{T}^{\text{in}} \\
\textit{match} &: x^{\text{in}} \equiv \mathcal{T} \ (\mathcal{T} \text{ contains both } \textit{in} \text{ and } \textit{out} \text{ variables}) \\
\textit{generator} &: x^{\text{out}} \equiv \mathcal{T}
\end{aligned}
$$

Mode Inference: Initialization

- Input variables: *ground* $\to$ *ground*
- Output variables: *free* $\to$ *ground*
- Other variables: *free* $\to$?

$$
\begin{array}{l}
\textbf{let rec } \mathtt{add}^o\ \mathtt{x}^{g\to g}\ \mathtt{y}^{g\to g}\ \mathtt{z}^{f\to g} = \textbf{conde} \\
\quad (\mathtt{x}^{g\to g} \equiv \mathtt{0} \ \wedge\ \mathtt{y}^{g\to g} \equiv \mathtt{z}^{f\to g});\\
\quad (\mathtt{x}^{g\to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f\to ?} \ \wedge \\
\quad\ \ \mathtt{add}^o\ \mathtt{x}_1^{f\to ?}\ \mathtt{y}^{g\to g}\ \mathtt{z}_1^{f\to ?} \ \wedge \\
\quad\ \ \mathtt{z}^{f\to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f\to ?})
\end{array}
$$

Mode Inference: Disjunction

Run inference on each disjunct independently

$$\mathtt{x}^{g \to g} \equiv \mathtt{0} \wedge \mathtt{y}^{g \to g} \equiv \mathtt{z}^{f \to g}$$

$$\begin{array}{l} \mathtt{x}^{g \to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f \to ?} \wedge \\ \mathtt{add}^o\ \mathtt{x}_1^{f \to ?}\ \mathtt{y}^{g \to g}\ \mathtt{z}_1^{f \to ?} \wedge \\ \mathtt{z}^{f \to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f \to ?} \end{array}$$

Mode Inference: Unification

Propagate the groundness information according to the 4 types of modded unifications

$$\mathtt{x}^{g \to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f \to ?} \Rightarrow \mathtt{x}^{g \to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f \to g}$$

$$\mathtt{z}^{f \to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f \to ?} \Rightarrow \mathtt{z}^{f \to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f \to g}$$

Mode Inference: Conjunction

Pick a conjunct according to the priority, propagate groundness

- Always deterministic
- Guard
- Assignment
- Matches
- Calls with some ground arguments
- Unifications-generators
- Calls with all free arguments

Mode Inference: Conjunction

$$\begin{array}{l}
\texttt{add}^o\ \ \texttt{x}_1^{f\to ?}\ \ \texttt{y}^{g\to g}\ \ \texttt{z}_1^{f\to ?}\ \ \wedge \\
\texttt{x}^{g\to g} \equiv \texttt{S}\ \ \texttt{x}_1^{f\to ?}\ \ \wedge \\
\texttt{z}^{f\to g} \equiv \texttt{S}\ \ \texttt{z}_1^{f\to ?}
\end{array}$$

Mode Inference: Conjunction

$$
\begin{aligned}
&\texttt{add}^o\ \ \texttt{x}_1^{f\to ?}\ \ \texttt{y}^{g\to g}\ \ \texttt{z}_1^{f\to ?}\ \ \wedge\\
&\texttt{x}^{g\to g} \equiv \texttt{S}\ \ \texttt{x}_1^{f\to ?}\ \ \wedge\\
&\texttt{z}^{f\to g} \equiv \texttt{S}\ \ \texttt{z}_1^{f\to ?}
\end{aligned}
$$

$$
\begin{aligned}
&\texttt{x}^{g\to g} \equiv \texttt{S}\ \ \texttt{x}_1^{f\to g}\ \ \wedge\\
&\texttt{add}^o\ \ \texttt{x}_1^{g\to g}\ \ \texttt{y}^{g\to g}\ \ \texttt{z}_1^{f\to ?}\ \ \wedge\\
&\texttt{z}^{f\to g} \equiv \texttt{S}\ \ \texttt{z}_1^{f\to ?}
\end{aligned}
$$

Mode Inference: Conjunction

$$\begin{array}{l} \mathtt{add}^o\ \mathtt{x}_1^{f\to ?}\ \mathtt{y}^{g\to g}\ \mathtt{z}_1^{f\to ?}\ \wedge \\ \mathtt{x}^{g\to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f\to ?}\ \wedge \\ \mathtt{z}^{f\to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f\to ?} \end{array}$$

$$\begin{array}{l} \mathtt{x}^{g\to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f\to g}\ \wedge \\ \mathtt{add}^o\ \mathtt{x}_1^{g\to g}\ \mathtt{y}^{g\to g}\ \mathtt{z}_1^{f\to ?}\ \wedge \\ \mathtt{z}^{f\to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f\to ?} \end{array}$$

$$\begin{array}{l} \mathtt{x}^{g\to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f\to g}\ \wedge \\ \mathtt{add}^o\ \mathtt{x}_1^{f\to g}\ \mathtt{y}^{g\to g}\ \mathtt{z}_1^{f\to g}\ \wedge \\ \mathtt{z}^{f\to g} \equiv \mathtt{S}\ \mathtt{z}_1^{g\to g} \end{array}$$

Order in Conjunctions

```
let rec multᵒ x y z = conde [
  ...
  (fresh (x₁ r')
    (x ≡ S x₁) ∧
    (addᵒ y r' z) ∧
    (multᵒ x₁ y r')
  )]
```

Order in Conjunctions: Slow Version

```
multIIO₁ :: Nat → Nat → Stream Nat
...
multIIO₁ (S x₁) y  = do
  (r₁, r) ← addX y
  multIII x₁ y r₁
  return r

multIII :: Nat → Nat → Nat → Stream ()
...
multIII (S x₁) y z = do
  z₁ ← multIIO₁ x₁ y
  addIII y z₁ z
multIII _ _ _ = Empty
```

Premature grounding of z_1 leads to generate-and-test behavior

Order in Conjunctions: Faster Version

```
multIIO :: Nat → Nat → Stream Nat
...
multIIO (S x₁) y = do
  r' ← multIIO x₁ y
  addXY y r'
```

Functional Conversion: Intermediate Language

$$
\begin{array}{lcll}
\mathcal{F}_V & = & \text{Return}\,[\mathcal{T}_V] & \text{return a tuple of terms} \\
 & \mid & \text{Match}_V\,(\mathcal{T}_V, \mathcal{F}_V) & \text{match a variable against a pattern} \\
 & \mid & \text{Bind}\,[([V]\,, \mathcal{F}_V)] & \text{monadic bind on streams} \\
 & \mid & \text{Sum}\,[\mathcal{F}_V] & \text{concatenation of streams} \\
 & \mid & \text{Guard}\,(V, V) & \text{equality check} \\
 & \mid & \text{Gen}_G & \text{generator} \\
 & \mid & R_i([V]\,, [G]) & \text{function call}
\end{array}
$$

Functional Conversion into Intermediate Language

- Disjunction $\to$ Sum $[\mathcal{F}_V]$
- Conjunction $\to$ Bind $[([V], \mathcal{F}_V)]$
- Relation call $\to R_i([V], [G])$
- Unification $\to$
    - Return $[\mathcal{T}_V]$
    - Match$_V$ $(\mathcal{T}_V, \mathcal{F}_V)$
    - Guard (V, V)
    - Gen$_G$

Functional Conversion into Haskell

- TemplateHaskell to generate code
- Stream monad
- do-notation

Functional Conversion into OCaml

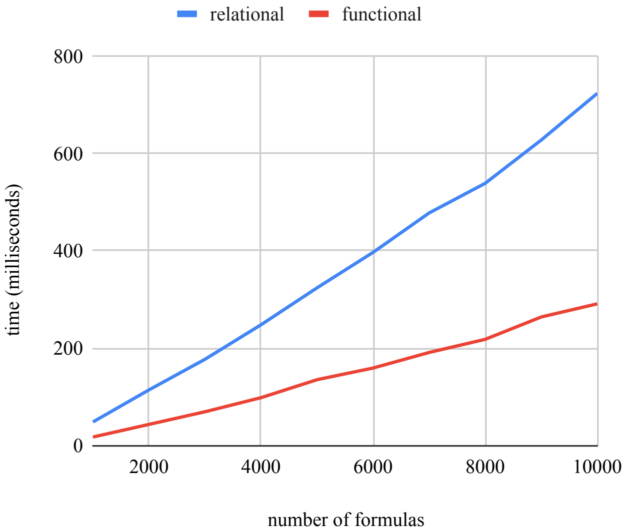
- Hand-crafted (not so) pretty-printer
- Stream monad
- let*
- Taking extra care to employ laziness

Evaluation

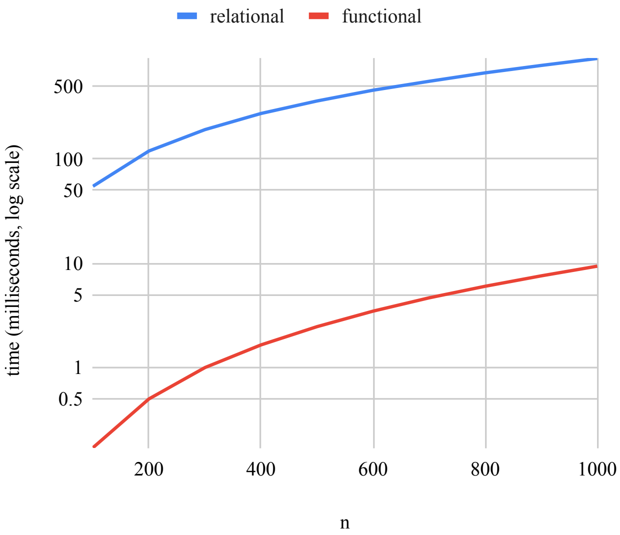
We converted relational interpreters and measured execution time

- Logic formulas generation
  - Inverse computation of an evaluator of logic formulas
  - Generating formulas which evaluate to **true**
- Multiplication relation
  - Forward direction: multiplication
  - Backward direction: division
  - Generation

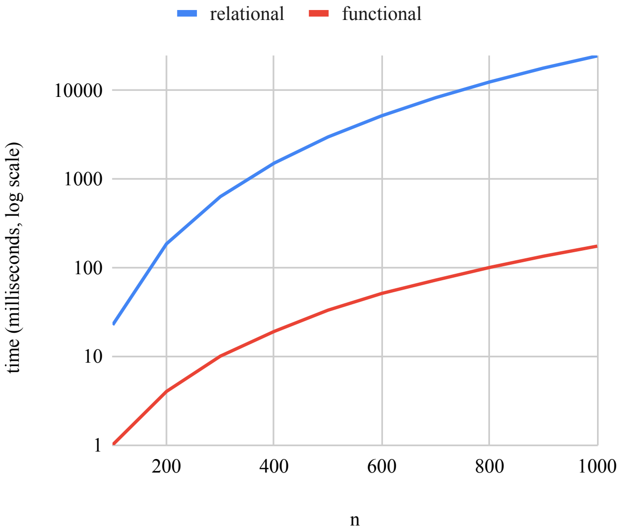
Generation of Logic Formulas: `evalo` [**true** ; **false** ; **true**] `q` **true**



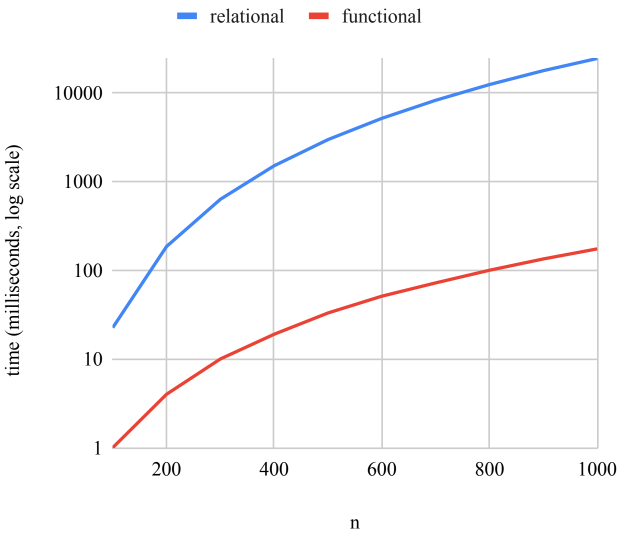
Multiplication: `mulo n 10 q`



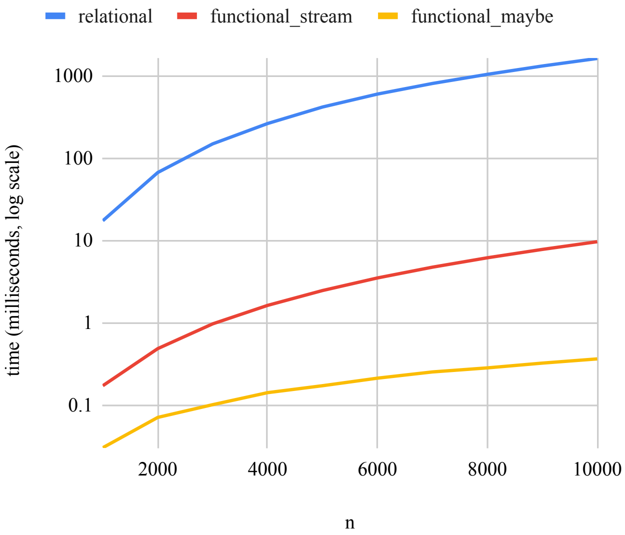
Division: `mulo (n/10) q n`



Multiplication Generation: `take n (mulo 10 q r)`



Need for Determinism Check: `mulo q 10 1000`



Need for Determinism Check

- Just replacing the monad Stream with the monad Maybe improves performance about 10 times for relations on natural numbers
  - The implementation stays the same!
- Pure (no monad) version is even faster
- Use determinism check to figure out when replacing Stream is feasible
- How to combine different monads naturally?

Need for Partial deduction

miniKanren can run a verifier backwards to get solver

run `q` (`eval`^{o} `q` **true**)

Augmenting functional conversion with partial deduction must be beneficial

Conclusion

Conclusion

- We presented a functional conversion scheme
- The conversion speeds up implementations considerably
- We implemented the conversion scheme in Haskell
- We found some way to order conjuncts

We are currently working on

- The integration with partial deduction
- The integration into the framework of using relational interpreters for solving