Semi-Automated Direction-Driven Functional Conversion

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Relational Programming

One relation to solve many problems

Nondeterminism

Completeness of search

Relational Conversion: Easy

Given a function

```
let rec add x y =
  match x with
  | 0 → y
  | S x₁ → S (add x₁ y)
```

generate miniKanren relation

```
let rec addᵒ x y z = conde [
  (x ≡ 0 ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁)) ]
```

Principal Directions of miniKanren Relations

Every argument of a relation can be either `in` or `out`

The 8 directions of the addition relation $\texttt{add}^o$ `x y z`:

Forward direction	$\texttt{add}^o$ `in in out`	addition
Backward direction	$\texttt{add}^o$ `out out in`	decomposition
Predicate	$\texttt{add}^o$ `in in in`	
Generator	$\texttt{add}^o$ `out out out`	
	$\texttt{add}^o$ `in out in`	subtraction
	$\texttt{add}^o$ `out in in`	subtraction
	$\texttt{add}^o$ `out in out`	
	$\texttt{add}^o$ `in out out`	

Each Direction is a Function

Each Direction is a Function (kind of)

Functions:

Forward direction	$\texttt{add}^o\texttt{in in out}$	addition
	$\texttt{add}^o\texttt{in out in}$	subtraction
	$\texttt{add}^o\texttt{out in in}$	subtraction
Predicate	$\texttt{add}^o\texttt{in in in}$	

Relations:

Backward direction	$\texttt{add}^o\texttt{out out in}$	decomposition
Generator	$\texttt{add}^o\texttt{out out out}$	
	$\texttt{add}^o\texttt{out in out}$	
	$\texttt{add}^o\texttt{in out out}$	

These relations are functions which return multiple answers (list monad)

miniKanren Comes with an Overhead

Unifications

Occurs-check

Scheduling complexity

Functional Conversion

Given a relation and a principal direction, construct a functional program that generates the same answers as miniKanren would

Preserve the completeness of the search

Both inputs and outputs are expected to be ground

Example: Addition in the Forward Direction

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁)) ]
```

```
addIIO :: Nat → Nat → Nat
addIIO x y =
  case x of
    O → y
    S x₁ → S (addIIO x₁ y)
```

Addition in the Backward Direction: Nondeterminism

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁)) ]
```

```
addOOI :: Nat → Stream (Nat, Nat)
addOOI z =
  return (O, z) `mplus`
  case z of
    O → Empty
    S z₁ → do
      (x₁, y) ← addOOI z₁
      return (S x₁, y)
```

Free Variables in Answers: Generators

```
let rec addᵒ x y z = conde [
  (x ≡ 0 ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧ z ≡ S z₁ ∧ addᵒ x₁ y z₁) ) ]
```

Free Variables in Answers: Generators

```
addIOO :: Nat → Stream (Nat, Nat)
addIOO x = case x of
  O → do
    z ← genNat
    return (z, z)
  S x1 → do
    (y, z1) ← addIOO x1
    return (y, S z1)

genNat :: Stream Nat
genNat = Mature O (S <$> genNat)
```

Predicates

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  (fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁)) ]
```

```
addIII :: Nat → Nat → Nat → Stream ()
addIII x y z =
  case x of
    O | y == z → return ()
      | otherwise → Empty
    S x₁ →
      case z of
        O → Empty
        S z₁ → addIII x₁ y z₁
```

Conversion Scheme

- Normalization
- Mode analysis
- Functional conversion

Normalization: Flat Term

Flat terms: a var or a constructor which takes *distinct* vars as arguments:

$$\mathcal{FT}_V = V \cup \{\mathcal{C}_i\left(x_1, \ldots, x_{k_i}\right) \mid x_j \in V, x_j - \mathit{distinct}\}$$

Examples:

$$\begin{aligned}
C\left(x_1, x_2\right) \equiv C\left(C\left(y_1, y_2\right), y_3\right) &\iff x_1 \equiv C\left(y_1, y_2\right) \wedge x_2 \equiv y_3 \\
C\left(C\left(x_1, x_2\right), x_3\right) \equiv C\left(C\left(y_1, y_2\right), y_3\right) &\iff x_1 \equiv y_1 \wedge x_2 \equiv y_2 \wedge x_3 \equiv y_3 \\
x \equiv C\left(y, y\right) &\iff x \equiv C\left(y_1, y_2\right) \wedge y_1 \equiv y_2
\end{aligned}$$

Normalization: Goal

$$
\begin{array}{llll}
\mathcal{K}_V^N & = & \bigvee(c_1,\ldots,c_n)\,,c_i \in \mathrm{Conj}_V & \text{normal form} \\
\mathrm{Conj}_V & = & \bigwedge(g_1,\ldots,g_n)\,,g_i \in \mathrm{Base}_V & \text{normal conjunction} \\
\mathrm{Base}_V & = & V \equiv \mathcal{FT}_V & \text{flat unification} \\
 & \mid & R_i\,(x_1,\ldots,x_{k_i})\,,x_j \in V, x_j - \mathit{distinct} & \text{flat call}
\end{array}
$$

Mode of a Variable

Mode of a variable: mapping between its instantiations

Ground term contains no fresh variables
Free variable: a fresh variable, no info about its instantiation

Once we know that a variable is *ground*, it stays *ground* in subsequent conjuncts

Mode *in*: *ground* $\rightarrow$ *ground*
Mode *out*: *free* $\rightarrow$ *ground*

Mercury uses more complicated modes

Modded Goal

Assign mode to every variable, make sure they are consistent

Modded Unification Types

$$
\begin{aligned}
\textit{assignment} &: x^{\text{out}} \equiv \mathcal{T}^{\text{in}} \text{ and } x^{\text{in}} \equiv y^{\text{out}} \\
\textit{guard} &: x^{\text{in}} \equiv \mathcal{T}^{\text{in}} \\
\textit{match} &: x^{\text{in}} \equiv \mathcal{T} \ (\mathcal{T} \text{ contains both } \textit{in} \text{ and } \textit{out} \text{ variables}) \\
\textit{generator} &: x^{\text{out}} \equiv \mathcal{T}
\end{aligned}
$$

Mode Inference: Initialization

- Input variables: *ground* $\to$ *ground*
- Output variables: *free* $\to$ *ground*
- Other variables: *free* $\to$?

$$
\begin{array}{l}
\textbf{let rec } \mathtt{add}^o\ \mathtt{x}^{g\to g}\ \mathtt{y}^{g\to g}\ \mathtt{z}^{f\to g} = \textbf{conde} \\
\quad (\mathtt{x}^{g\to g} \equiv \mathtt{0} \wedge \mathtt{y}^{g\to g} \equiv \mathtt{z}^{f\to g});\\
\quad (\mathtt{x}^{g\to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f\to ?} \wedge \\
\quad\ \ \mathtt{add}^o\ \mathtt{x}_1^{f\to ?}\ \mathtt{y}^{g\to g}\ \mathtt{z}_1^{f\to ?} \wedge \\
\quad\ \ \mathtt{z}^{f\to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f\to ?})
\end{array}
$$

Mode Inference: Disjunction

Run inference on each disjunct independently

$\texttt{x}^{g \to g} \;\equiv\; \texttt{0} \;\wedge\; \texttt{y}^{g \to g} \;\equiv\; \texttt{z}^{f \to g}$

$\texttt{x}^{g \to g} \;\equiv\; \texttt{S}\;\; \texttt{x}_1^{f \to ?} \;\;\wedge$ $\texttt{add}^o\;\; \texttt{x}_1^{f \to ?}\;\; \texttt{y}^{g \to g}\;\; \texttt{z}_1^{f \to ?} \;\;\wedge$ $\texttt{z}^{f \to g} \;\equiv\; \texttt{S}\;\; \texttt{z}_1^{f \to ?}$

Mode Inference: Unification

Propagate the groundness information according to the 4 types of modded unifications

$$\mathtt{x}^{g\to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f\to ?} \Rightarrow \mathtt{x}^{g\to g} \equiv \mathtt{S}\ \mathtt{x}_1^{f\to g}$$

$$\mathtt{z}^{f\to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f\to ?} \Rightarrow \mathtt{z}^{f\to g} \equiv \mathtt{S}\ \mathtt{z}_1^{f\to g}$$

Mode Inference: Conjunction

Pick a conjunct according to the priority, propagate groundness

1. Guard
2. Assignment
3. Match
4. Call with some ground arguments
5. Unification-generator
6. Call with all free arguments

Mode Inference: Conjunction

$$\begin{array}{l}
\texttt{add}^o\ \ \texttt{x}_1^{f\to ?}\ \ \texttt{y}^{g\to g}\ \ \texttt{z}_1^{f\to ?}\ \ \wedge \\
\texttt{x}^{g\to g} \equiv\ \texttt{S}\ \ \texttt{x}_1^{f\to ?}\ \ \wedge \\
\texttt{z}^{f\to g} \equiv\ \texttt{S}\ \ \texttt{z}_1^{f\to ?}
\end{array}$$

Mode Inference: Conjunction

$$\begin{array}{l} \texttt{add}^o\ \ \texttt{x}_1^{f\to ?}\ \ \texttt{y}^{g\to g}\ \ \texttt{z}_1^{f\to ?}\ \ \wedge \\ \texttt{x}^{g\to g}\ \equiv\ \texttt{S}\ \ \texttt{x}_1^{f\to ?}\ \ \wedge \\ \texttt{z}^{f\to g}\ \equiv\ \texttt{S}\ \ \texttt{z}_1^{f\to ?} \end{array}$$

$$\begin{array}{l} \texttt{x}^{g\to g}\ \equiv\ \texttt{S}\ \ \texttt{x}_1^{f\to g}\ \ \wedge \\ \texttt{add}^o\ \ \texttt{x}_1^{g\to g}\ \ \texttt{y}^{g\to g}\ \ \texttt{z}_1^{f\to ?}\ \ \wedge \\ \texttt{z}^{f\to g}\ \equiv\ \texttt{S}\ \ \texttt{z}_1^{f\to ?} \end{array}$$

Mode Inference: Conjunction

$$\begin{aligned}&\mathtt{add}^o\ \ \mathtt{x}_1^{f\to ?}\ \ \mathtt{y}^{g\to g}\ \ \mathtt{z}_1^{f\to ?}\ \ \wedge\\ &\mathtt{x}^{g\to g}\ \equiv\ \mathtt{S}\ \ \mathtt{x}_1^{f\to ?}\ \ \wedge\\ &\mathtt{z}^{f\to g}\ \equiv\ \mathtt{S}\ \ \mathtt{z}_1^{f\to ?}\end{aligned}$$

$$\begin{aligned}&\mathtt{x}^{g\to g}\ \equiv\ \mathtt{S}\ \ \mathtt{x}_1^{f\to g}\ \ \wedge\\ &\mathtt{add}^o\ \ \mathtt{x}_1^{g\to g}\ \ \mathtt{y}^{g\to g}\ \ \mathtt{z}_1^{f\to ?}\ \ \wedge\\ &\mathtt{z}^{f\to g}\ \equiv\ \mathtt{S}\ \ \mathtt{z}_1^{f\to ?}\end{aligned}$$

$$\begin{aligned}&\mathtt{x}^{g\to g}\ \equiv\ \mathtt{S}\ \ \mathtt{x}_1^{f\to g}\ \ \wedge\\ &\mathtt{add}^o\ \ \mathtt{x}_1^{f\to g}\ \ \mathtt{y}^{g\to g}\ \ \mathtt{z}_1^{f\to g}\ \ \wedge\\ &\mathtt{z}^{f\to g}\ \equiv\ \mathtt{S}\ \ \mathtt{z}_1^{g\to g}\end{aligned}$$

Order in Conjunctions

```
let rec multᵒ x y z = conde [
  ...
  (fresh (x₁ r₁)
    (x ≡ S x₁) ∧
    (addᵒ y r₁ z) ∧
    (multᵒ x₁ y r₁)
  )]
```

Order in Conjunctions: Slow Version

```
multIIO₁ :: Nat → Nat → Stream Nat
...
multIIO₁ (S x₁) y  = do
  (r₁, r) ← addIOO y
  multIII x₁ y r₁
  return r

multIII :: Nat → Nat → Nat → Stream ()
...
multIII (S x₁) y z = do
  z₁ ← multIIO₁ x₁ y
  addIII y z₁ z
multIII _ _ _ = Empty
```

Premature grounding of z_1 leads to *generate-and-test* behavior

Order in Conjunctions: Faster Version

```
multIIO :: Nat → Nat → Stream Nat
...
multIIO (S x₁) y = do
  r₁ ← multIIO x₁ y
  addIIO y r₁
```

Functional Conversion: Intermediate Language

$$
\begin{array}{rcll}
\mathcal{F}_V & = & \text{Return}\,[\mathcal{T}_V] & \text{return a tuple of terms} \\
 & | & \text{Match}_V\,(\mathcal{T}_V, \mathcal{F}_V) & \text{match a variable against a pattern} \\
 & | & \text{Bind}\,[([V]\,, \mathcal{F}_V)] & \text{monadic bind on streams} \\
 & | & \text{Sum}\,[\mathcal{F}_V] & \text{concatenation of streams} \\
 & | & \text{Guard}\,(V, V) & \text{equality check} \\
 & | & \text{Gen}_G & \text{generator} \\
 & | & R_i([V]\,, [G]) & \text{function call}
\end{array}
$$

Functional Conversion into Intermediate Language

- Disjunction $\rightarrow$ Sum $[\mathcal{F}_V]$
- Conjunction $\rightarrow$ Bind $[([V], \mathcal{F}_V)]$
- Relation call $\rightarrow R_i([V], [G])$
- Unification $\rightarrow$
  - Return $[\mathcal{T}_V]$
  - Match$_V$ $(\mathcal{T}_V, \mathcal{F}_V)$
  - Guard (V, V)
  - Gen$_G$

Functional Conversion: Generators

- In the untyped miniKanren it is only possible to generate *all terms*
- Instead pass generators to functions as additional arguments
  - It is up to the user what generator to pass

```
addIOO :: Nat → Stream Nat → Stream (Nat, Nat)
addIOO x gen_z = case x of
  O → do
    z ← gen_z
    return (z, z)
  S x₁ → do
    (y, z₁) ← addIOO x₁ gen_z
    return (y, S z₁)
```

Functional Conversion: Generators

- We pass a generator for every variable in *rhs* of a unification-generator
- Generators used in calls should be passed to the parent function
- In a typed version, it should be possible to automatically derive generators based on the type

```
multOIO :: Nat → Stream Nat → Stream Nat
multOIO y gen_add_z =
  return (0, 0) `mplus`
  do
    (z1 , z) ← addIOO y gen_add_z
    x ← muloOII y z1
    return (S x, z)
```

Functional Conversion into Haskell

- TemplateHaskell to generate code
- Stream monad
- do-notation

Functional Conversion into OCaml

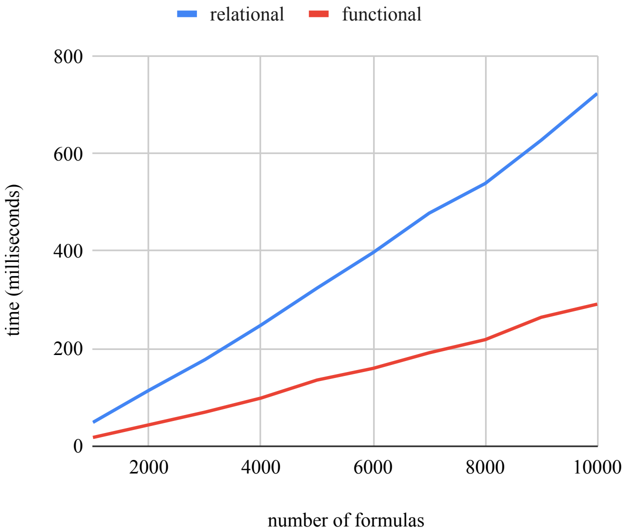
- Hand-crafted (not so) pretty-printer
- Stream monad
- let*
- Taking extra care to employ laziness

Evaluation

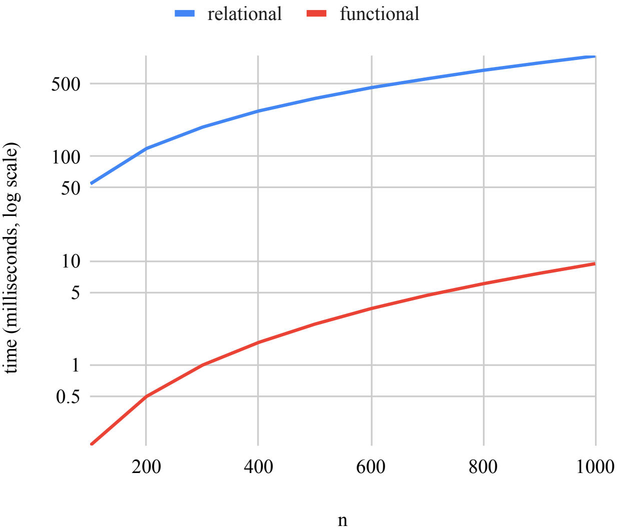
We converted relational interpreters and measured execution time

- Logic formulas generation
  - Inverse computation of an evaluator of logic formulas
  - Generating formulas which evaluate to **true**
- Multiplication relation
  - Forward direction: multiplication
  - Backward direction: division
  - Generation

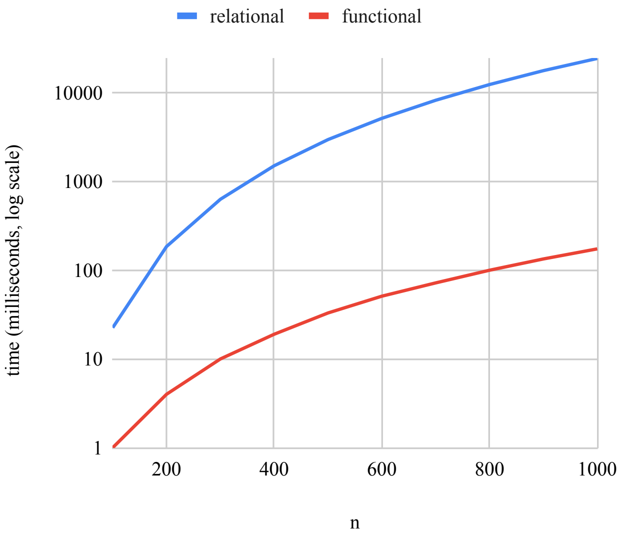
Generation of Logic Formulas: `evalo` [**true** ; **false** ; **true**] `q` **true**



Multiplication: `mulo n 10 q`



Division: `mulo (n/10) q n`



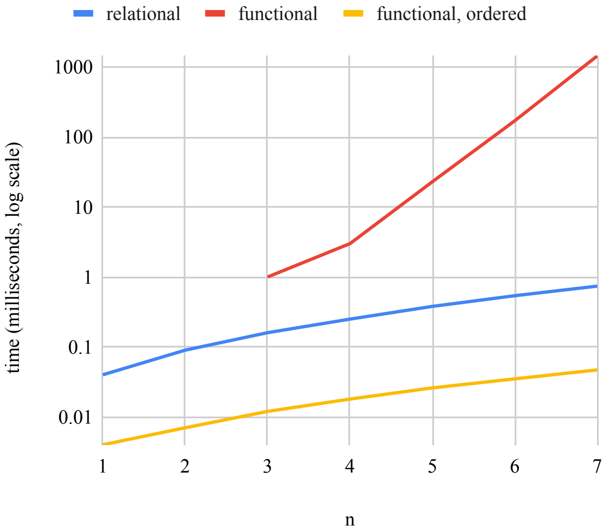
Maybe for Semi-Determinism

```
muloOII :: Nat → Nat → Stream Nat
muloOII x1 x2 = zero `mplus` positive
  where
    zero = do
      guard (x2 == O)
      return O
    positive = do
      x4 ← addoIOI x1 x2
      S <$> muloOII x1 x4
```

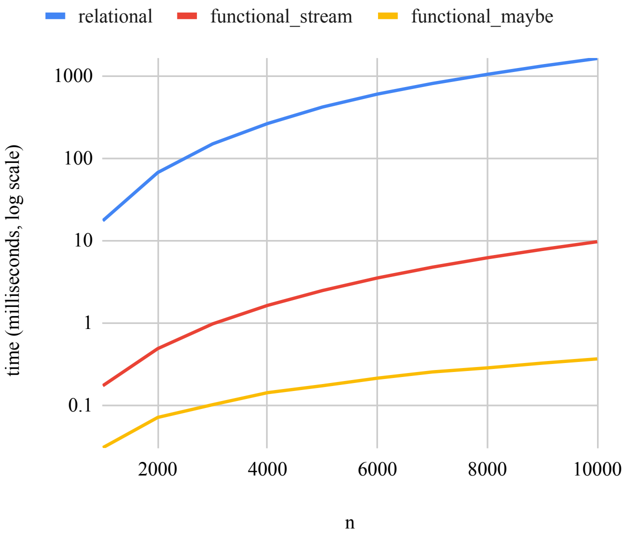
Maybe for Semi-Determinism

```
muloOII :: Nat → Nat → Maybe Nat
muloOII x1 x2 = zero `mplus` positive
  where
    zero = do
      guard (x2 == 0)
      return 0
    positive = do
      x4 ← addoIOI x1 x2
      S <$> muloOII x1 x4
```

Multiplication Generation: `take n (mulo 10 q r)`



Need for Determinism Check: `mulo q 10 1000`



Need for Determinism Check

- Just replacing the monad Stream with the monad Maybe improves performance about 10 times for relations on natural numbers
  - The implementation stays the same!
- Pure (no monad) version is even faster
- Use determinism check to figure out when replacing Stream is feasible
- How to combine different monads naturally?

Need for Partial Deduction

miniKanren can run a verifier backwards to get solver

$$\textbf{run}\ \ \mathtt{q}\ \ (\mathtt{eval}^{o}\ \mathtt{q}\ \textbf{true})$$

Augmenting functional conversion with partial deduction must be beneficial

Conclusion

Conclusion

- We presented a functional conversion scheme
- The conversion speeds up implementations considerably
- We implemented the conversion scheme in Haskell
- We found some way to order conjuncts

We are currently working on

- The integration with partial deduction
- The integration into the framework of using relational interpreters for solving