A Case Study in Functional Conversion and Mode Inference in miniKanren

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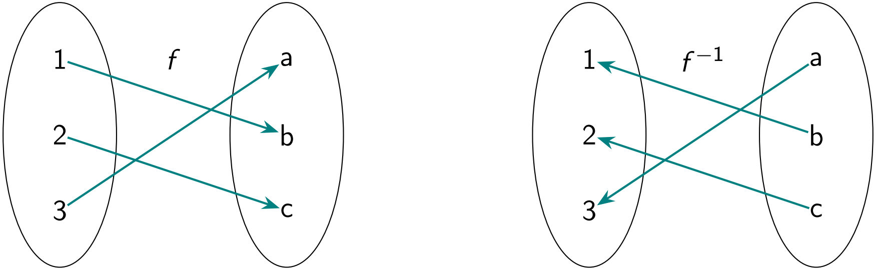
PEPM @ POPL 2024

January 16, 2024

Program Inversion



We can view programs as functions. Functions can be inversed. Some inversions solve more complicated tasks than the original programs.

`program evaluation`$^{-1} \approx$ `program syntesis`

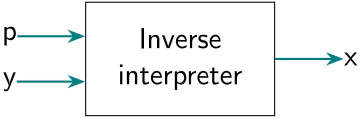
`type checking`$^{-1} \approx$ `type inference`

Inverse Interpreter

$$[\![p]\!](x) = y$$

$$[\![p^{-1}]\!](y) = x$$

$$[\![invInt]\!](p, y) = x$$



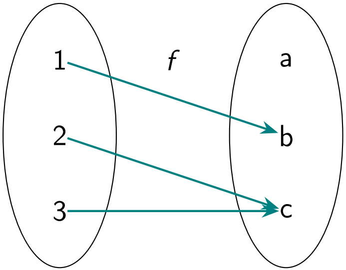
Verifiers for Search

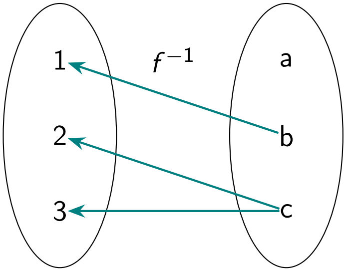
$$\texttt{verify} :: \texttt{a} \rightarrow \texttt{Bool}$$
$$\texttt{verify x} \equiv \texttt{True}, \text{if x -- solution}$$

$$\texttt{verify}^{-1} :: \texttt{Bool} \rightarrow [\texttt{a}]$$
$$\texttt{findSolution} = \texttt{verify}^{-1}\ \texttt{True}$$

Relational Programming for Inversion

Nondeterministic Inversion





Program inversion

10

$\Omega(n!)$

Program inversion

Many complicated programgs are inverse of simple ones

Type inference or habitation is inverse of type checking

Program inversion: Given a program f, produce inverse porgram f^{-1}

Given $\text{typecheck}(\text{program}, \text{types}) = \mathit{true}$, produce $\text{typecheck}^{-1}(\text{program}, \mathit{true}) = \text{types}$.

Relational inversion

A program is a relation between its inputs and outputs

miniKanren can evaluate relations in both directions

Write a simple verifier, convert to miniKanren, get a solver

Problem: miniKanren may be slow. So convert it back!

Relational Programming

Subset of logic programming, focus on pure relations

Extra-logical features (cuts, side-effects, search order manipulation) discouraged

Interleaving search guarantees that all answers are found

miniKanren

miniKanren is a simple relational language designed to be implemented as shallow embedding.

relation

$$
\begin{array}{l}
\textsf{let}\ \ \textsf{rec}\ \ \mathtt{add}^o\ \mathtt{x}\ \mathtt{y}\ \mathtt{z}\ =\ \textsf{conde}\ \ [\\
\quad (\mathtt{x}\ \equiv \mathtt{0}\ \ \wedge\ \ \mathtt{y}\ \equiv \mathtt{z});\\
\quad (\ \textsf{fresh}\ \ (\mathtt{x}_1\ \mathtt{z}_1)\\
\qquad (\mathtt{x}\ \equiv \mathtt{S}\ \ \mathtt{x}_1\ \wedge\\
\qquad\ \ \mathtt{add}^o\ \mathtt{x}_1\ \mathtt{y}\ \mathtt{z}_1\ \wedge\\
\qquad\ \ \mathtt{z}\ \equiv \mathtt{S}\ \ \mathtt{z}_1)\ )\ \ ]
\end{array}
$$

miniKanren is a simple relational language designed to be implemented as shallow embedding.

relation

disjunction

term unification

$$
\begin{array}{l}
\textsf{let}\ \ \textsf{rec}\ \ \mathtt{add}^o\ \mathtt{x}\ \mathtt{y}\ \mathtt{z}\ =\ \textsf{conde}\ \ [\\
\quad (\mathtt{x}\ \equiv \mathtt{O}\ \ \wedge\ \ \mathtt{y}\ \equiv \mathtt{z});\\
\quad (\ \textsf{fresh}\ \ (\mathtt{x}_1\ \mathtt{z}_1)\\
\qquad (\mathtt{x}\ \equiv \mathtt{S}\ \ \mathtt{x}_1\ \wedge\\
\qquad\ \mathtt{add}^o\ \mathtt{x}_1\ \mathtt{y}\ \mathtt{z}_1\ \wedge\\
\qquad\ \mathtt{z}\ \equiv \mathtt{S}\ \ \mathtt{z}_1)\ )\ \ ]
\end{array}
$$

conjunction

variable introduction

relation call

Stream

Stream is a list-like structure, representing nondetermistic computation with interleaving search

$$[1, 2, 3] >>= f = f(1) \langle|\rangle f(2) \langle|\rangle f(3)$$

$$[1, 2, 3] \langle|\rangle [a, b, c] = [1, a, 2, b, 3, c]$$

miniKanren implementation - Stream of substitutions
Conversion to functional language - *Stream* of values

Example: Addition in the Forward Direction

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  ( fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁) ) ]
```

$$\text{add}^\circ\ 1\ 2\ z = \{z \mapsto 3\}$$

```
addIIO :: Nat → Nat → Nat
addIIO x y =
  case x of
    O → y
    S x₁ → S (addIIO x₁ y)
```

$$\text{addIIO}\ 1\ 2 = 3$$

Functional Conversion

Given a relation and a principal direction, construct a functional program that generates the same answers as miniKanren would

Preserve the completeness of the search

Both inputs and outputs are expected to be ground

Speed improvement: up to 3 orders of magnitude on benchmark of mulltiplication

Addition in the Backward Direction: Nondeterminism

```
let rec addᵒ x y z = conde [
  (x ≡ 0 ∧ y ≡ z);
  ( fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁) ) ]
```

$$\langle \text{fresh } x, y \text{ in add}^{\circ}\ x\ y\ 2 \rangle = [\ \{x \to 0, y \to 2\}, \{x \to 1, y \to 1\}, \{x \to 2, y \to 0\}\ ]$$

```
addOOI :: Nat → Stream (Nat, Nat)
addOOI z =
  return (0, z) <|>
  case z of
    0 → Empty
    S z₁ → do
      (x₁, y) ← addOOI z₁
      return (S x₁, y)
```

$$\text{addOOI } 2 = [(0,2), (1,1), (2,0)]$$

Free Variables in Answers: Generators

$$
\begin{array}{l}
\textbf{let rec } add^o\ x\ y\ z = \textbf{conde } [\\
\quad (x \equiv O \wedge y \equiv z); \\
\quad (\textbf{fresh } (x_1\ z_1) \\
\qquad (x \equiv S\ x_1 \wedge \\
\qquad\ add^o\ x_1\ y\ z_1 \wedge \\
\qquad\ z \equiv S\ z_1))\ ]
\end{array}
$$

$$\langle \text{fresh } y, z \text{ in add}^\circ\ 1\ y\ z \rangle = [\{z \rightarrow S\ y\}]$$

$$\text{genNat} = [0, 1, 2, 3, \ldots]$$

$$\text{addIOO } 1 = [(0, 1), (1, 2), (2, 3), \ldots]$$

```
addIOO :: Nat → Stream (Nat, Nat)
addIOO x =
  case x of
    O → do
      z ← genNat
      return (z, z)
    S x₁ → do
      (y, z₁) ← addIOO x₁
      return (y, S z₁)

genNat :: Stream Nat
genNat =
    (return O) <|> (S <$> genNat)
```

Conversion Scheme

1. Normalization
2. Mode analysis
3. Functional conversion

Normalization: Flat Term

Eliminate nested constructors and repeated variables

$$\mathcal{FT} = V \cup \{\mathcal{C}\ x_0 \ldots x_k \mid x_j \in V, x_j - distinct\}$$

$$\begin{aligned} C\left(x, y\right) \equiv C\left(C\left(v, u\right), w\right) \quad &\Longleftrightarrow \quad x \equiv C\left(v, u\right) \wedge y \equiv w \\ add^{\circ}(x, x, z) \quad &\Longleftrightarrow \quad add^{\circ}\left(x, y, z\right) \wedge x \equiv y \end{aligned}$$

Normalization: Goal

Eliminate disjunctions within conjunctions

$\mathcal{K}^N$	$::=$	$c_1 \vee \ldots \vee c_n$	$c_i \in \text{Conj}$	normal form
Conj	$::=$	$g_1 \wedge \ldots \wedge g_n$	$g_i \in \text{Base}$	normal conjunction
Base	$::=$	$V \equiv \mathcal{FT}$		flat unification
	$\mid$	$R\, x_1 \ldots x_k$	$x_j \in V, x_j - \mathit{distinct}$	flat call

Mode of a Variable

Instantiation describes whether at a given point a variable has a known value:

Ground term	no fresh variables	`Cons O (Cons (S O) Nil)`
Free variable	a fresh variable	_.0

Once we know that a variable is ground, it stays ground in later conjuncts

Mode is a transition between instantiations, associated with each use of a variable

Mode `I`: `ground` $\to$ `ground`
Mode `O`: `free` $\to$ `ground`
Taken together, modes represent data flow.

Mercury uses more complicated modes

Modded Unification Types

$$
\begin{array}{rl}
\text{assignment} & x^{\mathtt{O}} \equiv \mathcal{T}^{\mathtt{I}} \\
\text{assignment} & x^{\mathtt{I}} \equiv y^{\mathtt{O}} \\
\text{guard} & x^{\mathtt{I}} \equiv \mathcal{T}^{\mathtt{I}} \\
\text{match} & x^{\mathtt{I}} \equiv \mathcal{T} \\
\text{generator} & x^{\mathtt{O}} \equiv \mathcal{T}
\end{array}
$$

$\mathcal{T}$ contains at least one f variable

Order in Conjunctions

```
let rec multᵒ x y z = conde [
  ( fresh (x₁ r₁)
    (x ≡ S x₁) ∧
    (addᵒ y r₁ z) ∧
    (multᵒ x₁ y r₁));
  ...]
```

```
multIIO :: Nat → Nat → Stream Nat
multIIO (S x₁) y = do
  r₁ ← multIIO x₁ y
  addIIO y r₁
...
```

10x faster

$O(xy)$

```
multIIO₁ :: Nat → Nat → Stream Nat
multIIO₁ (S x₁) y    = do
  (r₁, r) ← addIOO y
  multIII x₁ y r₁
  return r
...
multIII :: Nat → Nat → Nat → Stream ()
multIII (S x₁) y z = do
  z₁ ← multIIO₁ x₁ y
  addIII y z₁ z
multIII _ _ _ = Empty
...
```

$\Omega(x!)$

Order in Conjunctions: Slow Version

```
let rec multᵒ x y z = conde [
  ( fresh (x₁ r₁)
    (x ≡ S x₁) ∧
    (addᵒ y r₁ z) ∧
    (multᵒ x₁ y r₁));
  ...]
```

```
multIIO₁ :: Nat → Nat → Stream Nat
multIIO₁ (S x₁) y    = do
  (r₁, r) ← addIOO y
  multIII x₁ y r₁
  return r
 ...
multIII :: Nat → Nat → Nat → Stream ()
multIII (S x₁) y z = do
  z₁ ← multIIO₁ x₁ y
  addIII y z₁ z
multIII _ _ _ = Empty
 ...
```

Premature grounding of z_1 leads to the generate-and-test behavior
$\Omega(x!)$ complexity.

Order in Conjunctions: Faster Version

```
let rec multᵒ x y z = conde [
  ( fresh (x₁ r₁)
    (x ≡ S x₁) ∧
    (addᵒ y r₁ z) ∧
    (multᵒ x₁ y r₁));
  ...]
```

```
multIIO :: Nat → Nat → Stream Nat
multIIO (S x₁) y = do
  r₁ ← multIIO x₁ y
  addIIO y r₁
 ...
```

$O(xy)$ complexity, 10x faster than relational version

Mode Inference: Conjunction

Priority:

1. Guard
2. Assignment
3. Match
4. Recursion, same direction
5. Call, some args ground
6. Unification-generator
7. Call, all args free

$\mathcal{F}_V$	$=$	$\mathcal{F}_V <\mid> \cdots <\mid> \mathcal{F}_V$	interleaving
	$\mid$	$(\overline{V} \leftarrow \mathcal{F}_V)^*$	monadic bind on streams
	$\mid$	$\text{return}\, \mathcal{T}_V^*$	return a tuple of terms
	$\mid$	$V == \mathcal{T}_V$	equality check
	$\mid$	$\mathit{case}\ V\ \mathit{of}\ \overline{\mathcal{T}_V \to \mathcal{F}_V}$	match a variable against a pattern
	$\mid$	$R_i\ \overline{V}\ \overline{Gen_G}$	function call
	$\mid$	Gen_G	generator

Functional Conversion into Intermediate Language

$$
\begin{array}{rcl}
\text{Disjunction} & \rightarrow & <\,|\,> \mathcal{F}_V^* \\[2ex]
\text{Conjunction} & \rightarrow & \text{Bind}\,(V^*, \mathcal{F}_V)^* \\[2ex]
\text{Relation call} & \rightarrow & R_i(V^*, G^*) \\[2ex]
\text{Unification} & \rightarrow & \text{return}\,\mathcal{T}_V^* \\
& | & \text{Match}_V\,(\mathcal{T}_V, \mathcal{F}_V) \\
& | & \text{Guard}\,(V, \mathcal{T}_V) \\
& | & \text{Gen}_G
\end{array}
$$

Functional Conversion: Generators

In the untyped miniKanren it is only possible to generate all terms

Instead pass generators to functions as additional arguments
It is up to the user what generator to pass

```
addIOO :: Nat → Stream Nat → Stream (Nat, Nat)
addIOO x gen_z =
  case x of
    O → do
      z ← gen_z
      return (z, z)
    S x1 → do
      (y, z1) ← addIOO x1 gen_z
      return (y, S z1)
```

Functional Conversion: Generators

We pass a generator for every variable in rhs of a unification-generator

Generators used in calls should be passed to the parent function

In a typed version, it should be possible to automatically derive generators from types

```
multOIO :: Nat → Stream Nat → Stream Nat
multOIO y gen_add_z =
  return (O, O) <|>
  do
    (z1, z) ← addIOO y gen_add_z
    x ← muloOII y z1
    return (S x, z)
```

Functional Conversion into the Target Languages

Haskell	OCaml
TemplateHaskell to generate code	Hand-crafted (not so) pretty-printer
Stream monad	Stream monad
do-notation	let*
	Taking extra care to ensure laziness

Relational Sort

```
let rec sortᵒ x y = conde [
    (x ≡ [] ∧ y ≡ []);
    ( fresh (s xs xs₁)
      y ≡ s :: xs₁ ∧
       smallest ᵒ x s xs ∧
       sort ᵒ xs xs₁)]
```

- ✓ sorting
- ⏲ permutations

Only good for sorting:

`run q (sortᵒ xs q)`

```
let rec sortᵒ x y = conde [
    (x ≡ [] ∧ y ≡ []);
    ( fresh (s xs xs₁)
      y ≡ s :: xs₁ ∧
       sort ᵒ xs xs₁ ∧
       smallest ᵒ x s xs )]
```

- ⏲ sorting
- ✓ permutations

Only good permutation generation:

`run q (sortᵒ q xs)`

Relational Sort: Sorting

	Relation		Function
	sorto smallesto	smallesto sorto	
[3;2;1;0]	0.077s	0.004s	0.000s
[4;3;2;1;0]	timeout	0.005s	0.000s
[31;...;0]	timeout	1.058s	0.006s
[262;...;0]	timeout	timeout	1.045s

Relational Sort: Generating Permutations

	Relation		Function
	`smallesto` `sorto`	`sorto` smallesto	
`[0;1;2]`	0.013s	0.004s	0.004s
`[0;1;2;3]`	timeout	0.005s	0.005s
`[0;...;6]`	timeout	0.999s	0.021s
`[0;...;8]`	timeout	timeout	1.543s

Conclusion

Conclusion

- We presented a functional conversion scheme
- The conversion speeds up implementations considerably
- We implemented the conversion scheme in Haskell

We are currently working on

- Determinism check
- Integration with partial deduction
- Integration into the framework of using relational interpreters for solving

Maybe for Semi-Determinism

```
muloOII :: Nat → Nat → Stream Nat
muloOII x1 x2 =
    zero <|> positive
  where
    zero = do
      guard (x2 == 0)
      return 0
    positive = do
      x4 ← addoIOI x1 x2
      S <$> muloOII x1 x4
```

Maybe for Semi-Determinism

```
muloOII :: Nat → Nat → Maybe Nat
```
~~muloOII :: Nat → Nat → Stream Nat~~
```
muloOII x1 x2 =
    zero <|> positive
  where
    zero = do
      guard (x2 == 0)
      return 0
    positive = do
      x4 ← addoIOI x1 x2
      S <$> muloOII x1 x4
```

Need for Determinism Check

Simply replacing the type of monad from `Stream` to `Maybe` improves performance 10 times for relations on natural numbers

Pure (no monad) version is even faster

Use determinism check to figure out when replacing `Stream` is feasible

Need for Partial Deduction

Running a relational interpreter backwards fixes some arguments

```
run q (evalᵒ q true)
```

Augmenting functional conversion with partial deduction must be beneficial

Functional Conversion: Example

```
let rec addᵒ x y z = conde [
  (x ≡ O ∧ y ≡ z);
  ( fresh (x₁ z₁)
    (x ≡ S x₁ ∧
     addᵒ x₁ y z₁ ∧
     z ≡ S z₁) ) ]
```

```
data Term = O | S Term
addoIIO :: Term → Term → Stream Term
addoIIO x y = msum [
    do {
        guard (x == O);
        z ← return y;
        return z
    },
    do {
        S x₁ ← return x;
        z₁ ← addoIIO x₁ y;
        z ← return (S z₁);
        return z
    }
]
```