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In this paper we present a scheme of translation of MINIKANREN programs into a host functional programming language as a sequence of examples. Examples start from simplest translations and evolve to introduce different features of MINIKANREN which influence translation. Currently translation is not automated: everything is done manually. We believe the translation may be semi-automated: some decisions will still be left up to a programmer. Although this project is at the early state, evaluation demonstrates its usefulness by significantly speeding up such programs as computing a topological sorting of a graph and generating logic formulas which evaluate to the given value.

## 2 Examples of Translation

A relation  $\text{add}^0 x y z$  succeeds whenever  $z$  is a sum of  $x$  and  $y$ . An implementation of this relation which uses Peano numbers is shown in Listing 1.

One can run a relation in some *direction* by passing it *input* arguments. For example, executing  $\text{add}^0 (S\ 0)\ 0\ z$  finds the sum of the first two arguments and maps  $z$  to the sum  $S\ 0$ . We can also provide only the last argument:  $\text{add}^0 x y (S\ 0)$ . This computes all pairs of Peano numbers  $(x, y)$  which sum up to the given value  $z = S\ 0$  which are  $(0, S\ 0)$  and  $(S\ 0, 0)$ . Moreover, we can pass as input arguments not only *ground terms* but terms which contain fresh variables, such as  $\text{add}^0 x (S\ y)\ z$ . Executing this relation finds all triples  $(x, y, z)$  such that  $x + (y + 1) = z$ . Running in some directions can fail. For example  $\text{add}^0 (S\ x)\ y\ 0$  may never succeed, since  $(1 + x) + y$  can never be equal to **zero**.

There exists a multitude of different directions for each relation. In this paper we only consider directions in which input arguments are ground, i.e. do not contain any fresh variables, we will call them *principal directions*. We denote a principal direction by the name of a relation followed by specification of its arguments: in place of each argument we write either **in** when the argument is input or out if it is output. There are 8 principal directions for  $\text{add}^0 x y z$ :

- three directions with one input:  $\text{add}^0$  **in** out out,  $\text{add}^0$  out **in** out, and  $\text{add}^0$  out out **in**;
- three directions with two inputs:  $\text{add}^0$  **in in** out,  $\text{add}^0$  **in** out **in**,  $\text{add}^0$  out **in in**;
- one direction with no input arguments:  $\text{add}^0$  out out out;
- one direction when all arguments are input:  $\text{add}^0$  **in in in**;

When all arguments of a relation are input arguments, it works as a predicate, while passing no arguments corresponds to the generation of all valid values for all arguments of a relation.

In the rest of this section we describe a translation scheme which allows to implement principal directions of a relation as functions. Each direction we consider illustrates some aspect of the translation. Functional implementations of the

```
addXY :: Nat → Nat → Nat
addXY x y =
  case x of
    0 → y
    S x' → S (addXY x' y)
```

Listing 2. Function for addo **in in** out direction

```
addXY :: Nat → Nat → Stream Nat
addXY x y =
  case x of
    0 → return y
    S x' → S <$> addXY x' y
```

Listing 3. Using streams in a function for addo **in in** out direction

principal directions of the  $\text{add}^0 x y z$  relation which does not make into this section, may be found in Appendix.

### 2.1 Basic Translation

Consider  $\text{add}^0$  **in in** out. This direction can be expressed as a function presented in Listing 2. The relation  $\text{add}^0 x y z$  has two branches in a **conde**: one unifies  $x$  with **zero** and the other — with  $S\ x'$ . Since we know that  $x$  is always ground in this direction, we can replace unifications with a pattern-matching.

When  $x$  unifies with **zero**, the rest of the **conde** branch is the unification  $y \equiv z$ . This unification means that the output value of the direction is equal to  $y$ . Thus we can just return  $y$  as the result when  $x$  is pattern-matched with **zero**.

Now consider the **conde** branch in which  $x$  unifies with  $S\ x'$  where  $x'$  is a fresh variable. The variable  $x$  in this direction is always ground, thus  $x'$  is also ground after unification. This means, that the recursive call  $\text{add}^0 x' y z'$  is done in the direction  $\text{add}^0$  **in in** out and can be translated into a recursive call to the function  $\text{addXY}$ . This recursive call computes the value of  $z'$ , making it ground. The only thing that is left is to apply the constructor  $S$  to the result of the recursive call, since  $z \equiv S\ z'$ .

### 2.2 Nondeterministic Directions

Running a relation in a given direction may succeed with one or more possible answers or it may fail, i.e. it may run nondeterministically. It is natural to implement nondeterminism by using Streams which are at the core of MINIKANREN. Any deterministic directions can be trivially transformed to using streams as shown in Listing 3. One example in which there are multiple answers is  $\text{add}^0$  out out **in**. This direction corresponds to finding all pairs of numbers which sum up to the given  $z$  and can be implemented as shown in Listing 4.

In this case, the input variable  $z$  does not discriminate two branches of **conde**. Although the second branch of **conde**

```

221 addZ :: Nat → Stream (Nat, Nat)
222 addZ z =
223   return (0, z) `mplus`
224   case z of
225     0 → Empty
226     S z' → do
227       (x', y) ← addZ z'
228       return (S x', y)

```

**Listing 4.** Function for addo out out in direction

```

232 addX :: Nat → Stream (Nat, Nat)
233 addX x =
234   case x of
235     0 → do
236       z ← genNat
237       return (z, z)
238     S x' → do
239       (y, z') ← addX x'
240       return (y, S z')

```

```

242 genNat :: Stream Nat
243 genNat = Mature 0 (S <$> genNat)

```

**Listing 5.** Function for addo in out out direction

unifies  $z$  with a term  $S\ z'$ , the first branch unifies  $z$  with a free variable  $y$ . In this case we need to consider the two branches independently and then combine the results into a new stream.

The first **conde** branch produces a single answer in which  $x$  is **zero**, and  $y$  is equal to  $z$ . This single result is then wrapped into a singleton stream.

The second **conde** branch succeeds only if  $z$  is a successor of another value, thus when  $z$  is a **zero** we should fail. We express this by pattern-matching on  $z$  and returning an Empty stream when  $z$  is **zero**. Otherwise  $z$  unifies with  $S\ z'$ , which means that  $z'$  is ground, and the recursive call to the relation is done in the direction  $\text{add}^o \text{ out out in}$ . This recursive call returns a stream of pairs  $(x', y)$ , and by applying the constructor  $S$  to  $x'$  we get the value of  $x$ .

The two translated **conde** branches are then combined by using ``mplus``: the same combinator which is used in MINIKANREN for disjunctions. We use `do`-notation when translating the second branch of **conde** which is just a syntactic sugar for the monadic bind operation `>>=`. Binds implement conjunctions in MINIKANREN and it is no surprise they fit well into the functional implementation.

### 2.3 Free Variables in Answers

In some directions, there are infinitely many answers, such as in  $\text{add}^o \text{ in out out}$ . When only the second argument is

```

276 addXYZ :: Nat → Nat → Nat → Stream ()
277 addXYZ x y z =
278   case x of
279     0 | y == z → return ()
280     | otherwise → Empty
281   S x' →
282     case z of
283       0 → Empty
284       S z' → addXYZ x' y z'

```

**Listing 6.** Function for addo in in in direction

known, the answer is all pairs of numbers  $(y, z)$  which satisfy  $x + y = z$ . In MINIKANREN, this is expressed with help of free variables. Say  $x$  is  $S\ 0$ , then the stream of answers is represented as  $(\_ .0, S\ \_ .0)$ . This means that whatever the value of  $y$  is,  $z$  is just its successor. In our paper we only consider scenarios when the answers are ground, so we expect the stream of answers to be  $(0, S\ 0), (S\ 0, S\ (S\ 0)), \dots$ . To do it, we need to systematically generate a stream of ground values for  $y$  and  $z$ . Currently, we leave the generation up to the user, but generators may be automatically created from their types.

Listing 5 shows the functional implementation of the direction  $\text{add}^o \text{ in out out}$ . This direction is very similar to the  $\text{add}^o \text{ in in out}$ : we can pattern match on  $x$ , call the same function recursively in the second **conde** branch and construct the resulting value for  $z$  by applying the constructor  $S$ . But in the case when  $x$  is **zero**, the only thing we know about the values of  $y$  and  $z$  is that they are equal. In this case we can generate a stream of all Peano numbers for  $z$  (or  $y$ ) and use them in the returned result.

The generation of all numbers is done as shown in Listing 5, function `genNat`. The only thing one should be careful about, is to ensure lazy generation of the values, especially in case of an eager host language, such as OCAML.

### 2.4 Predicates

When all arguments of a relation are input, the direction serves as a predicate. Consider  $\text{add}^o \text{ in in in}$  and its functional implementation in Listing 6. In this case there is no actual answers we should return: the only thing that matters is whether the computation succeeded or failed. Failure is expressed with an empty stream and success — as a singleton stream with a unit value.

All arguments of the relation in this direction are ground. This means, that all unification can be replaced with either pattern-matching or simple equality check. When translating the first **conde** branch we pattern match on  $x$ , and then check if  $y$  and  $z$  are equal. The second **conde** branch introduces another pattern matching, this time on  $z$ , which ensures that  $z$  is not **zero**.

```

331 let rec multo x y z = conde [
332   (x ≡ 0 ∧ z ≡ 0);
333   (y ≡ 0 ∧ z ≡ 0);
334   (x ≡ S 0 ∧ z ≡ y);
335   (y ≡ S 0 ∧ z ≡ x);
336   (fresh (x' r')
337     (x ≡ S x') ∧
338     (add y r' z) ∧
339     (mult x' y r')
340   ) ]

```

Listing 7. Multiplication relation

```

344 multXY' :: Nat → Nat → Stream Nat
345 multXY' 0 y = return 0
346 multXY' x 0 = return 0
347 multXY' (S 0) y = return y
348 multXY' x (S 0) = return x
349 multXY' (S x') y = do
350   (r', r) ← addX y
351   multXYZ x' y r'
352   return r
353
354 multXYZ :: Nat → Nat → Nat → Stream ()
355 multXYZ 0 y 0 = return ()
356 multXYZ x 0 0 = return ()
357 multXYZ (S 0) y z | y == z = return ()
358 multXYZ x (S 0) z | x == z = return ()
359 multXYZ (S x') y z = do
360   z' ← multXY' x' y
361   addXYZ y z' z
362   multXYZ _ _ _ = Empty

```

Listing 8. Inefficient implementation of mult<sup>o</sup> in in out direction

```

367 multXY :: Nat → Nat → Stream Nat
368 multXY 0 y = return 0
369 multXY x 0 = return 0
370 multXY (S 0) y = return y
371 multXY x (S 0) = return x
372 multXY (S x') y = do
373   r' ← multXY x' y
374   addXY y r'

```

Listing 9. Efficient implementation of mult<sup>o</sup> in in out direction

## 2.5 Order within Conjunctions

Up until now we only seen examples with only one recursive call which is done to the same relation. Many programs in MINIKANREN use several relations in the same bodies, see for example Listing 7. The relation  $\text{mult}^o x y z$  relates

variables such that  $x * y = z$ . The base cases in this relation are when  $x$  or  $y$  are **zero** and  $S 0$ . When  $x$  unifies with a successor of another value, then we can use equalities  $(x' + 1) * y = x' * y + y$ . This is done by adding  $y$  to the intermediate result of multiplying  $x'$  by  $y$ .

When translating it into a function for the given direction, we need to make sure to call functional counterparts of  $\text{add}^o$  and  $\text{mult}^o$  in the right order which depends on the direction. Consider the direction  $\text{mult}^o$  in in out. The translation of base cases is done with the same principals as the previous examples. The last **conde** branch contains two call to two different relations:  $\text{add}^o$  and  $\text{mult}^o$ . Variables  $x'$  and  $y$  in this direction are ground, which impose possible directions on the relation calls. There are two ways we can do these calls.

One of them is to first call  $\text{add}^o$  in the direction  $\text{add}^o$  in out out since  $y$  is ground, while  $r$  and  $r'$  are to be computed. After this, all arguments in the call to  $\text{mult}^o$  are known, and it can be used as a predicate  $\text{mult}^o$  in in in. Finally, we return  $r$  if the predicate succeeds: see Listing 8. Unfortunately, this order proves to be too slow: it takes about half of a second to multiply 4 by 4, and more than 300 seconds to multiply 5 by 5. This can be explained by the fact that  $\text{add}^o$  in out out generates an infinite streams of answers, only one which succeeds in multiplication, but considering them all even to find the first (and only) answer to  $\text{multXY}'$  takes too much time.

Better and more efficient implementation of  $\text{mult}^o$  in in out is shown in Listing 9. Here, we first execute the recursive call of the direction  $\text{mult}^o$  in in out, and then use  $\text{add}^o$  in in out to compute the final result. None of these relations produce an infinite stream, and the function runs in a fraction of a second. You may note also that in this case there is no need to generate any additional functions for directions which are different from the one being translated.

In general, it is not clear how to choose the best order in which to translate calls within a conjunction. One heuristic is to favor calls which do not produce infinite streams, namely do not use generators for free variables.

## 3 Evaluation

To evaluate our proposed translation scheme, we manually rewritten several problems in different directions and compared their execution times with their relational counterparts. Here we showcase two relational programs and their translations.

### 3.1 Topologic sort

This program topologically sorts a directed graph. A graph is represented as a list of edges, where each edge is a pair of vertices. First vertex in a pair is the beginning of the edge, and the second vertex is the end of the edge. A vertex is a distinct Peano number in the range  $[0.. n-1]$  where  $n$  is



```

441 topsort graph numbering =
442   let n = S (numberOfNodes graph) in
443   go graph numbering n
444   where
445     go graph numbering n =
446       case graph of
447         [] → True
448         (b, e) : graph' →
449           let nb = lookup numbering b in
450           let ne = lookup numbering e in
451           less nb ne &&
452           less ne n &&
453           topsort graph' numbering

```

**Listing 10.** Functional interpreter for topologic sort of a graph

the number of edges. The vertices are sorted as a result of executing the program. The sort is represented as a list of length  $n$  in which the order of vertex  $i$  is the  $i$ -th element of the list. We call this list *numbering*. For example, numbering  $[2, 1, 0]$  means that the zeroth variable is the second, the first variable is the first, and the last variable is the zeroth in the ordering.

The relational program is generated from a functional interpreter [cite stuff](#). The functional interpreter takes a graph and a numbering and checks if the variables are indeed topologically sorted as shown in Listing 10. To do it, it checks all edges of the graph in order, finds the numbers which correspond to the vertices in the numbering, and ensures that the beginning comes before the end of the edge, and that the edge is not greater than the number of vertices in graph.

This simple predicate along with the other functions it uses is translated into a relational program shown in Listing 11. The relation is then specialized so that it searches for a correct topologic sort by fixing its last argument to **true**. The result of specialization is in Listing 12. Specialization removes any **conde** branches which are failing, i.e. unify the result  $r$  with **false**.

The specialized version is manually translated in a direction `topsorto in out`. This creates a function which constructs a numbering which topologically sorts vertices in a given graph. Most of the translation follows the principles outlined in [red section](#), but there are several notable details about this translation.

First of all, we translated all Peano numbers into `Ints` and all `MINIKANREN` boolean values into `Bools`. This can be done because of the groundness of variables in this direction. [Write something a little more convincing.](#)

Second of all, the relational interpreter contains two consecutive calls to `lookupo` relation, both of which has the same numbering passed to them. When translating them,

the first call is done in the `lookupo out in` out direction, since only the value of its second argument  $b$  is known to be ground. Calling this direction computes the numbering which is a list with only its  $b$ -th element fixed —  $nb$ . We generate values of  $nb$  with a generator, since  $nb$  is a free variable. The same goes for all other elements of the numbering. We restrict the amount of the generating lists by capping their length with `maxLength` and capping maximum value of an element with `maxInt`, both of which correspond to the number of vertices in the input graph.

Having now numbering ground, the second call to `lookupo` relation is done in the direction `lookupo in in out`. The second direction is much simpler as it does not involve generation of any new values for free variables. Translations of the both directions are in Listing 14.

Calls to `lesso x y r` relations are both done in direction `lesso in in out`, and their outputs must be **true**. To express this check we use `guard` which fails computation (i.e. returns an Empty stream) if its argument is false.

### 3.2 Logic Formulas Generation

In this example we translate an evaluator of logic formulas in a direction which generates formulas which evaluate to a given result. Logic formulas are values of type `Term` presented in Listing 15. A formula is either a boolean literal, a variable indexed by an integer number, a negation of another formula, a conjunction or disjunction of two formulas.

The relational interpreter is shown in Listing 16. The relation `evalo fm st r` computes the value  $r$  of a formula  $fm$  with a given variable mapping  $st$ . The boolean value  $v$  of a variable `Var i` is the  $i$ -th element of  $st$  which can be retrieved by means of the relation `elemo i st v`. The relation `evalo` uses relations `addo`, `oro`, and `noto` for boolean operations.

Translation of `evalo` relation in the direction `evalo out out in` is presented in Listing 17. As in the previous example, here relation `evalo` is called twice when formula is either a conjunction or a disjunction. The direction of the second call is different from the direction of the first call, as first call generates possible variable mappings. The implementation of the direction `evalo out in in` is shown in Listing 18. The implementations of the directions `addo in in out`, `oro in in out`, `noto in in out`, and `elemo in in out` are in Listing ??

### 3.3 Execution Time Comparison

In order to assess the usefulness of the proposed transformation scheme we compared execution times of `MINIKANREN` relations `topsorto` and `evalo` with their functional translations. All functional translations are done by hand, having a specific direction in mind. All implementations are written in OCAML language and can be found in [this repository](#).

For the evaluator of logic formulas, we run both implementations to search for 10000 formulas which evaluate to **True**. The functional implementation restricts the length of

---

```

551 let topsorto graph numbering r =
552   let rec topsorto graph numbering n r = conde [
553     (graph ≡ [] ∧ r ≡ true);
554     (fresh (b e graph')
555       (graph ≡ (b, e) : graph' ∧
556         (fresh (q47 nb ne)
557           (lookupo numbering b nb ∧
558             lookupo numbering e ne ∧
559             lesso nb ne q47 ∧
560             conde [
561               (q47 ≡ !! false ∧ r ≡ false);
562               (fresh (q43)
563                 (q47 ≡ true ∧
564                   lesso ne n q43 ∧
565                   conde [
566                     (q43 ≡ false ∧ r ≡ false);
567                     (q43 ≡ true ∧ topsorto graph' numbering n r)])))])))] in
568   (fresh (n n') (n' ≡ s n ∧ numberOfNodeso graph n ∧ topsorto graph numbering n' r))

```

---

Listing 11. Relational interpreter for topologic sort of a graph

---

```

572 let topsortoTrue graph numbering =
573   let rec topsorto graph numbering n = conde [
574     (graph ≡ []);
575     (fresh (b e graph')
576       (graph ≡ (b, e) : graph' ∧
577         (fresh (q47 q43 nb ne)
578           (lookupo numbering b nb ∧
579             lookupo numbering e ne ∧
580             lesso nb ne q47 ∧
581             q47 ≡ true ∧
582             lesso ne n q43 ∧
583             q43 ≡ true ∧
584             topsorto graph' numbering n)))))] in
585   (fresh (n n') (n' ≡ s n ∧ numberOfNodeso graph n ∧ topsortoTrue graph numbering n'))

```

---

Listing 12. Specialized relational interpreter for topologic sort of a graph

the variable mapping list, thus we also restricted the size of it in its relational counterpart. We averaged the execution time over 10 runs. The result are presented in table 1. “OCanren” contains execution time of relational implementation, and “Function” column contains execution time of the functional implementation. In our experiments, functional implementation outperforms the relational interpretation by 1.3-2.5 times.

We run topsort<sup>o</sup> on directed graphs with exactly one edge between each pair of edges. For example, graph with 4 vertices has the following edges: [(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)], which we sort lexicographically. We generated graphs for a given number of vertices and then executed both relational and functional implementations of topsort<sup>o</sup>. The correct numbering in this condition should map each vertex into

itself. We also run the same functions on the same graph, but with its list of edges reversed, i.e. [(2, 3), (1, 3), (1, 2), (0, 3), (1, 2), (0, 2), (0, 1)], where n is the number of vertices in the graph.

Execution times averaged over 10 runs are presented in table 2. Columns “Functional” and “Functional (r)” contain execution times of functional implementations when run on a graph and reversed graph correspondingly. Columns “OCanren” and “OCanren (r)” contain execution times of functional implementations when run on a graph and reversed graph correspondingly. Relational implementation took more than 300 seconds for a sorted graph with 7 vertices, thus we only consider graphs with up to 6 vertices. On all graphs, functional implementation much less time that the MINIKANREN program. Topologically sorting a reversed

---

```

661 topsortGraph :: Graph → Stream [Nat]
662 topsortGraph graph = do
663   n ← numberOfNodesG graph
664   go graph (n + 1) n (n + 1)
665   where
666     go graph n maxInt maxListLength =
667       case graph of
668         [] → return []
669         ((b, e) : graph') → do
670           (nb, numbering) ← lookupKey b maxInt maxListLength
671           ne ← lookupXsKey numbering e
672           q47 ← lessXY nb ne
673           guard q47
674           q43 ← lessXY ne n
675           guard q43
676           topsortGraphNumbering graph' numbering n

```

---

**Listing 13.** Functional implementation for a topsorttoTrue in out direction

---

```

680 lookupKey :: Int → Int → Int → Stream (Int, [Int])
681 lookupKey key maxInt maxListLength =
682   case key of
683     0 → fromList [(x, x:xs) | xs ← genList (genInt maxInt) (maxListLength - 1),
684                      x ← genInt maxInt]
685     _ | key > 0 → do
686       (value, tl) ← lookupKey (key - 1) maxInt (maxListLength - 1)
687       fromList [(value, y : tl) | y ← genInt maxInt]
688     _ → Empty
689 lookupXsKey :: [Int] → Int → Stream Int
690 lookupXsKey xs key =
691   case xs of
692     [] → Empty
693     (h : tl) → case key of
694       0 → return h
695       S key' → lookupXsKey tl key'

```

---

**Listing 14.** Functional implementations for a lookupo out in out and lookupo in in out directions

---

```

699 data Term = Lit Bool
700           | Var Int
701           | Neg Term
702           | Conj Term Term
703           | Disj Term Term

```

---

**Listing 15.** Term data type

graph takes much less time. This is caused by earlier rejection of candidate solutions, since vertex numbers are higher in the beginning of the list.

As a result of our evaluation, we can conclude that the translation of MINIKANREN program with a given direction into a function speeds up execution a lot and thus it is reasonable to continue working in this direction.

Topological Sort

## 4 Related Work

- Relational interpreters for context ([1])
- Mercury for mode analysis
- Curry as translation to Haskell

```

evalo st fm u =
  fresh (x y v w z) (conde [
    (fm ≡ Conj x y ∧ ando v w u ∧ evalo st x v ∧ evalo st y w);
    (fm ≡ Disj x y ∧ oro v w u ∧ evalo st x v ∧ evalo st y w);
    (fm ≡ Neg x ∧ noto v u ∧ evalo st x v);
    (fm ≡ Var z ∧ elemo z st u);
    (fm ≡ Lit u)])

ando x y b = conde [
  (x ≡ True ∧ y ≡ True ∧ b ≡ True);
  (x ≡ False ∧ y ≡ True ∧ b ≡ False);
  (x ≡ True ∧ y ≡ False ∧ b ≡ False);
  (x ≡ False ∧ y ≡ False ∧ b ≡ False)]

oro x y b = conde [
  (x ≡ True ∧ y ≡ True ∧ b ≡ True);
  (x ≡ False ∧ y ≡ True ∧ b ≡ True);
  (x ≡ True ∧ y ≡ False ∧ b ≡ True);
  (x ≡ False ∧ y ≡ False ∧ b ≡ False)]

noto x b = [
  (x ≡ True ∧ b ≡ False);
  (x ≡ False ∧ b ≡ True)]

elemo i st v =
  fresh (h t i') conde [
    (i ≡ 0 ∧ st ≡ (v : t));
    (i ≡ S i' ∧ st ≡ (h : t) ∧ elemo i' t v)]

```

Listing 16. Relational evaluator of logic formulas

**Table 1.** Execution times of the OCanren and functional implementations of eval<sup>o</sup>, search for 10000 formulas which evaluate to True

Var. mapping length	Function (sec.)	OCanren (sec.)
0	0.283	0.998
1	0.306	0.668
2	0.227	0.543
3	0.224	0.500
4	0.206	0.482
5	0.211	0.482
6	0.254	0.483
7	0.370	0.491
8	0.357	0.492
9	0.377	0.491

## 5 Future Work

- Research if there exists a way to automatically deduce better order of calls within a conjunction.
- Formalize translation scheme, prove its correctness
- Implement automatic translation

- Integrate the translator into a relational interpreters framework

## 6 Conclusion

### Acknowledgments

Here is where acknowledgments come

### A Principal Directions of the Addition Relation



---

```

881 evalR :: Bool → Int → Stream (Term, [Bool])
882 evalR result maxLength =
883   lit result `mplus`
884   var result `mplus`
885   neg result `mplus`
886   disj result `mplus`
887   conj result
888 where
889   conj result = do
890     (v, w) ← andR result
891     (y, st) ← evalR w maxLength
892     x ← evalStR st v
893     return (Conj x y, st)
894   disj result = do
895     (v, w) ← orR result
896     (y, st) ← evalR w maxLength
897     x ← evalStR st v
898     return (Disj x y, st)
899   neg result = do
900     v ← notR result
901     (x, st) ← evalR v maxLength
902     return (Neg x, st)
903   var result = do
904     (z, st) ← elemR result maxLength
905     return (Var z, st)
906   lit result =
907     if result
908     then return (Lit True, [])
909     else return (Lit False, [])

```

---

**Listing 17.** Functional implementation of the direction evalo out out in

**Table 2.** Execution times of the OCanren and functional implementations of topsorto

Number of vertices	Function (sec.)	OCanren (sec.)	Function (r) (sec.)	OCanren (r) (sec.)
3	0.000	0.001	0.000	0.001
4	0.000	0.015	0.000	0.012
5	0.001	0.346	0.000	0.107
6	0.021	14.309	0.003	0.764

---

```

991 evalStR :: [Bool] → Bool → Stream Term
992 evalStR st result =
993     lit st result `mplus`
994     var st result `mplus`
995     neg st result `mplus`
996     disj st result `mplus`
997     conj st result
998 where
999     conj st result = do
1000         (v, w) ← andR result
1001         y ← evalStR st w
1002         x ← evalStR st v
1003         return (Conj x y)
1004     disj st result = do
1005         (v, w) ← orR result
1006         y ← evalStR st w
1007         x ← evalStR st v
1008         return (Disj x y)
1009     neg st result = do
1010         v ← notR result
1011         x ← evalStR st v
1012         return (Neg x)
1013     var st result = do
1014         z ← elemStR st result
1015         return (Var z)
1016     lit st result =
1017         if result
1018         then return (Lit True)
1019         else return (Lit False)

```

---

**Listing 18.** Functional implementation of the direction evalo out in in

---

```

1101 andR :: Bool → Stream (Bool, Bool)
1102 andR result =
1103     if result
1104     then
1105         return (True, True)
1106     else
1107         return (True, False) `mplus`
1108         return (False, True) `mplus`
1109         return (False, False)
1110
1111 orR :: Bool → Stream (Bool, Bool)
1112 orR result =
1113     if result
1114     then
1115         return (True, False) `mplus`
1116         return (True, True) `mplus`
1117         return (False, True)
1118     else
1119         return (False, False)
1120
1121 notR :: Bool → Stream Bool
1122 notR result =
1123     if result
1124     then return False
1125     else return True
1126
1127 elemR :: Bool → Int → Stream (Int, [Bool])
1128 elemR _ maxLength | maxLength <= 0 = Empty
1129 elemR result maxLength =
1130     zero result `mplus` succ result
1131     where
1132         zero result =
1133             fromList [ (0, result : tl) | tl ← genList genBool (maxLength - 1) ]
1134         succ result = do
1135             (n', t) ← elemR result (maxLength - 1)
1136             fromList [(n' + 1, h : t) | h ← genBool ]

```

---

**Listing 19.** Functions used in logic formulas generation

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