



# A Case Study in Functional Conversion and Mode Inference in miniKanren

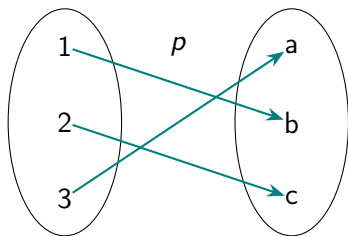
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JetBrains Research

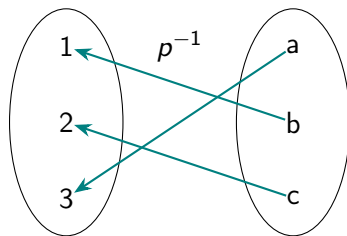
PEPM @ POPL 2024

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# Program Inversion



$$\llbracket p \rrbracket(x) = y$$



$$\llbracket p^{-1} \rrbracket(y) = x$$

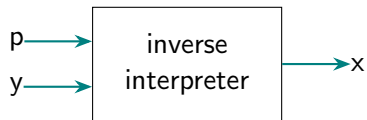
program synthesis:    program evaluation<sup>-1</sup>  
type inference:        type checking<sup>-1</sup>

# Inverse Interpreter

$$\llbracket p \rrbracket(x) = y$$

$$\llbracket p^{-1} \rrbracket(y) = x$$

$$\llbracket invInt \rrbracket(p, y) = x$$



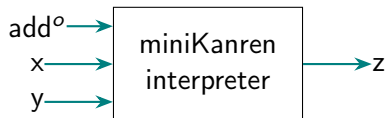
# miniKanren as an Inverse Interpreter

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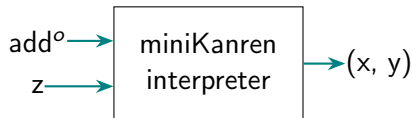
```
let rec addo x y z =  
  (x ≡ 0 ∧ y ≡ z) ∨  
  (fresh (x1 z1)  
   (x ≡ S x1 ∧  
    addo x1 y z1 ∧  
    z ≡ S z1))
```

---

`run`  $z$  ( $\text{add}^o$  0 1  $z$ ) =  $\{z \mapsto 1\}$



`run`  $x, y$  ( $\text{add}^o$   $x$   $y$  1) =  
 $[\{x \mapsto 0, y \mapsto 1\}, \{x \mapsto 1, y \mapsto 0\}]$



# Relational Interpreters for Search

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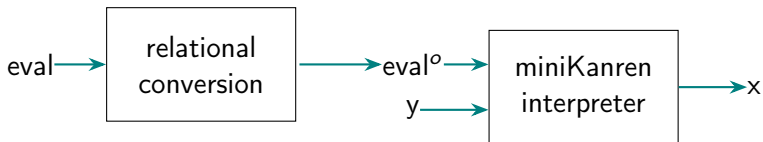
```
eval (Conj x y) =  
  eval x && eval y  
...
```

---

---

```
evalo fm u = fresh (x y v w)  
  (fm ≡ Conj x y ∧  
   ando v w u ∧  
   evalo x v ∧  
   evalo y w);  
...
```

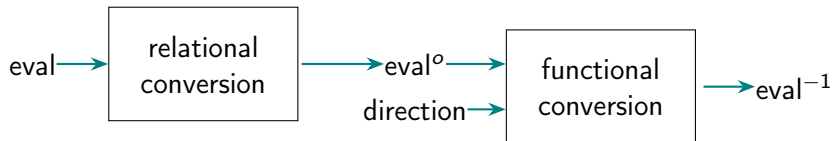
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# Relational Interpreters for Search: the Issue

It is slow


# Functional Conversion to the Rescue



- Generate the same answers as `MINIKANREN` would
- Inputs: ground
- Outputs: ground
- Hopefully faster

## MINIKANREN Syntax

relation



```
let rec addo x y z =  
  (x ≡ 0 ∧ y ≡ z) ∨  
  ( fresh (x1 z1)  
    (x ≡ S x1 ∧  
      addo x1 y z1 ∧  
      z ≡ S z1) )
```



## MINIKANREN Syntax

relation

```
let rec addo x y z =  
  (x ≡ 0 ∧ y ≡ z) ∨  
  ( fresh (x1 z1)  
    (x ≡ S x1 ∧  
      addo x1 y z1 ∧  
      z ≡ S z1) )
```

relation call

# MINIKANREN Syntax

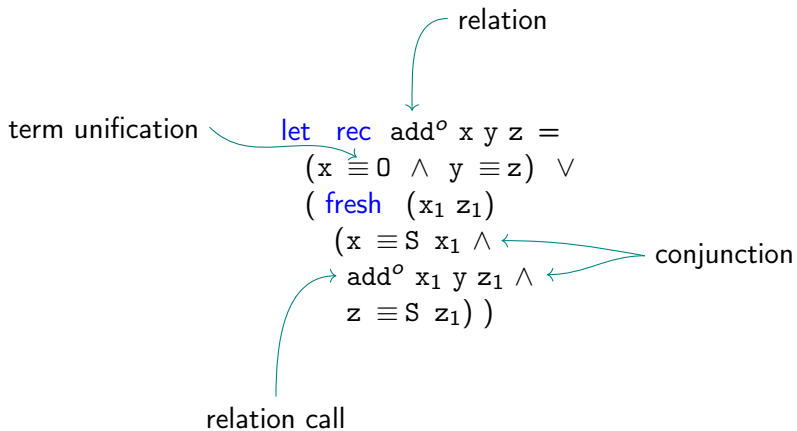
term unification

relation

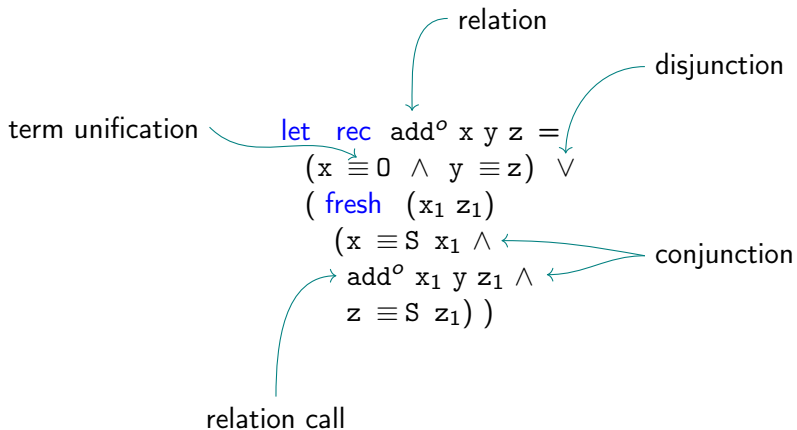
let rec  $\text{add}^o$  x y z =  
 (x  $\equiv$  0  $\wedge$  y  $\equiv$  z)  $\vee$   
 ( fresh (x<sub>1</sub> z<sub>1</sub>)  
 (x  $\equiv$  S x<sub>1</sub>  $\wedge$   
 add<sup>o</sup> x<sub>1</sub> y z<sub>1</sub>  $\wedge$   
 z  $\equiv$  S z<sub>1</sub>) )

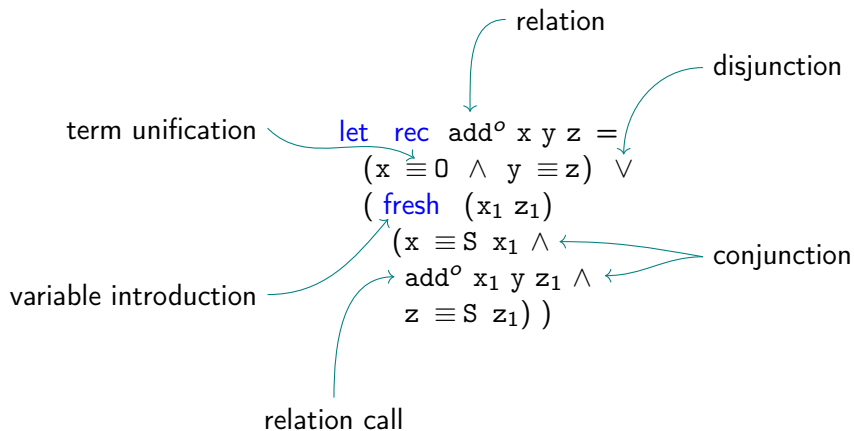
relation call

# MINIKANREN Syntax



# MINIKANREN Syntax





## Example: Addition in the Forward Direction

---

```
let rec addo x y z =  
  (x ≡ 0 ∧ y ≡ z) ∨  
  (fresh (x1 z1)  
   (x ≡ S x1 ∧  
    addo x1 y z1 ∧  
    z ≡ S z1))
```

---

$\text{add}^o\ 1\ 2\ z = \{z \mapsto 3\}$

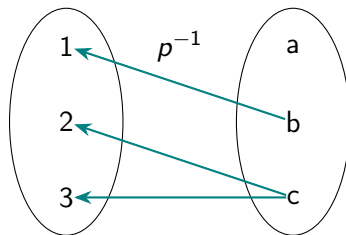
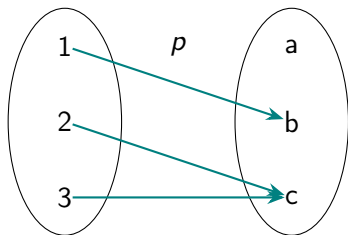
---

```
addII0 :: Nat → Nat → Nat  
addII0 x y =  
  case x of  
    0 → y  
    S x1 → S (addII0 x1 y)
```

---

$\text{addII0}\ 1\ 2 = 3$

# Nondeterministic Inversion



$$\text{add}^\circ \times y \ 1 = [\{x \mapsto 0, y \mapsto 1\}, \{x \mapsto 1, y \mapsto 0\}]$$

- Represents nondeterminism
- List-like
- Interleaving search

$$[1, 2, 3] >> f = f(1) <|> f(2) <|> f(3)$$

$$[1, 2, 3] <|> [a, b, c] = [1, a, 2, b, 3, c]$$

- MINIKANREN: Stream of substitutions
- Functional conversion: Stream of values



## Addition in the Backward Direction: Nondeterminism

---

```
let rec addo x y z =  
  (x ≡ 0 ∧ y ≡ z) ∨  
  (fresh (x1 z1)  
   (x ≡ S x1 ∧  
    addo x1 y z1 ∧  
    z ≡ S z1))
```

---

---

```
add00I :: Nat → Stream (Nat, Nat)  
add00I z =  
  return (0, z) <|>  
  case z of  
    0 → Empty  
    S z1 → do  
      (x1, y) ← add00I z1  
      return (S x1, y)
```

---

$\text{add}^o\ x\ y\ 1 = [\{x \mapsto 0, y \mapsto 1\}, \{x \mapsto 1, y \mapsto 0\}]$

$\text{add00I}\ 1 = [(0,1), (1,0)]$

## Free Variables in Answers: Generators

---

```
let rec addo x y z =  
  (x ≡ 0 ∧ y ≡ z) ∨  
  (fresh (x1 z1)  
   (x ≡ S x1 ∧  
    addo x1 y z1 ∧  
    z ≡ S z1 ) )
```

---

$\text{add}^o 1 y z = \{z \mapsto S y\}$

$\text{genNat} = [0, 1, 2, \dots]$

$\text{addI00 } 1 = [(0,1), (1,2), (2,3), \dots]$

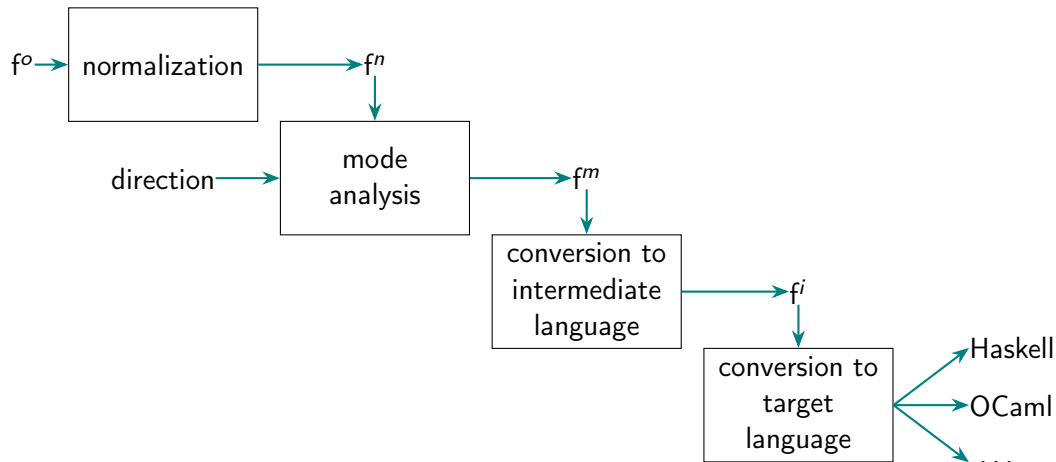
---

```
addI00 :: Nat → Stream (Nat, Nat)  
addI00 x =  
  case x of  
    0 → do  
      z ← genNat  
      return (z, z)  
    S x1 → do  
      (y, z1) ← addI00 x1  
      return (y, S z1)
```

```
genNat :: Stream Nat  
genNat =  
  (return 0) <|> (S <$> genNat)
```

---

# Conversion Scheme



# Normalization: Flat Term

Eliminate nested constructors and repeated variables

$$\mathcal{FT} = V \cup \{C\ x_0 \dots x_k \mid x_j \in V, x_j - \textit{distinct}\}$$

$$\begin{aligned} C(x, y) \equiv C(C(v, u), w) &\iff x \equiv C(v, u) \wedge y \equiv w \\ add^\circ(x, x, z) &\iff add^\circ(x, y, z) \wedge x \equiv y \end{aligned}$$

# Normalization: Goal

Eliminate disjunctions within conjunctions

$\mathcal{K}^N$	$::=$	$c_1 \vee \dots \vee c_n$	$c_i \in \text{Conj}$	normal form
Conj	$::=$	$g_1 \wedge \dots \wedge g_n$	$g_i \in \text{Base}$	normal conjunction
Base	$::=$	$V \equiv \mathcal{FT}$		flat unification
	$ $	$R\ x_1 \dots x_k$	$x_j \in V, x_j - \textit{distinct}$	flat call

Ground term     $s \ (s \ 0)$

Free variable     $x$

Once a variable is ground, it stays ground

Mode : Inst  $\mapsto$  Inst

Mode I:    ground  $\rightarrow$  ground

Mode 0:    free  $\rightarrow$  ground

# Modded Unification Types

assignment	$x^0 \equiv \mathcal{T}^I$
guard	$x^I \equiv \mathcal{T}^I$
match	$x^I \equiv \mathcal{T}$
generator	$x^0 \equiv \mathcal{T}$

# Order in Conjunctions

---

```
let rec multo x y z = conde [  
  (fresh (x1 r1)  
   (x ≡ S x1) ∧  
   (addo y r1 z) ∧  
   (multo x1 y r1));  
  ...]
```

---

---

```
multIIIO :: Nat → Nat → Stream Nat  
multIIIO (S x1) y = do  
  r1 ← multIIIO x1 y  
  addIIIO y r1  
  ...
```

---

$O(xy)$       10x faster

---

```
multIIIO1 :: Nat → Nat → Stream Nat  
multIIIO1 (S x1) y = do  
  (r1, r) ← addIOO y  
  multIII x1 y r1  
  return r  
  ...  
multIII :: Nat → Nat → Nat → Stream ()  
multIII (S x1) y z = do  
  z1 ← multIIIO1 x1 y  
  addIII y z1 z  
multIII _ _ _ = Empty  
  ...
```

---

generate-and-test

$\Omega(x!)$



Priority:

- ① Guard
- ② Assignment
- ③ Match
- ④ Recursion, same direction
- ⑤ Call, some args ground
- ⑥ Unification-generator
- ⑦ Call, all args free

## Ordering Heuristic: Example

---

```
let rec multo x y z = conde [  
  ( fresh (x1 r1)  
    (x ≡ S x1) ∧  
    (addo y r1 z) ∧  
    (multo x1 y r1));  
  ...]  
...
```

---

---

```
multII0 :: Nat → Nat → Stream Nat  
multII0 (S x1) y = do  
  r1 ← multII0 x1 y  
  addII0 y r1  
...  
...
```

---

# Functional Conversion: Intermediate Language

$\mathcal{F}$	$=$	$\mathcal{F} <   > \dots <   > \mathcal{F}$	interleaving
		$(\overline{V} \leftarrow \mathcal{F})^*$	monadic bind on streams
		return $\mathcal{T}^*$	return a tuple of terms
		$V == \mathcal{T}$	equality check
		case $V$ of $\mathcal{T} \rightarrow \mathcal{F}$	match a variable against a pattern
		$R \ \overline{V} \ \overline{Gen_G}$	function call
		$Gen_G$	generator

# Functional Conversion into Intermediate Language

Disjunction  $\rightarrow \mathcal{F} < | > \dots < | > \mathcal{F}$

Conjunction  $\rightarrow (\overline{V} \leftarrow \mathcal{F})^*$

Relation call  $\rightarrow R \ \overline{V} \ \overline{Gen_G}$

Unification  $\rightarrow$  return  $\mathcal{T}^*$   
|  $case \ V \ of \ \mathcal{T} \rightarrow \mathcal{F}$   
|  $V \equiv \mathcal{T}$   
|  $Gen_G$

# Functional Conversion: Generators

---

```
addI00 :: Nat → Stream Nat → Stream (Nat, Nat)
addI00 x geny =
  case x of
    0 → do
      y ← geny
      return (y, y)
    S x1 → do
      (y, z1) ← addI00 x1 geny
      return (y, S z1)
```

---

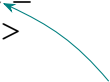
$$add^{\circ} 1 y z = [\{z \rightarrow S y\}]$$

# Functional Conversion: Generators

---

```
multOIO :: Nat → Stream Nat → Stream Nat
multOIO y gen_add_y =
  return (0, 0) <|>
  do
    (z1, z) ← addIOO y gen_add_y
    x ← muloOII y z1
    return (S x, z)
```

---



# Relational Sort

---

```
let rec sorto x y =  
  (x ≡ [] ∧ y ≡ []) ∨  
  (fresh (s xs xs1)  
   y ≡ s :: xs1 ∧  
   smallesto x s xs ∧  
   sorto xs xs1)
```

---

- ✓ sorting
- ⌚ permutations

---

```
let rec sorto x y =  
  (x ≡ [] ∧ y ≡ []) ∨  
  (fresh (s xs xs1)  
   y ≡ s :: xs1 ∧  
   sorto xs xs1 ∧  
   smallesto x s xs)
```

---

- ⌚ sorting
- ✓ permutations

# Relational Sort: Sorting

	Relation		Function
	sorto smallesto	smallesto sorto	
[3;2;1;0]	0.077s	0.004s	0.000s
[4;3;2;1;0]	⌚timeout	0.005s	0.000s
[31;...;0]	⌚timeout	1.058s	0.006s
[262;...;0]	⌚timeout	⌚timeout	1.045s



# Relational Sort: Generating Permutations

	Relation		Function
	smallesto sorto	sorto smallesto	
[0;1;2]	0.013s	0.004s	0.004s
[0;1;2;3]	⌚timeout	0.005s	0.005s
[0;...;6]	⌚timeout	0.999s	0.021s
[0;...;8]	⌚timeout	⌚timeout	1.543s

# Possible speedups

- Determinism
- Partial deduction

## Maybe for Semi-Determinism

```
mul00II :: Nat → Nat → Stream Nat
mul00II x1 x2 =
  zero <|> positive
  where
    zero = do
      guard (x2 == 0)
      return 0
    positive = do
      x4 ← addoIOI x1 x2
      S <$> mul00II x1 x4
```

## Maybe for Semi-Determinism

```
mul00II :: Nat → Nat → Maybe Nat
mul00II :: Nat → Nat → Stream Nat
mul00II x1 x2 =
  zero <|> positive
  where
    zero = do
      guard (x2 == 0)
      return 0
    positive = do
      x4 ← addoIOI x1 x2
      S <$> mul00II x1 x4
```



# Need for Partial Deduction

```
run q (evalo q true)
```

```
run q (eval_trueo q)
```

## Conclusion

- We presented a functional conversion scheme
- The conversion speeds up implementations considerably
- We implemented the conversion scheme in Haskell

We are currently working on

- Determinism check
- Integration with partial deduction
- Integration into the framework of using relational interpreters for solving

## Functional Conversion: Example

---

```
let rec addo x y z =  
  (x ≡ 0 ∧ y ≡ z) ∨  
  (fresh (x1 z1)  
   (x ≡ S x1 ∧  
    addo x1 y z1 ∧  
    z ≡ S z1))
```

---

---

```
data Term = 0 | S Term  
addoII0 :: Term → Term → Stream Term  
addoII0 x y = msum [  
  do {  
    guard (x == 0);  
    z ← return y;  
    return z  
  },  
  do {  
    S x1 ← return x;  
    z1 ← addoII0 x1 y;  
    z ← return (S z1);  
    return z  
  }  
]
```

---