Differential Equations
Part 2

Wednesday, October 21

Laplace Transforms

Chapter 7

Definition

Let f(t) be an integrable function on

$$[0, \infty) \tag{1}$$

. The Laplace transform of f(t) is defined by

$$\mathcal{L}(f) = F(s) = \int_0^\infty f(t)e^{-st}dt \tag{2}$$

and

$$\mathcal{L}(f(t)) = F(s) \tag{3}$$

if and only if

$$\mathcal{L}^{-1}(F(s)) = f(t) \tag{4}$$

$$\mathcal{L}(1) = \int_0^\infty e^{-st}dt = \frac{-e^{-st}}{s}|_0^\infty =^{s>0} \frac{-1}{s}(0-1) = \frac{1}{s} \tag{5}$$

$$= \frac{-1}{s}(0-0) - \frac{e^{-st}}{s^2}|_0^\infty = \frac{-1}{s^2}(0-1) = \frac{1}{s^2} \tag{6}$$

Inverse Laplace Transform

$$\begin{gathered} Given\ F(s). \\ \mathcal{L}^{-1}(F(s)) = f(t)\ if\ and\ only\ if \\ \mathcal{L}(f(t0) = F(s) \end{gathered}$$

$$\textbf{EX.}\ Find\ \mathcal{L}^{-1}(\frac{1}{s}), \mathcal{L}^{-1}(\frac{1}{s^2+a^2}) \\ and\ \mathcal{L}^{-1}(\frac{1}{s^n}), n \geq 1$$