

Differential Equations

Part 2

Wednesday, October 21

1 Laplace Transforms

Chapter 7

1.1 Definition

Let $f(t)$ be an integrable function on

$$[0, \infty) \tag{1}$$

. The Laplace transform of $f(t)$ is defined by

$$L(f) = F(s) = \int_0^{\infty} f(t)e^{-st} dt \tag{2}$$

and

$$L(f(t)) = F(s) \tag{3}$$

if and only if

$$L^{-1}(F(s)) = f(t) \tag{4}$$

$$L(1) = \int_0^{\infty} e^{-st} dt = \left. \frac{-e^{-st}}{s} \right|_0^{\infty} =_{s>0} \frac{-1}{s}(0 - 1) = \frac{1}{s} \tag{5}$$

$$= \frac{-1}{s}(0 - 0) - \left. \frac{e^{-st}}{s^2} \right|_0^{\infty} = \frac{-1}{s^2}(0 - 1) = \frac{1}{s^2} \tag{6}$$