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Comparison of Forecasting Volatility in the Mid-Cap-German Stock Market

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Abstract: The objective of this paper is to examine different GARCH models and assess their power in forecasting return's volatility of the mid-cap German stock market between the periods 4.1.2010 to 6.3.2023. GARCH, TGARCH, and EGARCH models were employed using normal, Student-t and generalized error distributions. All together, nine models were used to forecast the volatility of the MDAX based on its returns. The findings suggest that volatility in returns has significant persistence and asymmetric effects. After evaluating the forecasting ability of all nine models, the EGARCH model was found to have the most superior forecasting performance.

1. INTRODUCTION

Financial data typically exhibit features such as leptokurtosis, volatility clustering or pooling, and leverage effects, which cannot be explained by linear structural models or time series models alone. These features are defined as: a) distributions with fat tails and excess peakedness at the mean, b) volatility appearing in bunches, with large or small returns of either sign, and c) higher volatility following a large price drop than a price rise of the same magnitude. To model and forecast such volatility, non-linear models like the Autoregressive Conditional Heteroscedastic (ARCH) or Generalized Autoregressive Conditional Heteroscedastic (GARCH) have been proposed.

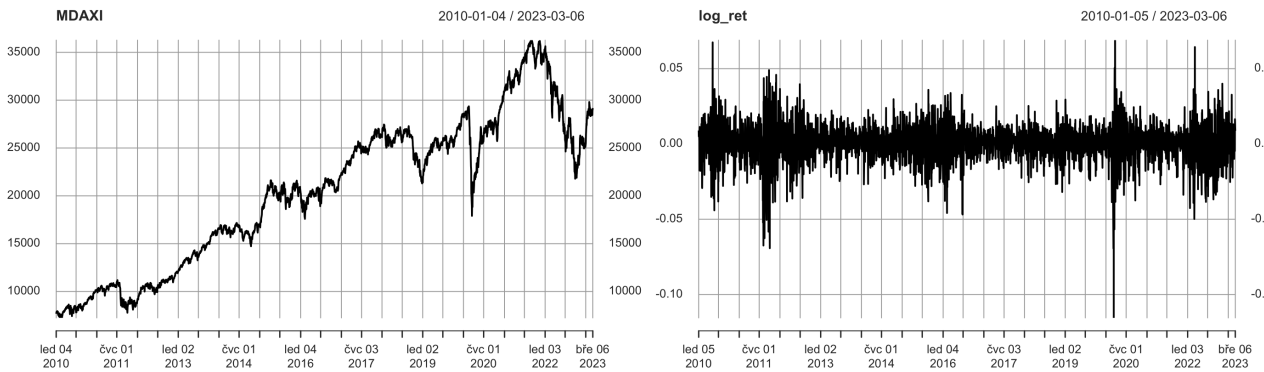
This paper aims to forecast stock market volatility for the German mid-cap stock market using GARCH, TGARCH, and EGARCH models and compare their forecasting performance. Several empirical studies have previously investigated volatility in the German stock market using DAX, the Deutsche Boerse AG German Stock index (Bluhm et al., 2001; Liang et al., 2021). DAX is a prominent benchmark for German and European stocks, listing major companies by liquidity and market capitalization, and an indicator of trends in Germany's economy. In our study, we focus on analyzing volatility in the mid-cap German stock market employing MDAX index. The objective of MDAX is to replicate the returns of an index consisting of 50 mid-sized German companies that are either listed on the Prime Standard Segment of the

Frankfurt Stock Exchange or have a significant presence in Germany. The companies in this index are considered mid-cap, meaning they have a market capitalization between small and large-cap companies. The fund aims to closely track the performance of this index by investing in a portfolio of securities that mimic its composition.

The paper proceeds as follows. The second section reviews time series data used for the analysis. The third section provides empirical application of GARCH estimation strategy and a brief overview of GARCH models, while the fourth section presents the estimation results. Finally, the fifth section summarizes and concludes the paper.

2. DATA

The dataset used for this analysis consist of over 3300 observations for daily closing price of MDAX Performance Index from 4 January 2010 to 6 March 2023. Graph of the period of interest is depicted in the Figure 1a. The first closing price of Mid-Cap-German stock index in our dataset starts at 7676,65 Euro, falling to its minimum on the 5th of February 2010 to 7243.13 Euro and reaching its maximum on the 2nd of September 2021 of 36275,62 Euro. From the graph is it also observable that there are two major breakpoints - first one due to COVID-19 pandemics, the second one is caused by Russian-Ukraine war.



(a) Prices (b) Logarithmic returns

Figure 1: Time series plots

As expected, the graph suggests that the prices are not stationary, that is why we firstly get rid of missing observations and then convert prices into logarithmic returns. The graph of computed logarithmic returns is depicted in Figure 1b. The graph suggests that the volatility clustering is present, especially over the past years caused by general uncertainty on the financial markets in the light of recent global crises. For modelling purposes, stationarity of the data must be achieved. Thus, we employ the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test to examine whether the logarithmic returns of our time series data are stationary. The ADF test rejected the null hypothesis of non-stationarity at all commonly used significance levels, which means we have evidence to support stationarity of the data. To further confirm these results, we also used the KPSS test, which showed that we cannot reject the null hypothesis of stationarity at any commonly used significance level. This reinforces our conclusion that our time series data is stationary.

3. EMPIRICAL METHODOLOGY

Methods used for the purposes of our analysis follow general GARCH estimation strategy.

3.1. Mean equation

We start with fitting the correct autoregressive moving average model (ARMA) as our mean equation for logarithmic returns. This is done by firstly visually checking autocorrelation function (ACF) and partial autocorrelation function (PACF) of the returns. According to the AFC and PACF depicted in the Figure 2, a pattern of first-order moving-average process MA(1) is observed - autocorrelation function drops towards 0 for second lag and higher while partial correlation function shows only a slow decay.

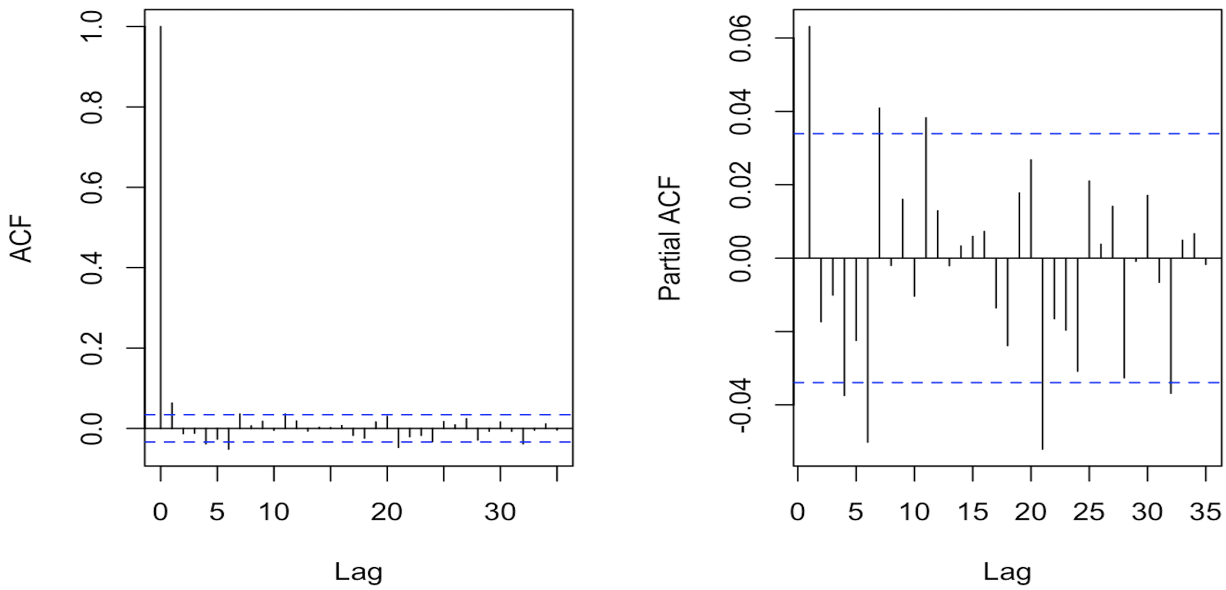


Figure 2: ACF and PACF of logarithmic returns

This is confirmed by selection procedure based on Akaike information criterion (AIC) and Bayesian Information Criteria (BIC), both of them implying ARMA(0,1) model for our mean equation. Thus, selected ARMA(0,1) model has the following form:

$$y_t = \mu + \theta_1 u_{t-1} + u_t$$

3.2. Mean equation residual diagnostic

Second step consists of checking how much of the variation in our time series has been explained by the selected mean equation. This is done by performing diagnostic of the ARMA(0,1) model's residuals. ARCH-LM test is employed to test for heteroscedasticity in the residuals from the model. Estimated p-values of ARCH-LM test strongly reject the null hypothesis that there is no heteroskedasticity in the residuals, giving us an evidence of their non-normality. Moreover, kurtosis of residuals exceeds 3, which suggest their leptokurtic distribution. Thus, Jarque-Bera test is applied to test normality of model's residuals. The test rejects the null hypothesis of normality at any commonly used level of significance, signalling that there is some conditional heteroskedasticity present in the data.

Before we start with estimating GARCH models, testing squared residuals might be handy for further evidence. Looking at the Figure 3, squared residuals are definitely not constant over time. This is confirmed also by employing Box-Ljung test that rejects the null hypothesis of independently distributed squared residuals.

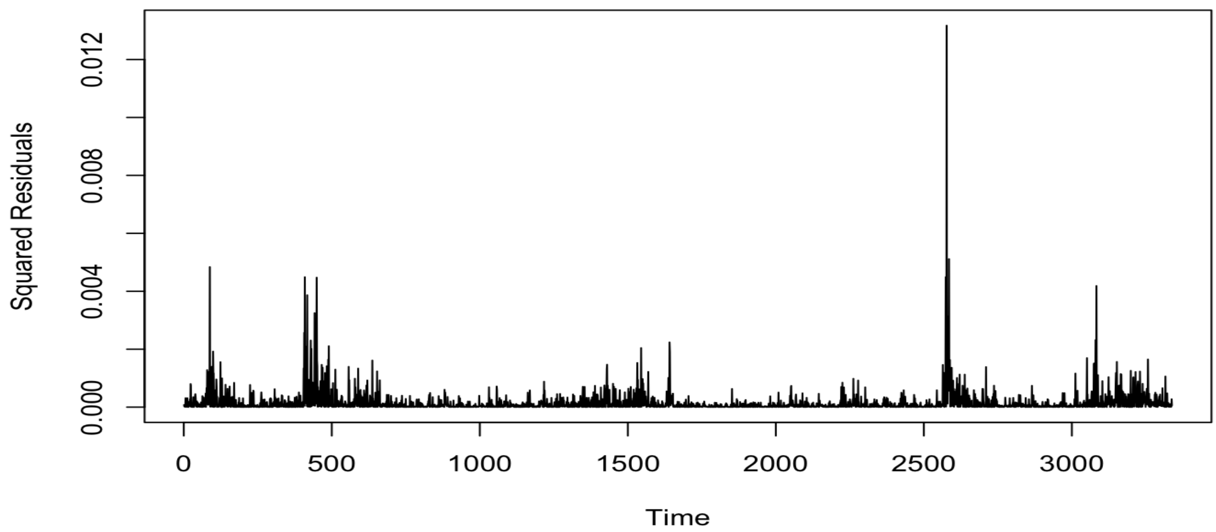


Figure 3: Squared residuals of ARMA(0,1) model

In addition, looking at ACF and PACF functions of squared residuals provided in the Figure 4, a strong serial correlation in squared residuals of the mean model is observed suggesting further evidence that there are some autoregressive conditional heteroskedastic (ARCH) effects present in the data, and a volatility model such as GARCH or EGARCH may be necessary to account for it.

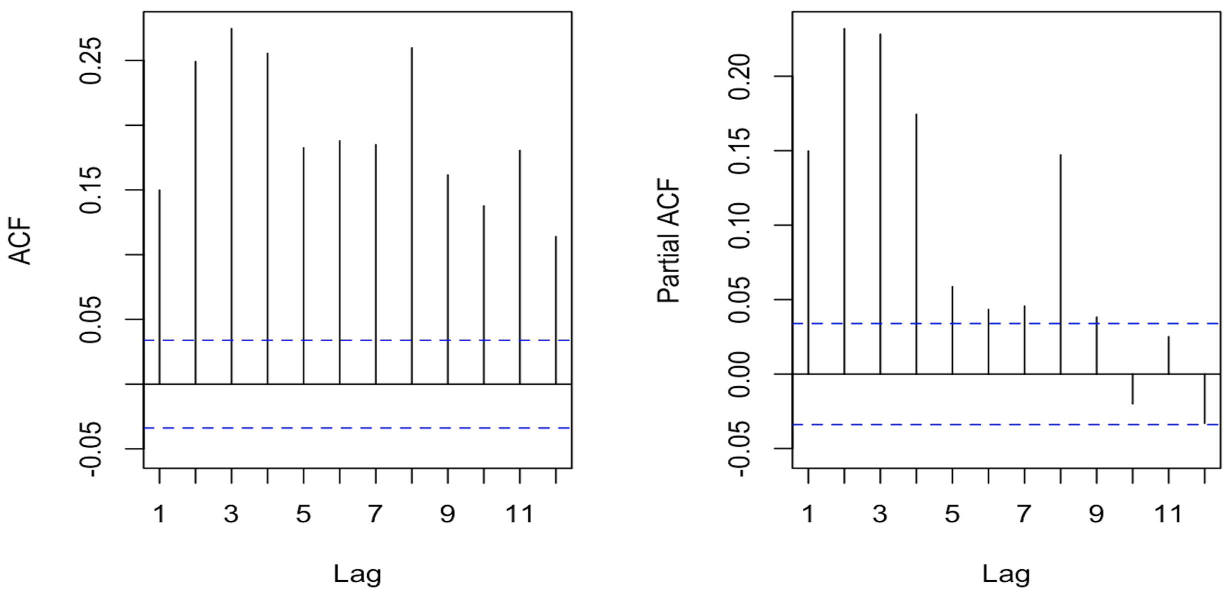


Figure 4: ACF and PACF of squared residuals of ARMA(0,1) model

3.3. GARCH estimation

Even though GARCH(1,1) is sufficient to accommodate most of the financial time series, optimal order of the model has been selected by checking other models of different orders as well. However, none of the other models outperformed GARCH(1,1) model, in which Bayesian information criterion (BIC) has been minimized. In addition, selection process confirms the selected orders of the model.

3.3.1. GARCH estimation

Since we obtained strong support for the usage of volatility models to assess conditional variance from our data, it is reasonable to use GARCH model. GARCH provides a flexible and powerful framework for modeling and forecasting volatility, and their use is widespread in finance and economics. GARCH model has been firstly introduced by Bollerslev (1986) as an extension of Engle's (1982) ARCH model. Notations of GARCH process differ across the literature, but the GARCH(1,1) process used in our analysis has the following form:

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \; with \; \epsilon_t = \sigma_t e_t$$

where σ_t^2 is conditional volatility, ϵ_{t-1}^2 are squared unexpected returns for the previous period derived from the mean equation, and thus α_1 captures effect of the previous period's return, β_1 captures effect of the previous period's volatility and ω weights long-run average variance. Stationarity constraints has to be satisfied for GARCH model, thus $\omega > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$.

The GARCH models that are commonly used assume that positive and negative error terms have an equal weight meaning that both good and bad news have the same effect on volatility. However, this assumption is often violated in practice, particularly in stock returns, thus extension of GARCH model might be useful.

3.3.2. TGARCH model

The leverage effect, first denoted by Black (1976), implies that negative returns cause higher subsequent volatility than positive returns. That is why recent research has proposed several parameterized extensions to the conventional GARCH model as the volatility appears to respond in an unequal manner to shocks of different signs. First of them is the TGARCH model initially introduced by Glosten et al. (1993) and Zakoian (1994) as an extension of the GARCH model, aiming to account for the asymmetric impacts that result in markets responding differently to substantial positive and negative shocks. The TGARCH(1,1) model is defined as follows:

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \gamma_1 \epsilon_{t-1}^2 I_{\epsilon_{t-1}<0} + \beta_1 \sigma_{t-1}^2, \; with \; \epsilon_t = \sigma_t e_t$$

where in addition to α_1 and β_1 coefficients, γ_1 as a leverage term is employed and $I_t = 0$ if $e_t < 0$ and 0 otherwise. In this model, the effect of good news shows their impact by α_1, while bad news shows their impact by $\alpha_1 + \gamma_1$. In addition, if γ_1 is differentiable from 0 news impact is asymmetric and $\gamma_1 > 0$ shows that leverage effect exists. To satisfy non-negativity condition coefficients would be $\omega > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and $\gamma_1 {+}_1 \geq 0$. According to Brooks (2008), the model is still acceptable even if $\gamma_1 < 0$, provided that $\gamma_1 + \alpha_1 \geq 0$.

3.3.3. EGARCH model

Exponential GARCH, introduced by Nelson (1991), is given by

$$log_e(\sigma_t^2) = \omega + \alpha_1 z_{t-1} + \gamma_1(|z_{t-1}| - E[z_{t-1}]) + \beta_1 log_e(\sigma_{t-1}^2),\ with\ z_t = \frac{e_t}{\sqrt{\sigma_t^2}}$$

where α_1 captures the sign effect, γ_1, also called leverage parameter, captures the size effect. The parameter γ_1 is thus typically positive and α_1 is negative. As suggested, parameters of the EGARCH model are not restricted to positive values. z_t is the standardized innovation, which characterizes EGARCH model is an explicit multiplicative function of lagged innovations, in contrast to the standard GARCH, in which the volatility is modeled as an additive function of the lagged error terms.

When applying GARCH models to return series, it is often found, similarly to our time series, that GARCH residuals still tend to be heavy tailed (Mittnik, Paolella and Rachev, 2002, Nelson, 1991). To accommodate this, not only the normal distribution but also the Student-t and GED distribution are used in estimating GARCH type models.

3.4. Forecasting performance estimation

The in-sample evidence refers to the historical performance of the models. We estimate the variance for all models throughout the entire sample period and compare their forecasting accuracy. The accuracy of the forecasts is evaluated using four measures employed in the research of Wang and Wu (2012): the Mean Square Error (MSE) and the Mean Absolute Error (MAE), which are presented in the following equations:

$$MSE_1 = n^{-1} \sum_{t-1}^{n} (\sigma_t^2 - \hat{\sigma_t}^2)^2$$

$$MSE_2 = n^{-1} \sum_{t-1}^{n} (\sigma_t - \hat{\sigma}_t)^2$$

$$MAE_1 = n^{-1} \sum_{t-1}^{n} |\sigma_t^2 - \hat{\sigma_t}^2|$$

$$MAE_2 = n^{-1} \sum_{t-1}^{n} |\sigma_t - \hat{\sigma}_t|$$

where n is the number of forecasts, σ_t^2 is the actual volatility and $\hat{\sigma_t}^2$ is the volatility forecast at day t.

4. RESULTS

Different processes such as GARCH, TGARCH, and EGARCH models using normal, Student-t, and generalized error distributions have been applied to fit our data and model volatility of returns.

The results are demonstrated in Tables 4, 5, and 6. Combined values of coefficients α_1 and β_1 are less than 1 for all distributions in GARCH and TGARCH models. For all three GARCH(1,1) models, sign bias remains significant at 90% significance level, suggesting leverage effect to be present. Thus, TGARCH model is used. In all three TGARCH models the parameter γ_1 is significantly greater than 0, which suggests asymmetric news impact. However, the estimated coefficient of α_1 in TGARCH models using normal and Student-t distributions has been proven to be insignificant which suggest not very good fit and leverage effect might be rather exponential than quadratic. To accommodate this, EGARCH model has been fitted. EGARCH model shows a positive and significant leverage effect parameter as well. All in all, after fitting EGARCH the sign bias is no longer significant, suggesting a good fit for our time series data. Among the three EGARCH models, the one with standardized Student-t distribution has the lowest BIC. The conditional volatility estimated using this model can be found in 6.

When looking at the tables 4, 5, and 6, δ_1 is positive and significant in all of the models, which means that past errors have a positive impact on the current variance of returns. ω is the constant term in the conditional variance equation and represents the long-run average variance. In GARCH and TGARCH models it shows no effect, however, after fitting EGARCH, ω becomes significantly negative, which means that the conditional variance is mean-reverting. Similarly for α_1, which turns to be negative in EGARCH, due to model's benefit in no parameter restriction. Thus, negative value of α_1 indicates that past squared errors have a negative effect on the current variance. β_1 parameter measures the persistence of the conditional variance. In all models, it is close to 1, which means that shocks to the variance are long-lasting and take a long time to dissipate. Estimated parameter on shape represents the skewness of the distribution of the standardized residuals. Its positive value in all models with Student-t and generalized error distributions indicates that the distribution of the standardized residuals is positively skewed.

In all models, we moreover tested null hypothesis of Weighted ARCH LM Test, which suggests that the ARCH process is adequately fitted. The results indicate that the null hypothesis cannot be rejected across all models, however, in the GARCH model the null hypothesis was rejected at a 10% level of significance.

The performance of GARCH, TGARCH, and EGARCH models in forecasting is presented in Table 1, 2 and 3 respectively. It has been shown that the EGARCH model exhibits the highest level of forecasting accuracy based on all four measures.

Comparing results only for EGARCH, we can conclude that the lowest values of all four measures exhibited EGARCH under standardized normal distribution of the conditional density used for innovations, even though estimated BIC and AIC are the lowest for Student-t distribution. This might be also visible from the Figures 5 that depict standardized residu-

Table 1: Comparison forecasting performance for normal distribution

Normal Distribution			
Measure	GARCH	TGARCH	EGARCH
MSE_1	5.056e-08	7.092e-08	**4.371e-08**
MSE_2	1.405e-04	1.443e-04	**1.344e-04**
MAE_1	1.491e-04	1.537e-04	**1.431e-04**
MAE_2	1.099e-02	1.094e-02	**1.079e-02**

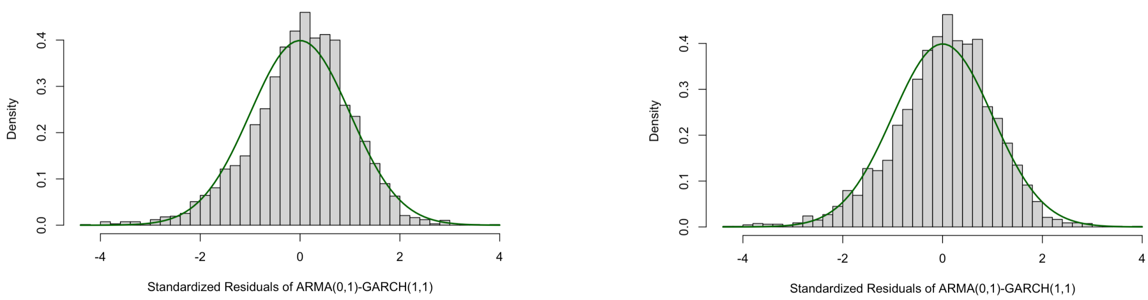
Table 2: Comparison forecasting performance for Student-t distribution

Student-t Distribution			
Measure	GARCH	TGARCH	EGARCH
MSE_1	5.395e-08	7.701e-08	**4.780e-08**
MSE_2	1.438e-04	1.478e-04	**1.375e-04**
MAE_1	1.523e-04	1.574e-04	**1.463e-04**
MAE_2	1.108e-02	1.101e-02	**1.086e-02**

Table 3: Comparison forecasting performance for generalized error distribution

Generalized Error Distribution			
Measure	GARCH	TGARCH	EGARCH
MSE_1	5.211e-08	7.372e-08	**4.555e-08**
MSE_2	1.417e-04	1.459e-04	**1.357e-04**
MAE_1	1.504e-04	1.554e-04	**1.444e-04**
MAE_2	1.102e-02	1.096e-02	**1.082e-02**

als of EGARCH estimated using standardized normal distribution (5a) and EGARCH using standardized Student-t distribution (5b). When comparing two histograms, using standardized normal distribution of the conditional density used for the innovations seems to accommodate fat tails of the data better than Student-t distribution.



(a) Normal Distribution (b) Student-t Distribution

Figure 5: Histograms of residuals

5. CONCLUSION

The study analyzed the predictive ability of three GARCH models - GARCH, TGARCH, and EGARCH - using different distribution types (normal, Student-t, and generalized error distributions) to assess the volatility of returns of MDAX index for the mid-cap German stock market. The findings revealed that significant GARCH effects were present in the data, indicating persistent volatility and asymmetric effects in MDAX index returns. Furthermore, the research compared the in-sample forecasting accuracy of the nine models during the period from 6.1.2010 to 6.3.2023, and the results showed that the EGARCH model had the best predictive performance based on real data.

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Appendix

Table 4: GARCH estimation results

	NORM	STD	GED
μ	0.001***	0.001***	0.001***
δ_1	0.043**	0.036**	0.030*
ω_1	0.000**	0.000	0.000*
α_1	0.117***	0.113***	0.114***
β_1	0.859***	0.872***	0.866***
Shape	-	8.014***	1.472***
AIC	-6.2602	-6.2836	-6.2835
BIC	-6.2511	-6.2726	-6.2725
LogLikelihood	10453.34	10493.31	10493.12

Notes: *p<0.1; **p<0.05; ***p<0.01

Table 5: TGARCH estimation results

	NORM	STD	GED
δ_1	0.064***	0.064***	0.058***
ω_1	0.000***	0.000***	0.000
α_1	0.005	0.001	0.003***
β_1	0.865***	0.869***	0.868***
γ_1	0.201***	0.211***	0.204***
Shape	-	1.669***	1.593***
AIC	-6.2948	-6.3094	-6.3072
BIC	-6.2856	-6.2984	-6.2963
LogLikelihood	10511.03	10536.45	10532.8

Notes: *p<0.1; **p<0.05; ***p<0.01

Table 6: eGARCH estimation results

	NORM	STD	GED
δ_1	0.057***	0.063***	0.054***
ω_1	-0.369***	-0.338***	-0.354***
α_1	-0.151***	-0.158***	-0.153***
β_1	0.959***	0.963***	0.961***
γ_1	0.169***	0.167***	0.168***
Shape	-	11.016***	1.608***
AIC	-6.2999	-6.3132	-6.3112
BIC	-6.2908	-6.3022	-6.3002
LogLikelihood	10519.56	10542.7	10539.36

Notes: *p<0.1; **p<0.05; ***p<0.01

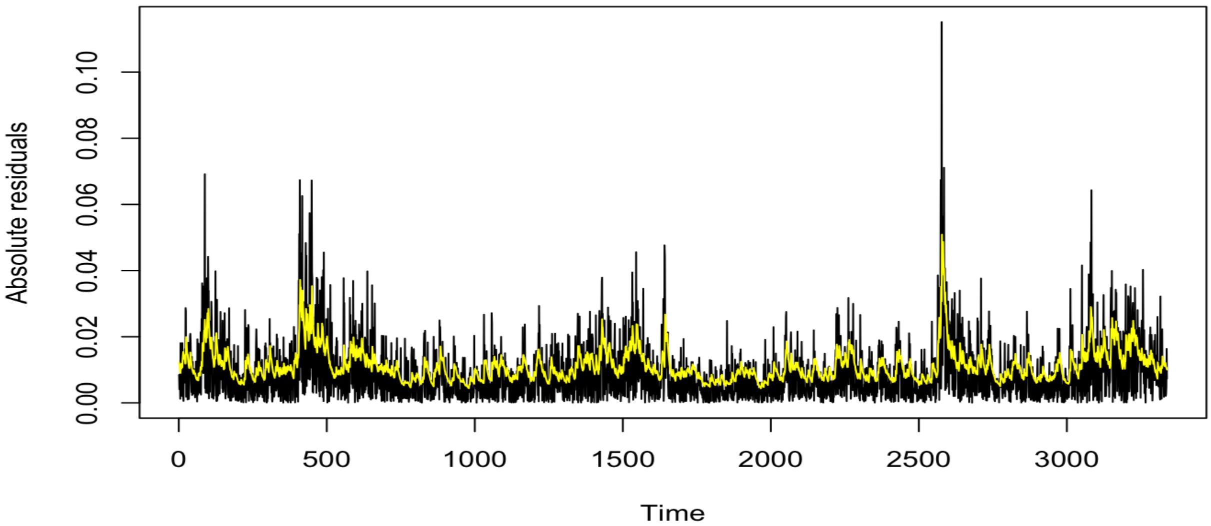

Figure 6: Conditional volatility estimated using EGARCH Student-t distr. - yellow colour