Lecture 4

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4.1 Properties of subspaces

- V(F) is also a subspace, so is $\{0\}$.
- Let V be the vector space $\mathbb{R}^3$. Then the set W consisting of those vectors whose third component is zero, i.e. $w = \{a, b, 0 : a, b \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$
- Let V be the vector space of all $n \times n$ matrices. Then the set W consisting of these matrices $A = [a_{ij}]$ for which $a_{ji} = a_{ij}$ (Symmetric matrices) is a subspace of V.
- Let V be the vector space of polynomials. Then the set W consisting of polynomials with degree $\leq n$, for a fixed n, is a subspace of V.
- Let V be the vector space of all functions for a non-empty set X into the real field $\mathbb{R}$. Then the set consisting of all bounded functions in V is a subspace of V.
  - A function $f \in V$ is bounded iff $\exists M \in R$ such that $|f(x)| \leq M$
- Let S be a non-empty subset of V(F), The set of all linear comination of vectors in S, denoted by $L(S)$, is a subspace of V containing S.
- Furthermore, if W is any other subspace of V containing S, then $L(S) \leq W$.
- The solution space of a system of linear equations $\subseteq R^{n\times 1}$
- In F^n, the set of all n-tuples $(x_1, x_2, ..., x_n)$ with $x_1 = 0$ is a subspace.
- The set of all hermitian matrices is **NOT** a subspace of the space of all $n \times n$ matrices over $\mathbb{C}$. The set of all $n \times n$ complex hermitian matrices is a vector space over the field of real numbers.
- Suppose W_1 and W_2 are subspaces of a vector space V(F)
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