MTL104 : Linear Algebra	**Spring 2020-21**
Lecture 5	
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5.1 Direct Sum

5.1.1 Direct sum of subspaces

Theorem : $V = W_1 \oplus W_2 \iff V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$

Proof($\implies$) :

- Suppose $V = W_1 \oplus W_2$
- $V = W_1 \oplus W_2 \implies V = W_1 + W_2$
- To prove : $W_1 \cap W_2 = \{0\}$
- Let $z = W_1 \cap W_2$ where $z \neq 0$
- Then : $z = \mathbf{0} + z = z + \mathbf{0}$ where blue elements belong to W_1 and green elements belong to W_2
- As z has been expressed in two ways, therefore the representation is not unique.
- Therefore z has to be **0**. Hence proved.

Proof($\impliedby$):

- Suppose $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$
- To Prove : $V = W_1 \oplus W_2$
- let $z \in V$.
- Let blue represent the fact that the element belongs to W_1 and green elements belong to W_2
- Suppose the following two representations of z exist:
  - $z = x_1 + y_1$
  - $z = x_2 + y_2$
- Then $x_1 + y_1 = x_2 + y_2 = z$
- $x_1 - x_2 = y_1 - y_2$

- Clearly LHS = RHS
- But as $W_1 \cap W_2 = \{0\}$
- Therefore, $x_1 - x_2 = 0$ and $y_1 - y_2 = 0$
- Therefore $x_1 = x_2$ and $y_1 = y_2$
- Therefore each $z \in V$ is uniquely represented as $x + y$
- Therefore $V = W_1 \oplus W_2$

5.1.2 Extended direct sum