

# CS458: Introduction to Information Security

## Notes 5: Digital Signatures

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Slides: Modified from [Christof Paar and Jan Pelzl](#) & Cryptography and Network by Behrouz Forouzan, the McGraw-Hill Companies

- Security services
- The principle of digital signatures
- The RSA digital signature scheme

# Attacks, Services and Mechanisms

- Security Attack

- Any action that compromises the security of information.

- Security Mechanisms

- A mechanism that is designed to detect, prevent, or recover from a security attack.
- A mechanism might operate by itself, or with others, to provide a particular service.
- Examples of common security mechanisms:
  - Cryptography, Message digests and digital signatures, Digital certificates, Public Key Infrastructure (PKI)

- Security Services

- Refer to the different services available for maintaining the security and safety of an organization
- Intended to counter security attacks, and they make use of one or more security mechanisms to provide the service.
- They help in preventing any potential risks to security
- Implement security policies and are implemented by security mechanisms.

# Core Security Services

- The objectives of security system are called security services.
  1. **Confidentiality**: Information is kept secret from all but authorized parties
  2. **Message/Data Integrity**: Ensures that a message has not been modified in transit.
  3. **Message/Data Origin Authentication**: Ensures that the sender of a message is authentic.
  4. **Non-repudiation**: Ensures that the sender of a message can not deny the creation of the message. (e.g., order of a pink car)
- Confidentiality is provided by using primarily symmetric ciphers and less frequently asymmetric encryption.
- Integrity and message authentication are provided by digital signatures and message authentication codes.
- Non-repudiation can be achieved with digital signatures.

# Additional Security Services<sup>1</sup>

5. **Identification/Peer entity authentication**: Establishing and verification of the identity of an entity, e.g. a person, a computer, ...
  - **who are you?**
6. **Access control/Authorization**: Restricting access to the resources to privileged entities. (decide **who can do what?**)
7. **Auditing**: Provides evidences about security-relevant activities, e.g., by keeping logs about certain events. (provide a proof **who did what?**)
8. **Availability**: System is accessible and usable on demand by authorized users according to intended goal
9. **Physical security**: Providing protection against physical tampering and/or responses to physical tampering attempts
10. **Anonymity/privacy**: Providing protection against discovery and misuse of identity. (what if we don't want to be identified?)

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<sup>1</sup>Slide partial credit: Jia Wang, IIT

# Introduction to Digital Signatures

- Goal:
  - Signature-like function for the electronic world that mimic the conventional (paper) signature
- Conventional (paper) Signature
  - In the physical world, it is common to use handwritten signatures on handwritten or typed messages.
  - They are used to bind signatory to the message.
- Digital Signatures
  - Similarly, a digital signature is a technique that binds a person/entity to the digital data.
  - This binding can be independently verified by receiver as well as any third party.

# Motivation

- Consider the real-life example where a person pays by credit card and signs a bill; the seller verifies that the signature on the bill is the same with the signature on the card
- Contracts are valid if they are signed.
- Signatures provide non-repudiation.
  - ensuring that a party in a dispute cannot repudiate, or refute the validity of a statement or contract.
- *Can we have a similar service in the electronic world?*

- *Why not use symmetric cryptography?*
- Discuss a setting in which symmetric cryptography fails to provide a desirable security function.
- Odd Colors for Cars
  - **Bob** orders a pink car from the car dealer **Alice**
  - After seeing the pink car, **Bob** states that he has never ordered it
  - How can **Alice** prove towards a judge that **Bob** has ordered a pink car?  
(And that she did not fabricate the order herself)
    - Symmetric cryptography fails because both **Alice** and **Bob** can be malicious
    - Can be achieved with public-key cryptography



# Comparison

- Let us begin by looking at the differences between conventional signatures and digital signatures.
  - Inclusion
  - Verification Method
  - Relationship
  - Duplicity

# Comparison: Inclusion

- A conventional signature is included in the document; it is part of the document.
  - include message and signature in the same document.
- But when we sign a document digitally, we send the signature as a separate document.
  - i.e., digital signature: the signature will be attached to the document, is not a part of the document

# Comparison: Verification Method

- For a conventional signature, when the recipient receives a document, she compares the signature on the document with the signature on file.
- For a digital signature
  - The recipient receives the message and the signature.
  - The recipient needs to apply a verification technique to the combination of the message and the signature to verify the authenticity.

# Comparison: Relationship

- Compare based on the relationship between the signature and the documents.
- For a conventional signature, there is normally a one-to-many relationship between a signature and documents.
  - i.e., all documents would have the same signature if it signed by the same person/entity
- For a digital signature, there is a one-to-one relationship between a signature and a message.
  - if we modify even one bit, we will have a different signature for this message.

# Comparison: Duplicity

- In conventional signature, a copy of the signed document can be distinguished from the original one on file.
- In digital signature, there is no such distinction unless there is a factor of time on the document.

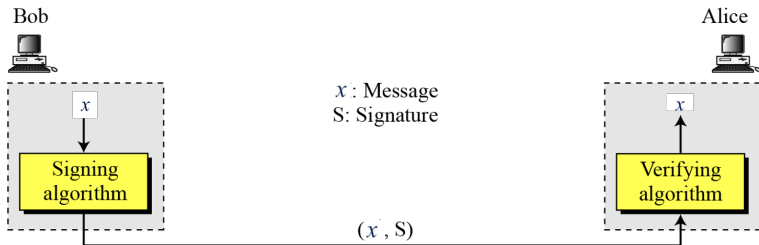
# Digital Signatures

- Aim of a signature
  - Prove to anyone that a message originated at (or is approved by) a particular user
- Symmetric key cryptography
  - Two users, **Alice** and **Bob**, share a secret key **K**
  - Receiver of message (user **Alice**) can verify that message came from the other user (**Bob**)
  - User **C** cannot prove that the message came from **Bob** (it may also have came from **Alice**)
- Public key cryptography can provide signature
  - Only one user has the private key

# Digital signature process

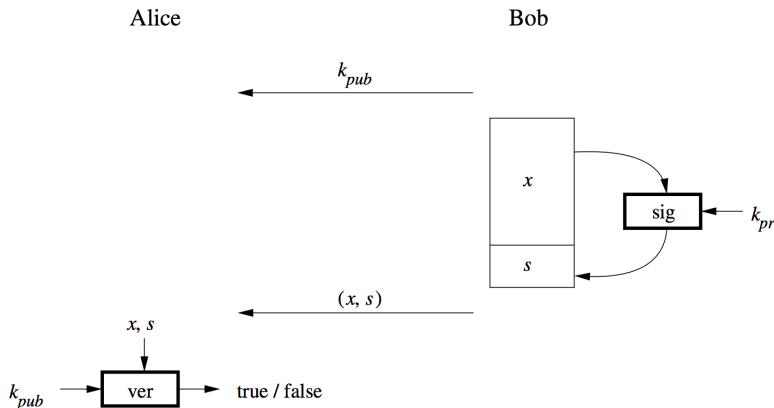
- The sender uses a [signing algorithm](#) to sign the message.
- The message and the signature are sent to the receiver.
- The receiver receives the message and the signature and applies the [verifying algorithm](#) to the combination.
  - If the result is true, the message is accepted;
  - otherwise, it is rejected.

# Digital signature process





# Basic Protocol for Digital Signature



- A digital signature needs a public-key system.
- The signer signs with his/her private key; the verifier verifies with the signer's public key.
  - *The person who signs the message uses a private key, and the receiving party uses the matching public key.*

# Main Idea

- For a given message  $x$ , a digital signature is appended to the message.
- Only the person with the private key should be able to generate the signature.
- The signature must change for every document.
- The **signature** is realized as a function with the message  $x$  and the private key as input.
- The public key, signature  $s$ , and the message  $x$  are the inputs to the **verification function**.

# Digital Signature Operations (Concept)

- **Signing**

- Bob signs a message by encrypting with own private key
  - $s = E_{k_{pr,B}}(x)$
- Bob attaches signature to message

- **Verification**

- Alice verifies a message by decrypting signature with signer's (Bob) public key
  - $x' = D_{k_{pub,B}}(s)$
- Alice then
  - compares received message  $x$  with decrypted  $x'$
  - if identical, signature is verified
  - otherwise, rejected

- We discussed several security services including message confidentiality, message authentication, message integrity, and nonrepudiation.
- A digital signature can directly provide the last three; for message confidentiality we still need encryption/decryption.

# Message Authentication

- A secure digital signature scheme, like a secure conventional signature can provide message authentication.
- *When the verifier validates the digital signature using the public key of the sender, he is assured that the signature has been created only by the sender who possesses the corresponding secret private key and no one else.*

# Message Integrity

- The integrity of the message is preserved even if we sign the whole message because we cannot get the same signature if the message is changed.
- *In case an attacker has access to the data and modifies it, the digital signature verification at the receiver end fails. The digital signature of modified data and the output provided by the verification algorithm will not match.*
  - ⇒ The receiver can safely deny the message assuming that data integrity has been breached*

# Nonrepudiation

- Nonrepudiation is the assurance that someone cannot deny something.
- Typically, nonrepudiation refers to the ability to ensure that a party to a contract or a communication cannot deny the authenticity of their signature on a document or the sending of a message that they originated.
- Alice can take  $x$ , and signature  $s$  to court and prove that Bob signed  $x$
- *Since it assumed that only the signer has the knowledge of the signature key, he can only create unique signature on a given data*
- $\Rightarrow$  *The receiver can present data and the digital signature to a third party as evidence if any dispute arises in the future*

- A digital signature does not provide data confidentiality.
- If there is a need for confidentiality, another layer of encryption/decryption must be applied.



- Several digital signature schemes have evolved during the last few decades.
  - RSA Digital Signature Scheme
  - ElGamal Digital Signature Scheme
  - Schnorr Digital Signature Scheme
  - Digital Signature Standard (DSS)
  - Elliptic Curve Digital Signature Scheme

# Main idea of the RSA signature scheme

- To generate the private and public key
  - Use the same key generation as RSA encryption.
- To generate the signature
  - “**encrypt**” the message  $x$  with the private key.  
 $s = \text{sig}_{K_{pr}}(x) \equiv x^d \bmod n$
  - Append  $s$  to message  $x$
- To verify the signature
  - “**decrypt**” the signature with the public key
  - $\text{ver}_{K_{pub}}(x, s)$ 
    - $x' \equiv s^e \bmod n$
    - If  $x \equiv x'$ , the signature is valid
    - If  $x \not\equiv x'$ , the signature is invalid

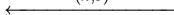
# The RSA signature Protocol

**Alice**

**Bob**

$$k_{pr} = d, k_{pub} = (n, e)$$

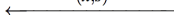
$(n, e)$



compute signature:

$$s = \text{sig}_{k_{pr}}(x) \equiv x^d \pmod n$$

$(x, s)$



verify:  $\text{ver}_{k_{pub}}(x, s)$

$$x' \equiv s^e \pmod n$$

$$x' \begin{cases} \equiv x \pmod n & \implies \text{valid signature} \\ \not\equiv x \pmod n & \implies \text{invalid signature} \end{cases}$$

- Alice can conclude from the valid signature that Bob generated the message and that it was not altered in transit
  - message authentication, non-repudiation and message integrity are given.
- If confidentiality is required, the message  $x$  and signature  $s$  can be encrypted, e.g., using AES.
- *Signature verification is very efficient as a small number can be chosen for the public key.*

# The RSA signature Protocol: Example

**Alice**

**Bob**

1. choose  $p = 3$  and  $q = 11$
2.  $n = p \cdot q = 33$
3.  $\Phi(n) = (3 - 1)(11 - 1) = 20$
4. choose  $e = 3$
5.  $d \equiv e^{-1} \equiv 7 \pmod{20}$

$\longleftarrow (n,e)=(33,3)$

compute signature for message

$x = 4$ :

$$s = x^d \equiv 4^7 \equiv 16 \pmod{33}$$

$\longleftarrow (x,s)=(4,16)$

verify:

$$x' = s^e \equiv 16^3 \equiv 4 \pmod{33}$$

$x' \equiv x \pmod{33} \implies$  valid signature

# Existential Forgery Attack

- Creation by an adversary of at least one message/signature pair  $(x, s)$ , where  $x$  has never been signed by the legitimate signer.
- The adversary can choose  $x$  freely;  $x$  need not have any particular meaning; the message content is irrelevant
- As long as the pair  $(x, s)$  is valid, the adversary has succeeded in constructing an existential forgery

# Existential Forgery Attack Against RSA Digital Signature

- Attacker generates a valid signature for a random message  $x$ .

**Alice**

**Eve**

**Bob**

$$k_{pr} = d$$
$$k_{pub} = (n, e)$$

$$\xleftarrow{(n, e)}$$

$$\xleftarrow{(n, e)}$$

1. choose signature:

$$s \in \mathbb{Z}_n$$

2. compute message:

$$x \equiv s^e \bmod n$$

$$\xleftarrow{(x, s)}$$

verification:

$$s^e \equiv x' \bmod n$$

since  $x' = x$

$\Rightarrow$  valid signature!

# Existential Forgery and Padding

- An attacker can generate valid message-signature pairs  $(x, s)$
- Limitations of this attack
  - Attacker can not directly control the semantics of the message  $x$ 
    - But attacker can only choose signature  $s$  and NOT the message  $x$
    - Attacker cannot generate messages like “Transfer \$1000 into Oscar’s account”
- The fact that an automated verification process does not recognize the forgery is certainly not a desirable feature
- For this reason, schoolbook RSA signature is rarely used in practice, and padding schemes are applied in order to prevent this and other attacks
  - Formatting the message  $x$  according to a padding scheme can be used to make sure that an attacker cannot generate valid  $(x, s)$  pairs.
  - A messages  $x$  generated by an attacker during an Existential Forgery Attack will not coincide with the padding scheme.

# Lessons Learned

- Digital signatures provide message integrity, message authentication and nonrepudiation.
- RSA is currently the most widely used digital signature algorithm.
- Competitors are the Digital Signature Standard (DSA) and the Elliptic Curve Digital Signature Standard (ECDSA).
- RSA verification can be done with short public keys  $e$ . Hence, in practice, RSA verification is usually faster than signing.
- In order to prevent certain attacks, RSA should be used with padding. The modulus of the RSA signature schemes should be at least *2048-bits* long.



- Understanding Cryptography: A Textbook for Students and Practitioners - *available online*
  - Chapter 10: Digital Signatures