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Assignment-0

Q.1. $a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $c = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

Ans. (1) $2a - b$.

$$2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$\therefore$ $2a - b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

(2) $\hat{a}$, a unit vector in the direction of a.

$\|a\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$\hat{a} = \dfrac{a}{\|a\|} = \dfrac{1}{\sqrt{14}} \langle 1, 2, 3 \rangle$

$\therefore \hat{a} = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix} \underset{\text{rationalizing}}{=} \begin{bmatrix} \sqrt{14}/14 \\ \sqrt{14}/7 \\ 3\sqrt{14}/14 \end{bmatrix}$

(3) $\|a\|$ and the angle of a relative to the positive x axis $\|a\| = \sqrt{1^2+2^2+3^2}$.

$\|a\| = \sqrt{14}$.

unit vector for positive x-axis $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\cancel{\cos\theta = \frac{x \cdot A}{\|A\|\|a\|} =}$$

$$\cos\theta = \frac{x \cdot a}{\|a\|\,\|x\|} = \frac{1+0+0}{\sqrt{14}\cdot\sqrt{1}} = \frac{1}{\sqrt{14}} = \frac{\sqrt{14}}{14}$$

$$\therefore \theta = \cos^{-1}\left(\frac{\sqrt{14}}{14}\right) \text{ or}$$

$$\theta = \arccos(\sqrt{14}/14)$$

4) Direction cosines of a.

$\|a\| = \sqrt{14}$.

Then direction cosines are

$$\cos\alpha = \frac{1}{\sqrt{14}} = \frac{\sqrt{14}}{14}$$

$$\cos\beta = \frac{2}{\sqrt{14}} = \frac{\sqrt{14}}{7}$$

$$\cos\gamma = \frac{3}{\sqrt{14}} = \frac{3\sqrt{14}}{14}.$$

5) the angle between a & b.

$$a.b = (1*4)+(2*5)+(3*6)$$

$$a.b = 4+10+18 = 32$$

$$\|a\| = \sqrt{14}$$

$$\|b\| = \sqrt{4^2+5^2+6^2} = \sqrt{77}$$

$$\cos\theta = \frac{32}{\|a\|\,\|b\|} = \frac{32}{\sqrt{14}\sqrt{77}} = \frac{32}{\sqrt{1078}}$$

(6) $a \cdot b$ and $b \cdot a$.

$\therefore$ $a \cdot b = 4 + 10 + 18 = 32$

$b \cdot a = 4 + 10 + 18 = 32$

(7) $a \cdot b$ by using angle between a & b.

$$\cos\theta = \frac{32}{\sqrt{1078}}$$

$$\therefore \cos\theta = \frac{a \cdot b}{\|a\|\|b\|}$$

$$\frac{32}{\sqrt{1078}} = \frac{a \cdot b}{\sqrt{14}\sqrt{77}}$$

$\therefore$ $a \cdot b = 32$

(8) Scalar projection of b onto $\hat{a}$

$$\hat{a} \cdot b = \left(\frac{\sqrt{14}}{14} \times 4\right) + \frac{\sqrt{14}}{\cancel{14}\,7} \times 5 + \frac{3\sqrt{14}}{14} \times 6$$

$$= \frac{2\sqrt{14}}{7} + \frac{5\sqrt{14}}{7} + \frac{9\sqrt{14}}{7}$$

$$\hat{a} \cdot b = \frac{16\sqrt{14}}{7}$$

$$\|\hat{a}\| = \sqrt{\left(\frac{\sqrt{14}}{14}\right)^2 + \left(\frac{2\sqrt{14}}{14}\right)^2 + \left(\frac{3\sqrt{14}}{14}\right)^2}$$

$$= \sqrt{\frac{14}{196} + \frac{14 \times 4}{\cancel{49}\,196} + \frac{\cancel{9\times}\,126}{\cancel{14256}\,196}}$$

$$= \sqrt{\frac{14+56+126}{196}}$$

$$= \sqrt{\frac{196}{196}}$$

$$= 1.$$

$\|\hat{a}\| = 1.$

$\therefore$ scalar proj of b onto $\hat{a}$ $= \frac{a.b}{\|a\|}$

$$= \frac{16\sqrt{14}}{7}.$$

(9) a vector which is perpendicular to a.

a_p = vector $\perp$ to a.

$\therefore \quad a \cdot a_p = 0.$

let a_p be (a_1, b_1, c_1).

so, $a \cdot a_p = (1 * a_1 + 2 * b_1 + 3 * c_1) = 0$

$a \cdot a_p = a_1 + 2b_1 + 3c_1 = 0.$

$\cancel{a_1}$ $2b_1 + 3c_1 = -a_1$

$\therefore$ ~~2b~~ $2b_1 + 3$

One of the solutions for a_p is $(1, 1, -1)$

(10). $a \times b$ & $b \times a$.

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$\therefore a \times b = \hat{i}(12-15) - \hat{j}(6-12) + \hat{k}(5-8)$

$= -3\hat{i} + 6\hat{j} - 3\hat{k}$

$\therefore$ coefficients are $(-3, +6, -3)$.

$|a \times b| = \sqrt{3^2 + 6^2 + 3^2} = \sqrt{9 + 36 + 9}$

$|a \times b| = \sqrt{54}$

$$b \times a = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix}$$

$b \times a = \hat{i}(15-12) - \hat{j}(12-6) + \hat{k}(8-5)$

$= 3\hat{i} - 6\hat{j} + 3\hat{k}$

coeffs are $(3, -6, 3)$.

$|b \times a| = \sqrt{54}$.

(11) vector perpendicular to both a & b

$\therefore$ let a_p be the vector perpendicular to both a & b.

$\therefore \quad a \cdot a_p = 0$ & $b \cdot a_p = 0$

$a_p = (a_1, b_1, c_1)$

So, $a.a_p = a_1 + 2b_1 + 3c_1$ —(1)

$b.a_p = 4a_1 + 5b_1 + 6c_1$ —(2)

multiplying (1) by 2.

$2a_1 + 4b_1 + 6c_1 = 0$

$4a_1 + 5b_1 + 6c_1 = 0$

$- \quad - \quad -$

$-2a_1 - b_1 = 0$

$\therefore 2a_1 = -b_1.$

~~One solution can be~~ $\cancel{(\tfrac{1}{2}, -2, 0)}$

$-b + 4b_1 + 6c_1 = 0$

$\therefore 3b_1 + 6c_1 = 0$

$\therefore 6c_1 = -3b_1$

$\cancel{c_1} \quad b_1 = -2c_1.$

$\therefore$ One solution can be $(1, -2, 1)$.

(12) the linear dependency between a, b, c

$$\cancel{\begin{vmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{vmatrix}}$$

$$\cancel{= 1(15-6) - 4(6-3) - 1(12-15)}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix}$$

$$= 1\underbrace{(15-6)}_{9} - 2(12+6) + 3(4+5)$$

$= 9 - 36 + 27$

$\cancel{+6} \; \cancel{+0} \quad = 0$

As the determinant is ~~not~~ equal to 0, the linear dependency between a, b, c does ~~not~~ exist i.e. they are dependent of each other.

(13) $a^T b$ & ab^T $\quad a^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

$$a^T b = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$= (1 \times 4) + (2 \times 5) + (3 \times 6)$$

$$= 4 + 10 + 18$$

$$a^T b = 32$$

$$b^T = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \; 1 \times 3$$

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \; 3 \times 1$$

$$ab^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 4 & 1 \times 5 & 1 \times 6 \\ 2 \times 4 & 2 \times 5 & 2 \times 6 \\ 3 \times 4 & 3 \times 5 & 3 \times 6 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

B]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix} \quad d = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Ans. (1) $2A - B$

$$2A - B = 2\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 4-2 & 6-1 \\ 8-2 & -4-1 & 6-4 \\ 0-3 & 10+2 & -2-1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$$

(2) AB

$$A.B = \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10-3 & 0+5+2 & 0-20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

(2) $B.A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+8+0 & 2-4+5 & 3+6-1 \\ 2+4+0 & 4-2-20 & 6+3+4 \\ 3-8+0 & 6+4+5 & 9-6-1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$

(3) $(AB)^T$ and $B^T A^T$.

$(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+4+9 & 4\cancel{8}-4+9 & 0+10-3 \\ 2+2-6 & 8-2-6 & 0+5+2 \\ 1-8+3 & 4+8+3 & 0-20-1 \end{bmatrix}$

$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$

4. $|A|$ and $|C|$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix}$$

$= 1(2-15) - 2(-4) + 3(20)$

$= -13 + 8 + 60$

$= \cancel{-21+60}\ 55$

$|A| = \cancel{39}.\ 55.$

$$|B| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{vmatrix}$$

$= 1(1-8) - 2(2+12) + 1(-4-3)$

$= -7\ \cancel{-28}\ -7$

$= -7 - 28 - 7$

$= -42$

5. the matrix (A, B or C) in which row vectors form an orthogonal set.

Orthogonal sets are linearly indepen

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix} = 39 \neq 0$$

The row vectors of Matrix A form an orthogonal set

$|B| = -42 \neq 0$ $\therefore$ the rows are linearly independent

The row vectors of matrix B form an orthogonal set

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix}$$

$= 1(15-6) - 2(12+6) + 3(4+5)$

$= 9 - 36 + 27 = 0$

$= 0$ $\therefore$ the rows are linearly dependent

The row vectors of matrix C form an orthogonal set.

⑥ A^{-1} and B^{-1}

$|A| = \cancel{39}\ 55$

$|B| = -42.$

$$A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix} \qquad B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}$$

Adjucate (A) = ?

$A_{11} = 2 - 15 = -13$ $\qquad A_{31} = 20$

$A_{12} = -2 - 15 = -17$ $\qquad A_{32} = 5$

$A_{13} = 6 + 6 = 12.$ $\qquad A_{33} = -2 - 8 = -10$

$A_{21} = -4$

$A_{22} = -1$

$A_{23} = 3 - 12 = -9$

$$\text{Adjucate (A)} = \begin{bmatrix} -13 & +17 & 12 \\ +4 & -1 & +9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -13/39 & 17/39 & 12/39 \\ 4/39 & -1/39 & 9/39 \\ 20/39 & -5/39 & -10/39 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -13/55 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 20/55 & -5/55 & -10/55. \end{bmatrix}$$

Adjucate(B) = ?

$B_{11} = 1 \cancel{-} 8 = 8-7$

$B_{12} \quad 2+2 = 4$

$B_{13} = -8-1 = -9$

$B_{21} = 2+12 = 14$

$B_{22} = 1-3 = -2$

$B_{23} = -4-2 = -6$

$B_{31} = -4-3 = -7$

$B_{32} = -2 \cancel{-} 6 = -8$

$B_{33} = 1-4 = -3$

Adjucate(B) = $\begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & +6 \\ -7 & +8 & -3 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} -7/-42 & -4/-42 & -9/-42 \\ -14/-42 & -2/-42 & 6/-42 \\ -7/-42 & 8/-42 & -3/-42 \end{bmatrix}$

(7) C^{-1}

$|C| = 0$

As C is a linearly dependent matrix, it does not have an inverse matrix

(8)

Ad.

$Ad = \underset{3\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$

$$Ad = \begin{bmatrix} 1+4+9 \\ 4-4+9 \\ 0+10-3 \end{bmatrix}$$

$$Ad = \begin{bmatrix} 14 \\ 9 \\ 7 \end{bmatrix}$$

9) the scalar projection of the rows of A onto the vector d with normalizing normalizing vector d:

$$\|d\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$\therefore d = \begin{bmatrix} \cancel{14} \; 1/\sqrt{14} \\ 2\sqrt{14} \\ 3\sqrt{14} \end{bmatrix}$$

$\cancel{d.a}$ $$\text{proj}_d a = \left(\frac{d.a}{|d|}\right)\frac{d}{|d|}$$

$$\therefore \text{proj}_d a_1 = \frac{1}{\cancel{\sqrt{14}}}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= 1+4+9.$$

$$= \frac{14}{\sqrt{14}}.$$

$$\text{proj}_d\, a_2 = \frac{1}{\sqrt{14}} \begin{bmatrix} 4 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \frac{1}{\sqrt{14}} (4 - 4 + 9)$$

$$= \frac{9}{\sqrt{14}}$$

$$\text{proj}_d\, a_3 = \frac{1}{\sqrt{14}} \begin{bmatrix} 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \frac{1}{\sqrt{14}} (10 - 3) = \frac{7}{\sqrt{14}}$$

(10) the vector projection of the rows of A onto the vector d with normalizing d.

$$\text{proj}_d\, a_1 = \frac{14}{(\sqrt{14})^2} \langle 1, 2, 3 \rangle$$

$$= \langle 1, 2, 3 \rangle$$

$$\text{proj}_d\, a_2 = \frac{9}{(\sqrt{14})^2} \langle 4, -2, 3 \rangle$$

$$= \left\langle \frac{36}{14}, \frac{-18}{14}, \frac{27}{\sqrt{14}} \right\rangle$$

$$\text{proj}_d a_3 = \frac{7}{(\sqrt{14})^2} \langle 0, 5, -1 \rangle$$

$$= \left\langle 0, \frac{35}{14}, \frac{-7}{14} \right\rangle$$

(11) linear combination of the columns of A using elements of d

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix}$$

$$+ 3 \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix}$$

(12) solution x for the equation $Bx = d$

$$\underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{d}$$

$$x_1 + 2x_2 + x_3 = 1$$

$$2x_1 + x_2 - 4x_3 = 2$$

$$3x_1 - 2x_2 + x_3 = 3.$$

$$x_1 + 2x_2 + x_3 = 1$$
$$3x_1 - 2x_2 + x_3 = 3$$
$$- \quad + \quad - \quad -$$

$$-2x_1 + 4x_2 = -2$$
$$-x_1 + 2x_2 = -1$$
$$x_1 = 1 + 2x_2.$$
$$1 + 2x_2 + 2x_2 + x_3 = 1$$
$$1 + 4x_2 + x_3 = 1$$
$$x_3 = -4x_2.$$
$$\text{If } x_2 = 1$$
$$x_1 = 3$$
$$x_3 = -4.$$

$\therefore$ solution for x is $\begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$

(13) solution x for the equation $Cx = d$.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x_1 + 2x_2 + 3x_3 = 1$$
$$4x_1 + 5x_2 + 6x_3 = 2$$
$$-x_1 + x_2 + 3x_3 = 3.$$

$$x_1 + 2x_2 + 3x_3 = 1$$
$$-\ x_1 + x_2 + 3x_3 = 3$$
$$3x_2 + 6x_3 = 4$$

$$3x_3 = 2 - \frac{3}{2}x_2.$$

$$x_1 + 2x_2 + 2 - \frac{3}{2}x_2 = 1$$

$$x_1 = -1 - \frac{x_2}{2}$$

If $x_2 = 2$, then

$$x_3 = \frac{-1}{3}$$

$$\therefore x_1 = -1 - \frac{2}{2} = -2.$$

$\therefore$ one solution for x is $\begin{bmatrix} -2 \\ 2 \\ -1/3 \end{bmatrix}$

c) $D = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, $E = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$ $F = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(i) eigen values & eigen vectors of D.

$$|A - \lambda I| = 0 \qquad (A - \lambda I)x = 0$$

$$\therefore \left| \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} \right| = 0$$

$$(1-\lambda)(2-\lambda) - 6 = 0$$

$$2 - 2\lambda - \lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda_1 = 4 \quad \text{or} \quad \lambda_2 = -1$$

for $\lambda_1 = 4$.

$$\therefore \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-3x_1 + 2x_2 = 0$$

$$3x_1 = 2x_2$$

$$x_1 = \frac{2}{3} x_2$$

so one solution for x for $\lambda_1 = 4$ is,

$$x = \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$$

for $\lambda_2 = -1$

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$2x_1 + 2x_2 = 0$$

$$x_1 = -x_2$$

$$\therefore x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

② dot product between the eigenvectors of

$$\begin{bmatrix} 2/3 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= -\frac{2}{3} + 1 = -\frac{2}{3} + 1 = \frac{1}{3}$$

③ dot product between the eigenvectors of E

$$\left| \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix} \right| = 0$$

$$(2-\lambda)(5-\lambda) - 4 = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$(\lambda - 6)(\lambda - 1) = 0$$

$$\lambda_1 = 6 \quad \text{or} \quad \lambda_2 = 1.$$

for $\lambda = 6$

$$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-4x_1 - 2x_2 = 0 \qquad -2x_1 - x_2 = 0$$

$$x_2 = -2x_1$$

so $x = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\lambda = 1$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$x_1 - 2x_2 = 0$. $\quad -2x_1 + 4x_2 = 0$.

$\therefore x_1 = 2x_2$.

So, $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

dot product $= (1 * 2) + (-2 * 1)$

$= 2 - 2 = 0$.

(4) the property of eigenvectors of E and its reason

The eigen vectors of E are orthogonal.

(5) $Fx = 0$. As it is trivial soln, $x = 0$

(6) $$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.$$

$x_1 + 2x_2 = 0$. $\quad x_1 = -2x_2$.

$2x_1 + 4x_2 = 0$.

So, $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ are non-trivial

solutions.

$\lambda = 1$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$x_1 - 2x_2 = 0$. $\quad -2x_1 + 4x_2 = 0$.

$\therefore \quad x_1 = 2x_2$.

So, $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

dot product $= (1 * 2) + (-2 * 1)$

$= 2 - 2 = 0$.

(4) the property of eigenvectors of E and its reason

The eigen vectors of E are orthogonal.

(5) $Fx = 0$. As it is trivial soln, $x = 0$

(6)
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.$$

$x_1 + 2x_2 = 0$. $\quad x_1 = -2x_2$.

$2x_1 + 4x_2 = 0$.

So, $\cancel{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ are non-trivial solutions.

(7). the only sollution x to the equatio $Dx = 0$ and the reason for having a single solution

$$\overset{D}{\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}} \overset{x}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = 0$$

$$x_1 + 2x_2 = 0$$
$$3x_1 + 2x_2 = 0$$
$$- \quad -$$
$$-2x_1 = 0 \qquad x_1 = 0$$

$|D| \neq 0$. $\quad x + 2x = 0$.

$$3x = 0$$
$$x = 0.$$

$x = 0$. is the solution for it. i.e a trivial solution.

D] $f(x) = x^2 + 3$ $\quad g(x) = x^2$

$$g(x,y) = x^2 + y^2.$$

(1) $f'(x) = 2x$

$$f''(x) = 2.$$

(2) $\frac{\partial q}{\partial x} = 2x$

$\frac{\partial q}{\partial y} = 2y$.

(3) $\nabla g(x, y) = \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right)$

$= (2x, 2y)$.

(4) $\frac{d}{dx} f(g(x))$

$f(x) = x^2 + 3$

replacing $g(x)$, we get

$f(g(x)) = (x^2)^2 + 3$.

$= x^4 + 3$.

$\frac{d}{dx} f(g(x)) = \cancel{3x^3}\ 4x^3$