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Assignment -1

Q1. Geometric image formation

a) $f = 10$

$p = (3, 2, 1)$ $(x, y, z) = (3, 2, 1)$.

Co-ordinate of p when projecting on image

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z} \begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \end{bmatrix}$$

b) When the image plane in the pinhole camera model is behind the centre of projection, an inverted image is projected on the image plane. When the image plane is in front of the center of projection, the image projected on the image plane is not inverted. The model where image plane lies behind the centre of projection is the one which corresponds better to a physical pinhole camera model. We use the other model to mimic the behaviour of a real camera and avoid inversion of the image.

c) When the focal length gets bigger, the projection of the object on the image plane also gets bigger / larger (zoom).
When the distance to the object gets bigger, the projection of the object on the image plane will be smaller.

d) $(x, y) = (1, 1)$
co-ordinates in homogeneous co-ordinates?
2D $\longrightarrow$ 2DH $(x, y, 1)$.
$(1, 1) \longrightarrow (1, 1, 1)$
Another point in 2DH corresponding to the above point is $(2, 2, 2)$.

e) 2DH point $\Rightarrow (1, 1, 2)$, 2D point = ?
2D point $\Rightarrow (\frac{1}{2}, \frac{1}{2})$ $\quad (\because \frac{x}{w}, \frac{y}{w})$
2D point $\Rightarrow (0.5, 0.5)$

f) 2DH point $\Rightarrow (1, 1, 0)$.
Here, w is 0 so it shows direction of the point $(1, 1)$. When $w = 0$, it means point in infinity. So, $(1, 1, 0)$ represents point at infinity.

g) The co-ordinates of the image point in the non-linear projection equation is a 2 vector point. To make it a linear equation in homogeneous coordinates, we simply add a third vector coordinate and assign it to one.

h) $M = K[I|0]$

dimensions of $K \Rightarrow 3\times3$

$I \Rightarrow 3\times3$

$0 \Rightarrow 3\times1$

$M \Rightarrow 3\times4$.

i) $M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ $P = [1, 2, 3]$

2DH 3DH

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \underset{3\times4}{\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1+4+9+4 \\ 5+12+21+8 \\ 1+4+3+2 \end{bmatrix} = \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix}$$

$$2D \Rightarrow \begin{bmatrix} 18/10 \\ 46/10 \end{bmatrix} = \begin{bmatrix} \frac{9}{5} \\ \frac{\cancel{18}\,23}{\cancel{10}\,5} \end{bmatrix} = \begin{bmatrix} 9/5 \\ 23/5 \end{bmatrix}$$

2D point is $\begin{bmatrix} 9/5 \\ 23/5 \end{bmatrix}$.

Q 2. Modeling transformations:

a) $(x, y) \Rightarrow (1, 1)$. Translation by $(2, 3)$.

$t_x = 2 \quad t_y = 3$.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cancel{2}\,1 \\ \cancel{3}\,1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2 \\ 1+3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$x' = 3 \quad y' = 4 \quad \therefore (x', y') = (3, 4)$

b) $(x, y) \Rightarrow (1, 1)$ Scaling by: $(2, 2)$

$S_x = 2 \quad S_y = 2$.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

∴ After scaling, the co-ordinates of the point will be (2, 2).

c) (1, 1), rotation by 45 degrees.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} - 1/\sqrt{2} \\ 1/\sqrt{2} + 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}$$

~~d) (2, 2), rotation by 45 degrees.~~

$$\cancel{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}}$$

$$\cancel{= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\sqrt{2} \end{bmatrix}}$$

d) (1, 1), translation to (2, 2), rotation by 45°

$R_{P,u}(\theta) = T(P)\, R_u(\theta)\, T(-P)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 1 & 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} + 1/\sqrt{2} \\ -1/\sqrt{2} - 1/\sqrt{2} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2-\sqrt{2} \\ 1 \end{bmatrix}$$

$\therefore (x', y') = (2, 2-\sqrt{2})$

e) ~~M = TRP~~

$\underline{P}' = T . R . \underline{P}$

f) $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $s_x = 3$

$s_y = 2.$

Can be represented as

$\underline{P}' = \left[\begin{array}{c|c} S & 0 \\ \hline 0^T & 1 \end{array}\right] \underline{P}$

So, M is a scaling matrix

g) $M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

can be represented as

$\underline{P}' = \left[\begin{array}{c|c} I & t \\ \hline 0^T & 1 \end{array}\right] \underline{P}$

So, M is a translation matrix.

h) $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

M is a scaling matrix.

So, ~~the~~ to reverse the effects of scaling.

we need to do inverse of the scale.

$S^{-1}(S_x, S_y) = S\left(\frac{1}{S_x}, \frac{1}{S_y}\right)$.

$$\therefore \cancel{M^{-1} = \begin{bmatrix} 3/3 & 0 & 0 \\ 0 & 2/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$M^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$M \cdot M^{-1} = I$.

i) $M = R(45)\,T(1,2)$

$p' = RTP$ then $P = \cancel{R}\,T^{-1} R^{-1} p'$

~~So~~ $M = RT$

then $M^{-1} = (RT)^{-1} = T^{-1} R^{-1}$

$$M = \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$R^{-1}(45) = R(-45)$

$T^{-1}(1,2) = T(-1,-2)$

$$\therefore M^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e $M^{-1} = \cancel{R}\,T^{-1}(1,2)\,R^{-1}(45)$

j) vector (1,3).

If two vectors are perpendicular, then their dot product is 0.

If $V_1 \perp V_2$ then $V_1 . V_2 = 0$.

Let $V_1 \Rightarrow (1,3)$ $V_2 \Rightarrow (x,y)$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$x + 3y = 0$. $\therefore$ $x = -3y$.

If $y = 1$, then $x = -3$.

So, $(-3, 1)$ is one of the vectors which is perpendicular to $(1,3)$.

k) As the direction of the point $(2,5)$ $\bar{Q}$ is given, point $(2,5)$ will be $(2,5,0)$. vector $(1,3)$ $\bar{P}$ will be $(1,3,1)$.

Projection of a vector on another vector can be given by in direction of the vector

$$\text{proj}_{\bar{p}}\bar{q} = \frac{\bar{p}.\bar{q}}{|\bar{q}|^2} \bar{q} = \frac{1.2 + 3.5}{|\sqrt{2^2+5^2}|^2} \bar{q} = \frac{17}{|\sqrt{29}|^2}(2,5)$$

$$= \frac{17}{29}(2,5).$$

Q 3. General camera model.

a) So far, the projection equation that we have seen ~~to~~ relates 3D points to 2D projections all in the same co-ordinate system. But, world, camera ~~or~~ and image, all three have different co-ordinate systems, Image co-ordinates are in units of pixels & camera has units in mms. As we want to relate 3D points in world co-ordinate system to 2D points in image co-ordinate system, we need a general projection matrix that uses different co-ordinate system for camera & image...

b) $M = TR$ $\quad p^{(c)} = M_{c \leftarrow w}\, p^{(w)}$

$\cancel{M_{w}}$ $M_{c \leftarrow w} = (TR)^{-1}$

$= R^{-1} T^{-1}$

$$= \left[\begin{array}{c|c} R & 0 \\ \hline 0 & 1 \end{array}\right]^{-1} \left[\begin{array}{c|c} I & T \\ \hline 0 & 1 \end{array}\right]^{-1} \qquad R^{-1} = R^T, \quad T^{-1} = -T$$

$$= \left[\begin{array}{c|c} R^T & 0 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} I & -T \\ \hline 0 & 1 \end{array}\right] = \begin{bmatrix} R^T & -R^T T \\ 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{c|c} R^* & T^* \\ \hline 0 & 1 \end{array}\right]$$

R^* & T^* are translation & rotation wrt to world.

c) $\hat{x}, \hat{y}, \hat{z}$

Rotation of camera w.r.t the world is

$$R^T = \begin{bmatrix} \hat{x}_c \\ \hat{y}_c \\ \hat{z}_c \end{bmatrix}$$

d) R^* & T^* are extrinsic camera parameters where R^* is rotation & T^* is translation w.r.t the world

$R^* = R^T$ $T^* = -R^T T$.

e) Transformation matrix M will be

$$M = \begin{bmatrix} ku & 0 & u_0 \\ 0 & kv & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} ku & 0 & 512 \\ 0 & kv & 512 \\ 0 & 0 & 1 \end{bmatrix}$$

whole eqn.

$$p^{(i)} = \begin{bmatrix} ku & 0 & 512 \\ 0 & kv & 512 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} [I|0] \left[\begin{array}{c|c} R^* & T^* \\ \hline 0 & 1 \end{array}\right]$$

f) $K^* \rightarrow$ intrinsic parameters

where (ku, kv) are pixels per mm in X & Y direction respectively & (u_0, v_0) ~~is optical center~~.

R^* and $T^* \rightarrow$ extrinsic parameters

rotation translation.

g) A 2D skew parameter in the camera model can change the shape of an existing object in a 2D plane. The object size can be changed or the image can be stretched along a particular direction. It can also be used to make model more accurate.

h) Due to radial distortion, straight lines in real world appear to be curved in the image. The points in the real world are moved in the radial direction from their correct position in the image.

$$\lambda = 1 + \underbrace{k_1}_{\text{linear distortion coeff}} d + \underbrace{k_2}_{\text{quadratic distortion coeff}} d^2$$

d = distance from the center

$$P_E^{(I)} = \begin{bmatrix} 1/\lambda & & \\ & 1/\lambda & \\ & & 1 \end{bmatrix} K^* [R^* | T^*] \underline{P}^{(W)}$$

λ, d will make camera model to ~~scale~~ shrink away from the center.
The bigger the distance (d), the more will be the distortion.

(i) Weak perspective camera will not induce any perspective distortions. In a real cameras, distant objects appear smaller. In weak perspective camera, there is no foreshortening & Distant objects don't appear smaller, they appear the same size.
The weak perspective camera is correct when depth variation in the scene is small compared with distance from camera.
Affine model is similar to weak perspective model and is used as a computational model. Affine camera is a zero-order or a first order approximation of full perspective projection. It is valid when the depth variation of the object is small compared to the distance from camera to the object.

Q4. Color & photometric image formation

a) Surface radiance ($L(P)$) is power of light per unit area reflected from surface.
Image radiance ($E(P)$) is power of light per unit area received at the image.

b) $$E(P) = L(P) \frac{\pi}{4} \left(\frac{d}{f}\right)^2 (\cos\alpha)^4$$

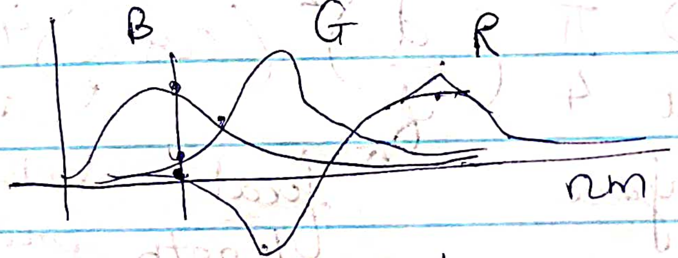
$E(P)$: light at image
$L(P)$: light at surface
f: focal length
α: angle between principal axis & surface normal

c) Albedo of a surface is the fraction of the incident sunlight that the surface reflects. The surface absorbs the radiation that is not reflected.

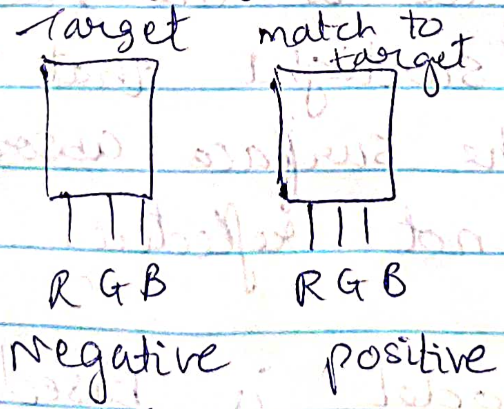
d) The RGB color model is based on the theory that all colours which are visible ~~to~~ can be created using red, green and blue. The human vision is sensitive to these three colours & see everything in RGB

e) The colors along the line that connects (0,0,0) with (1,1,1)^T are all the gray colors between black and white also called as ~~gr~~ gray axis.

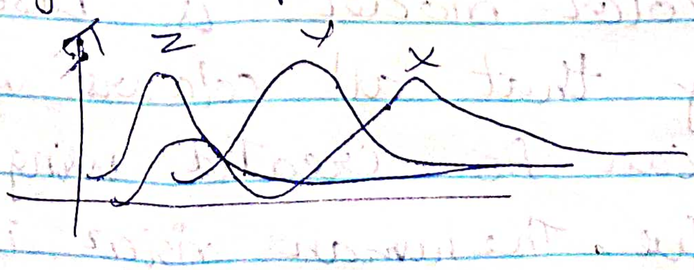
f) RGB colors are mapped to real world colors using CIE tables. The wavelength of colors are ~~mat~~ mapped to RGB intensities.



The RGB intensities are matched to the target colour. Negative values are added to the target to help match.



We can use X, Y, Z instead of R, G, B so that negative weights are not necessary



CIE XYZ system

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.813 & 0.011 \\ 0.000 & 0.010 & 0.990 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}.$$

g) For any given Y value, the XZ plane will contain all possible chromaticities at that luminance. The Y luminance component represents the brightness of a color. The luminance component represents the intensity of the image and looks like a gray scale version.

h) The LAB colour space allows you to quantify the color utilizing an independent color space. The LAB color space can be used to make colors in your photos look more vibrant and lively without moving the saturation slider.