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CS512 Spring 2022.

Assignment 5

Q1. Camera Calibration :

Q1a) $p = MP$

In forward projection, we have a given world point P and ~~translation~~ projection matrix M and we have to convert it to image point p ~~(3DH - 2DH)~~.

$$\underbrace{p}_{\substack{\text{image pt}\\ \text{2DH}}} = \underbrace{M}_{\substack{\text{projection}\\ \text{matrix}\\ 3\times4}}\underbrace{P}_{\substack{\text{world point}\\ \text{3DH}}}$$

In camera calibration, we estimate the intrinsic and extrinsic parameters of the camera.

Given corresponding points in real world and image, find parameters

corresponding points $\{P_i\}_{i=1}^{m} \longleftrightarrow \{p_i\}_{i-1}^{m}$

In reconstruction, we are given an image point & we have to compute the world point from it. While doing 3DH to 2DH, we loose a lot of info so reconstructing 3DH from 2DH is difficult.

Therefore, according to the above definitions, forward projection is the easiest whereas reconstruction is the most difficult.

(2)

(Q1.b)

Ans. For camera calibration, we need ~~tw~~ ~~correspoint~~ corresponding world & image points which will be our input for the calibration algorithm.

given corresponding points. $\{p_i\}_{i=1}^{m}$ pixels $\longleftrightarrow$ $\{P_i\}_{i=1}^{m}$ ~~world~~ meters

Then we need to find the intrinsic and extrinsic parameters like

K^* : $f \rightarrow$ focal length, $u_0, v_0 \rightarrow$ optical center

$s \rightarrow$ scale, $k_u k_v \rightarrow$ skew

$R^* T^*$: $R \rightarrow$ rotation, $T \rightarrow$ translation

Q1c)

Ans. 1) First step in non-coplanar calibration is estimating the projection matrix

$$P_i' = M P_i$$

$$\begin{bmatrix} x_i' \\ y_i' \\ w_i' \end{bmatrix} = \begin{bmatrix} - & m_1^T & - \\ - & m_2^T & - \\ - & m_3^T & - \end{bmatrix} P_i$$

For m points

$$\begin{bmatrix} P_1^T & 0 & -x_1 P_1^T \\ 0 & P_1^T & -y_1 P_1^T \\ \vdots \\ P_m^T & 0 & -x_m P_m^T \\ 0 & P_m^T & -y_m P_m^T \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$$Ax = 0$$

To solve $Ax=0$, use SVD

$$A = UDV^T$$

2) Next find parameters from M

$$M = K^* [R^* | T^*] = \rho \hat{M}$$

$$[K^* R^* | K^* T^*] = \rho \hat{M}$$

$$\begin{bmatrix} \alpha_u r_1^T + s r_2^T + u_0 r_3^T & \\ \alpha_v r_2^T + v_0 r_3^T & K^* T^* \\ r_3^T & \end{bmatrix} = \rho \hat{M} = \rho \begin{bmatrix} - a_1^T - & \\ - a_2^T - & b \\ - a_3^T - & \end{bmatrix}$$

$$\rho = \frac{1}{|a_3|} \qquad u_0 = |\rho|^2 a_1 . a_3$$

$$v_0 = |\rho|^2 a_2 . a_3 \qquad \alpha_v = \sqrt{|\rho|^2 a_2 . a_3 - v_0^2}$$

$$s = \frac{1}{\alpha_v} |\rho|^4 (a_1 \times a_3) \cdot (a_2 \times a_3)$$

$$\alpha_u = \sqrt{|\rho|^2 a_1 \cdot a_1 - s^2 - u_0^2}$$

$$K^* = \begin{bmatrix} \alpha_u & s & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\epsilon = \mathrm{sgn}(b_3)$$

$$T^* = \epsilon |\rho| (K^*)^{-1} b$$

$$r_3 = \cancel{\rho |\rho|} \; \epsilon |\rho| a_3$$

$$r_1 = \frac{1}{\alpha_v} |\rho|^2 a_2 \times a_3$$

$$r_2 = r_3 \times r_1$$

$$R^* = [r_1^T \; r_2^T \; r_3^T]^T$$

Q 1. C)

Ans. Steps in non-coplanar calibration algorithm:-

① Find projection matrix M

② Find parameters (extrinsic, intrinsic) from M.

Q . d)

Ans. $P = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 18 \\ 14 \\ 7 \end{bmatrix}$

$\therefore \quad x = \dfrac{18}{7} \qquad y = \dfrac{14}{7}$

$\therefore$ 2D image co-ordinates are $\begin{bmatrix} 18/7 \\ 2 \end{bmatrix}$

(3)

Q 1.e)

Ans. $(1,2,3) \longleftrightarrow (100, 200)$

Projection matrix $M = ?$

~~A~~ = ?

$$\begin{bmatrix} P_1^T & 0 & -x_1 P_1^T \\ 0 & P_1^T & -y_1 P_1^T \\ \vdots & & \\ \vdots & & \\ P_m & 0 & -x_m P_m^T \\ 0 & P_m^T & -y_m P_m^T \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad Ax = 0$$

substituting the given points in above matrix ~~M~~

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & -100 & -200 & -300 & -100 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 1 & -200 & -400 & -600 & -200 \end{bmatrix}$$

Q 1.f)

Ans. To find a unique solution for M, we need atleast 6 points. There are 11 unknown parameters which are ~~a1, a2, a3, b~~, $r_1, r_2, r_3, T^*, K^*, \theta, \alpha_u, \alpha_v, u_0, v_0, s$

We need atleast 12 equations for this so we need 6 points to get 12 equations.

We estimate the projection matrix as mentioned below:-

Q 1.g)

$$\begin{bmatrix} P_1^T & 0 & -x_1 P_1^T \\ 0 & P_1^T & -y_1 P_1^T \\ \vdots & & \\ P_m^T & 0 & -x_m P_m^T \\ 0 & P_m^T & -y_m P_m^T \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \boxed{Ax = 0}$$

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Then, to solve this, we use SVD
$A = UDV^T$
Solution = column of V belonging to zero singular value

$$\hat{\alpha} = \begin{bmatrix} \hat{m}_1 \\ \hat{m}_2 \\ \hat{m}_3 \end{bmatrix} \Rightarrow \hat{M} = \begin{bmatrix} - \hat{m}_1^T - \\ - \hat{m}_2^T - \\ - \hat{m}_3^T - \end{bmatrix}$$

The solution is not unique if $A\hat{x} = 0$
We also have $A(\rho \hat{x}) = 0 \Rightarrow \rho\hat{x}$ is a solution.

Q 1.g)
Ans.

$$\hat{M} = \begin{bmatrix} - \hat{m}_1^T - \\ - \hat{m}_2^T - \\ - \hat{m}_3^T - \end{bmatrix} \quad 3\times 3 \text{ matrix}$$

The solution for $\hat{M}$ is not unique
So, we find ρ so that

$$M = K^* [R^* | T^*] = \rho \hat{M} \qquad \rho = ?$$

We find parameters & ρ from M

$$K^* [R^* | T^*] = \rho \hat{M}$$
$$[K^* R^* \mid K^* T^*] = \rho \hat{M}$$

$$\underbrace{\begin{bmatrix} \alpha_u r_1^T + s r_2^T + u_0 r_3^T & \\ \alpha_v r_2^T + v_0 r_3^T & K^* T^* \\ r_3^T & \end{bmatrix}}_{\text{unknowns}} = \rho \hat{M} = \underbrace{\rho}_{\text{unknowns}} \begin{bmatrix} - a_1^T - & \\ - a_2^T - & b \\ - a_3^T - & \end{bmatrix}$$

~~unknowns~~

(4a).

Ans) contd. To extract parameters using orthogonality of $r_1\ r_2\ r_3$

$r_1 . r_2 = 0$ $r_2 . r_3 = 0$ $r_1 . r_3 = 0$

$r_1 \times r_2 = r_3$ $r_2 \times r_3 = r_1$ $r_3 \times r_1 = r_2$

(5)

Q1.h)
Ans. Given $\{P_i\}_{i=1}^{m} \leftrightarrow \{p_i\}_{i=1}^{m}$ and estimated

$$M = \begin{bmatrix} - m_1^T - \\ - m_2^T - \\ - m_3^T - \end{bmatrix}$$

The quality of fit is assessed using :-

$$E = \frac{1}{m}\sum_i \left(\left\| x_i - \frac{m_1^T P_i}{m_3^T P_i} \right\|^2 + \left\| y_i - \frac{m_2^T P_i}{m_3^T P_i} \right\|^2 \right)$$

Distance between known & predicted positions.

Q1.i)
Ans. ① For planar camera calibration, we first need to estimate 2D homography (projective map) between calibration plane and image (for several images)
② Estimate intrinsic parameters
③ Compute extrinsic parameters for view of interest.

Planar camera calibration solves 2DH points whereas non-planar solves 3DH points. ~~For~~ For planar, points should be on the same plane whereas for non-planar, points will be on different planes. For planar, single image with different views is used for camera calibration whereas 3D image is used for calibration in non-planar

(6)

Q1. j)

Ans. Homography (H) 2D projective map is used for transformation from one plane to other and it is 3×3. Projection matrix is used to project 3DH points to get 2DH image points and it is 3×4.

~~For pr~~

$$\underset{2DH}{P_i'} = K^*\underset{3\times4}{[R^* \mid T^*]}\ \underset{3DH}{\underline{P_i}}$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = K^* [r_1\ r_2\ r_3\ T^*] \begin{bmatrix} x_i \\ y_i \\ 0 \\ 1 \end{bmatrix}$$

To get a 2D projective map we assume

$$\{\underline{P_i}\}_z = 0 \iff \underline{P_i} = (x_i, y_i, 0)$$

z coordinate is zero

$$\therefore P_i' = K^*(x_i r_1 + y_i r_2 + T^*)$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \underbrace{K^* [r_1\ r_2\ T^*]}_{3\times3} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

2D projection map. Homography   2DH

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = H\underline{P_i}^* = \begin{bmatrix} - h_1^T - \\ - h_2^T - \\ - h_3^T - \end{bmatrix} \underline{P_i}^*$$

$$x_i = \frac{u_i}{w_i} = \frac{h_1^T P_i^*}{h_3^T \underline{P_i}^*} \Rightarrow h_1^T \underline{P_i}^* - x_i h_3^T \underline{P_i}^* = 0$$

$$y_i = \frac{v_i}{w_i} = \frac{h_2^T \underline{P_i}^*}{h_3^T \underline{P_i}^*} \Rightarrow h_2^T \underline{P_i}^* - y_i h_3^T \underline{P_i}^* = 0.$$

Q 2. Camera calibration 2.

Ans a)

$(1,2) \leftrightarrow (3,4,5)$

Projection matrix = ?

$$\begin{bmatrix} P_1^T & 0 & -x_1 P_1^T \\ 0 & P_1^T & -y_1 P_1^T \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 & 5 & 1 & 0 & 0 & 0 & 0 & -3 & -4 & -5 & -1 \\ 0 & 0 & 0 & 0 & 3 & 4 & 5 & 1 & -6 & -8 & -10 & -2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix}.$$

Q 2. b)

Ans.

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

$$\rho \hat{M} \equiv K^* [R^* \mid T^*]$$

$$\rho \hat{M} \equiv [K^* R^* \mid K^* T^*]$$

$$\left[\begin{array}{c|c} \alpha_u r_1^T + s r_2^T + u_0 r_3^T & \\ \alpha_v r_2^T + v_0 r_3^T & K^* T^* \\ r_3^T & \end{array}\right] \equiv \rho \left[\begin{array}{c|c} -a_1^T- & \\ -a_2^T- & b \\ -a_3^T- & \end{array}\right]$$

$u_0 = |\rho|^2 a_1 . a_3$

$v_0 = |\rho|^2 a_2 . a_3$.

$$M = \left[\begin{array}{c|c} -a_1 T- & \\ -a_2 T- & b \\ -a_3 T- & \end{array}\right]$$

$\therefore \quad a_1^T = [1 \quad 2 \quad 3]$

$a_2^T = [2 \quad 3 \quad 4]$

$a_3^T = [3 \quad 4 \quad 5]$

$|\rho|^2 = \dfrac{1}{|a_3|^2}$

$|a_3|^2 = \left(\sqrt{3^2+4^2+5^2}\right)^2$

$|a_3|^2 = \left(\sqrt{9+16+25}\right)^2$

$|a_3|^2 = \left(\sqrt{50}\right)^2 = 50$

$\rho^2 = \dfrac{1}{50}$

$\therefore \quad u_0 = \rho^2 \, a_1 . a_3$

$= \dfrac{1}{50} (1 \quad 2 \quad 3) . (3 \quad 4 \quad 5)$

$= \dfrac{1}{50} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} . \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

$= \dfrac{1}{50} (3 + 8 + 15)$

$u_0 = \dfrac{26}{50}$

$v_0 = \rho^2 \, a_2 . a_3$

$= \dfrac{1}{50} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} . \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

$= \dfrac{1}{50} (6 + 12 + 20)$

$v_0 = \dfrac{38}{50}$

$\therefore \quad (u_0, v_0) = \left(\left(\dfrac{26}{50}\right), \left(\dfrac{38}{50}\right) \right)$

Q2.c)

Ans. $(1,2) \longleftrightarrow (3,4,5)$

image p world.

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \begin{array}{l} \rightarrow m_1^T \\ \rightarrow m_2^T \\ \rightarrow m_3^T \end{array}$$

$$E = \frac{1}{m} \sum_i \left(\left\| x_i - \frac{m_1^T p_i}{m_3^T p_i} \right\|^2 + \left\| y_i - \frac{m_2^T p_i}{m_3^T p_i} \right\|^2 \right)$$

$m = 1$ as number of points is 1.

$$m_1^T p = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}$$

$$= 3 + 8 + 15 + 4 = 30$$

$$m_2^T p = \begin{bmatrix} 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}$$

$$= 6 + 12 + 20 + 5 = 43$$

$$m_3^T p = \begin{bmatrix} 3 & 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}$$

$$= 9 + 16 + 25 + 6$$

$$= 56.$$

$$E = \left\| 1 - \frac{30}{56} \right\|^2 + \left\| 2 - \frac{43}{56} \right\|^2$$

$$= \left\| \frac{56 - 30}{56} \right\|^2 + \left\| \frac{112 - 43}{56} \right\|^2$$

$$= \left\| \frac{26}{56} \right\|^2 + \left\| \frac{69}{56} \right\|^2$$

$$= (0.4642)^2 + (1.2321)^2$$

$$= 0.2155 + 1.5181$$

$$E = 1.7336$$

Q2.d) $I + Q$.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$I + Q =$
i.e. R^*
$$\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^* = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$T^* = -RT^*$
$\therefore T = -(R^*)^T T^*$

$\therefore$ Rotation & Translation = w.r.t world
$$- \underset{4\times 4}{\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}}$$

$$= - \begin{bmatrix} 6 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -2 \\ -3 \\ -1 \end{bmatrix}$$

Q2.c) ~~I~~ $(1,2) \longleftrightarrow (3,4,0)$.

$\underset{\text{Image}}{(1,2)}$ $\quad$ $\underset{\text{world}}{(3,4,0)}$

world point $= \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$

2DH.

$$\begin{bmatrix} \underline{P}_1^{*T} & 0^T & -x_1 \underline{P}_1^{*T} \\ 0 & \underline{P}_1^{*T} & -y_1 \underline{P}_1^{*T} \\ \vdots & & \\ \underline{P}_m^{*T} & 0^T & -x_m \underline{P}_m^{*T} \\ 0 & P_m^{*T} & -y_m \underline{P}_m^{*T} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 & 0 & 0 & 0 & 0 & -3 & -4 & 0 \\ 0 & 0 & 0 & 3 & 4 & 0 & -6 & -8 & 0 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Q3. Multiple view geometry
a)

Ans a) Dense stereo matching is finding corresponding points for each pixel from two or more images at slightly different viewpoints. This produces more points and is correlation, SSD based. Sparse stereo matching is feature based and can handle large disparities. So, advantage of sparse stereo is that it can used in frames with very high disparity between each frame. Advantage of dense approach is that it would work for frames with very less disparities, ~~but would fail for frames, with high disparity~~ The disadvantage of sparse stereo approach is that it would not work on frames with little or no difference between them. The disadvantage of dense approach is that it would fail for frames with high disparities.

Ans. b) Correlation can be calculated as

$$\Psi(w_1, w_2) = \sum_i w_1(x_i, y_i) \cdot w_2(x_i, y_i)$$

SSD can be calculated as

$$\Psi(w_1, w_2) = -\sum_i (w_1(x_i, y_i) - w_2(x_i, y_i))^2$$

where high value means good correspondence. To remove dependency on absolute value, we normalize window values (subtract mean and divide by standard deviation)

NCC computes normalized ~~cor~~ cross correlation of two windows whereas SSD computes the

sum of square distance between the windows.
If the search space is allowed to be the entire image, there might be mis mathing points since the chances of error are more.
The risk is that the chances of error making increases as there will be a lot of options corresponding to ~~the~~ a point in an image.
The search space is hence reduced by only considering epipolar line instead of whole image.

Q3.c)

Ans.
$$z = f\frac{T}{d}$$

$$x_l = (100, 200)$$
$$x_r = (103, 200)$$
$$f = 10$$
$$T = 100$$

$$\text{depth } z = f\frac{T}{d}$$

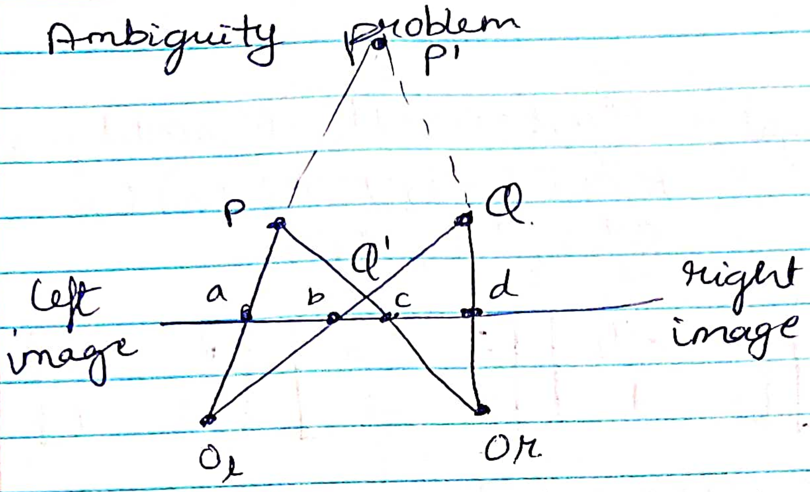
$$d = 103 - 100 = 3.$$

$$\therefore \text{depth } z = \frac{10 \times 100}{3}$$

$$\text{depth } z = \frac{1000}{3} = 333.33$$

Q3. d

Ans d) Ambiguity problem



A small change / error in correspondence can make large errors in reconstruction So, even if we make a mistake with two points, the incorrect points are very from the original points.

~~Idealt~~

Ideally, if correspondence is done correctly then points a & c are matched & resulting P point is obtained. same is done when b & d are matched and we get Q. But if we make a mistake & match a with d & b with c, we get wrong points P' & Q' during reconstruction.

Correct points : P, Q

Incorrect : P', Q'

Q 3.e) Rotation Translation

Ans e) R_L, T_L

R_r, T_r

$M_{left \leftarrow right} = M_{left \leftarrow world} \; M_{world \leftarrow right}$

$$M_{L \leftarrow r} = \cancel{R_L^T \, T(-T_L) \, T(T_r) \, R_r}$$

$$= R_L^T \, T_L^{-1} \, T_r \, R_r$$

$$= \begin{bmatrix} R_L^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & -T_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & T_r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_r & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_L^T & -R_L^T T_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_r & T_r \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_L^T R_r & R_L^T (T_r - T_L) \\ 0 & 1 \end{bmatrix}$$

$$= \boxed{R = R_L^T R_r} \qquad \boxed{T = R_L^T (T_r - T_L)}$$

Multiple view geometry 2.

Q 4.a)
Ans.

$f = 10$ mm

$T = 20$

disparity $= 30$ mm

$Z = f \frac{T}{d}$

$\therefore \quad Z = 10 \times \frac{20}{30}$

$\Rightarrow Z = 6.66$ mm

Q 4.b)
Ans.

$A \equiv (1, 2, 3)$

$B \equiv (2, 3\,4)$

$$A \times B = [A]_{\times} B \equiv \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Rank 2 skew symmetric matrix

$$\text{Matrix} \equiv \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & \cancel{-}1 & 0 \end{bmatrix}$$

Above matrix if multiplied by B will result in the cross product $A \times B$.

Q 4.c)
Ans.

$$F = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$P_l = (1, 2)$$
$$P_r = (2, 3)$$

$$P_r^T F p_l = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 17 & 23 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= 11 + 34 + 23$$

$$P_r^T F p_l = 68$$

Q4. d)

$P_L = (1,2)$
$P_r = (2,3)$

∴ Fundamental matrix

$= \bar{P}_r^T \left(K_r^{*-T} E \, K_l^{*-1}\right) \bar{P}_L$

$P_r^T F P_L = 0.$

$\searrow$ fundamental matrix

$$[x_i, y_i, 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = 0.$$

$$[2, 3, 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 0.$$