Identification in detector data of overlapping gravitational-wave signals from compact binary coalescences

Project Report

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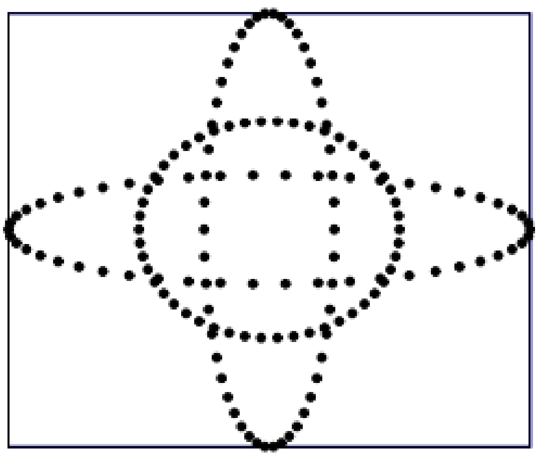
1 Acknowledgement

I would like to take this opportunity to express my sincere gratitude and appreciation to Prof.Shasvath Kapadia for his guidance and support throughout the duration of the "Identification in detector data of overlapping gravitational-wave signals from compact binary coalescences" project.I would also like to extend my thanks to the pioneers in the subject without whom I would not have had the purpose for the project,and for creating the references that played an important role in the completion of the project. Finally, I would like to acknowledge the support and resources provided by the institution, which have enabled us to undertake this project and helped us achieve our research objectives.

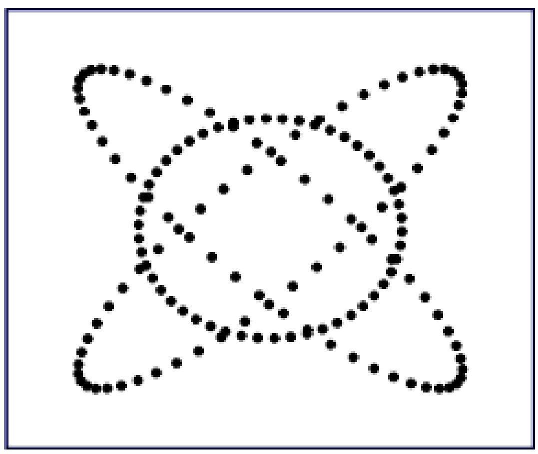
2 Introduction

Gravitational waves (GW) are ripples in spacetime. They are caused by massive objects moving with extreme acceleration. They are extremely weak to detect as a direct consequence of the weak interaction of gravity compared to other interactions.The sources of GW are compact binary coalescence. Compact object is defined as an object for which the ratio ($\frac{GM}{Rc^2}$) is a significant fraction of unity. When these two bodies orbit each other,energy is radiated in the form of GW,and the orbit starts to decay, they continue orbiting each other with great velocities till they merge into one single massive object. The three phases of such an event are : Inspiral, Merger, and Ringdown. The Compact Binary Coalescences include: Binary Black Hole(BBH),Binary Neutron Star(BNS),Neutron Star-Black Hole(NS-BH).

GW has two polarisations: a plus and a cross. The polarisation refers to change in the geometric shape of spacetime due to its stretching and squeezing as a gravitational wave passes through. The polarisations can be explained by how the gravitational wave interacts with a circular ring of masses, the plus polarisation causes the circular ring of masses to oscillate in perpendicular direction and the cross polarisation causes it to oscillate diagonally by an angle of 45°.



(a) Plus Polarisation [2]



(b) Cross Polarisation [2]

Figure 1

The detection of gravitational waves is done by LIGO detectors,which operate on the principle of interferometry. GWs produce strain on material objects.The strain is of the order of around 10^{-22}.The amplitude of the GW decreases with increasing distance of the source. The data from the detector is a source of various noise. The noise is roughly of the same order of the GW signal. The data as a function of time can be given as:

$$s(t) = n(t) + h(t)$$

where this h(t) is a linear combination of the plus and cross polarisation waves given by :

$$h(t) = f_+ \times h_+ + f_\times \times h_\times$$

f_+ and $f_\times$ are called the antenna function patterns ,they refer to the sensitivity of the detector. They have different shapes depending upon the location of the source of the gravitational waves. A technique called 'Matched Filtering' is used to detect these gravitational wave signals from binary mergers.

The central idea of this project is that if the detector data contains an overlapping pair of signals i.e. gravitational wave signals from two binary mergers separated in their merger time by certain critical separation time,the recovery of these signals can be improved by subtracting one signal.

The recovery process of gravitational wave signals is performed using a pipeline developed in Python using the package PYCBC,which provides the tools necessary for recoveries.

3 Matched Filtering

In matched filtering technique the detector data is correlated with a bank of templates and Signal-to-Noise Ratio(SNR) is computed.Template waveform is a known signal.Since these template waveforms are well modelled ,the search pipeline uses matched filtering technique to recover these signals in the detector noise.The matched filter is constructed by projecting the detector data signal against the template waveform.The matched filter consists of a weighted inner product in the frequency domain to construct the SNR time series $\rho(t)$.The inner product is given by [1]:

$$(s|h)(t) = 4Re\int_{f_{low}}^{f_{high}} \frac{\tilde{s}(f)\times\tilde{h^{*}}(f)}{S_n(f)} e^{2\pi ift}\, df$$

where $\tilde{s}(f)$ denotes the Fourier transformed detector data,given by [1]

$$\tilde{s}(f) = \int_{-\infty}^{+\infty} s(t)e^{-2\pi ift}\, dt$$

and $\tilde{h}(f)$ denotes the Fourier transformed template waveform and $S_n(f)$ is the PSD (Power Spectral Density) of detector noise.

This matched filtering is a cyclic process.The template is slid across the entire detector data and dot product is calculated at each point,if the detector data contains a signal same as that of the template waveform, the dot product will be maximum at that time stamp when integrated over and the SNR time series will have a maxima at that time stamp.

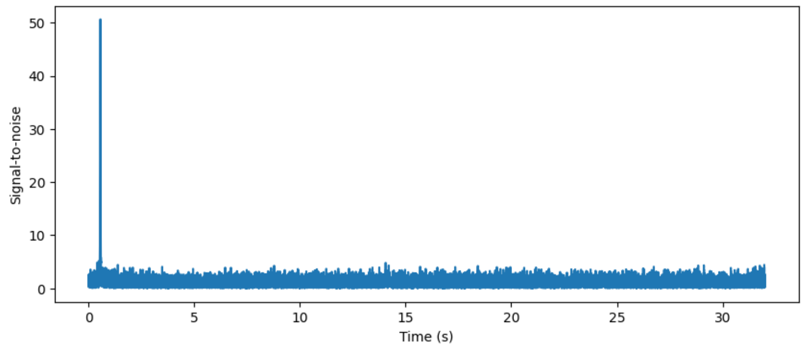


Figure 2: Signal-to-Noise Ratio Time Series

4 Chi-Squared Test

Triggers (maxima in SNR time series)generated by matched filtering the data against the template-bank* are subject to a chi-squared that determines if time-frequency distribution of power in the data is in agreement with the expected power in the matching template waveform.To construct this test ,the template is split into p frequency bins. These bins are constructed so that each contributes an equal amount of power to the total matched filter SNR. The matched filter SNR,ρ_i, is constructed for each of these p bins. For a real signal, ρ_i should contain 1/p of the total power. The χ^2 statistic compares the expected to measured power in each bin as per [1]

$$\chi^2 = p \times \sum_{i=1}^{p} [(\frac{\rho_{cos}{}^2}{p} - \rho_{cos,i})^2 + (\frac{\rho_{sin}{}^2}{p} - \rho_{sin,i})^2]$$

where ρ_{cos}^2 and ρ_{sin}^2 are the SNRs of two orthogonal phases of the matched filter. Lower mass binary systems such as binary neutron stars, lose energy to gravitational waves more slowly than higher mass systems. As a consequence, waveforms of lower mass systems are longer, having more gravitational wave cycles in the sensitive band of the detector. The number of bins(p) can be specified as a function of the intrinsic parameters of the template waveform. This allows the search to utilise more bins in the chi-squared test for longer templates, hence making the test more effective. Larger values of χ^2 indicate a higher likelihood of noise as opposed to a signal. For signals,the reduced chi-squared [1]

$$\chi^2{}_r = \frac{\chi^2}{(2p-2)}$$

should be near unity.

The re-weighted SNR called as the new SNR is calculated by taking in the arguments as old SNR time series and chi-squared; this new SNR is the refined version of the previous SNR time series as it eliminates any peaks which are not caused due to the presence of the signal.

5 Methodology

Procedure followed using PYCBC pipeline:

1. Generating Noise : In order to replicate the LIGO's data,a gaussian noise is generated and is coloured with a given PSD.

2. Generating Strain : Gravitational wave signals i.e. the strain waveform is generated using different models available in the package.
   For eg: BNS signal is generated using model 'TaylorF2'(Fig (b)),BBH signal is generated using model 'IMRPhenomPv2'(Fig (c)).

3. Combining the signals : The signals generated from two different binary mergers are combined in such a way that they form an overlapping pair of signals separated in their merger times by some critical separation time (t_c)(Fig (d)).

4. Injecting strain into noise : The overlapping signal is injected into the noise ,this thus forms the detector data (Fig (a)).

5. Generating Template : A template waveform i.e. a known signal is generated which is used for matched filtering. This known signal is being searched for in the detector data. In real,the signal present in the data is completely unknown,so a dense search needs to be implemented, for which a template bank is constructed which consists of template waveforms with different intrinsic parameters such as mass,distance,etc (Fig (e,f)).

6. Generating SNR Time Series : Matched Filtering is used to correlate the template and the detector data,if there is a signal present in the data,the SNR times series will have maxima corresponding to the template (Fig (g,h,i,j)).

7. Chi-Squared Test : To check the goodness of the fit of the template with the detector data ,this test is performed (Fig (k,l,m,n)).

8. Calculating New SNR : The re-weighted SNR time series is calculated by considering the previously generated SNR series and the chi-squared. This is the refined version of the SNR time series (Fig (o,p)).

9. Subtracting first detected signal : The first recovered signal from the overlapping pair of signal is subtracted from the data.

10. Recovery of second signal : After subtracting the first detected signal from the data,matched filtering is again carried out to recover the second signal present.

The code for the above methodology is available at [3]

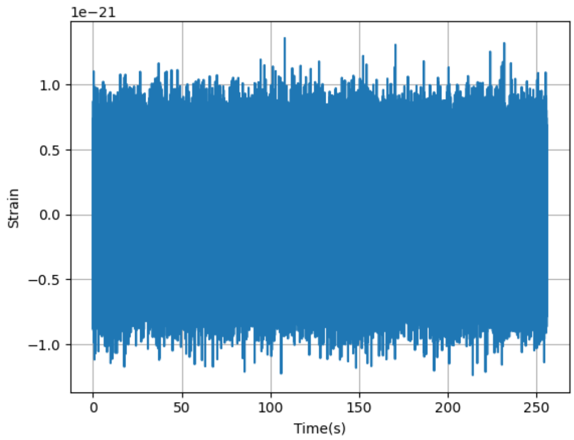


Figure (a) : Detector Data (Noise + Signal)

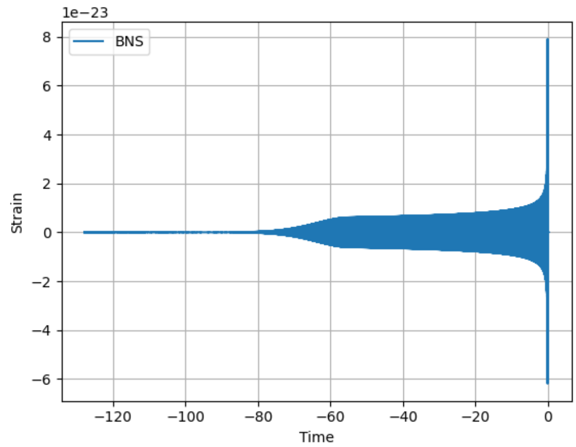


Figure (b) : BNS waveform

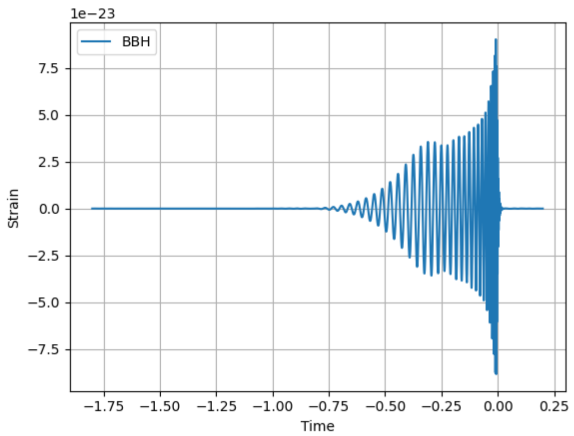


Figure (c) : BBH waveform

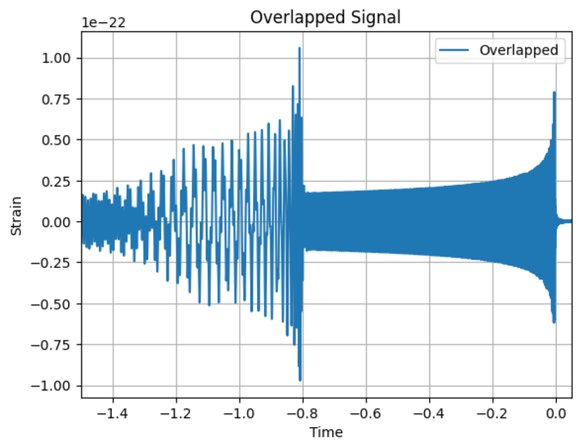


Figure (d) : Overlapped Signal

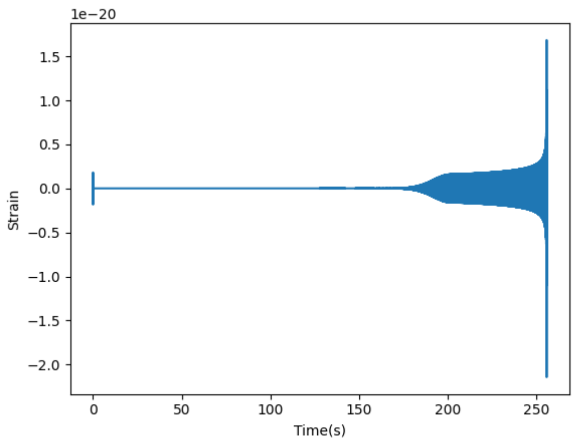


Figure (e) : BNS Template

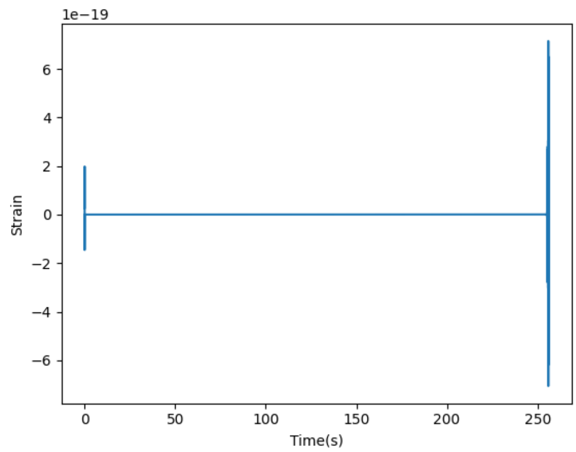


Figure (f) : BBH Template

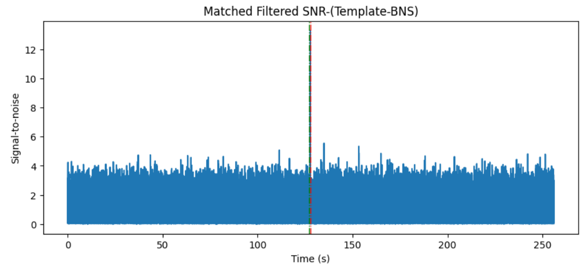


Figure (g) : SNR Time Series using BNS template for matched filtering

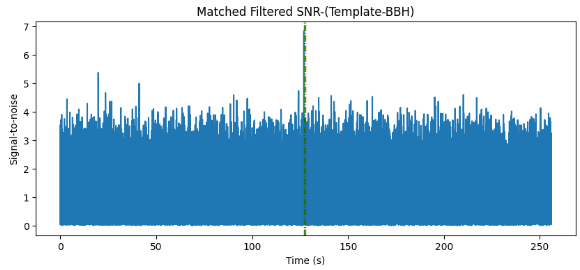


Figure (h) : SNR Time Series using BBH template for matched filtering

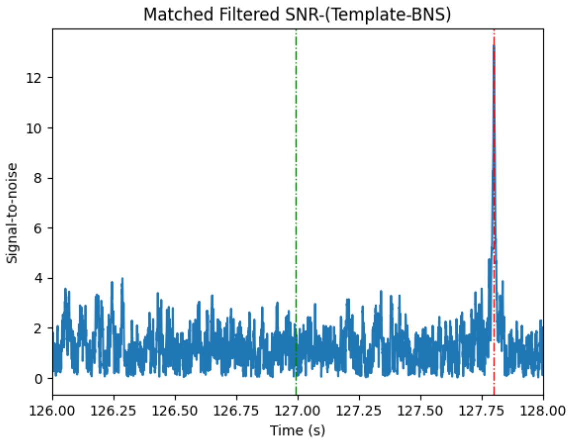


Figure (i) : Zoomed version of Figure(g). Green line & Red line denote the timestamp of injected BBH Signal & BNS Signal respectively.

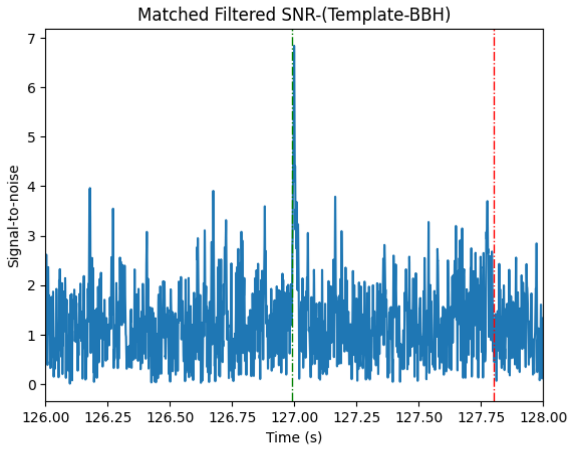


Figure (j) : Zoomed version of Figure(h). Green line & Red line denote the timestamp of injected BBH Signal & BNS Signal respectively.

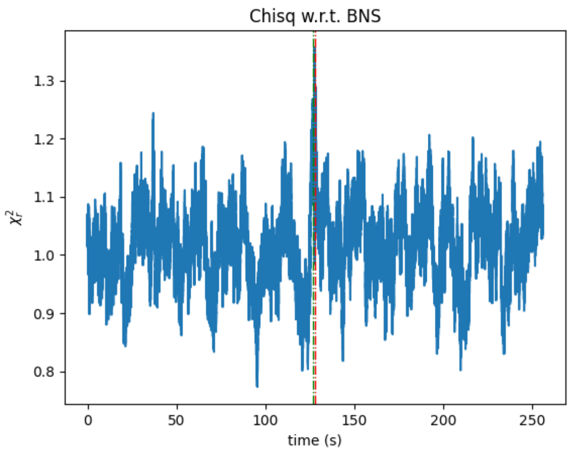


Figure (k) : Chi-Square w.r.t BNS template

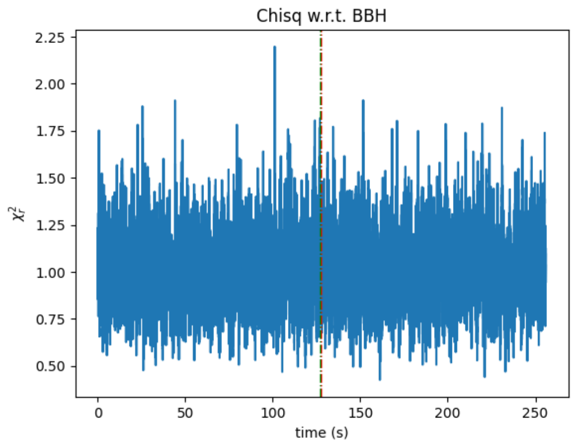


Figure (l) : Chi-Square w.r.t BBH template

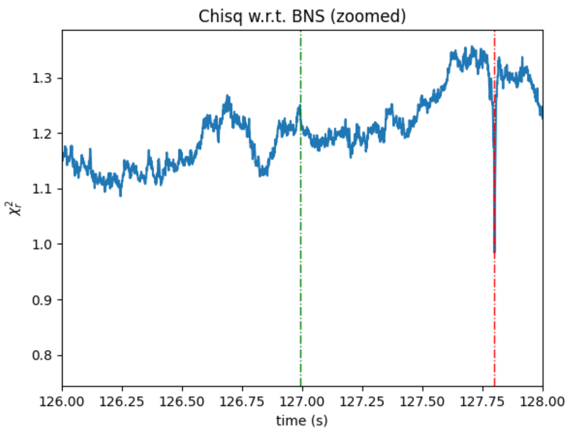


Figure (m) : Zoomed version of Figure(k).
Green line & Red line denote the timestamp of injected BBH Signal & BNS Signal respectively. Chi-Square value at the timestamp of BNS is near unity and that for BBH is higher than that of BNS.

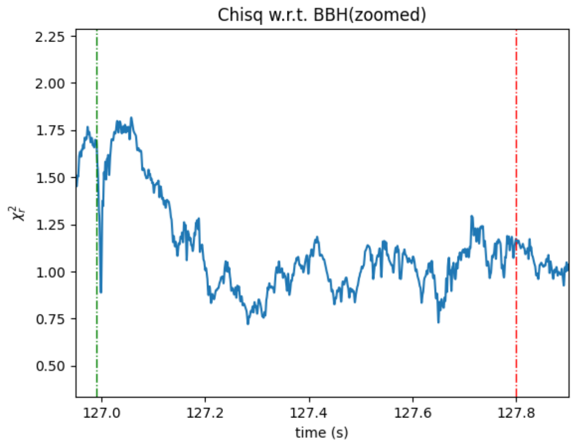


Figure (n) : Zoomed version of Figure(l).
Green line & Red line denote the timestamp of injected BBH Signal & BNS Signal respectively. Chi-Square value at the timestamp of BBH is near unity and that for BNS is higher than that of BBH.

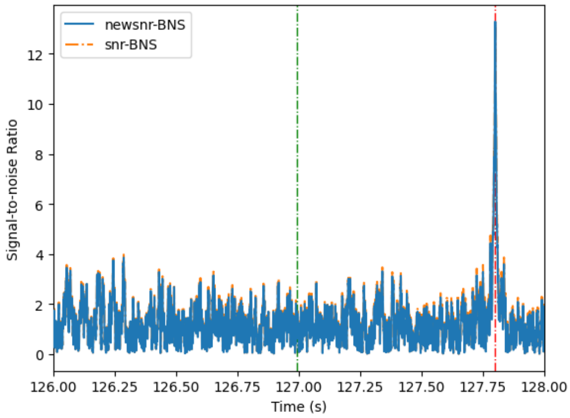


Figure (o) : Old SNR & New SNR w.r.t BNS template.
The noise gets reduced,refining the recovery.

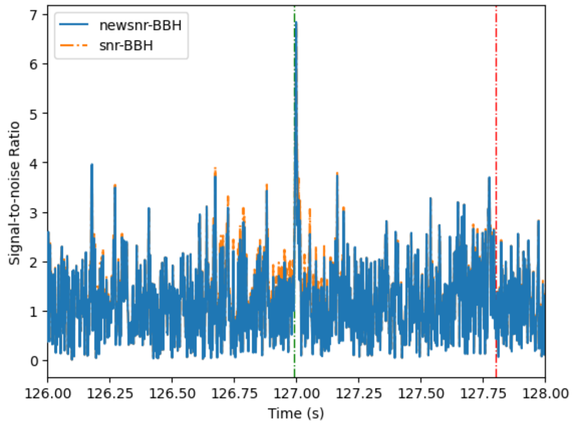


Figure (p) : Old SNR & New SNR w.r.t BBH template.
The noise gets reduced,refining the recovery.

6 Iterative Signal Recovery Process

1. An overlapping pair of signals is created :BBH (20,30)-BBH(30,40).

   [(20,30)and (30,40) refer to the masses of the binary Black Holes in the unit of solar mass ($1M_{\odot} = 2 \times 10^{30} kg$) corresponding to the two signals respectively.]

2. A template bank of 100 templates each is created corresponding to both the signals,in such a way that the masses cover the region around both signals.

3. Out of these a random pair is selected for injection. The separation time between the merger times is also randomly sampled.

4. The recovery process is initiated by carrying out Matched filtering on this data and the approximate first loud signal is captured.

5. Having this signal subtracted,the recovery process is again implemented to find the second loudest signal.

6. Now this second signal is subtracted from the original data where both injections are present.

7. This procedure is repeated to refine the recovery of louder injection and so on.Through this iterative procedure,the estimations of both injections are refined.

Three cases were observed for this iterative signal recovery process:

1. The injection parameters for both the signals were recovered successfully.

2. The injection parameters for only one signal was recovered successfully.

3. The injection parameters of one/both signals were recovered successfully in the initial iteration but were altered at the final iteration.

The new SNR value improved and the chi-squared value reduced after subtraction of one of the signals from the data.

The code for the above process is available at [3]

Following are the plots of the collected data:

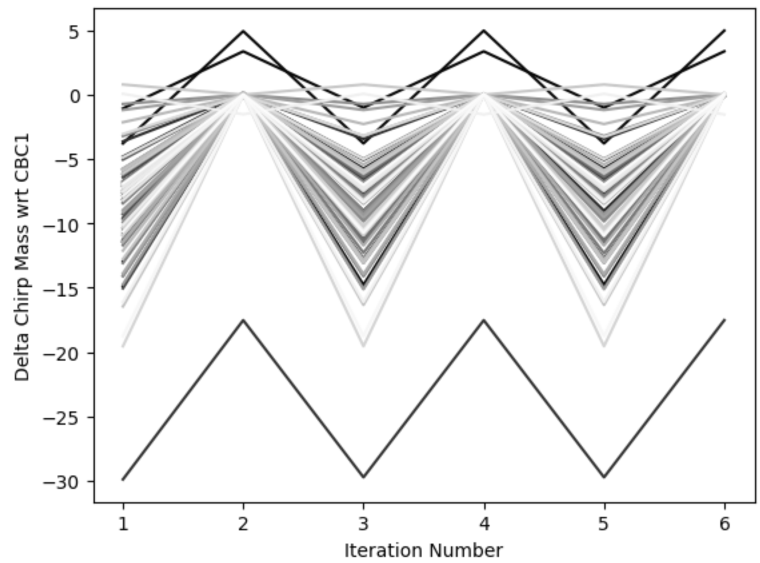


Figure 3: Delta Chirp Mass(CBC1) Vs Iteration Number

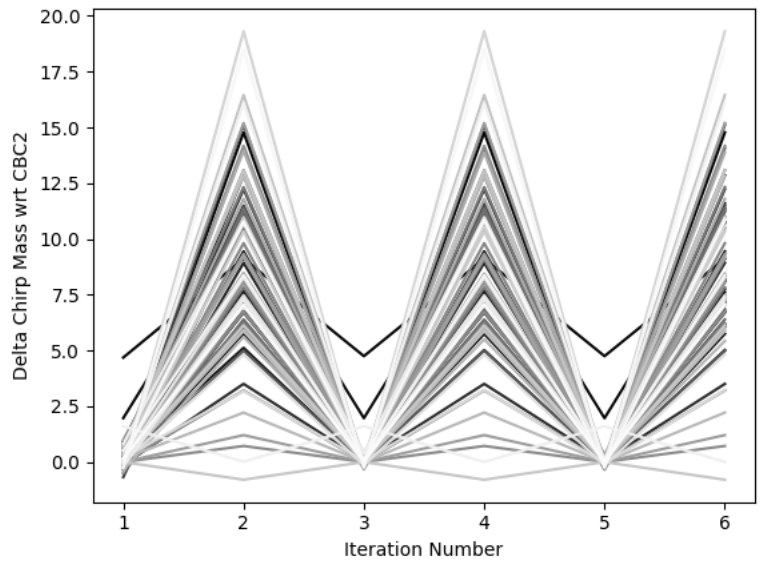


Figure 4: Delta Chirp Mass(CBC2) Vs Iteration Number

Chirp Mass is computed as : $\mathcal{M} = \frac{(m_1 \times m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$, where m_1 and m_2 are masses of the compact binaries.
Delta chirp mass is given by : Injected Chirp Mass - Recovered Chirp Mass

The biases are found to be oscillating.

7 References

1. S.A.Usman,and A.H.Nitz, PACS 04.30.-w,04.25.-g.
2. B.S. Sathyaprakash,and B.F.Schutz,"Physics,Astrophysics and Cosmology with Gravitational Waves".
3. https://github.com/kajolshelke/Project