Introduction to Neural Networks and Deep Learning

Introduction to Neural Networks

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Outline

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What are Neural Networks? [1]

Basic Intuition

The human brain is a highly complex, nonlinear and parallel computer

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Basic Intuition

The human brain is a highly complex, nonlinear and parallel computer

It is organized as a

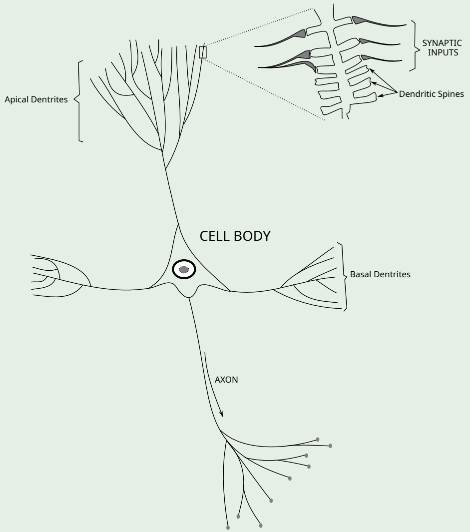
Network with (Ramon y Cajal 1911)

1. Basic Processing Units $\approx$ Neurons
2. Connections $\approx$ Axons and Dendrites

Outline

Example

The Neural Structure



Silicon Chip Vs Neurons

Speed Differential

1. Speeds in silicon chips are in the nanosecond range (10^{-9} s).
2. Speeds in human neural networks are in the millisecond range (10^{-3} s).

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1. 10 billion neurons in the human cortex.
2. 60 trillion synapses or connections

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High Energy Efficiency

1. Human Brain uses 10^{-16} joules per operation.
2. Best computers use 10^{-6} joules per operation.

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Pigeon Experiment

Watanabe et al. 1995 [2]

Pigeons as art experts

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Watanabe et al. 1995 [2]

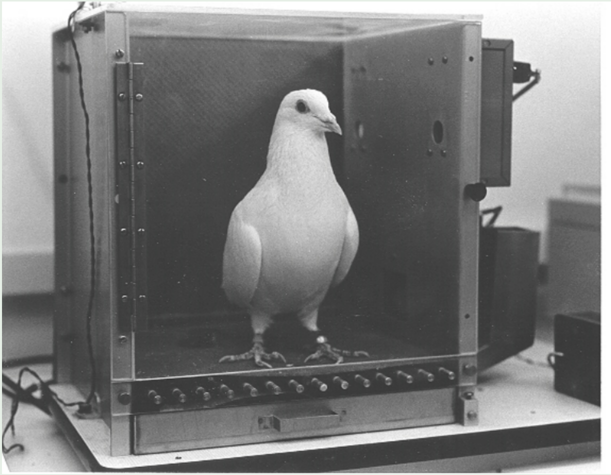
Pigeons as art experts

Experiment

- Pigeon is in a Skinner box
- Then, paintings of two different artists (e.g. Chagall / Van Gogh) are presented to it.
- A Reward is given for pecking when presented a particular artist (e.g. Van Gogh).

The Pigeon in the Skinner Box

Something like this



Results

Something Notable

- Pigeons were able to discriminate between Van Gogh and Chagall with 95% accuracy (when presented with pictures they had been trained on).

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- Pigeons were able to discriminate between Van Gogh and Chagall with 95% accuracy (when presented with pictures they had been trained on).
- Discrimination still 85% successful for previously unseen paintings of the artists.

Thus

1. Pigeons do not simply memorize the pictures.
2. They can extract and recognize patterns (the 'style').
3. They generalize from the already seen to make predictions.
4. This is what neural networks (biological and artificial) are good at (unlike conventional computer).

Outline

Formal Definition [1]

Definition

An **artificial neural network** is a massively parallel distributed processor made up of simple processing units. It resembles the brain in two respects:

1. Knowledge is acquired by the network from its environment through a learning process.
2. Inter-neuron connection strengths, known as synaptic weights, are used to store the acquired knowledge.

Inter-neuron connection strengths?

How do the neuron collect this information?

Some way to aggregate information needs to be devised...

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A Classic

Use a summation of product of weights by inputs!!!

Inter-neuron connection strengths?

How do the neuron collect this information?

Some way to aggregate information needs to be devised...

A Classic

Use a summation of product of weights by inputs!!!

Something like

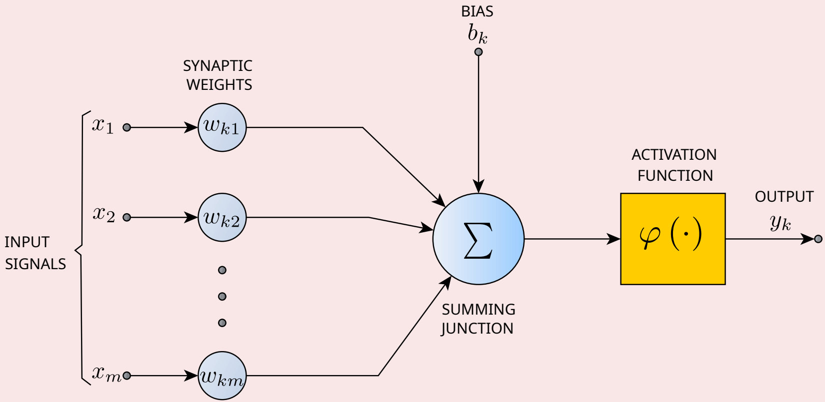
$$\sum_{i=1}^{m} w_i \times x_i$$

Where: w_i is the strength given to signal x_i

However: We still need a way to regulate this "aggregation" (Activation function)

The Model of a Artificial Neuron

Graphical Representation for neuron k



The use of Differential Equations in Neural Networks

It is not a well a known fact

- But the first proposed Neural Network was designed as combination of Differential Equations

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- But the first proposed Neural Network was designed as combination of Differential Equations

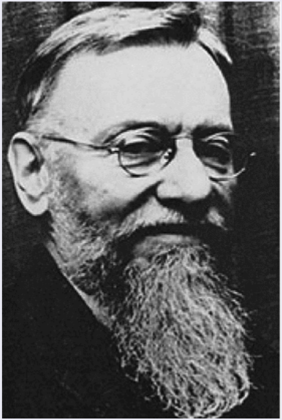
That McCulloch-Pitts model

- It is actually a discrete paraphrasing of such initial idea!!!

History

The study of Neurodynamics began in the 1930's

- With the work of **Nicolas Rashevsky** [3].



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Who he was?

- American theoretical physicist who was one of the pioneers of mathematical biology.

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And in 1933

- He proposed the first neural network architecture

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A simple Neural Network

Rashevsky proposed a Neural Network based in differential equations

$$
\begin{aligned}
\frac{de}{dt} =& A\boldsymbol{x}\left(t\right) - ae\\
\frac{dj}{dt} =& B\boldsymbol{x}\left(t\right) - bj\\
Output =& Heaviside\left(e - j - \theta\right)
\end{aligned}
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Something Notable

- Walter Pitts was his student, and together with Warren McCulloch rephrased the previous networks in a discrete version.

Into Big Data

He noticed something quite interesting [4]

- "in physics, one often averages over a large set of discrete events to obtain a continuous model"
  - This represents the large scale behavior of a system...

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What do we do in Deep Learning with Big Data?

- **Our results are done over million of samples as training sets to get an average training!!!**

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Basic Elements of an Artificial Neuron (AN) Model

Set of Connecting links

- A signal x_j, at the input of synapse j connected to neuron k is multiplied by the synaptic weight w_{kj}.
- The weight may lie in a negative or positive range.
  - What about the real neuron? In classic literature you only have positive values.

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A Complex Aggregation Function

- An aggregation function for the input signals, weighted by the respective synapses of the neuron.

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Activation function (Squashing function)

- It limits the amplitude of the output of a neuron.
- It maps the permissible range of the output signal to an interval.

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Mathematically

Adder

$$u_k = \sum_{j=1}^{m} w_{kj} x_j \tag{1}$$

Mathematically

Adder

$$u_k = \sum_{j=1}^{m} w_{kj} x_j \tag{1}$$

1. $x_1, x_2, ..., x_m$ are the input signals.
2. $w_{k1}, w_{k2}, ..., w_{km}$ are the synaptic weights.
3. It is also known as "Affine Transformation."

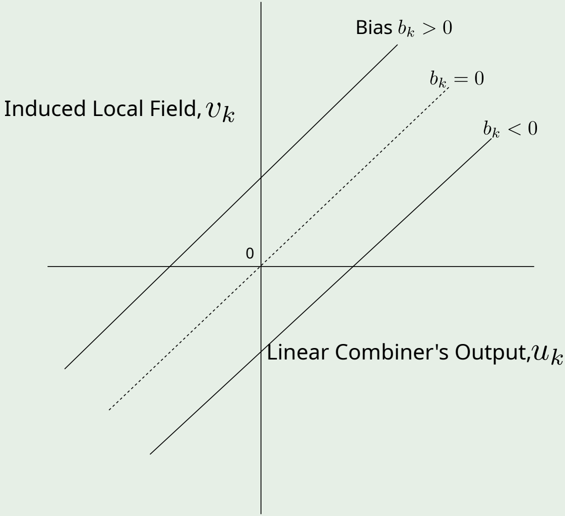
Activation function

$$y_k = \varphi\left(u_k + b_k\right) \tag{2}$$

1. y_k output of neuron.
2. φ is the activation function.

Integrating the Bias

The Affine Transformation



Thus

Final Equation

$$
\begin{aligned}
v_k &= \sum_{j=0}^{m} w_{kj} x_j \\
y_k &= \varphi\left(v_k\right)
\end{aligned}
$$

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Thus

Final Equation

$$v_k = \sum_{j=0}^{m} w_{kj} x_j$$
$$y_k = \varphi\left(v_k\right)$$

With

$x_0 = 1$ and $w_{k0} = b_k$

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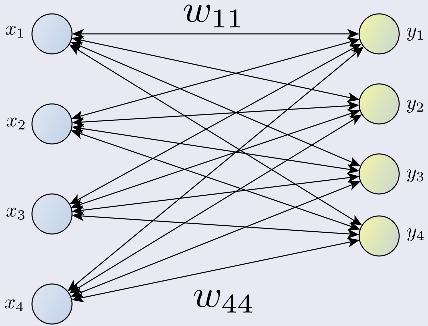
Energy Based Network

Bidirectional Associative Memory (BAM) [5]



A Little of Linear Algebra

Here, we can denote the weights as $n \times k$ matrix $\boldsymbol{W}$

- The n corresponds to the n dimensional vector $\boldsymbol{x}_0$
- The k corresponds to the k dimensional vector $\boldsymbol{y}_0$

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Here, we can denote the weights as $n \times k$ matrix $\boldsymbol{W}$

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- The k corresponds to the k dimensional vector $\boldsymbol{y}_0$

Therefore the mapping is build in the following way given the feedback

$$
\begin{aligned}
\boldsymbol{y}_0 &= \text{sgn}\left(\boldsymbol{x}_0 \boldsymbol{W}\right) \\
\boldsymbol{x}_1^T &= \text{sgn}\left(\boldsymbol{W} \boldsymbol{y}_0\right) \\
\boldsymbol{y}_1 &= \text{sgn}\left(\boldsymbol{x}_1 \boldsymbol{W}\right) \\
\ldots
\end{aligned}
$$

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This is done until a stable state is reached

Meaning

$$\boldsymbol{y} = \text{sgn}\left(\boldsymbol{x}\boldsymbol{W}\right)$$
$$\boldsymbol{x}^T = \text{sgn}\left(\boldsymbol{W}\boldsymbol{y}\right)$$

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Meaning

$$\boldsymbol{y} = \text{sgn}\left(\boldsymbol{x}\boldsymbol{W}\right)$$
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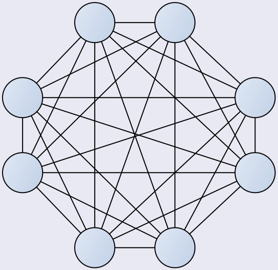
A Notable Example

- The Hopfield Networks



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Types of Activation Functions I

Threshold Function

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases} \text{ (Heaviside Function)} \quad (3)$$

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Threshold Function

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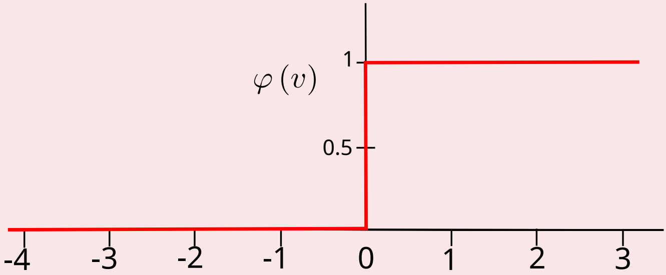
In the following picture



Thus

We can use this activation function

- To generate the first Neural Network Model

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- To generate the first Neural Network Model

Clearly

- The model uses the summation as aggregation operator and a threshold function.

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McCulloch-Pitts model [6]

McCulloch-Pitts model (Pioneers of Neural Networks in the 1940's)

$$\text{Output } y_k = \begin{cases} 1 & \text{if } v_k \geq \theta \\ 0 & \text{if } v_k < \theta \end{cases} \tag{4}$$

McCulloch-Pitts model [6]

McCulloch-Pitts model (Pioneers of Neural Networks in the 1940's)

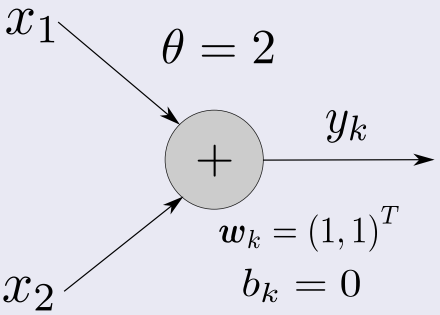
$$\text{Output } y_k = \begin{cases} 1 & \text{if } v_k \geq \theta \\ 0 & \text{if } v_k < \theta \end{cases} \tag{4}$$

with induced local field $\boldsymbol{w}_k = (1, 1)^T$

$$v_k = \sum_{j=1}^{m} w_{kj} x_j + b_k \tag{5}$$

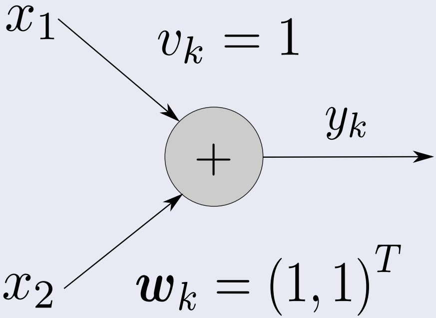
It is possible to do classic operations in Boolean Algebra

AND Gate



In the other hand

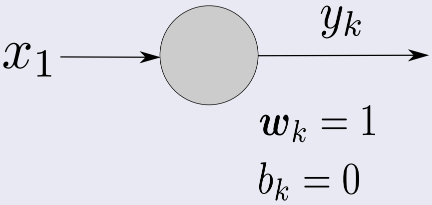
OR Gate



Finally

NOT Gate

$$y_k = \begin{cases} 1 & \text{if } v_k \leq \theta = 0 \\ 0 & \text{if } v_k > \theta = 0 \end{cases}$$



And the impact is further understood if you look at this paper

Claude Shannon

- "A Symbolic Analysis of Relay and Switching Circuits"
  - Shannon proved that his switching circuits could be used to simplify the arrangement of the electromechanical relays
  - These circuits could solve all problems that Boolean algebra could solve.

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  - Shannon proved that his switching circuits could be used to simplify the arrangement of the electromechanical relays
  - These circuits could solve all problems that Boolean algebra could solve.

Basically, he proved that computer circuits

- They can solve computational complex problems... then neural networks can simulate them...

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More advanced activation function

Piecewise-Linear Function

$$\varphi(v) = \begin{cases} 1 & \text{if } v_k \geq \frac{1}{2} \\ v & \text{if } -\frac{1}{2} < v_k < \frac{1}{2} \\ 0 & \text{if } v \leq -\frac{1}{2} \end{cases} \quad (6)$$

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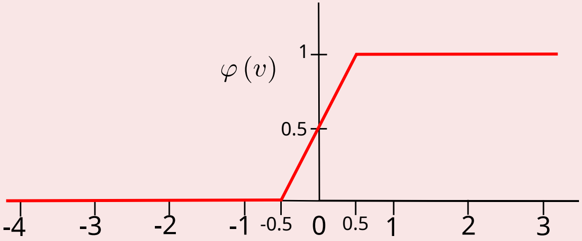
Example



Remarks

Notes about Piecewise-Linear function

The amplification factor inside the linear region of operation is assumed to be unity.

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Special Cases

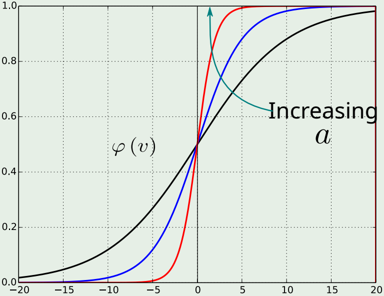
- A linear combiner arises if the linear region of operation is maintained without running into saturation.
- The piecewise-linear function reduces to a threshold function if the amplification factor of the linear region is made infinitely large.

A better choice!!!

Sigmoid function

$$\varphi(v) = \frac{1}{1 + \exp\{-av\}} \tag{7}$$

Where a is a slope parameter.



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The Problem of the Vanishing Gradient

When using a non-linearity

- However, there is a drawback when using Back-Propagation (As we saw in Machine Learning) under a sigmoid function

$$s(x) = \frac{1}{1+e^{-x}}$$

The Problem of the Vanishing Gradient

When using a non-linearity

- However, there is a drawback when using Back-Propagation (As we saw in Machine Learning) under a sigmoid function

$$s(x) = \frac{1}{1 + e^{-x}}$$

Because if we imagine a Deep Neural Network as a series of layer functions f_i

$$y(A) = f_t \circ f_{t-1} \circ \cdots \circ f_2 \circ f_1(A)$$

- With f_t is the last layer.

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Then, using the Chain Rule

Therefore, we finish with a sequence of derivatives

$$\frac{\partial y\left(A\right)}{\partial w_{1i}}=\frac{\partial f_t\left(f_{t-1}\right)}{\partial f_{t-1}}\cdot\frac{\partial f_{t-1}\left(f_{t-2}\right)}{\partial f_{t-2}}\cdot\cdots\cdot\frac{\partial f_2\left(f_1\right)}{\partial f_2}\cdot\frac{\partial f_1\left(A\right)}{\partial w_{1i}}$$

Therefore

Given the commutativity of the product

- You could put together the derivative of the sigmoid's

$$f(x) = \frac{ds(x)}{dx} = \frac{e^{-x}}{\left(1 + e^{-x}\right)^2}$$

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$$f(x) = \frac{ds(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$$

Therefore, deriving again

$$\frac{df(x)}{dx} = -\frac{e^{-x}}{(1+e^{-x})^2} + \frac{2(e^{-x})^2}{(1+e^{-x})^3}$$

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After making $\frac{df(x)}{dx} = 0$

- We have the maximum is at $x = 0$

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Therefore

The maximum for the derivative of the sigmoid

- $f(0) = 0.25$

Therefore

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Therefore, Given a **Deep** Convolutional Network

- We could finish with

$$\lim_{k \to \infty} \left(\frac{ds(x)}{dx} \right)^k = \lim_{k \to \infty} (0.25)^k \to 0$$

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$$\lim_{k \to \infty} \left(\frac{ds(x)}{dx} \right)^k = \lim_{k \to \infty} (0.25)^k \to 0$$

A Vanishing Derivative or Vanishing Gradient

- Making quite difficult to do train a deeper network using this activation function for Deep Learning and even in Shallow Learning

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Thus

The need to introduce a new function

$$f(x) = x^{+} = \max(0, x)$$

Thus

The need to introduce a new function

$$f\left(x\right)=x^{+}=\max\left(0,x\right)$$

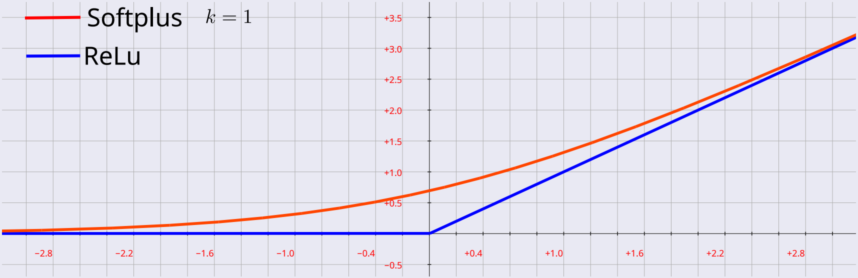
It is called ReLu or Rectifier

With a smooth approximation (Softplus function)

$$f\left(x\right)=\frac{\ln\left(1+e^{kx}\right)}{k}$$

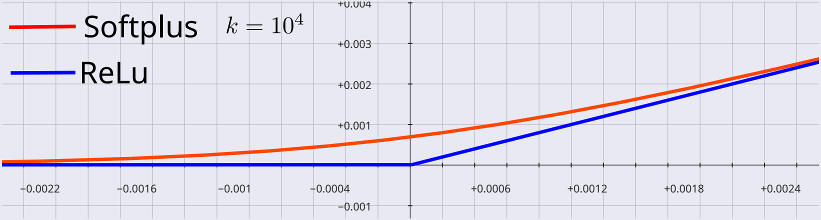
We have

When $k = 1$



Increase k

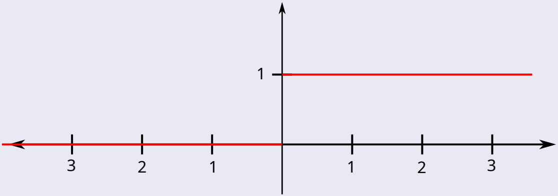
When $k = 10^4$



However, it seems to be

People are using the following derivative

$$\frac{d}{dx}ReLu\left(x\right)=\begin{cases}0 & x<0\\1 & x\geq 0\end{cases}$$

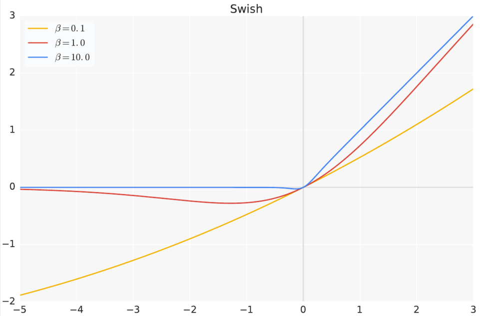


Example at Google Brain

"SWISH: A SELF-GATED ACTIVATION FUNCTION" [7]

$$S(x) = \frac{x}{1 + \exp\{-\beta x\}}$$

- Here β is a trainable parameter



Some Properties of the Swish

If $\beta = 1$

- We have the Sigmoid-weighted Linear Unit (SiL), proposed in reinforcement learning

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As $\beta \to \infty$

- The sigmoid component approaches a 0-1 function, Then what?

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If $\beta = 1$

- We have the Sigmoid-weighted Linear Unit (SiL), proposed in reinforcement learning

As $\beta \to \infty$

- The sigmoid component approaches a 0-1 function, Then what?

When $\beta = 0$

$$S(x) = \frac{x}{2}$$

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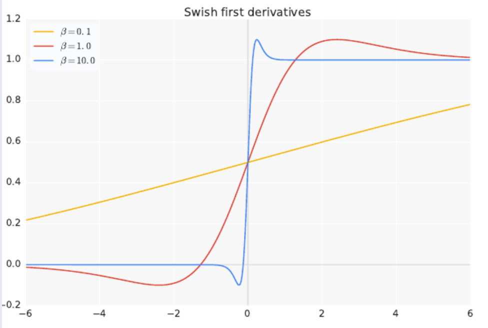
Thus

Swish interpolate Between the linear function and the ReLU function

- Not only that but at the derivatives



However

We need to analyze more activation functions

- So, we reach the objective of finding one that has smooth derivatives.

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- So, we reach the objective of finding one that has smooth derivatives.

And Several Derivatives

- It is going to be nice...

Final Remarks

Although, ReLu functions

- They can handle the problem of vanishing problem

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Although, ReLu functions

- They can handle the problem of vanishing problem

However, as we will see, saturation starts to appear as a problem

- As in Hebbian Learning!!!

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Neural Network As a Graph [1]

Definition

A neural network is a directed graph consisting of nodes with interconnecting synaptic and activation links.

Neural Network As a Graph [1]

Definition

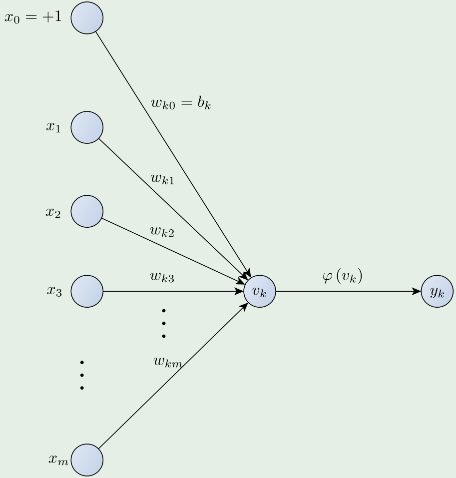
A neural network is a directed graph consisting of nodes with interconnecting synaptic and activation links.

Properties

1. Each neuron is represented by an function
2. Each link represent a weight.
3. The weighted sum of the input signals defines the local field.
4. The activation function maps local field to an output.

Example

Simple Perceptron



Some Observations

Observation

- A partially complete graph describing a neural architecture has the following characteristics:
  - Source nodes supply input signals to the graph.
  - Each neuron is represented by a single node called a computation node.
  - The communication links provide directions of signal flow in the graph.

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However

Other Representations exist!!!

Three main representations ones

- Block diagram, providing a functional description of the network.
- Signal-flow graph, providing a complete description of signal flow in the network.
  - Then one we plan to use.
- Architectural graph, describing the network layout.

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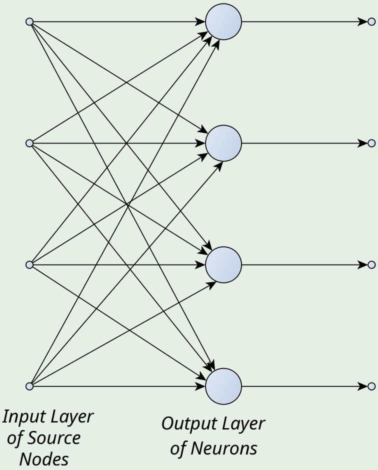
Single-Layer Feedforward Networks

We begin with something quite simple



Observations

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This network is know as a strictly feed-forward or acyclic type.

Outline

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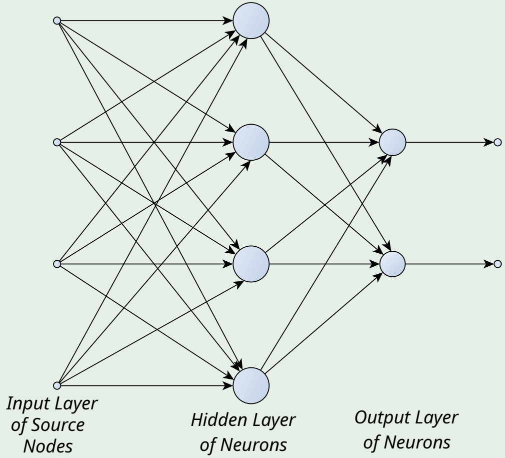
Multilayer Feedforward Networks

Stacking Layers



Observations

Observations

1. This network contains a series of hidden layer.

Observations

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1. This network contains a series of hidden layer.
2. Each hidden layers allows for classification of the new output space of the previous hidden layer.

Outline

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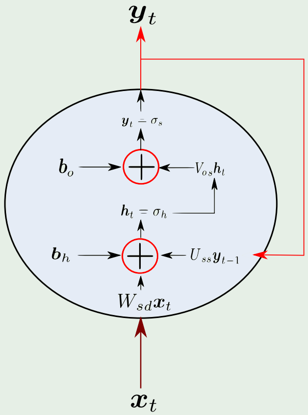
Recurrent Networks

Connecting the back with the front



Observations

Observations

1. This network has not self-feedback loops.

Observations

Observations

1. This network has not self-feedback loops.
2. It has something known as unit delay operator $B = z^{-1}$.

Outline

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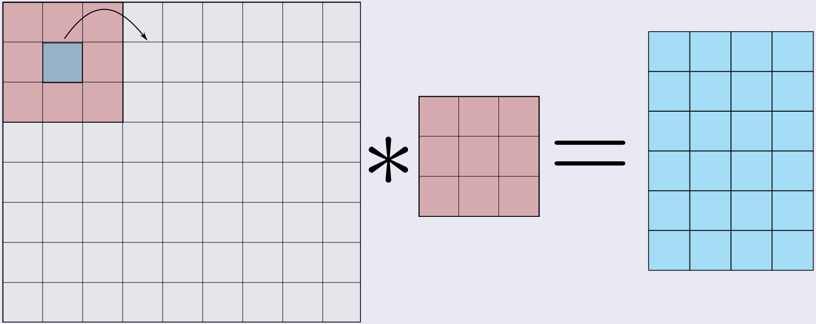
Convolutional Deep Learners

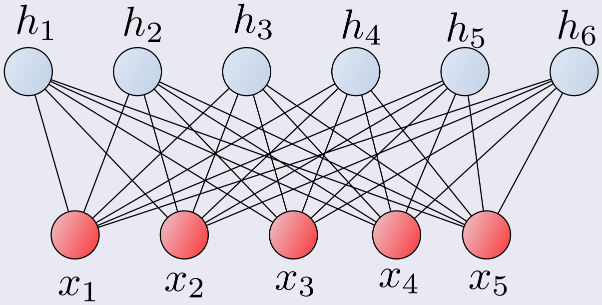
Using the principle of locality [8, 9]

Horizontal Stride $r = 2$



Restricted Boltzmann Machines

Energy Based Architectures [10]



Outline

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Knowledge Representation

Definition

- By Fischler and Firschein, 1987
  - “Knowledge refers to stored information or models used by a person or machine to interpret, predict, and appropriately respond to the outside world. ”

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It consists of two kinds of information:

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Observations can be

1. Labeled
2. Unlabeled

Outline

Set of Training Data

Training Data

- It consist of input-output pairs (x, y)
  - $x=$ input signal
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Thus, we have the following phases of designing a Neuronal Network

1. Choose appropriate architecture
2. Train the network - learning.
   1. Use the Training Data!!!
3. Test the network with data not seen before
   1. Use a set of pairs that where not shown to the network so the y component is guessed.
4. Then, you can see how well the network behaves - Generalization Phase.

Outline

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Representing Knowledge in a Neural Networks

Notice the Following

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   1. However, they are even designed!!!

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However: Pattern Classifiers Vs Neural Networks

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2. Neural Networks learns the environment by using the data from it!!!
   1. However, they are even designed!!!

Kurt Hornik et al. proved (1989)

"Standard multilayer feedforward networks with as few as one hidden layer using arbitrary squashing functions are capable of approximating any **Borel measurable function**" (Basically many of the known ones!!!)

Rules Knowledge Representation

Rule 1

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Examples

1. $d\left(\boldsymbol{x_i}, \boldsymbol{x_j}\right) = \left\|\boldsymbol{x_i} - \boldsymbol{x_j}\right\|$ (Classic Euclidean Metric).
2. $d_{ij}^2 = \left(\boldsymbol{x}_i - \boldsymbol{\mu}_i\right)^T \sum^{-1} \left(\boldsymbol{x}_j - \boldsymbol{\mu}_j\right)$ (Mahalanobis distance) where
   1. $\boldsymbol{\mu}_i = E\left[\boldsymbol{x}_i\right]$.
   2. $\sum = E\left[\left(\boldsymbol{x}_i - \boldsymbol{\mu}_i\right)\left(\boldsymbol{x}_i - \boldsymbol{\mu}_i\right)^T\right] = E\left[\left(\boldsymbol{x}_j - \boldsymbol{\mu}_j\right)\left(\boldsymbol{x}_j - \boldsymbol{\mu}_j\right)^T\right]$.

More

Rule 2

- Items to be categorized as separate classes should be given widely different representations in the network.

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Rule 3

- If a particular feature is important, then there should be a large number of neurons involved in the representation.

Rule 4

- Prior information and invariance should be built into the design:
  - Thus, simplify the network by not learning that data.

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