Introduction to Machine Learning

Notes in Linear Search

Andres Mendez-Vazquez

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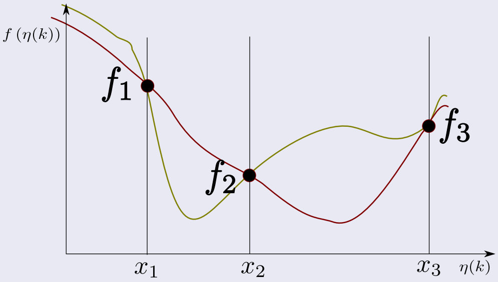
Outline

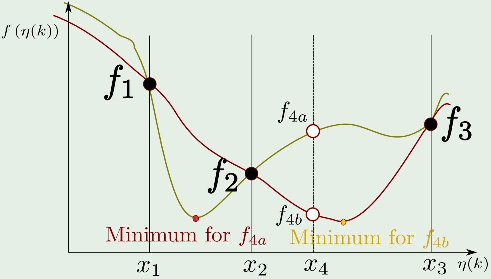
Gold Section

We have $f(\eta(k)) = J(\boldsymbol{w}(k) - \eta(k)\nabla J(\boldsymbol{w}(k)))$



Golden Section

Thus the idea is to use an evaluation f_4 to decide which subsection to drop



What is the Golden Ratio Idea?

Basically, given an interval $[x_1, x_3]$

Then, we select a point x_2 and x_3 such that we have a two possible intervals of search for the minimum

1. $[x_1, x_4]$
2. $[x_2, x_3]$

The Golden Linear Search requires these intervals be equal!!!

If they are not,

- You could run to a series of search wider intervals slowing down the rate of convergence.

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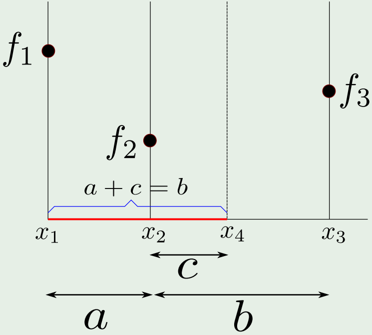
How?

By the equality $b = a + c$



Therefore

We have the following question?

Where do you place x_2? Thus you can generate x_4

You want to avoid

- x_2 to close to x_1 or x_3

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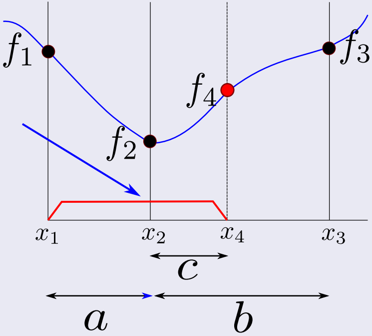
The process is as follow

We define

- $f_1 = f(x_1)$
- $f_2 = f(x_2)$
- $f_3 = f(x_3)$
- $f_4 = f(x_4)$

Two Cases

If $f_2 < f_4$ then the minimum lies between x_1 and x_4 and the new triplet is x_1, x_2 and x_4.



Here,we have the realization that

We have interval size reduction

$$x_4 - x_1 = \varphi\left(x_3 - x_1\right) \longmapsto x_4 = x_1 + \varphi x_3 - \varphi x_1$$

Then

$$x_4 = \left(1 - \varphi\right) x_1 + \varphi x_3$$

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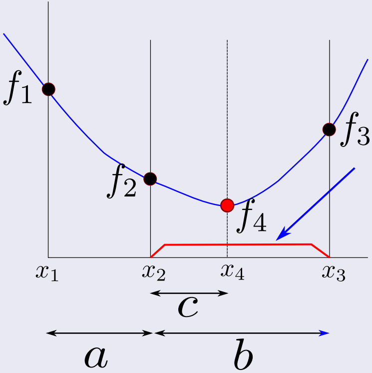

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$$x_3 - x_2 = \varphi\left(x_3 - x_1\right) \longmapsto -x_2 = \varphi x_3 - \varphi x_1 - x_3$$

Therefore

$$x_2 = \varphi x_1 + (1-\varphi) x_3$$

Thus, once we obtain φ, we get x_2 and x_4

- For this, we make the following assumption $[x_1, x_3] = [0, 1]$

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$$x_2 = 1 - \varphi$$

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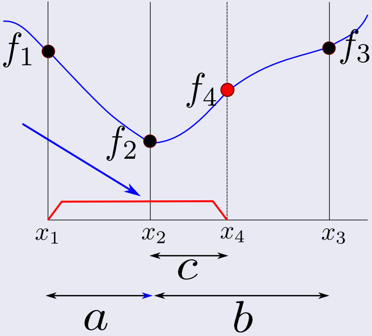

With a Little Algebra

Then, x_2 is between the the interval $[0, \varphi]$ and assume is a convex combination of such values

$$1 - \varphi = (1 - \varphi)\, 0 + \varphi\varphi \longmapsto \boldsymbol{\varphi^2 + \varphi - 1 = 0}$$

With Solution

$$\varphi = \frac{-1 + \sqrt{5}}{2} = 0.6180$$

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Finally, we have the algorithm

Golden Ratio

INPUT: $x_1, x_3, \tau, \varphi, f$

OUTPUT: $\frac{x_3 - x_1}{2}$

1. $x_2 = \varphi x_1 + (1 - \varphi)x_3$
2. $x_4 = (1 - \varphi)x_1 + \varphi x_3$
3. while $|x_3 - x_1| > \tau\,(|x_2| + |x_4|)$
4. if $f(x_2) < f(x_4)$:
5. $x_3 = x_4$
6. $x_4 = x_2$
7. $x_2 = \varphi x_1 + (1 - \varphi)x_3$
8. else
9. $x_1 = x_2$
10. $x_2 = x_4$
11. $x_4 = (1 - \varphi)x_1 + \varphi x_3$
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Repeat the procedure!!!

Until a error threshold is reached.

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There are better versions

Take a look

The papers at the repository.