Introduction to Algorithms

Locality Sensitive Hashing

Andres Mendez-Vazquez

January 26, 2023

Outline

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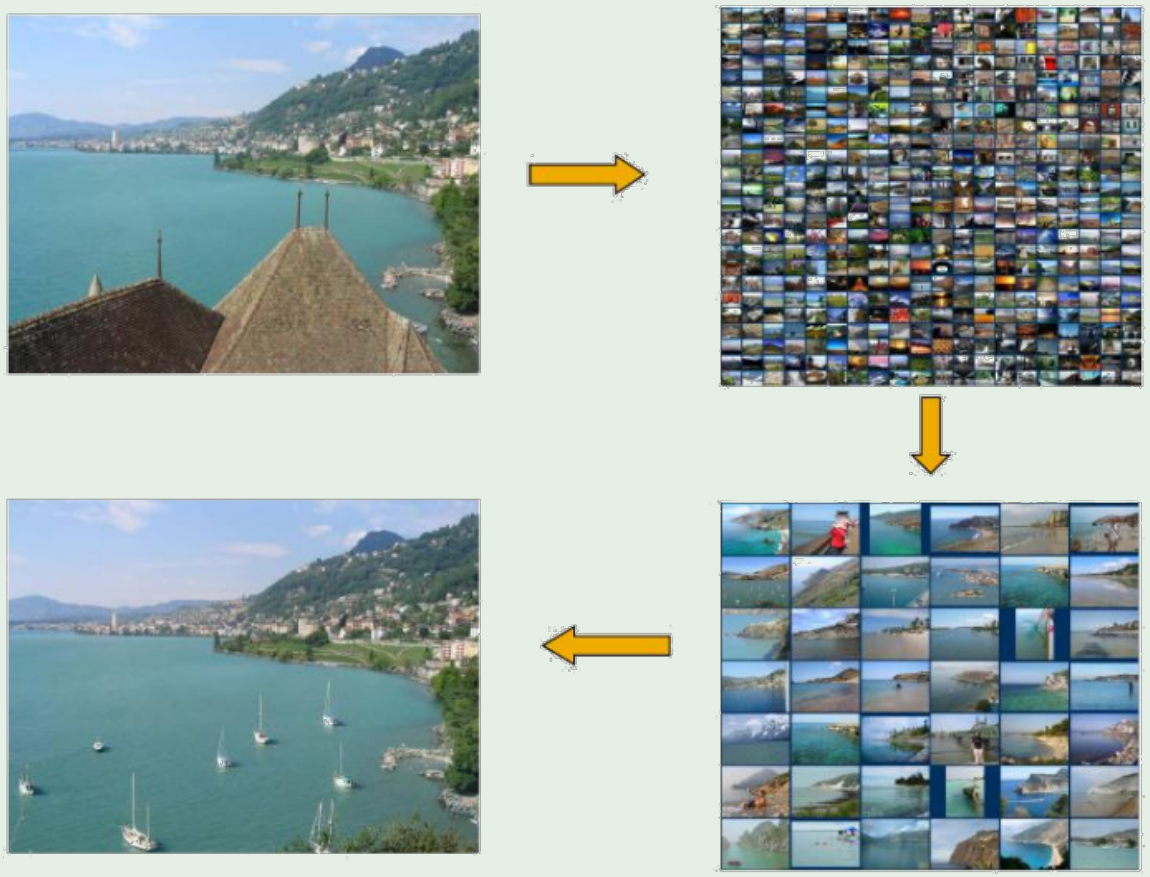
Scene Completion Problem

Example



Scene Completion Problem

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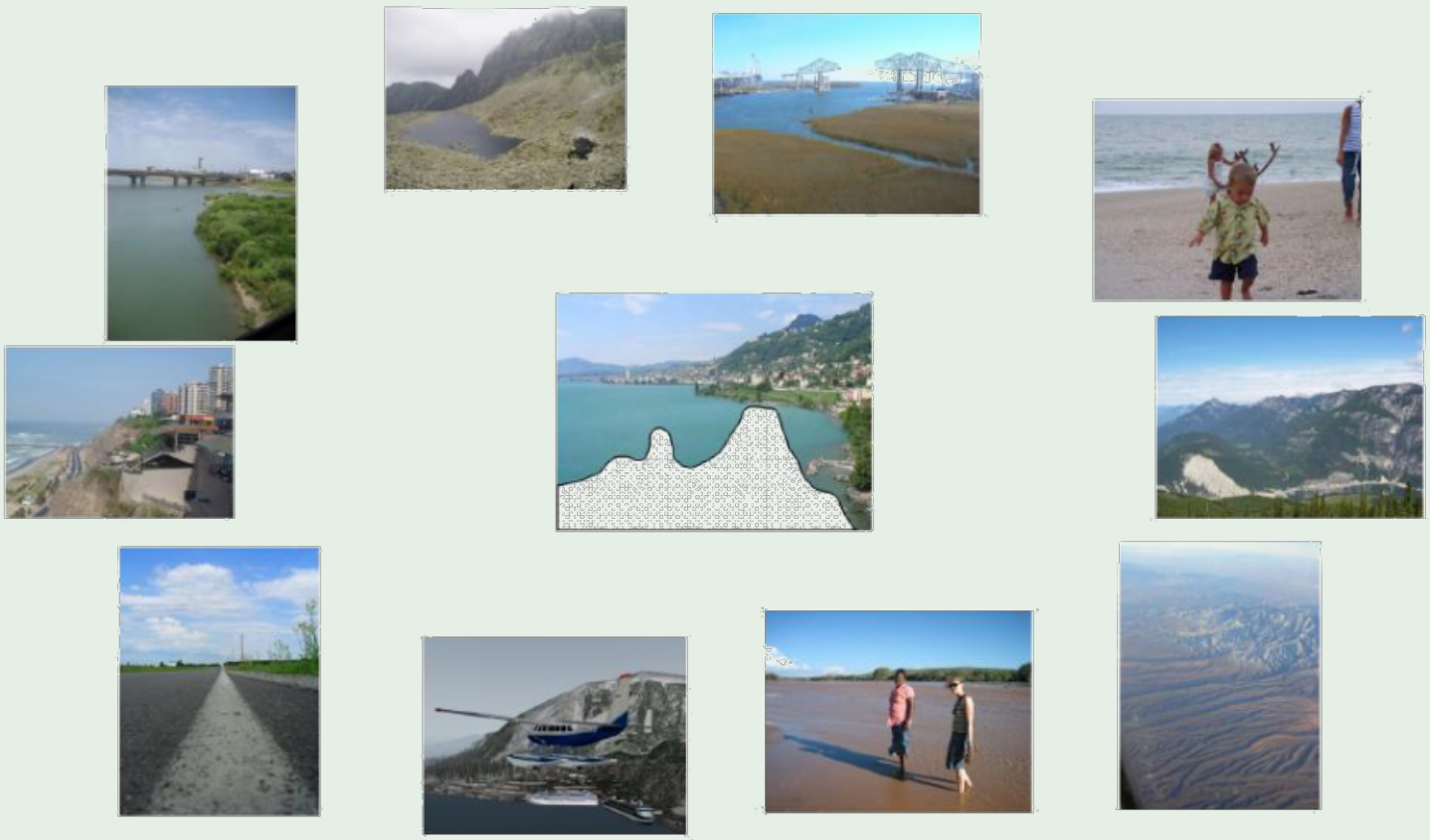
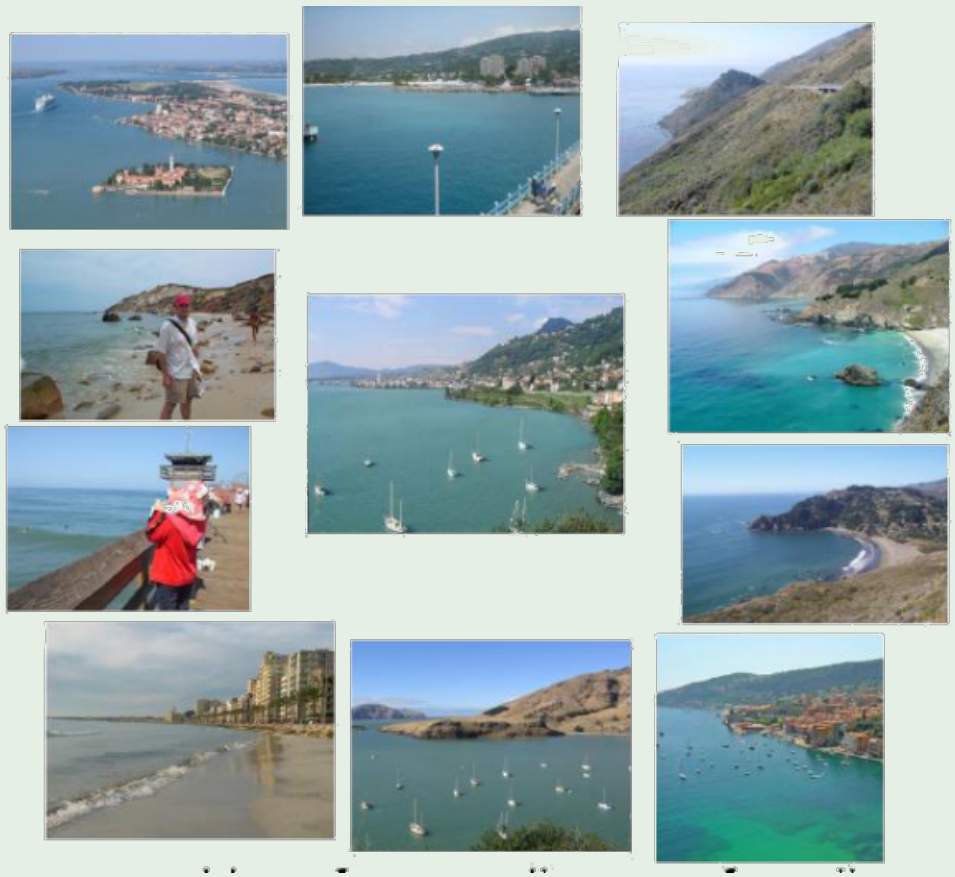
Scene Completion Problem

10 nearest neighbors from a collection of 20,000 images



Scene Completion Problem

10 nearest neighbors from a collection of 2 million images



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A Common Idea

Problems

- Many problems can be expressed as finding "similar" sets:
  - Find near-neighbors in **high-dimensional** space

Examples

- Pages with similar words
  - For duplicate detection, classification by topic
- Customers who purchased similar products
  - Products with similar customer sets
- Images with similar features
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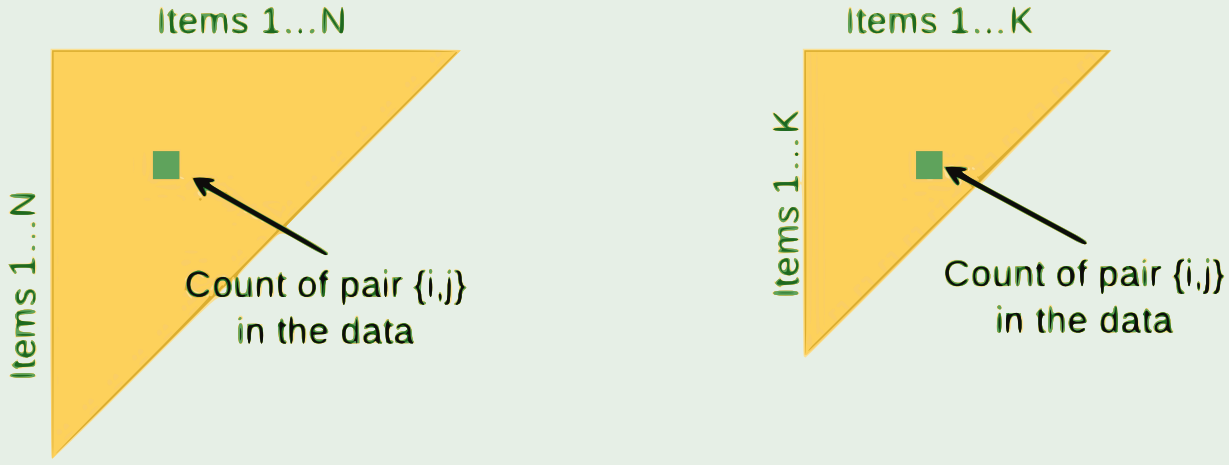
Relation to Previous Lecture

Last time: Finding frequent pairs



We had the Naïve solution

Single pass but requires space quadratic in the number of items:

- N = number of distinct items
- K =number of items with support $\geq s$

However the A-priori Algorithm

- First pass: Find frequent singletons
  - For a pair to be a candidate for a frequent pair, its singletons have to be frequent!
- Second pass:
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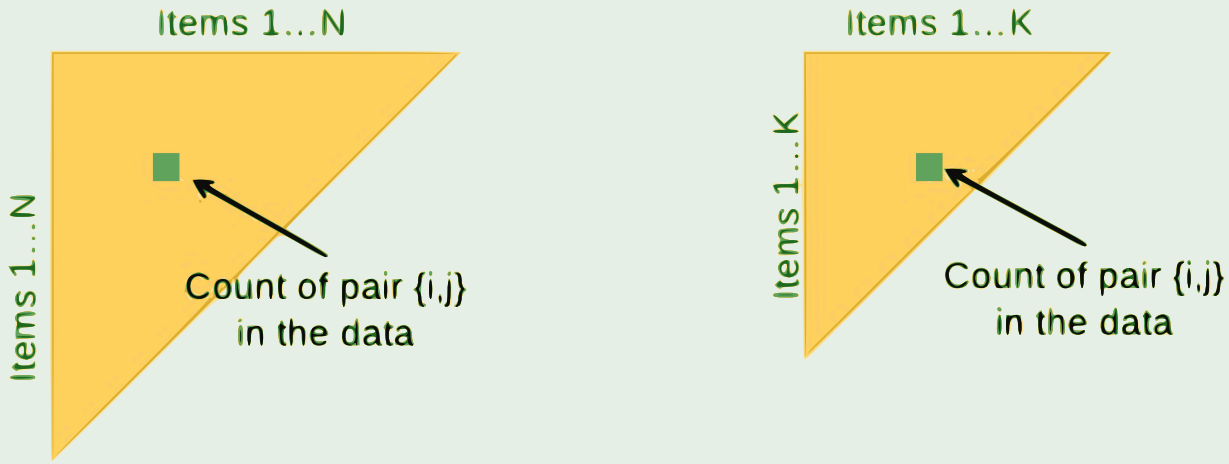

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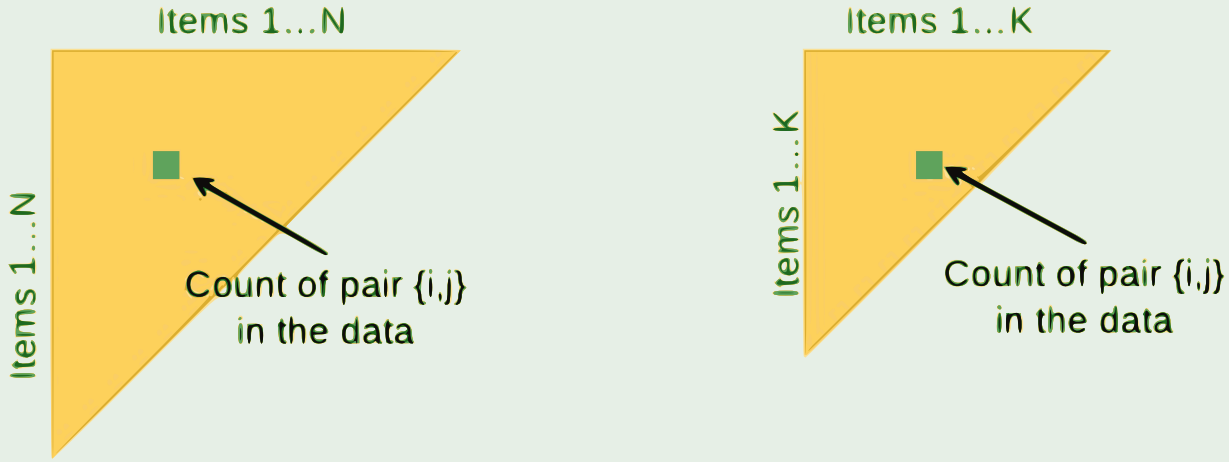

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Finding frequent pairs.

Further improvement using PCY

- Pass 1:
  - Count exact frequency of each item:

    Items $1 \ldots N$

  - Take pairs of items $\{i, j\}$, hash them into B buckets and count of the number of pairs that hashed to each bucket:

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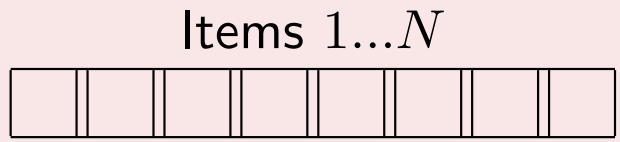

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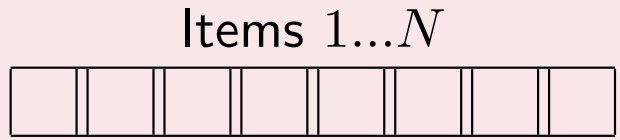

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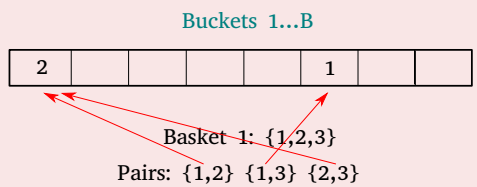

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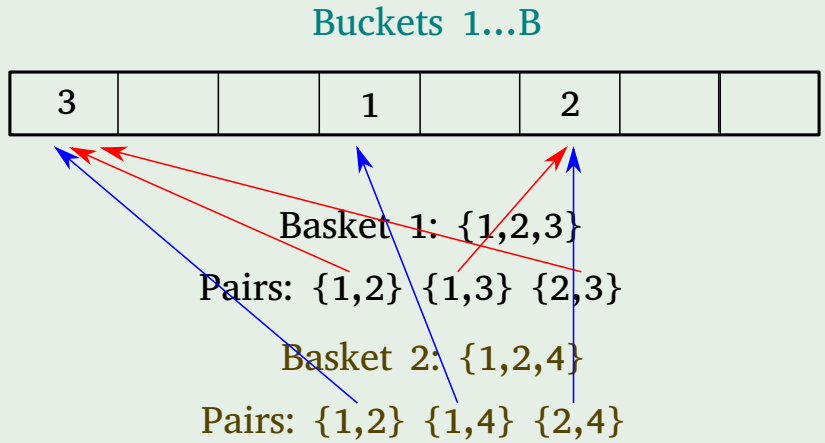
Pass 2:

- For a pair $\{i, j\}$ to be a candidate for a frequent pair, its singletons have to be frequent and it has to hash to a frequent bucket!



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Previous Lecture: A-Priori

- Main Idea: Candidates
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Today's Lecture

- Main Idea: Candidates
  - Pass 1: Take documents and has them to buckets such that **documents that are similar hash to the same bucket.**
  - Pass 2: Only compare documents that are candidates (Hashed into the same bucket)
- Thus, we need $O(N)$ instead of $O(N^2)$.

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Distance Measures

Goal

- Find near-neighbors in high-dim. space
  - We formally define "near neighbors" as points that are a "small distance" apart.

Application

- For each application, we first need to define what "distance" means

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Today: Jaccard distance (/similarity)

- The **Jaccard Similarity/Distance** of two **sets** is the size of their intersection / the size of their union:
  - $sim(C_1, C_2) = |C_1 \bigcap C_2| / |C_1 \bigcup C_2|$
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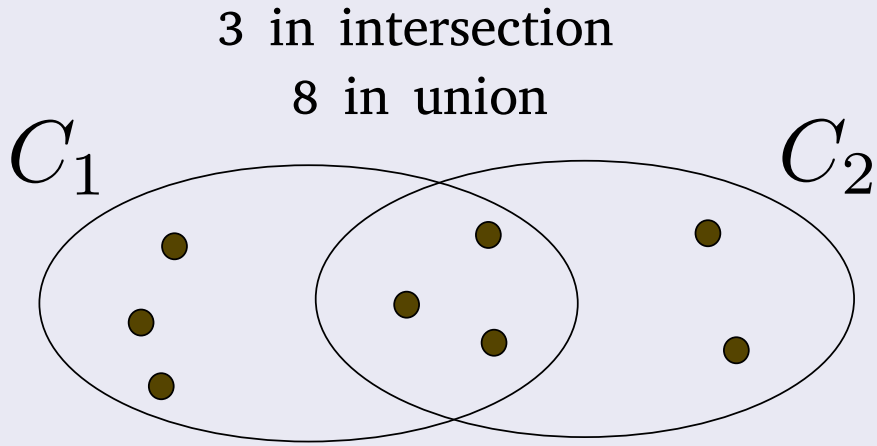

$$sim\,(C_1, C_2) = \frac{3}{8}$$

$$d\,(C_1, C_2) = \frac{5}{8}$$

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Finding Similar Documents

Goal

- Given a large number (N in the millions or billions) of text documents, find pairs that are "near duplicates."

Applications

- Mirror websites, or approximate mirrors.
  - We do not want to show both of them in a search.
- Similar news articles at many news sites.
  - Cluster articles by "same story."

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3 Essential Steps for Similar Docs

Step 1: Shingling

Convert documents to sets

Step 2: Minhashing

Convert large sets to short signatures, while preserving similarity

Locality-sensitive hashing

Focus on pairs of signatures likely to be from similar documents

- Candidate pairs!

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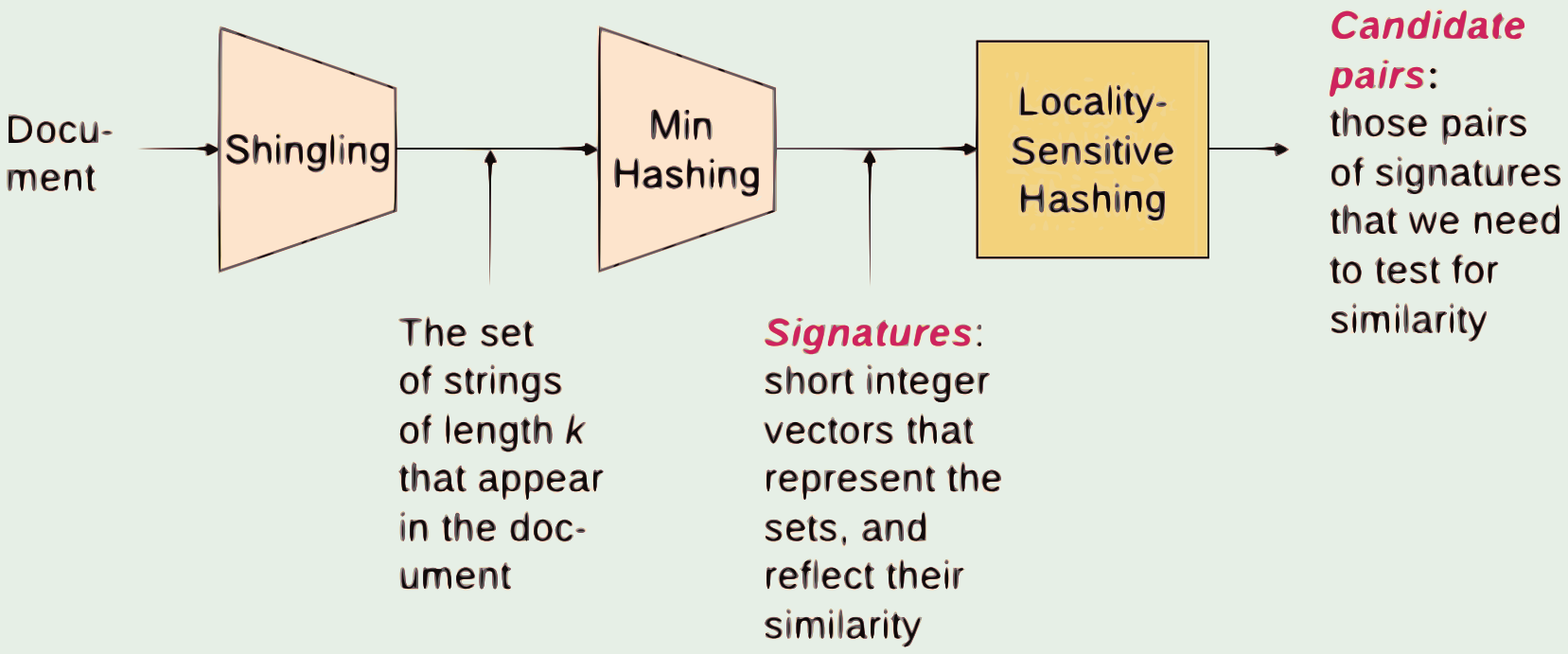
The Big Picture

The Process of Identification



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Documents as High-Dimensional Data

Step 1: Shingling

Convert documents to sets.

Simple approaches

- Document = set of words appearing in document.
- Document = set of "important" words.
- Don't work well for this application. Why?

We want to avoid to get tangled in the text structure

- Need to account for ordering of words!
- A different way: Use Shingles!!!

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k-shingle

- A k-shingle (or k-gram) for a document is a sequence of k tokens that appears in the doc.
  - Tokens can be characters, words or something else, depending on the application.
  - Assume tokens = characters for the examples.

Example

- $k = 2$; document $D_1 = abcab$ Set of 2-shingles: $S(D_1) = \{ab, bc, ca\}$
  - One possible option: Shingles as a bag (multiset). Thus, count ab twice: $S'(D_1) = \{ab, bc, ca, ab\}$

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Compressing Shingles

Compress

- To compress long shingles, we can hash them to (say) 4 bytes.

Represent a doc

- Represent a doc by the set of hash values of its k-shingles.
  - Idea: Two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared.

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Similarity Metric for Shingles

Document

- Document D_1 = set of k-shingles $C_1 = S(D_1)$

0/1 vector

- Equivalently, each document is a 0/1 vector in the space of k-shingles
  - Each unique shingle is a dimension.
  - Problem!!! Vectors are very sparse.
    - ⋆ We need a measure that can handle this situation.

A natural similarity measure is the Jaccard similarity

$$sim\left(D_1, D_2\right) = \frac{|D_1 \cap D_2|}{|D_1 \cup D_2|} \tag{1}$$

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Remember the SWAR-Popcount

Code - SWAR-Popcount - Divide and Conquer

```
// This works only in 32 bits
int product(int row,int vector){

  int i = row & vector;

  i = i - ((i >> 1) & 0x55555555);
  i = (i & 0x33333333) + ((i >> 2) & 0x33333333);
  i = (((i + (i >> 4)) & 0x0F0F0F0F) * 0x01010101) >> 24;

  return i & 0x00000001;

}
```

We can use this

Together with AND and OR to implement the Jaccard similarity

Remember the SWAR-Popcount

Code - SWAR-Popcount - Divide and Conquer

```
// This works only in 32 bits
int product(int row,int vector){

  int i = row & vector;

  i = i - ((i >> 1) & 0x55555555);
  i = (i & 0x33333333) + ((i >> 2) & 0x33333333);
  i = (((i + (i >> 4)) & 0x0F0F0F0F) * 0x01010101) >> 24;

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Together with AND and OR to implement the Jaccard similarity

Working Assumption

Similar text

- Documents that have lots of shingles in common have similar text, even if the text appears in different order.

Caveat

- You must pick k large enough, or most documents will have most shingles.
- It seems to be that
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Imagine the following

We need to find near-duplicate documents among $N = 1,000,000$ documents.

Compute pairwaise Jaccard similarites

- Naïvely, we would have to compute pairwaise Jaccard similarites for every pair of docs.
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  - At 10^5 secs/day and 10^6 comparisons/sec, it would take 5 days.

For something larger

For $N = 10$ million, it takes more than a year...

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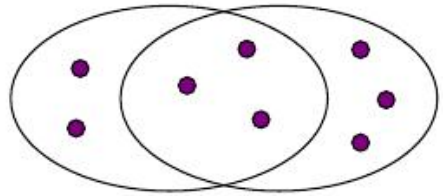
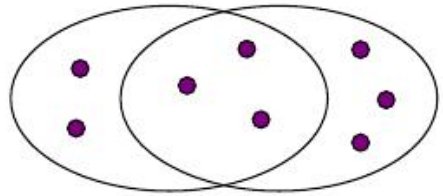


Encoding Sets as Bit Vectors



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- Encode sets using 0/1 (bit, boolean) vectors.
  - One dimension per element in the universal set.
- Interpret set intersection as bitwise **AND**, and set union as bitwise **OR**.

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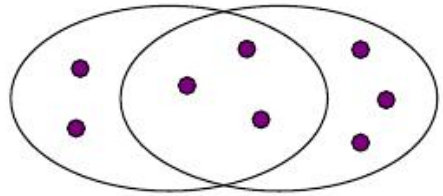

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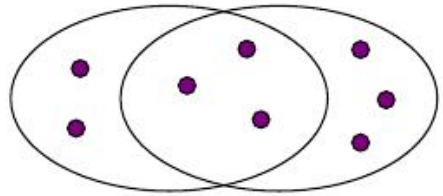

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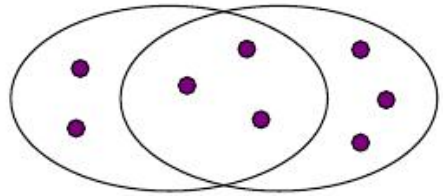

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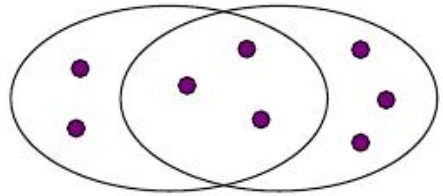

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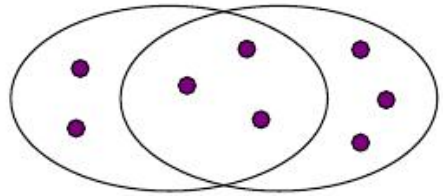

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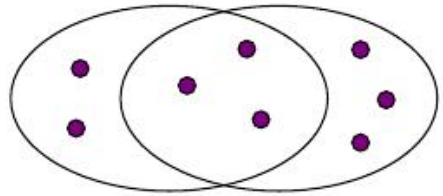

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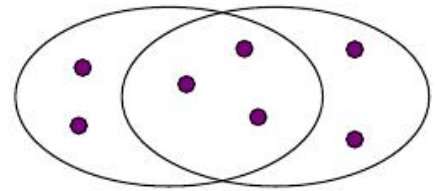
From Sets to Boolean Matrices

Rows

- Rows are equal to elements (shingles)



Columns

- The Columns are equal to sets (documents)
  - 1 in row e and column s if and only if e is a member of s
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value 1)
  - **Typical matrix is sparse!**

1	1	1	0
1	1	0	1
0	1	0	1
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Each document is a column

- Example: $sim(C_1, C_2) = ?$
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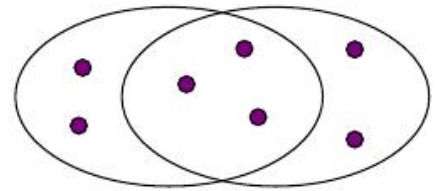

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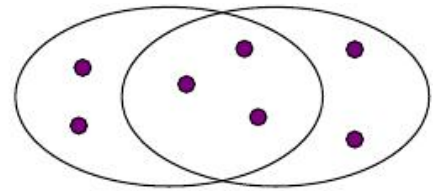

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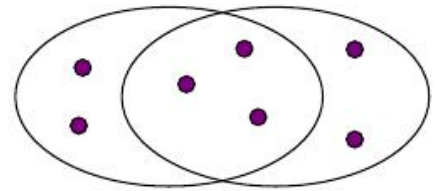

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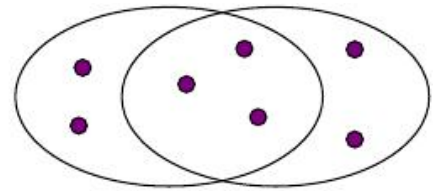

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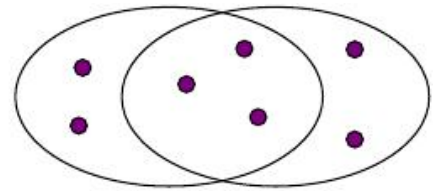

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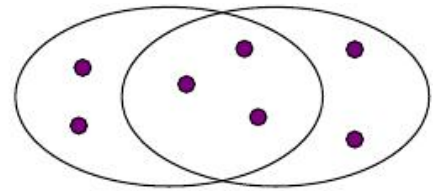

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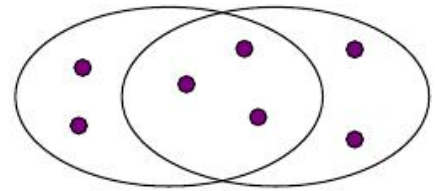

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Outline: Finding Similar Columns

So far and next goal

- So far:
  - Documents → Sets of shingles
  - Represent sets as boolean vectors in a matrix
- Next Goal: Find similar columns, Small signatures

Approach

1. Signatures of columns: small summaries of columns
2. Examine pairs of signatures to find similar columns
   - Essential: Similarities of signatures & columns are related
3. Optional: Check that columns with similar signatures are really similar

Warnings

- Comparing all pairs may take too much time: Job for Locality Sensitive Hashing (LSH)
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1. Signatures of columns: small summaries of columns
2. Examine pairs of signatures to find similar columns
   - Essential: Similarities of signatures & columns are related
3. Optional: Check that columns with similar signatures are really similar

Warnings

- Comparing all pairs may take too much time: Job for Locality Sensitive Hashing (LSH)
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So far and next goal

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  - Represent sets as boolean vectors in a matrix
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Hashing Columns (Signatures)

Key idea

- "Hash" each column C to a small signature $h(C)$, such that:
  - (1) $h(C)$ is small enough that the signature fits in RAM.
  - (2) $sim(C_1, C_2)$ is the same as the "similarity" of signatures $h(C_1)$ and $h(C_2)$.

Goal

- Find a hash function $h(\cdot)$ such that:
  - if $sim(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$.
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Similarity metric

- Clearly, the hash function depends on the similarity metric:
  - Not all similarity metrics have a suitable hash function.

Hash function

- There is a suitable hash function for Jaccard similarity: **Min-hashing**.

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Random permutation

Imagine the rows of the boolean matrix permuted under random permutation π .

"Hash" function $h_\pi(C)$

- Define a "hash" function $h_\pi(C) =$ the number of the first (in the permuted order π) row in which column C has value 1:

$$h_\pi(C) = min_\pi \pi(C)$$

What can we do?

- Use several (e.g., 100) independent hash functions to create a signature of a column

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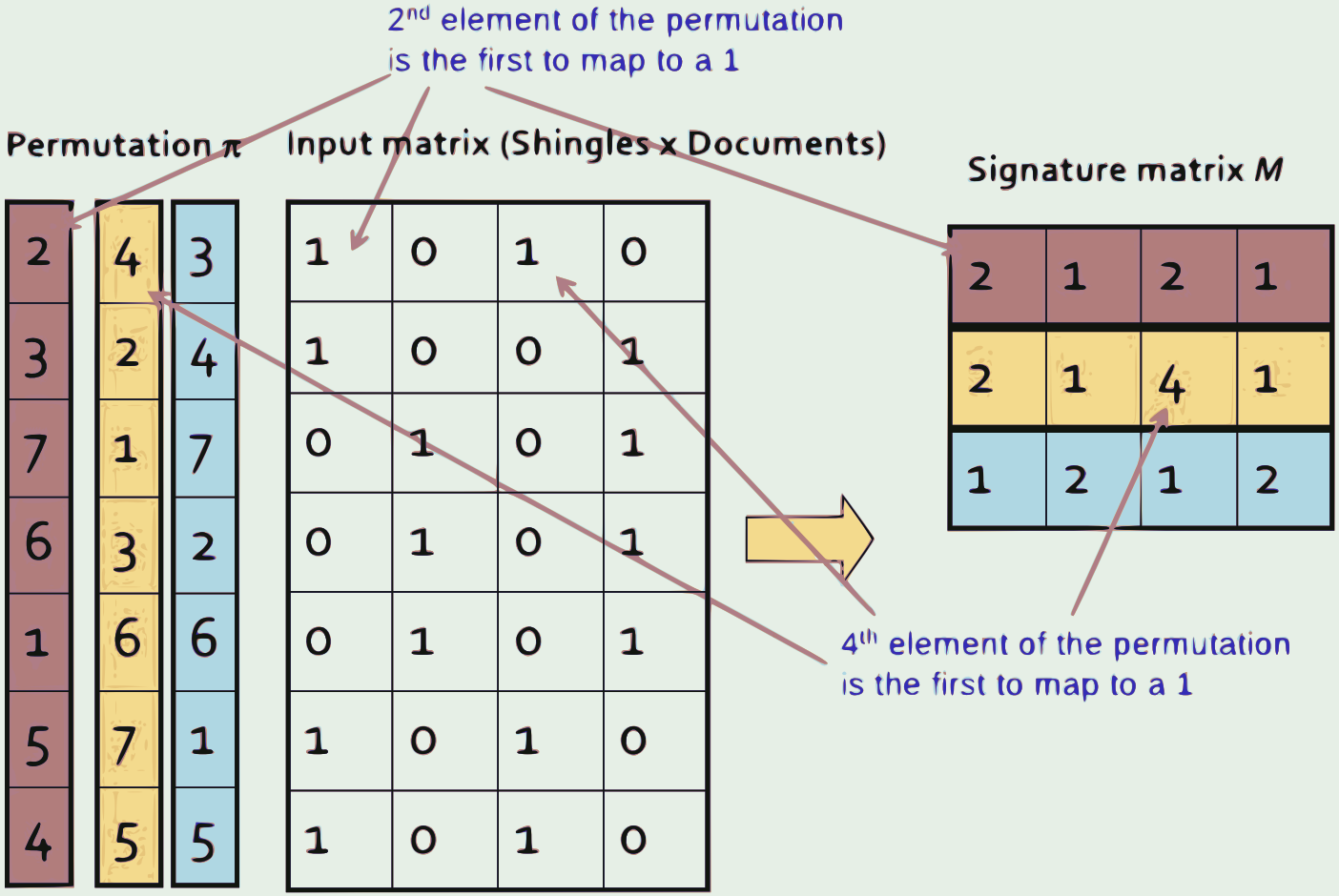
Min-Hashing Example

Note: Another (equivalent) way is to store row indexes:

1	5	1	5
2	3	1	3
6	4	6	4

Something Notable



Surprising Property

0	0
0	0
1	**1**
0	0
0	1
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- Choose a random permutation π
- Claim: $Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$ Why?

Why?

- Let X be a document (set of shingles)
- Then: $Pr[\pi(x) = min(\pi(X))] = 1/|X|$
- It is equally likely that any $x \in X$ is mapped to the min element
- Let x be s.t. $\pi(x) = min(\pi(C_1 \bigcup C_2))$
- Then either: $\pi(x) = min(\pi(C_1))$ if $x \in C_1$, or $\pi(x) = min(\pi(C_2))$ if $x \in C2$
  - One of the two cols had to have 1 at position x
- So the prob. that both are true is the prob. $x \in C_1 \bigcap C_2$

$$Pr[min(\pi(C_1)) = min(\pi(C_2))] = \frac{|C_1 \bigcap C_2|}{|C_1 \bigcup C_2|} = sim(C_1, C_2) \quad (2)$$

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Four Types of Rows between two Documents

Given cols C_1 and C_2, rows may be classified as

	C_1	C_2
A	1	1
B	1	0
C	0	1
D	0	0

$a = \#$rows of type A, etc.

Note

$$sim(C_1, C_2) = \frac{a}{a+b+c} \quad (3)$$

Then

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Similarity for Signatures

We know

- $Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Now generalize to multiple hash functions

Similarity

- The similarity of two signatures is the fraction of the hash functions in which they agree

Note

- Because of the minhash property, the similarity of columns is the same as the expected similarity of their signatures

Similarity for Signatures

We know

- $Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
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Min-Hashing Example

Note: Another (equivalent) way is to store row indexes:

1	5	1	5
2	3	1	3
6	4	6	4

Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix M

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

MinHash Signatures

- Pick $K = 100$ random permutations of the rows
- Think of $sig(C)$ (Signature of C) as a column vector
- $sig(C)[i] =$according to the i-th permutation, the index of the first row that has a 1 in column C

$$sig(C)[i] = min(\pi i(C))$$

- Note: The sketch (signature) of document C is small – ~ 100 bytes!
- We achieved our goal! We "compressed" long bit vectors into short signatures

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Outline

Implementation Trick

- Permuting rows even once is prohibitive

Row hashing!

- Pick $K = 100$ hash functions k_i
- Ordering under k_i gives a random row permutation!

One-pass implementation

- For each column C and hash-function k_i keep a "slot" for the min-hash value
  - Initialize all $sig(C)[i] = \infty$
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    - Suppose row j has 1 in column C
    - Then for each k_i:

If $k_i(j) < sig(C)[i]$, then $sig(C)[i] \leftarrow k_i(j)$

How to pick a random hash function $h(x)$?
Universal hashing:
$h_{a,b}(x) = ((a \cdot x + b) \bmod p) \bmod N$
where:
$a, b \ldots$ random integers
$p \ldots$ prime number $(p > N)$

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Trying to define LSH

2	1	4	1
1	2	1	2
2	1	2	1

Goal

- Find documents with Jaccard similarity at least s (for some similarity threshold, e.g., $s = 0.8$)

LSH – General idea

- Use a function $f(x, y)$ that tells whether x and y is a candidate pair: a pair of elements whose similarity must be evaluated.

For MinHash matrices

- Hash columns of signature matrix M to many buckets.
- Each pair of documents that hashes into the same bucket is a candidate pair.

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- Columns x and y of M are a candidate pair if their signatures agree on at least fraction s of their rows:
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- Hash columns of signature matrix M several times

Likely to hash

- Arrange that (only) similar columns are likely to hash to the same bucket, with high probability

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- Candidate pairs are those that hash to the same bucket

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Partition M into b Bands

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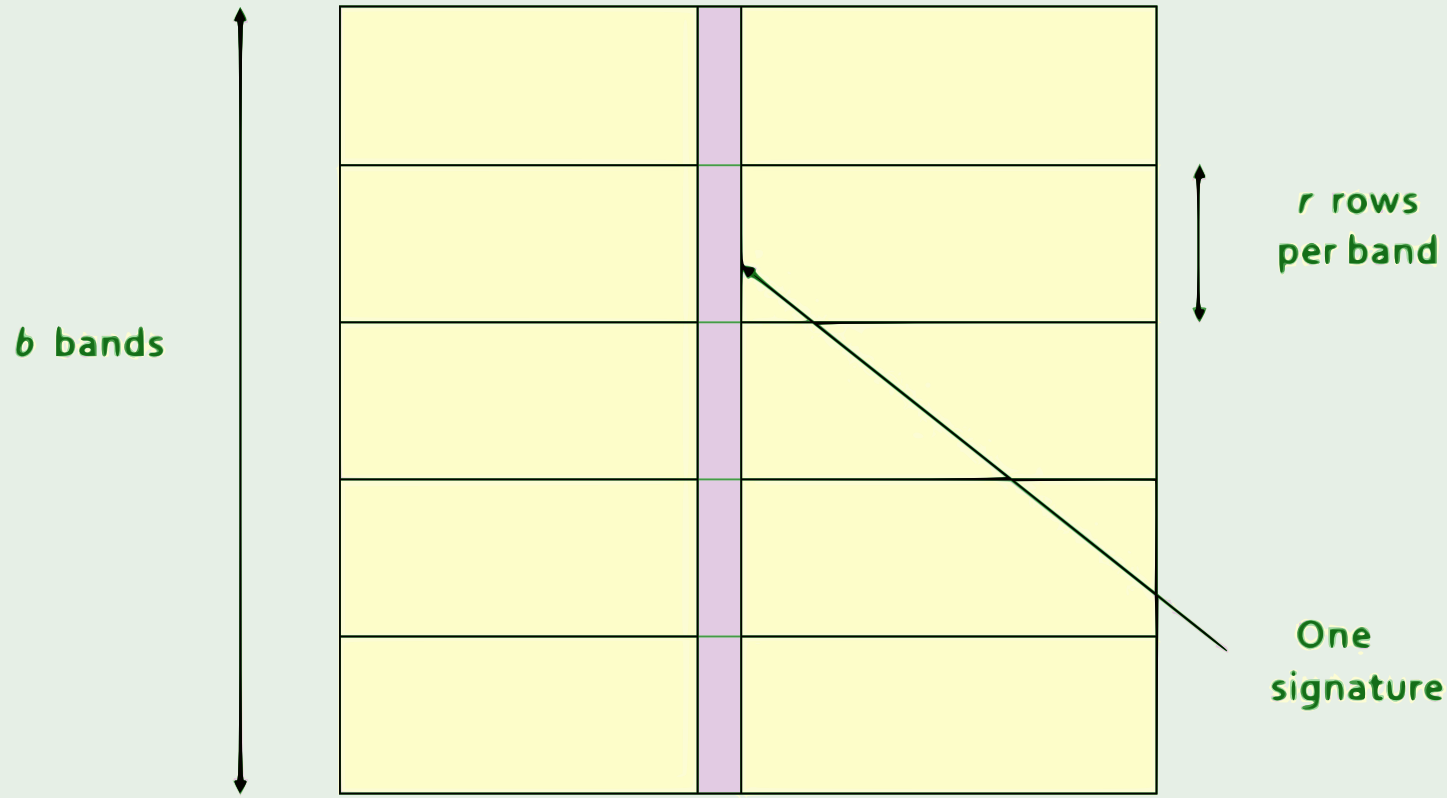

Partition M into b Bands

Divide Matrix

- Divide matrix M into b bands of r rows.
- For each band, hash its portion of each column to a hash table with k buckets.
  - Make k as large as possible.

Candidate

- Candidate column pairs are those that hash to the same bucket for ≥ 1 bands.

Catch most similar pairs

- Tune b and r to catch most similar pairs, but few non-similar pairs.

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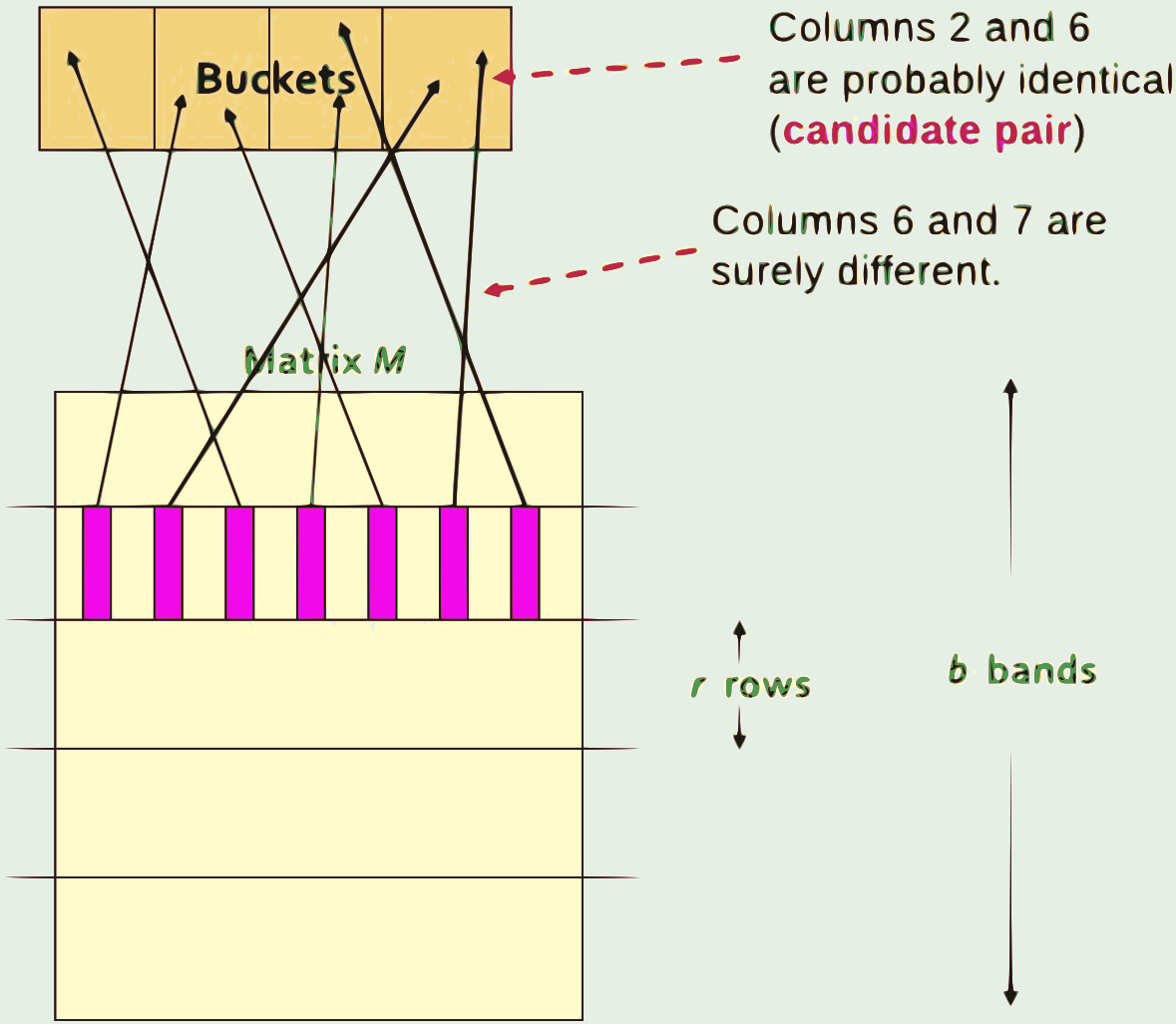

Simplifying Assumption

Identical

- There are enough buckets that columns are unlikely to hash to the same bucket unless they are identical in a particular band

Same bucket

- Then, we assume that "same bucket" means "identical in that band"

Not for correctness

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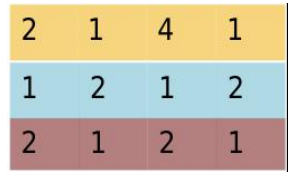

Assume the following case

- Suppose $100,000$ columns of M ($100k$ docs)
- Signatures of 100 integers (rows)
- Therefore, signatures take $40Mb$
- Choose $b = 20$ bands of $r = 5$ integers/band

Goal

- Find pairs of documents that are at least $s = 0.8$ similar

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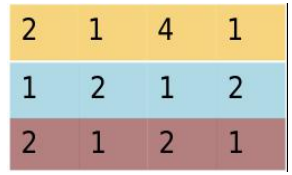

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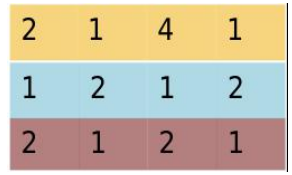

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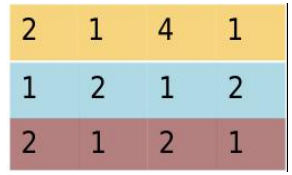
- Suppose $100,000$ columns of M ($100k$ docs)
- Signatures of 100 integers (rows)
- Therefore, signatures take $40Mb$
- Choose $b = 20$ bands of $r = 5$ integers/band

Goal

- Find pairs of documents that are at least $s = 0.8$ similar

Example of Bands



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Now, if C_1,C_2 are 80% Similar

2	1	4	1
1	2	1	2

Assume

- Find pairs of $\geq s = 0.8$ similarity, set $b = 20$, $r = 5$
- Assume: $sim(C_1, C_2) = 0.8$
  - Since $sim(C_1, C_2) \geq s$, we want C_1, C_2 to be a candidate pair: We want them to hash to at least 1 common bucket (at least one band is identical)

In one particular band

- Probability C_1, C_2 identical in one particular band: $(0.8)^5 = 0.328$

What is the Probability of not being similar at all?

- Probability C_1, C_2 are not similar in all of the 20 bands: $(1 - 0.328)^{20} = 0.00035$
  - i.e., about $1/3000$th of the 80%-similar column pairs are false negatives (we miss them)
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Identical in one particular band

- Probability C_1, C_2 identical in one particular band: $(0.3)^5 = 0.00243$.

Properties

- Probability C_1, C_2 identical in at least 1 of 20 bands: $1 - (1 - 0.00243)20 = 0.0474$.
  - In other words, approximately 4.74% pairs of docs with similarity 0.3% end up becoming candidate pairs.
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You need to pick

- The number of minhashes (rows of M).
- The number of bands b.
- The number of rows r per band to balance false positives/negatives.

Example

- if we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

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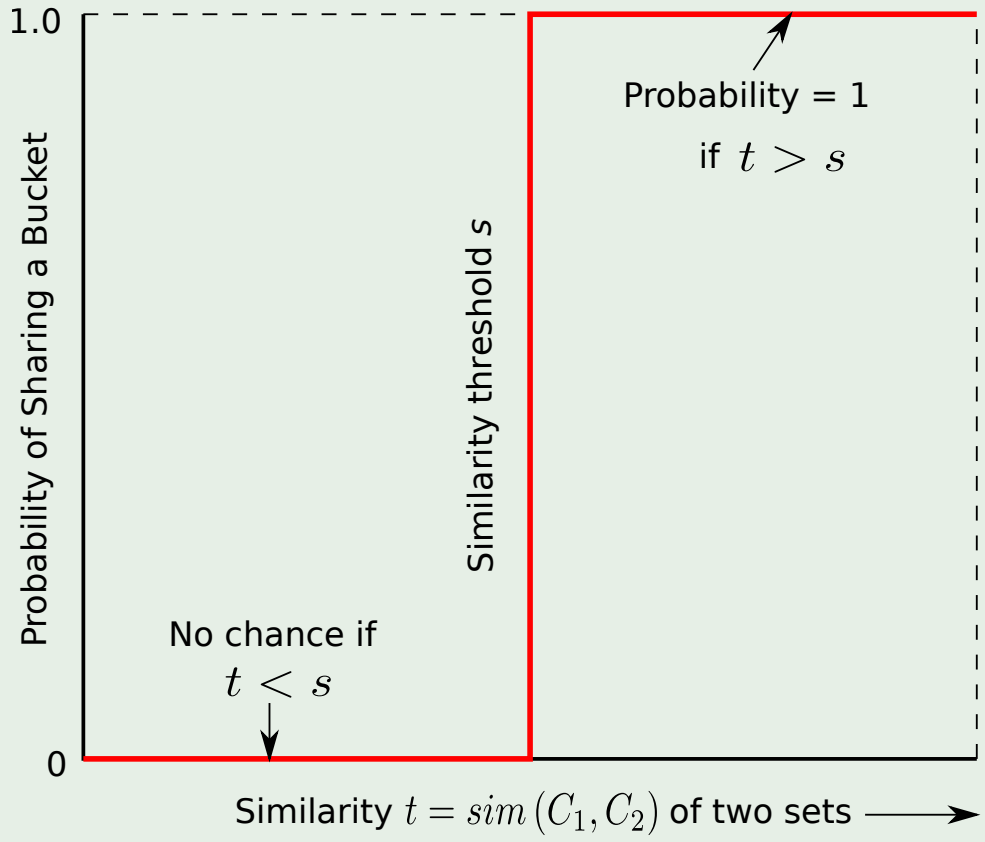
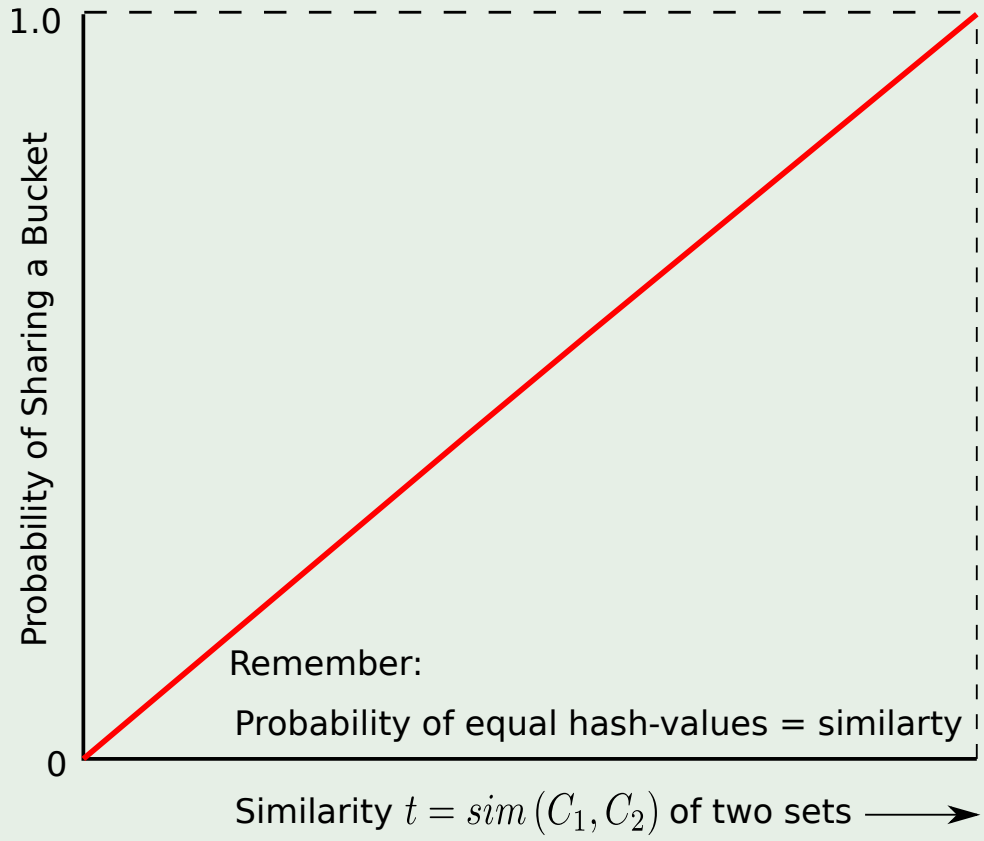
Analysis of LSH - What We Want

The Ideal detection of similar objects



What 1 Band of 1 Row Gives You

Not so great



Given that probability of two documents aggree in a row is s

We can calculate the probability that these documents become a candidate pair as follows

1. The probability that the signatures agree in all rows of one particular band is s^r.
2. The probability that the signatures disagree in at least one row of a particular band is $1 - s^r$.
3. The probability that the signatures disagree in at least one row of each of the bands is $(1 - s^r)^b$.
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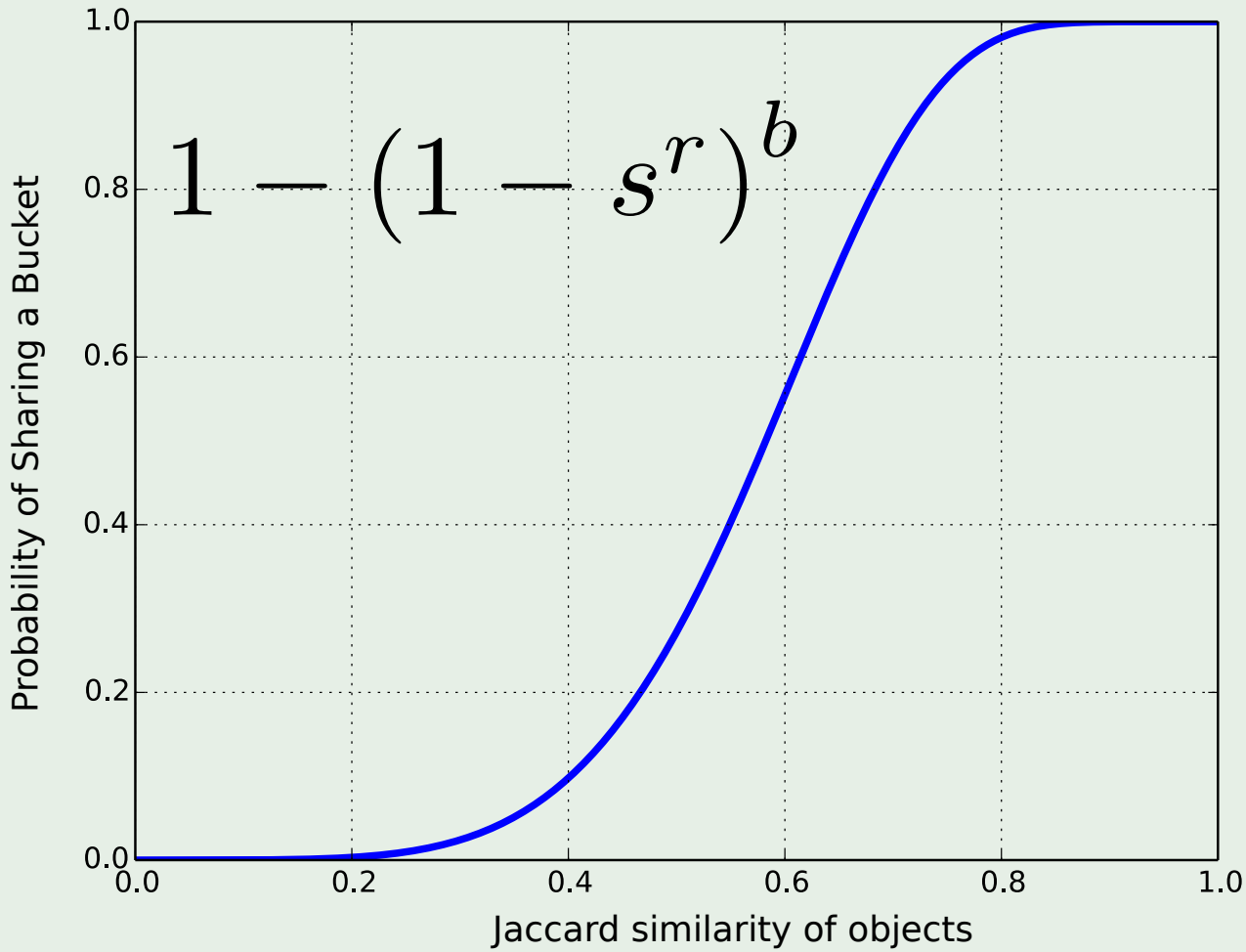
If you fix r and b

Something Notable



Example: $b = 20$; $r = 5$

Given

- Similarity threshold s

Similarity threshold s Prob. that at least 1 band is identical

s	$1 - (1 - s^r)^b$
.2	0.006
.3	0.047
.4	0.186
.5	0.470
.6	0.802
.7	0.975
.8	0.9996

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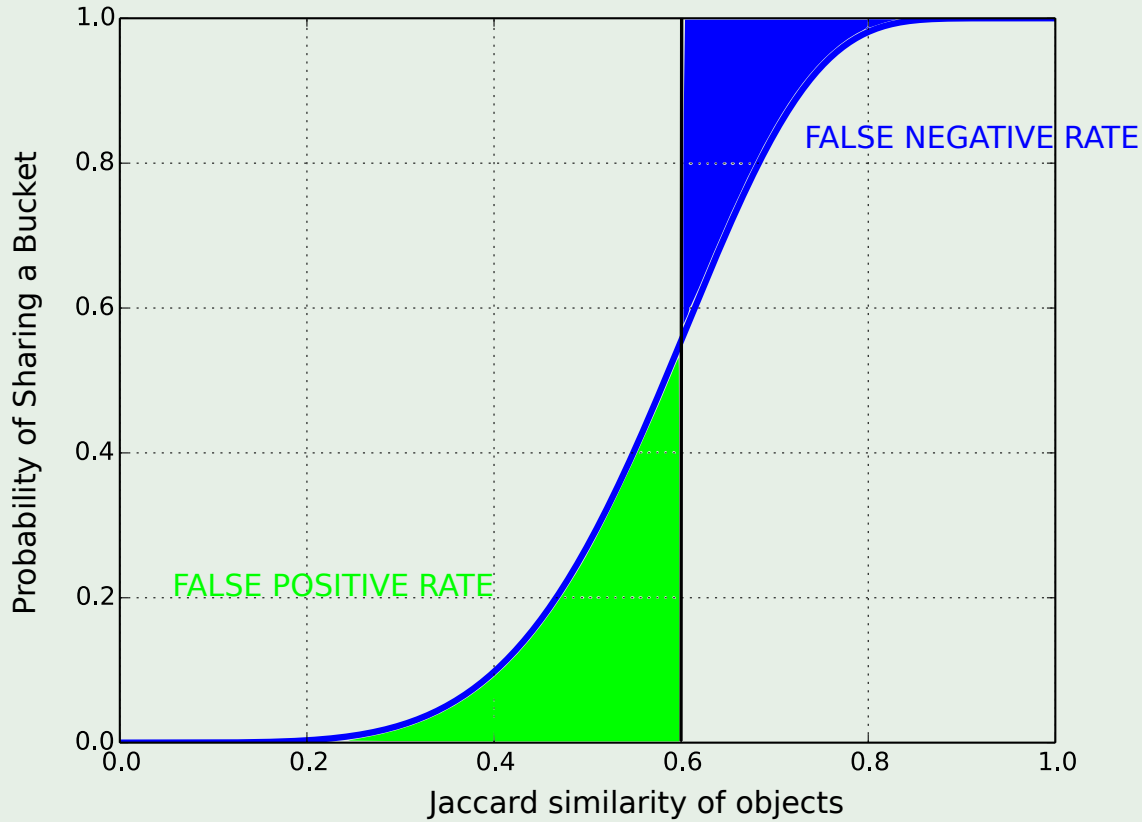
Picking r and b: The S-curve

Picking r and b to get the best S-curve

- 50 hash-functions $(r = 5, b = 10)$



LSH Summary

Tune M, b, r

- Tune M, b, r to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures

Check in main memory

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Shingling

- Convert documents to sets
  - We used hashing to assign each shingle an ID Min-hashing: Convert large sets to short

Min-hashing

- Convert large sets to short signatures, while preserving similarity.
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Locality-Sensitive Hashing

- Focus on pairs of signatures likely to be from similar documents.
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