Introduction to Machine Learning

A Basic Introduction to Learning

Andres Mendez-Vazquez

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Outline

1. Learning in the World
   - Introduction
   - What do we want?
   - What type of Variables do we have?
2. Regression as Controlled Overfitting
   - Polynomial Curve Fitting
   - A Loss Function for Learning
   - “Extreme” Cases of Fitting
3. Example of Approaches to Prediction
   - Two Simple Models
     - Linear Models
     - Nearest-Neighbor Methods
   - Many Methods are Variants of Them
   - Statistical Decision Theory
     - Loss Function
     - Nearest Neighborhood Example
     - Nearest Neighborhood vs Liner Regression
4. Supervised Learning as a Function Approximation
   - Statistical Model for $P\left(X,Y\right)$
   - Supervised Learning
   - Function Approximation
     - Parameters in Function Approximation
5. Some Classes of Estimators
   - Roughness Penalty and Bayesian Methods
   - Kernel Methods and Local Regression
   - Basis Functions and Dictionary Methods
6. Conclusions
   - A Vast Field

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Statistical Learning

Clearly, there are many problems important for us

- Predict whether a patient, hospitalized due to a heart attack, will have a second heart attack,
- Predict the price of a stock in 6 months from now,
- Given a market population what products to recommend to them,
- How to recognize in a video a car or person,
- How to predict maintenance in a factory,
- etc.

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Example

Given a sample on frequency of the most common words in a series of 4601 emails

	george	you	your	hp	free	hpl	!	our	re	edu
Spam	0.00	2.26	1.38	0.002	0.52	0.01	0.51	0.51	0.13	0.01
email	1.27	1.27	0.44	0.90	0.07	0.43	0.11	0.18	0.42	0.29

We want to design a series of rules to guess when you have a Spam or a genuine email

$$f_1\left(message\right) = \begin{cases} \%george < 0.6 \text{ and } \%you > 1.5 & spam \\ \text{Otherwhise} & email \end{cases}$$

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Therefore

Let $X \in \mathbb{R}^d$ a real valued random input and $Y \in \mathbb{R}$ a real valued output

With joint distribution $P(X, Y)$

We are looking for a function that takes the variables in X to map them into Y

$f(X)$ predicting Y

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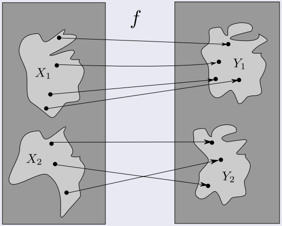

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We have two main types

Quantitative Data

- They are measures of values or counts and are expressed as numbers.
  - Quantitative data are data about numeric variables (e.g. how many; how much; or how often).

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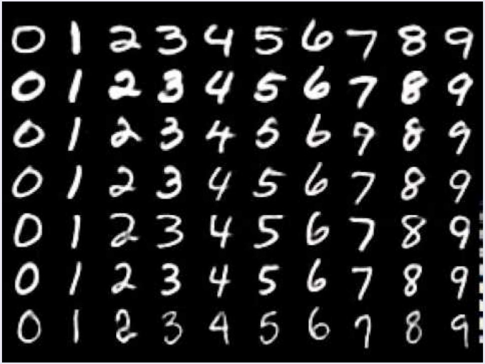
For Example (In the case of Outputs)

If we are classifying digits



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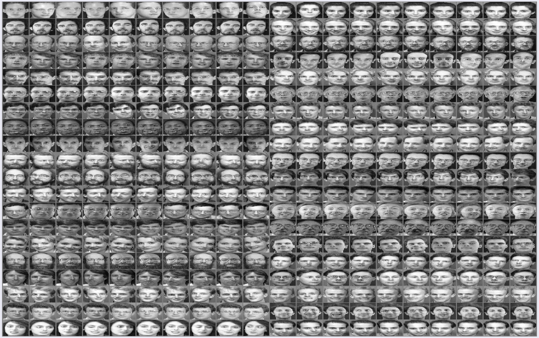

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$$\{\text{Andres,Fabiola}\} = \text{People that can drive a certain car}$$

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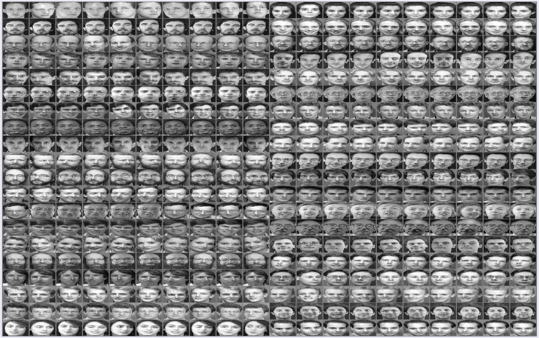

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The Basic Problem

Suppose

- We observe a real-valued input variable $x \in \mathbb{R}$

We are looking to predict

- The value of a real valued variable $y \in \mathbb{R}$

Thus, we have the following training data set of size N

$$\boldsymbol{x} \equiv (x_1, x_2, \cdots, x_N)^T$$

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Note: *We need data to construct prediction rules, often a lot of it.*

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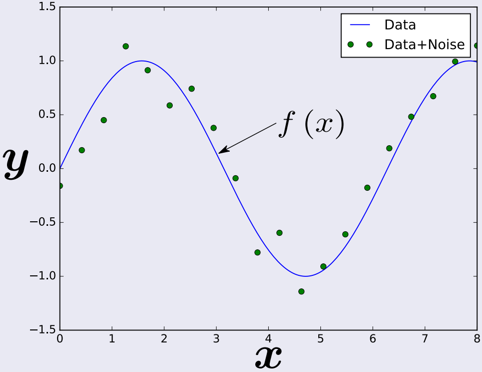
For Example

We have the function $g(x) = f(x) + \alpha U(0, 1)$ with the real function $f(x) = \sin\{x\}$



What is our Goal?

Our goal is to exploit this training set

- We want to make predictions of the value $\widehat{y}$ (pronounced y-hat) given a new value $\widehat{x}$ (y-hat).

What can we use first?

$$y = g(x, w) = w_0 + w_1 x + w_2 x^2 + \cdots + w_M x^d = \sum_{i=0}^{d} w_i x^i$$

Where

- d is the order of the polynomial.
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Further

These functions are linear at the parameter $\boldsymbol{w}$

- They are quite important and are called ***linear models!!!***

How do we guess these values?

- By fitting the polynomial to the training data.

How do we do this?

- This can be done by minimizing an error function or loss function measuring, ϵ:
  - The difference between the function $g(x, \boldsymbol{w})$, for any given value of $\boldsymbol{w}$, and the training set data points.

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Definition of “Learning.

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- Given that the information of an object has been summarized by d features comprised as a feature vector $\boldsymbol{x} \in \mathbb{R}^d$, and each of these objects has been labeled by elements in a set $\{y_i \in \mathbb{R}\}$.
- This allows to split the set of object into a series classes, as for example $y_i \in \{-1, 1\}$.
- Then, the process of learning is the generation of a mapping $f : \mathbb{R}^d \mapsto \{y_i\}$ such that, **for example**, the squared error estimation of the class label of a new sample is minimized:

$$\min_{\widehat{f}} R\left(\widehat{f}\right) = \min_{\widehat{f}} E_{\mathcal{X},\mathcal{Y}}\left[\left(\widehat{f}(x) - y\right)^2 | x \in \mathcal{X} \subseteq \mathbb{R}^d, y \in \mathcal{Y} \subseteq \mathbb{R}\right]$$

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- Given a sequence of data samples, $x_1, x_2, ..., x_N$ sampled iid from a distribution $P(x|\Theta)$, and an hypothesis function $f : X \mapsto Y$ that allows to map the samples x_i into a particular output y_i.
- A measure of the risk of missing the estimation, $f(x)$, is found by using a function, called loss function, measuring the difference between the desired output y_i and the estimation $f(x_i)$.
- Thus, the Empirical Risk is defined as the expected value of the loss function based in the joint distribution $P(x,y)$.

$$R(h) = E_{X,Y}\left[L(f(x), y)\right] = \int_{X,Y} L(f(x), y)\, p(x,y)\, dx dy$$

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This is the important part!!!

In general

- The risk $R(f)$ cannot be computed because the distribution $P(\boldsymbol{x}, y)$ is unknown to the learning algorithm

However, we can compute an approximation

- Called empirical risk, by averaging the loss function on the training set:

$$R_{emp}(f) = \frac{1}{N}\sum_{i=1}^{N} L(f(\boldsymbol{x}_i), y_i)$$

The Empirical Risk Minimization Principle

- It states that the learning algorithm should choose a hypothesis $\hat{f}$ which minimizes the empirical risk:

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$$\widehat{f} = \arg\min_{f \in \mathcal{F}} R_{emp}(f)$$

One simple choice of error function

The Average of the Sum of the Squares of the Errors

$$E(\boldsymbol{w}) = \frac{1}{N}\sum_{i=1}^{N}\left[g\left(x_i, \boldsymbol{w}\right) - y_i\right]^2$$

Something Notable

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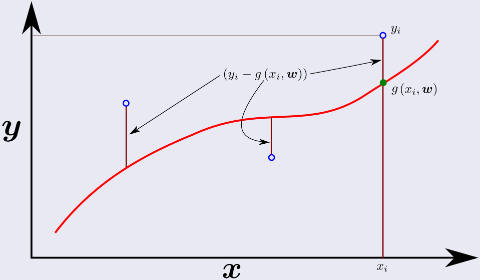

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   - Introduction
   - What do we want?
   - What type of Variables do we have?

2. Regression as Controlled Overfitting
   - Polynomial Curve Fitting
   - A Loss Function for Learning
   - “Extreme” Cases of Fitting

3. Example of Approaches to Prediction
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     - Nearest-Neighbor Methods
   - Many Methods are Variants of Them
   - Statistical Decision Theory
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5. Some Classes of Estimators
   - Roughness Penalty and Bayesian Methods
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Case 1

Choose the estimate of $f(x)$, $g(x, \boldsymbol{w})$, to be independent of $\mathcal{D}$

For example, $g(x, \boldsymbol{w}) = w_1 x + w_0$

We call this **HIGH BIAS**

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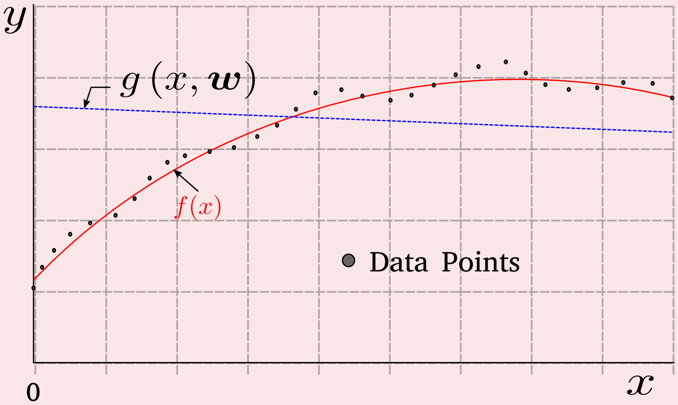

Case 2

In the other hand

Now, $g(x, \boldsymbol{w})$ corresponds to a polynomial of high degree so it can pass through each training point.

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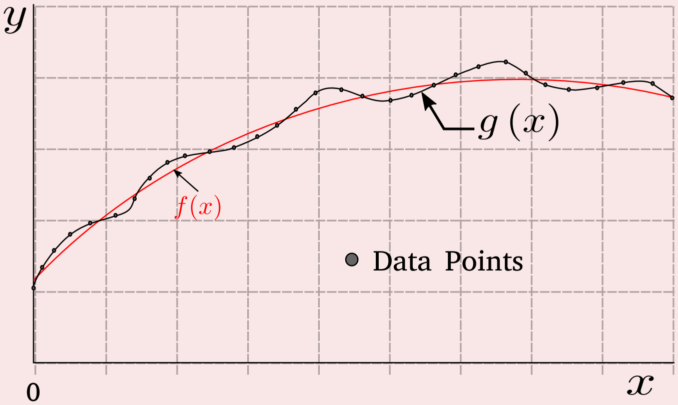

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Our General Case

Our Data Set

1. A Series of $X \in \mathbb{R}^d$ of real valued random input vector.

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix}$$

   - Here, each variable X_i is Quantitative or Qualitative variables in the correct numeric representation.

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Linear Models

We have the following model

- The linear model has been a mainstay of statistics for the past 30 years.

The Model looks like on an input $X^T = (X_1, X_2, \ldots, X_d)$

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It is many times convenient

To use the dot product in Linear Algebra

$$\widehat{Y} = (1, X_1, X_2, \dots, X_d) \begin{pmatrix} \widehat{w}_0 \\ \widehat{w}_1 \\ \vdots \\ \widehat{w}_d \end{pmatrix} = X^T \widehat{\boldsymbol{w}}$$

Furthermore, $\widehat{Y}$ could be a constant or a N vector

$$\widehat{Y} = \begin{pmatrix} \widehat{Y}_1 \\ \widehat{Y}_2 \\ \vdots \\ \widehat{Y}_N \end{pmatrix} = \begin{pmatrix} 1 & X_1^{(1)} & X_2^{(1)} & \cdots & X_d^{(1)} \\ 1 & X_1^{(2)} & X_2^{(2)} & \cdots & X_d^{(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_1^{(N)} & X_2^{(N)} & \cdots & X_d^{(N)} \end{pmatrix} \begin{pmatrix} \widehat{w}_0 \\ \widehat{w}_1 \\ \vdots \\ \widehat{w}_d \end{pmatrix} = Xw$$

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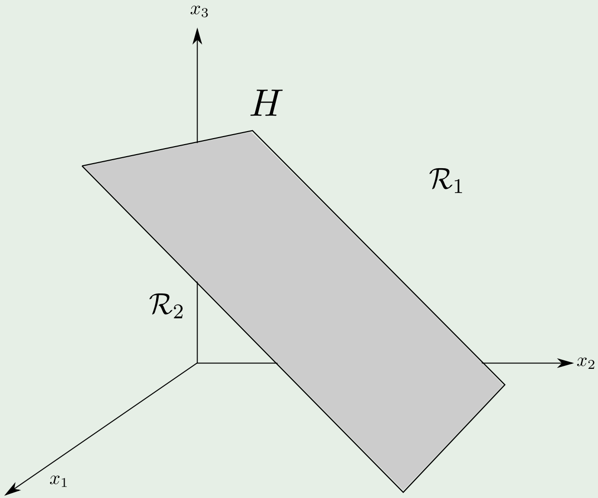
This basically define an hyperplane

The space is split in two regions (Example in $\mathbb{R}^3$) by the hyperplane H



A Convenient Loss Functions

Thus, we look for a Loss function (A convenient one the LSE)

$$L\left(\boldsymbol{w}\right)=\sum_{i=1}^{N}\left(\boldsymbol{y}_i-\boldsymbol{x}_i^T\boldsymbol{w}\right)^2$$

Then

It is possible to get a unique solution

$$\boldsymbol{w} = \left(\boldsymbol{X}^T \boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \boldsymbol{y}$$

Then, it is possible to fit the linear model to the following data

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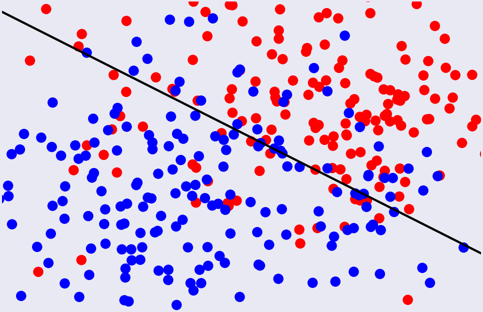

How do we do classification here?

Given

1. $Y = -1$ for the **blue** data set.
2. $Y = 1$ for the **red** data set.

Then, the fitted values $\hat{Y}$ are converted to a fitted class variable $\hat{G}$ according

$$\hat{G} = \begin{cases} \text{red} & \text{if } \hat{Y} > 0 \\ \text{blue} & \text{if } \hat{Y} \leq 0 \end{cases}$$

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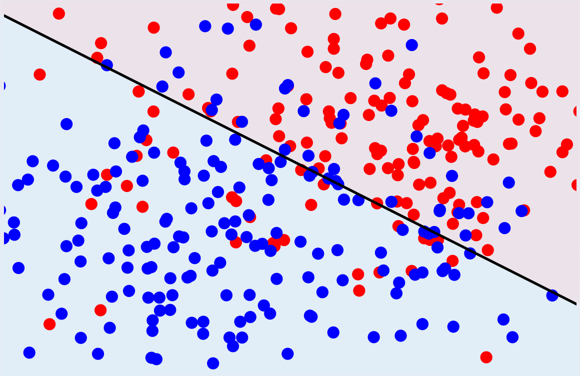
Decision Boundary

The two predicted classes are separated

Decision Boundary $\left\{ \boldsymbol{x} | \boldsymbol{x}^T \widehat{\boldsymbol{w}} = 0 \right\}$



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We do not know the underlaying models that generates the data.

Scenario 1

- The training data in each class were generated from bivariate Gaussian distributions with uncorrelated components and different means.

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Nearest-Neighbor Methods

Nearest-neighbor methods use those observations in the training set

- Which are closets in the input space to a sample x to from $\widehat{Y}$.

K-Nearest Formulation

$$\widehat{Y}(x) = \frac{1}{k} \sum_{x_i \in N_k(x)} y_i$$

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Clearly $N_k(\boldsymbol{x})$ requires a distance

Implies a Distance!!! Which one?

$$d_2(\boldsymbol{x},\boldsymbol{y}) = \sqrt{\boldsymbol{x}^T\boldsymbol{y}} \leftarrow\!-- \text{ Euclidean Distance}$$

$$d_1(\boldsymbol{x},\boldsymbol{y}) = \sum_{i=1}^{d} |x_i - y_i| \leftarrow\!-- \text{ Manhattan Distance}$$

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Furthermore

Given a Data Matrix $\boldsymbol{X}$ and the Mean Data Matrix $\overline{\boldsymbol{X}}$

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We generate the variance-covariance matrix

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Then, we have

The Mahalanobis Distance

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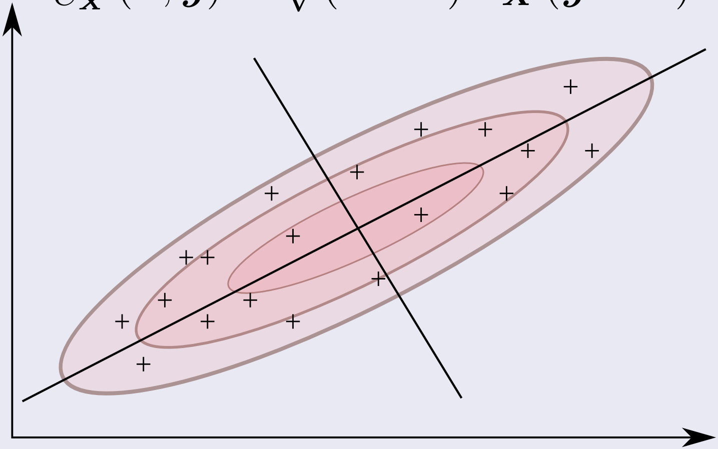

Therefore

we find the k observations

With x_i closest to x in input space, and average their responses.

And Again

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Example

We have only five neighbor, $k = 5$

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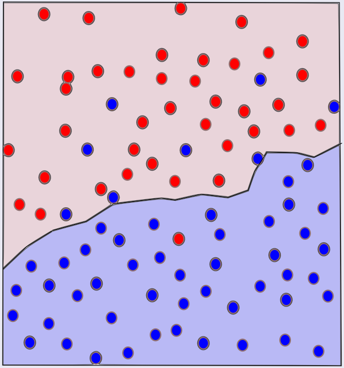
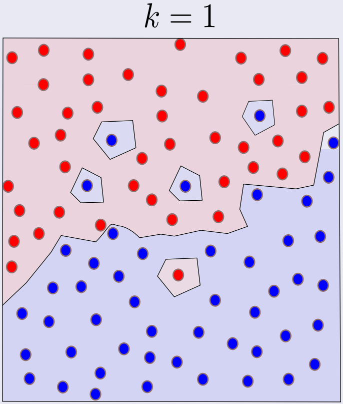


Example - Actually The Voronoi Tessellation of the Training Data

We have only one neighbor, $k = 1$



Note: Each point x_i has an associated tile bounding the region for which it is the closest input point.

Therefore

$K = 1$ Vs. $K = 5$

For $K = 5$, we see that far fewer training observations are misclassified when compared with the Linear Model

With $K = 1$

None of the training data are misclassified!!!

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For example

Kernel methods

- They use weights that decrease smoothly to zero with distance from the target point,
    - Quite different rather from using 0/1 weights used by k-nearest neighbors.

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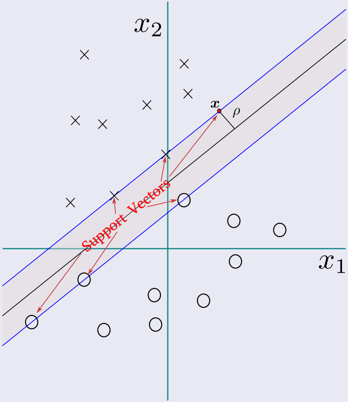
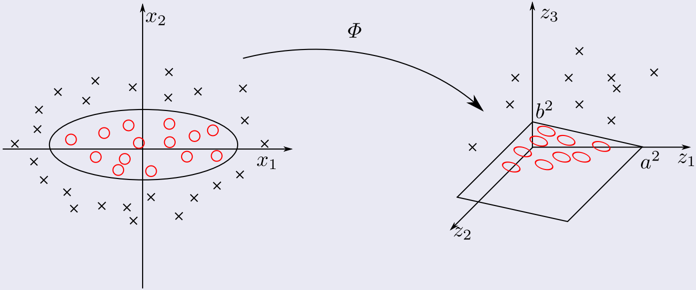


Furthermore

Something Notable

- In High-Dimensional spaces the distance kernels are modified to obtain better classifications.



$$\Phi : (x_1, x_2) \rightarrow \left(x_1^2, \sqrt{2}x_1x_2, x_2^2\right)$$

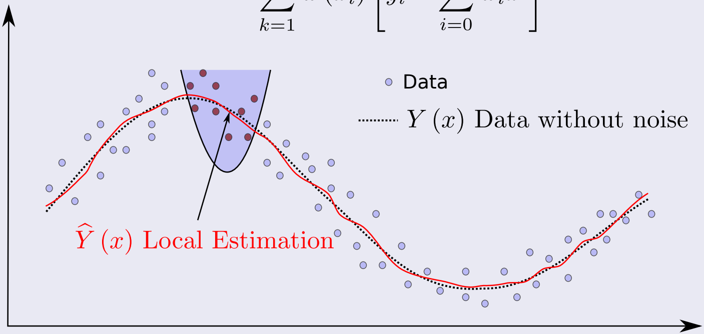
$$\left(\frac{x_1}{a}\right)^2 + \left(\frac{x_2}{b}\right)^2 = 1 \rightarrow \frac{z_1}{a^2} + \frac{z_3}{b^2} = 1$$

Example

Local Regression

Local regression fits linear models by locally weighted least squares.

$$\sum_{k=1}^{N} w\left(x_i\right)\left[y_i - \sum_{i=0}^{d} w_i x^i\right]^2$$


Data
$Y\left(x\right)$ Data without noise
$\widehat{Y}\left(x\right)$ Local Estimation

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The Samples as Random Variables

As Always Probability

We first consider:

- $X \in \mathbb{R}^d$ denote a real valued input vector
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Therefore, we have a Joint Distribution $P(X, Y)$ and we seek

$$f(X) \text{ predicting } Y$$

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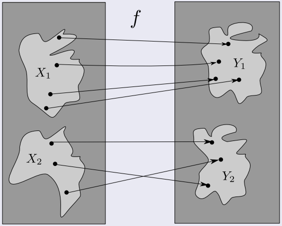

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We require a Loss Function

A convenient one is the Squared Error Loss

$$L\left(Y, f\left(X\right)\right) = \left(Y - f\left(X\right)\right)^2$$

There is a relation to noise $\epsilon \sim N\left(0, 1\right)$

$$Y_{noise}\left(X\right) = f\left(X\right) + \epsilon$$

The Squared Error Loss

- It tries to minimize the quadratic error $\epsilon = Y - f\left(X\right)$!!!

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A convenient one is the Squared Error Loss

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There is a relation to noise $\epsilon \sim N\left(0, 1\right)$

$$Y_{noise}\left(X\right) = f\left(X\right) + \epsilon$$

The Squared Error Loss

- It tries to minimize the quadratic error $\epsilon = Y - f\left(X\right)$!!!

We require a Loss Function

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This leads us to a criterion for choosing f

The Expected Prediction Error (EPE)

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\begin{aligned}
EPE &= E\left(Y - f\left(X\right)\right)^2 \\
&= \int \left[y - f\left(x\right)\right]^2 p_{xy}\left(x, y\right) dx dy
\end{aligned}
$$

Now, we can condition the probability density function with respect to X

$$p\left(X, Y\right) = p\left(Y|X\right) p\left(X\right)$$

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Thus

We have

$$\int \left[y - f\left(x\right)\right]^2 p_{xy}\left(x, y\right) dxdy = \int_X \int_Y \left[y - f\left(x\right)\right]^2 p_{y|x}\left(y|x\right) p_x\left(x\right) dxdy$$

$$= \int_X \left[\int_Y \left[y - f\left(x\right)\right]^2 p_{y|x}\left(y|x\right) dy\right] dx$$

$$= E_X \left[\int_Y \left[y - f\left(x\right)\right]^2 p_{y|x}\left(y|x\right) dy\right]$$

$$= E_X E_{Y|X} \left[\left(Y - f\left(X\right)\right)^2 |X\right]$$

What happens if we fix X?

$$EPE\left(f\right)_{X=x} = E_{Y|X=x} \left[\left(Y - f\left(x\right)\right)^2 |X = x\right]$$

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$$\begin{aligned}\int [y-f(x)]^2 p_{xy}(x,y)\,dxdy &= \int_X \int_Y [y-f(x)]^2 p_{y|x}(y|x)\,p_x(x)\,dxdy \\ &= \int_X \left[\int_Y [y-f(x)]^2 p_{y|x}(y|x)\,dy\right] dx \\ &= E_X \left[\int_Y [y-f(x)]^2 p_{y|x}(y|x)\,dy\right] \\ &= E_X E_{Y|X}\left[(Y-f(X))^2 |X\right]\end{aligned}$$

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We can optimize the function

By a Simple Analysis

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E_{Y|X=\boldsymbol{x}}\left[(Y-f(\boldsymbol{x}))^{2}|X=\boldsymbol{x}\right] = E_{Y|X=\boldsymbol{x}}\left[\left(Y+\overline{Y}-\overline{Y}-f(\boldsymbol{x})\right)^{2}|X=\boldsymbol{x}\right] \\
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Further, we have

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$$E_{Y|X=\boldsymbol{x}}\left[\left(Y-\overline{Y}\right)|X=\boldsymbol{x}\right]=E_{Y|X=\boldsymbol{x}}\left[Y\right]-E_{Y|X=\boldsymbol{x}}\left[\frac{1}{N}\sum_{i=1}^{N}Y_i\right]$$

$$=\mu_Y-\frac{1}{N}\sum_{i=1}^{N}E_{Y|X=\boldsymbol{x}}\left[Y_i\right]$$

$$=\mu_Y-\frac{N\mu_Y}{N}$$

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Then

We have that we can optimize point-wise

Then, if we choose

$$f(X) = \overline{Y} \approx E_Y [Y | X = \boldsymbol{x}]$$

- The conditional expectation, also known as the **regression function**!!!

Additionally, we can analyze Y

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The variance for Y that can be approximated by

$$\widehat{\sigma}_Y^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left(Y_i - \overline{Y} \right)^2$$

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Thus, the best prediction of Y at any point $X = x$ the regression function for LSE

- It is the conditional mean.

$$E_Y \left[Y | X = \boldsymbol{x} \right]$$

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Now Nearest Neighborhood

At each point $\boldsymbol{x}$

The method calculates the average of all those $y_i's$ with input $\boldsymbol{x}_i = \boldsymbol{x}$

$$\frac{1}{n_{\boldsymbol{x}_i=\boldsymbol{x}}}\sum_{\boldsymbol{x}_i=\boldsymbol{x}} y_i$$

Or in other way, an estimation based in the average

$$\widehat{f}\left(\boldsymbol{x}\right) = Ave\left(y_i | \boldsymbol{x}_i \in N_k\left(\boldsymbol{x}\right)\right)$$

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Two things happen here

- Expectation is approximated by averaging over sample data

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Thus, conditioning

- It is relaxing to some region **"close"** to the target point

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For large training sample size N

- The points in the neighborhood are likely to be close to $\boldsymbol{x}$.
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It is more under regularity conditions on $P(X,Y)$

- One can for that as $N \to \infty$ and $k \to \infty$ such that $k/N \to 0$

$$\hat{f}(x) \to E(Y|X=x)$$

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We often do not have very large number of samples!!!

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As the dimension d gets large

Thus, the metric size of the k-nearest neighborhood also gets larger.

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How does Linear Regression fit into this framework?

The regression function $f(x)$ is approximately linear in its arguments

$$f(\boldsymbol{x}) = \boldsymbol{x}^T \boldsymbol{w}$$

Plugging this linear model for $f(x)$ into EPE and differentiating

$$w = \left[E\left(XX^T\right)\right]^{-1} E(XY)$$

Note

- Note we have not conditioned on X.
- We have used our knowledge of the functional relationship.
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The least squares solution

- It amounts to replacing the expectation in

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by averages over the training data.

Then, we have that

k-nearest neighbors and least squares end up approximating conditional expectations by averages.

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We have the following differences

- Least squares assumes $f(x)$ is well approximated by a globally linear function.
- k-nearest neighbors assumes $f(x)$ is well approximated by a locally constant function.

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Some Times

We take the following assumption about the data

$$Y = f(X) + \epsilon$$

Where

- The Random Error has $E[\epsilon] = 0$
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Under this model, we have already a solution

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- There will be other non measured variables that also contribute to Y

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- Error in the measurement of the system error!!!

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It is natural to use

- Least Squares as a data criterion for model estimation!!!

Additionally, we can modify the independence assuming

$$Var\left(Y|X=x\right)=\sigma\left(x\right)$$

Then

- Both the mean and variance depend on X

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In general the conditional distribution $P\left(Y|X\right)$

- It can depend on X in complicated ways... and thus, the simplification models!!!

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Now

Given the model $Y = f(X) + \epsilon$

- Supervised Learning tries to learn f by data from a teacher.

Thus

- It is necessary to observe the system
- Collect data from it!!!
- Assemble a training set of observations

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This training set is feed into a learning algorithm

This system produces an output

$$\widehat{f}\left(x_i\right)$$

Something Notable

The Learning algorithm has the ability to modify its input/output relationship $\widehat{f}$ based on the difference $y_i - \widehat{f}\left(x_i\right)$.

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- At Applied Mathematics and Statistics the input $\mathcal{D}$ are viewed as points in $(d+1)-$dimensional space

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The function $f(x)$

Domain

- The domain of a function is the complete set of possible values of the independent variable.
- In our case, the $d-$dimensional subspace.

Range

- The range of a function is the complete set of all possible resulting values of the dependent variable.
- In our case, the output of $y_i's$ of our training data set.

That, we relate by the following function

$$y_i = f(x_i) + \epsilon_i$$

Assuming linear additivity structure between noise input and outputs.

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- It is to obtain a useful approximation (fitting) to $f(x)$ for all x in some region of $\mathbb{R}^d$, given the representations in $\mathcal{D}$.

You can think as no so glamorous than the learning paradigm

- But using this approach, we can use all the tools generated in the last 200 years for function approximation!!!

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Parameters in the Approximations

For example, in the linear model $f(x) = \boldsymbol{x}^T w$

- There is a parameter for approximation $\theta = w$

In another example, using linear basis expansion

$$f_\theta(\boldsymbol{x}) = \sum_{k=1}^{K} h_k(\boldsymbol{x})\,\theta_k$$

Traditional examples of these functions

- $x_1^2, x_1 x_2^2, \cos(x_1)$
- An also

$$h_k(\boldsymbol{x}) = \frac{1}{1 + \exp\{-\boldsymbol{x}^T\theta_k\}}$$

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Residual Sum of Squares (RSS)

Here, the general structure for the RSS(f) under a Penalty/Regularization

$$PRSS\left(f,\lambda\right)=RSS\left(f\right)+\lambda J\left(f\right)$$

For Example, we have Ridge Regression

$$\sum_{i=1}^{N}\left(y_i-\boldsymbol{x}^T\right)^2+\lambda\sum_{i=1}^{d}w_i^2 \tag{1}$$

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Kernel Methods

You can think on these methods as

- They try to estimate the regression function or conditional expectation by specifying:
  - The properties of the local Neighborhood,
  - The class of regular functions fitted locally.

For this, they use kernels as

$$K_{\lambda}(x, x_0) = \frac{1}{\lambda} \exp\left\{ -\frac{\|x - x_0\|^2}{2\lambda} \right\}$$

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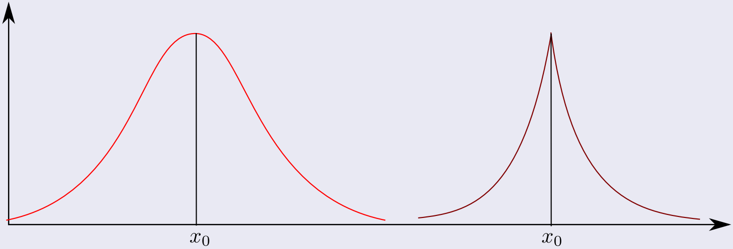
What happens here?

We have the following



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As in Regression

We can define a way of doing estimation

$$RSS\left(f_{\boldsymbol{w}}, \boldsymbol{x}_0\right) = \sum_{i=1}^{N} K_{\lambda}\left(\boldsymbol{x}_i, \boldsymbol{x}_0\right)\left(y_i - f_{\boldsymbol{w}}\left(\boldsymbol{x}_i\right)\right)^2$$

Where f_w

1. $f_w(x) = w_0$ the constant function (Nadaraya–Watson Estimate).
2. $f_w(x) = \sum_{i=0}^{d} x_i w_i$ the classic local linear regression models.

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For Example

Nearest-Neighbor Methods

It can be thought as a kernel method with a data dependent metric:

$$K_k\left(\boldsymbol{x}, \boldsymbol{x}_0\right) = I\left[\left\|\boldsymbol{x} - \boldsymbol{x}_0\right\| \leq \left\|\boldsymbol{x}_{(i)} - \boldsymbol{x}_0\right\| | i = 1, 2, \ldots, k\right]$$

Where

- $x_{(i)}$ is the training observation ranked i^{th} in distance from x_0.
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A more wide variety of flexible models

For Example, Linear and Polynomial Expansions

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Other Examples

Something Notable

- Tensor products of spline bases can be used for inputs with dimensions larger than one - CART and MARS models

Radial basis functions

$$f_w(x) = \sum_{m=1}^{M} w_m K_{\lambda_m}(\mu_m, x) \text{ with } K_\lambda(\mu, x) = \exp\left\{-\frac{\|x-\mu\|^2}{2\lambda}\right\}$$

A single-layer feed-forward neural network

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Machine Learning is a quite wide and vast field

- It requires Time
- It requires Effort
- It can be sometimes hard!!!

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