Introduction to Machine Learning

Introduction to Linear Classifiers

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Outline

1. Introduction
   - Introduction
   - The Simplest Functions
   - Splitting the Space
   - Defining the Decision Surface
   - Properties of the Hyperplane $\boldsymbol{w}^T\boldsymbol{x} + w_0$
   - Augmenting the Vector

2. Developing a Solution
   - Least Squared Error Procedure
     - The Geometry of a Two-Category Linearly-Separable Case
   - The Error Idea
   - The Final Error Equation
   - Geometric Interpretation
   - Issues with Least Squares!!!
     - Problem with Outliers
     - Problem with High Number of Dimensions

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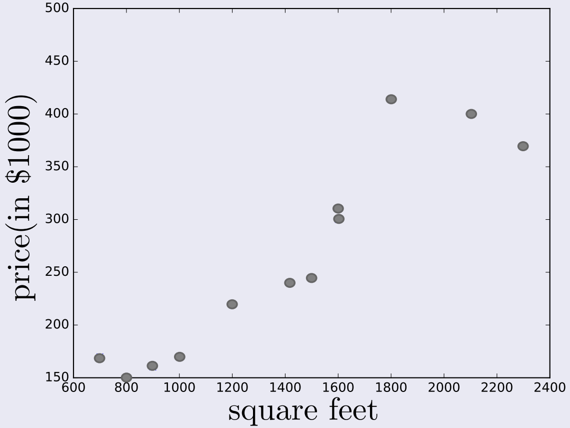
Many Times

We have this kind of data sets (House Prices)

$$\begin{pmatrix} \text{Squared Feet} \\ \text{Price} \end{pmatrix} \rightarrow \begin{pmatrix} 2104 \\ 400 \end{pmatrix} \begin{pmatrix} 1800 \\ 460 \end{pmatrix} \begin{pmatrix} 1600 \\ 300 \end{pmatrix} \begin{pmatrix} 2300 \\ 370 \end{pmatrix} \cdots$$



Thus

We can adjust a line/hyperplane to be able to forecast prices

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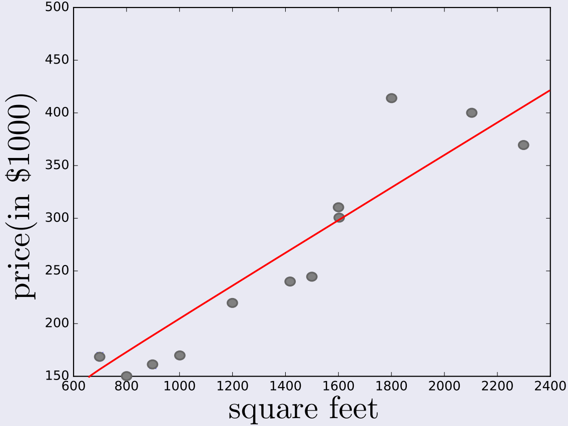

Thus, Our Objective

To find such hyperplane

To do forecasting on the prices of a house given its surface!!!

Here, where "Learning" Machine Learning style comes around

Basically, the process defined in Machine Learning!!!

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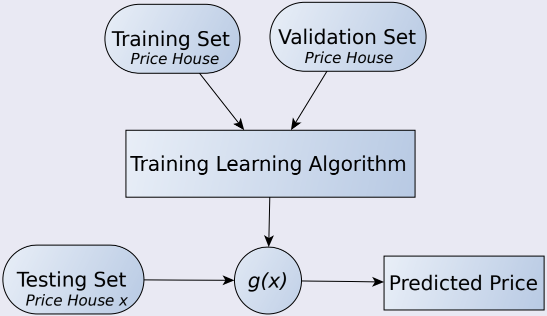
Then, in Supervised Training

We have the following process



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What is it?

First than anything, we have a parametric model!!!

Here, we have an hyperplane as a model:

$$g(\boldsymbol{x}) = \boldsymbol{w}^T\boldsymbol{x} + w_0 \tag{1}$$

Note: $\boldsymbol{w}^T\boldsymbol{x}$ is also know as dot product

In the case of $\mathbb{R}^2$

We have:

$$g(x) = (w_1, w_2)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + w_0 = w_1x_1 + w_2x_2 + w_0 \tag{2}$$

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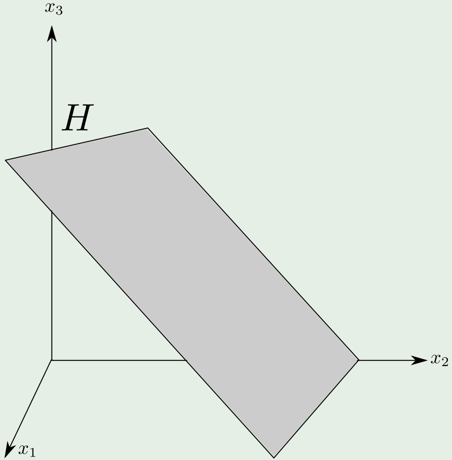
Example

Hyperplane in $\mathbb{R}^3$



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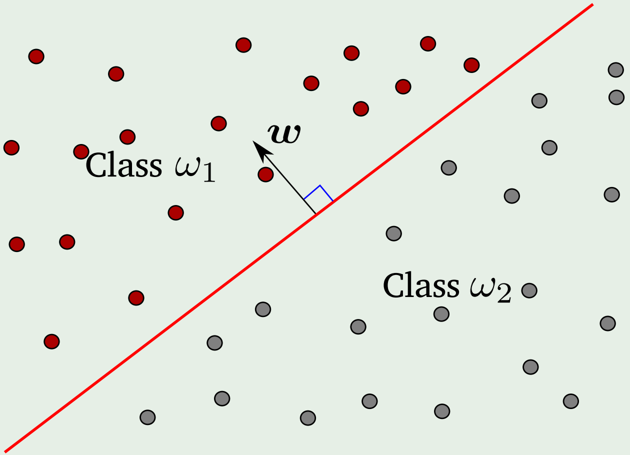
Splitting The Space $\mathbb{R}^2$

Using a simple straight line (Hyperplane)



Splitting the Space?

For example, assume the following vector $\boldsymbol{w}$ and constant w_0

$$\boldsymbol{w} = (-1, 2)^T \text{ and } w_0 = 0$$

Hyperplane

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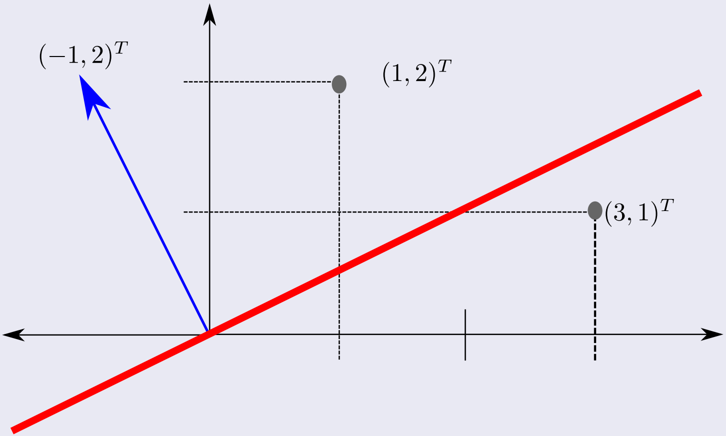

Then, we have

The following results

$$g\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right) = (-1, 2)\begin{pmatrix} 1 \\ 2 \end{pmatrix} = -1 \times 1 + 2 \times 2 = 3$$

$$g\left(\begin{pmatrix} 3 \\ 1 \end{pmatrix}\right) = (-1, 2)\begin{pmatrix} 3 \\ 1 \end{pmatrix} = -1 \times 3 + 2 \times 1 = -1$$

YES!!! We have a positive side and a negative side!!!

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The Decision Surface

The equation $g(x) = 0$ defines a decision surface

Separating the elements in classes, ω_1 and ω_2.

When $g(x)$ is linear the decision surface is an hyperplane

Now assume x_1 and x_2 are both on the decision surface

$$w^T x_1 + w_0 = 0$$
$$w^T x_2 + w_0 = 0$$

Thus

$$w^T x_1 + w_0 = w^T x_2 + w_0 \quad (3)$$

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$$\boldsymbol{w}^T \boldsymbol{x}_1 + w_0 = \boldsymbol{w}^T \boldsymbol{x}_2 + w_0 \tag{3}$$

Defining a Decision Surface

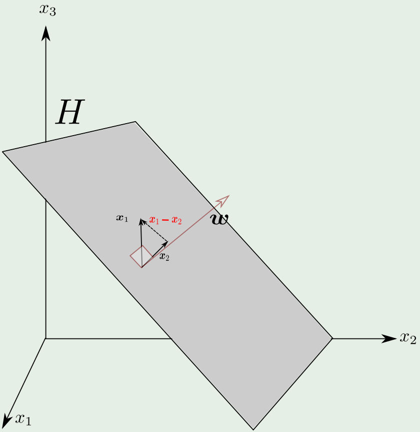
Then

$$\boldsymbol{w}^T \left(\boldsymbol{x}_1 - \boldsymbol{x}_2\right) = 0 \tag{4}$$

Therefore

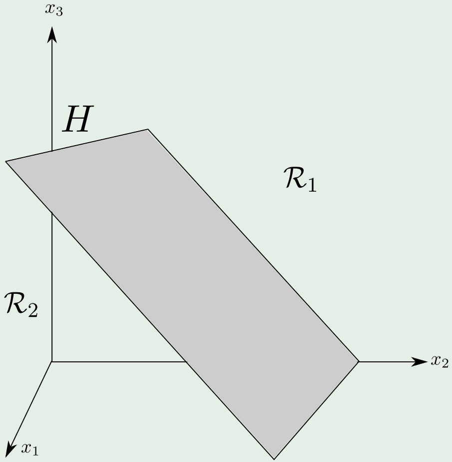
$\boldsymbol{x}_1 - \boldsymbol{x}_2$ lives in the hyperplane i.e. it is perpendicular to $\boldsymbol{w}^T$

- Remark: any vector in the hyperplane is a linear combination of elements in a basis
- **Therefore any vector in the plane is perpendicular to w^T**



Therefore

The space is split in two regions (Example in $\mathbb{R}^3$) by the hyperplane H



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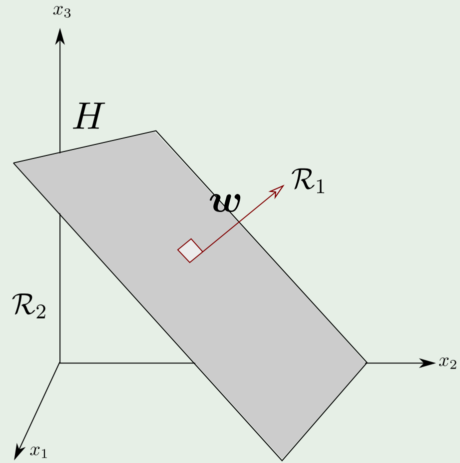
Some Properties of the Hyperplane

Given that $g(\boldsymbol{x}) > 0$ if $\boldsymbol{x} \in \mathcal{R}_1$



It is more

We can say the following

- Any $\boldsymbol{x} \in \mathcal{R}_1$ is on the positive side of H.
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In addition, $g(x)$ can give us a way to obtain the distance from x to the hyperplane H

First, we express any x as follows

$$x = x_p + r\frac{w}{\|w\|}$$

- r is the desired distance
  - Positive, if x is in the positive side
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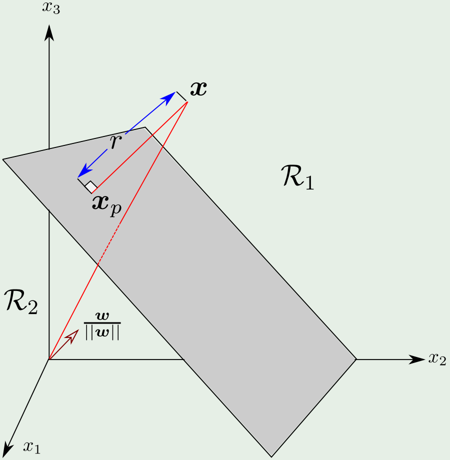
We have something like this

We have then



Now

Since $g\left(\boldsymbol{x}_p\right) = 0$

We have that

$$
\begin{aligned}
g\left(\boldsymbol{x}\right) &= g\left(\boldsymbol{x}_p + r\frac{\boldsymbol{w}}{\left\Vert \boldsymbol{w}\right\Vert}\right)\\
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&= \boldsymbol{w}^T\boldsymbol{x}_p + w_0 + r\frac{\boldsymbol{w}^T\boldsymbol{w}}{\left\Vert \boldsymbol{w}\right\Vert}\\
&= g\left(\boldsymbol{x}_p\right) + r\frac{\left\Vert \boldsymbol{w}\right\Vert^2}{\left\Vert \boldsymbol{w}\right\Vert}\\
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$$r = \frac{g\left(\boldsymbol{x}\right)}{\left\Vert \boldsymbol{w}\right\Vert} \tag{5}$$

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In particular

The distance from the origin to H

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Remarks

- If $w_0 > 0$, the origin is on the positive side of H.
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- If $w_0 = 0$, the hyperplane has the homogeneous form $\boldsymbol{w}^T\boldsymbol{x}$ and hyperplane passes through the origin.

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We want to solve the independence of w_0

We would like w_0 as part of the dot product by making $x_0 = 1$

$$g(\boldsymbol{x}) = w_0 \times 1 + \sum_{i=1}^{d} w_i x_i = w_0 \times x_0 + \sum_{i=1}^{d} w_i x_i = \sum_{i=0}^{d} w_i x_i \quad (7)$$

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By making

$$x_{aug} = \begin{pmatrix} 1 \\ x_1 \\ \vdots \\ x_d \end{pmatrix} = \begin{pmatrix} 1 \\ \boldsymbol{x} \end{pmatrix}$$

x_{aug} is called an augmented feature vector.

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- The resulting x_{aug} vectors, all lie in a d-dimensional subspace which is the x-space itself.

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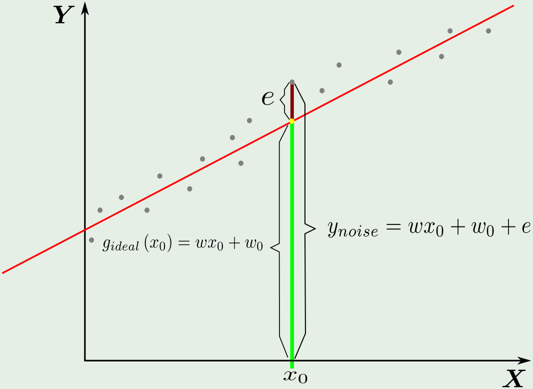

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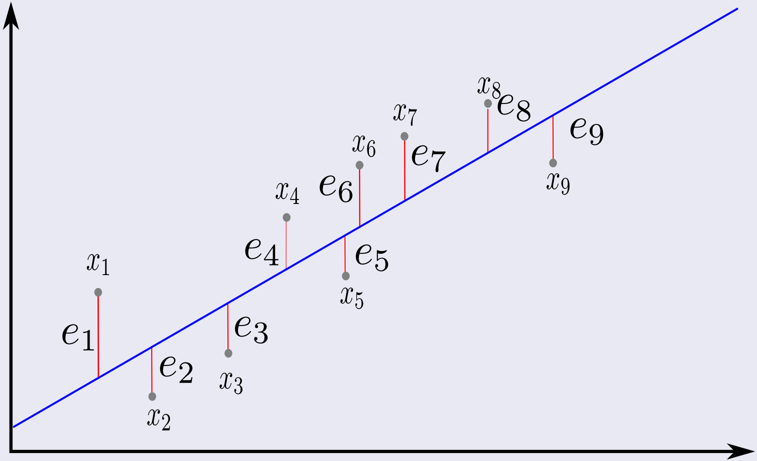
Here, we have multiple errors

What can we do?



Sum Over All the Errors

We can do the following

$$J(\boldsymbol{w}) = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} \left(y_i - g_{ideal}(\boldsymbol{x}_i)\right)^2 \tag{13}$$

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The following function

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We can rewrite the error equation as

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Furthermore

Then stacking all the possible estimations into the product Data Matrix and weight vector

$$\boldsymbol{X}\boldsymbol{w} = \begin{pmatrix} 1 & (\boldsymbol{x}_1)_1 & \cdots & (\boldsymbol{x}_1)_j & \cdots & (\boldsymbol{x}_1)_d \\ \vdots & & & \vdots & & \vdots \\ 1 & (\boldsymbol{x}_i)_1 & & (\boldsymbol{x}_i)_j & & (\boldsymbol{x}_i)_d \\ \vdots & & & \vdots & & \vdots \\ 1 & (\boldsymbol{x}_N)_1 & \cdots & (\boldsymbol{x}_N)_j & \cdots & (\boldsymbol{x}_N)_d \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_{d+1} \end{pmatrix}$$

Note about other representations

We could have $\boldsymbol{x}^T = (x_1, x_2, ..., x_d, 1)$ thus

$$\boldsymbol{X} = \begin{pmatrix} (\boldsymbol{x}_1)_1 & \cdots & (\boldsymbol{x}_1)_j & \cdots & (\boldsymbol{x}_1)_d & 1 \\ & & \vdots & & \vdots & \vdots \\ (\boldsymbol{x}_i)_1 & & (\boldsymbol{x}_i)_j & & (\boldsymbol{x}_i)_d & 1 \\ & & \vdots & & \vdots & \vdots \\ (\boldsymbol{x}_N)_1 & \cdots & (\boldsymbol{x}_N)_j & \cdots & (\boldsymbol{x}_N)_d & 1 \end{pmatrix} \tag{15}$$

Then, we have the following trick with $\boldsymbol{X}$

With the Data Matrix

$$\boldsymbol{X}w = \begin{pmatrix} \boldsymbol{x}_1^T\boldsymbol{w} \\ \boldsymbol{x}_2^T\boldsymbol{w} \\ \boldsymbol{x}_3^T\boldsymbol{w} \\ \vdots \\ \boldsymbol{x}_N^T\boldsymbol{w} \end{pmatrix} \tag{16}$$

Therefore

We have that

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_4 \end{pmatrix} - \begin{pmatrix} \boldsymbol{x}_1^T \boldsymbol{w} \\ \boldsymbol{x}_2^T \boldsymbol{w} \\ \boldsymbol{x}_3^T \boldsymbol{w} \\ \vdots \\ \boldsymbol{x}_N^T \boldsymbol{w} \end{pmatrix} = \begin{pmatrix} y_1 - \boldsymbol{x}_1^T \boldsymbol{w} \\ y_2 - \boldsymbol{x}_2^T \boldsymbol{w} \\ y_3 - \boldsymbol{x}_3^T \boldsymbol{w} \\ \vdots \\ y_4 - \boldsymbol{x}_N^T \boldsymbol{w} \end{pmatrix}$$

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$$\sum_{i=1}^{N}\left(y_i - \boldsymbol{x}_i^T\boldsymbol{w}\right)^2 = (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w})^T(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2 \quad (17)$$

The Final Discriminant Function

Very Simple!!!

$$g(\boldsymbol{x}) = \boldsymbol{x}^T \boldsymbol{w} = \boldsymbol{x}^T \left(\boldsymbol{X}^{\boldsymbol{T}} \boldsymbol{X}\right)^{-1} \boldsymbol{X}^{\boldsymbol{T}} \boldsymbol{y} \tag{18}$$

Pseudo-inverse of a Matrix

Definition

Suppose that $X \in \mathbb{R}^{m \times n}$ and $rank\left(X\right) = m$. We call the matrix

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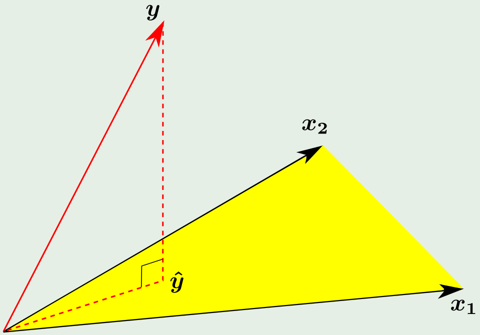
Geometrically

Given a $\boldsymbol{y}$, you obtain a projected $\widehat{\boldsymbol{y}}$ through the process $\boldsymbol{X}^T\boldsymbol{y}$



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With the Labels being chosen at the beginning

Question? Did you noticed the following?

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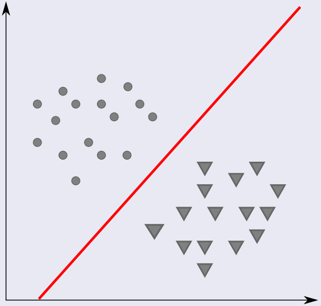

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Issues with Least Squares

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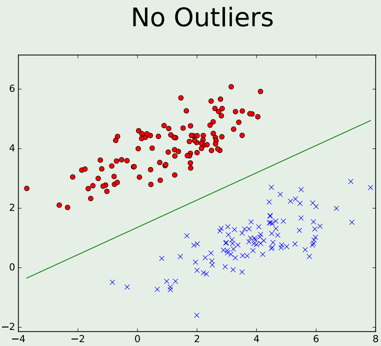

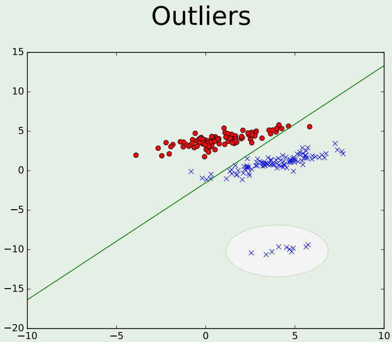

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