Analysis of Algorithms

Sorting in linear time

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Outline

About Sorting

Until now we had the following constraint

Given two numbers x and y, we compare to know if

$$\left.\begin{array}{l} x < y \\ x > y \\ x = y \end{array}\right\} \text{The Trichotomy Law} \quad (1)$$

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How to avoid to compare?

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2. You can use the idea of histograms!!!

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Counting to find the Position

Imagine the Following

4	9	2	0	2	0	1	4	6	2	9	10	11	6	6	7

Put the Blocks with Equal Elements Together in Clusters

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Imagine the Following

4	9	2	0	2	0	1	4	6	2	9	10	11	6	6	7

Put the Blocks with Equal Elements Together in Clusters

		2		6				
0		2	4	6		9		
0	1	2	4	6	7	9	10	11

Accumulate to Know the Clusters of Numbers

Accumulate Them

								11
							10	10
						9	9	9
						9	9	9
					7	7	7	7
				6	6	6	6	6
				6	6	6	6	6
				6	6	6	6	6
			4	4	4	4	4	4
			4	4	4	4	4	4
		2	2	2	2	2	2	2
		2	2	2	2	2	2	2
		2	2	2	2	2	2	2
	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

We need to store this information efficiently!!!

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						9	9	9
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				6	6	6	6	6
			4	4	4	4	4	4
			4	4	4	4	4	4
		2	2	2	2	2	2	2
		2	2	2	2	2	2	2
		2	2	2	2	2	2	2
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0	0	0	0	0	0	0	0	0
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2	1	3	0	2	0	3	1	0	2	1	1
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			4	4	4	4	4	4
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Then, we have that

Accumulated Array

2	3	6	6	8	8	11	12	12	14	15	16
0	1	2	3	4	5	6	7	8	9	10	11

We have the basic process there

We only need to be smart about it!!!

Then, we have that

Accumulated Array

2	3	6	6	8	8	11	12	12	14	15	16
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We have the following limitations

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Stable Property

Counting sort is stable; it keeps records in their original order.

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Algorithm

Counting sort algorithm - **Assume indexing starting at 1**

Counting-Sort(A, B, k)

1. let $C[0..k]$ be a new array
2. **for** $i = 0$ **to** k
3. $\quad C[i] = 0$
4. **for** $j = 1$ **to** $A.length$
5. $\quad C[A[j]] = C[A[j]] + 1$
6. $//C[i]$ contains the number of elements equal to i
7. **for** $i = 1$ **to** k
8. $\quad C[i] = C[i] + C[i-1]$
9. $//\ C[i]$ contains the number of elements$\leq i$
10. **for** $j = A.length$ **to** 1
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Remarks

You can use it for

It is efficient if the range of input data is not significantly greater than the number of objects to be sorted.

It is STABLE

A sorting algorithm is stable if whenever there are two records R and S with the same key and with R appearing before S in the original list, R will appear before S in the sorted list.

This why

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How?

It sorts each digit (or field/column) separately. Example:

$$\begin{matrix} 3 & 4 & \mathbf{5} & 1 \\ 1 & 2 & \mathbf{2} & 4 \\ 7 & 8 & \mathbf{9} & 1 \\ 1 & 2 & \mathbf{2} & 5 \end{matrix} \quad \Longrightarrow \quad \begin{matrix} 1 & 2 & \mathbf{2} & 4 \\ 1 & 2 & \mathbf{2} & 5 \\ 3 & 4 & \mathbf{5} & 1 \\ 7 & 8 & \mathbf{9} & 1 \end{matrix}$$

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Radix sort must use a stable sort.

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It starts with the least-significant digit

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Outline

It is based in the following idea

First

Every number can be represented in each base. For example:

$$1024 = 1 \times 10^3 + 0 \times 10^2 + 2 \times 10^1 + 4 \times 10^0 \tag{2}$$

Thus in general, given a radix b

$$x = x_d b^d + x_{d-1} b^{d-1} + \ldots + x_0 b^0 \tag{3}$$

It can be proved inductively

We can sort by using Least-Significative to Most-significative Order and we keep an stable sort using this order

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However

Remark 1

A most significant digit (MSD) radix sort can be used to sort keys in lexicographic order.

Remark 2

Unlike a least significant digit (LSD) radix sort, a most significant digit radix sort does not necessarily preserve the original order of duplicate keys.

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\begin{array}{ccc}
3 & 2 & 9 \\
4 & 5 & 7 \\
6 & 5 & 7 \\
8 & 3 & 9 \\
4 & 3 & 6 \\
7 & 2 & 0 \\
3 & 5 & 5
\end{array}
\Longrightarrow
\begin{array}{ccc}
7 & 2 & 0 \\
3 & 5 & 5 \\
4 & 3 & 6 \\
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6 & 5 & 7 \\
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8 & 3 & 9
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\begin{array}{ccc}
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\end{array}
\Longrightarrow
\begin{array}{ccc}
7 & \mathbf{2} & 0 \\
3 & \mathbf{2} & 9 \\
4 & \mathbf{3} & 6 \\
8 & \mathbf{3} & 9 \\
3 & \mathbf{5} & 5 \\
4 & \mathbf{5} & 7 \\
6 & \mathbf{5} & 7
\end{array}
\Longrightarrow
\begin{array}{ccc}
3 & 2 & 9 \\
3 & 5 & 5 \\
4 & 3 & 6 \\
4 & 5 & 7 \\
6 & 5 & 7 \\
7 & 2 & 0 \\
8 & 3 & 9
\end{array}
$$

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\begin{array}{ccc} 3 & 2 & 9 \\ 4 & 5 & 7 \\ 6 & 5 & 7 \\ 8 & 3 & 9 \\ 4 & 3 & 6 \\ 7 & 2 & 0 \\ 3 & 5 & 5 \end{array}
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$$

Radix Sort: Algorithm Using the Least Significative Digit

Algorithm

Radix-Sort(A, d)

1. **for** $i = 1$ **to** d
2. use a stable sort to sort array A on digit i

Outline

Iterative version using queues

How? First, assume Radix $= 10$

The integers are enqueued into an array of ten separate queues based on their digits from right to left.

Example: 170, 045, 075, 090, 002, 024, 802, 066

$$
\begin{array}{rc}
0: & 170, 190 \\
1: & none \\
2: & 002, 802 \\
3: & none \\
4: & 024 \\
5: & 045, 075 \\
6: & 066 \\
7: & none \\
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\end{array}
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$$\begin{aligned} 0 &: \quad 17\mathbf{0}, 19\mathbf{0} \\ 1 &: \quad none \\ 2 &: \quad 00\mathbf{2}, 80\mathbf{2} \\ 3 &: \quad none \\ 4 &: \quad 02\mathbf{4} \\ 5 &: \quad 04\mathbf{5}, 07\mathbf{5} \\ 6 &: \quad 06\mathbf{6} \\ 7 &: \quad none \\ 8 &: \quad none \\ 9 &: \quad none \end{aligned}$$

Iterative version using queues

Then, the queues are dequeued back into an array of integers, in increasing order

$$170, 090, 002, 802, 024, 045, 075, 066$$

Then

You repeat again!!!

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Radix Sort: Proving the Complexity

Lemma 1

Given n d-digit numbers in which each digit can take on up to k possible values, RADIX-SORT correctly sorts these numbers in $\Theta(d(n+k))$ time.

Proof

Quite simple!!!

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Thanks Carlos Alcala Oracle Class 2014 for the Example

Imagine the following

That you have a sequence of n IP Addresses. For example:

$$n \text{ IP addresses} \begin{cases} 192.168.45.120 \\ 192.128.15.120 \\ 100.192.168.45 \\ \vdots \\ 92.16.4.120 \end{cases}$$

Thus, Why not use the chunks in the IP to sort the IP addresses (Think Columns)

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- After all each chunk is a number between 0 and 255 i.e between 0 to $2^8 - 1 \Rightarrow$ size chunks is $r = 8$
- We have then for each IP address a size of $b = 32$ bits

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Given n b-bit numbers and any positive ingeter $r \leq b$, RADIX-SORT correctly sort these numbers in $\Theta(\frac{b}{r}(n+2^r))time.$

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- For more, look at V. J. Duvanenko, "In-Place Hybrid Binary-Radix Sort", Dr. Dobb's Journal, 1 October 2009

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You have n of them

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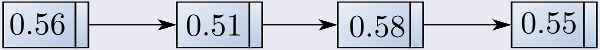

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Create n linked lists (buckets) to divide interval $[0, 1)$ into subintervals of size $1/n$.

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Add each input element to the appropriate bucket.

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Algorithm assuming

Buket-Sort(A)

1. let $B\,[0..n-1]$ be a new array
2. $n = A.length$
3. for $i = 0$ to $n-1$
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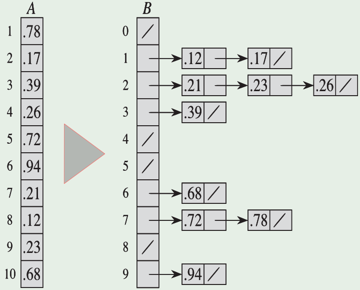

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We have the following

We need to analyze the algoritm

But we have an insertion sort at each bucket!!!

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Let n_i the random variable on the size of the bucket.

We get the following complexity function

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Final Complexity is

After using the expected value

$$E\left(T\left(n\right)\right)=\Theta\left(n\right)$$

Outline

Exercises

From Cormen's book solve the following

- 8.1-1
- 8.1-3
- 8.2-2
- 8.2-3
- 8.3-2
- 8.3-4
- 8.4-2