Analysis of Algorithms

Heapsort and Quicksort

Arturo Calderón

September 17, 2020

Heapsort

Heap

A heap is a binary tree where:

1. Every parent is greater or equal than it's children.
2. This binary tree must be complete except for its last level where every children are grouped to one side.

Heapsort

Heapsort is a sorting algorithm based on elements comparison that uses a heap to sort. This algorithm has a running time bounded by $O(n \log n)$.

Heapsort

Heap

A heap is a binary tree where:

1. Every parent is greater or equal than it's children.
2. This binary tree must be complete except for its last level where every children are grouped to one side.

Heapsort

Heapsort is a sorting algorithm based on elements comparison that uses a heap to sort. This algorithm has a running time bounded by $O(n \log n)$.

Heapsort

Heapsort intuition

Heapsort proceeds by storing every element of the vector in a heap and then extracts the root in sucessive iterations until the sorted vector is obtained. Heapsort relies on the heap property of having the biggest (or smallest) element of the vector as the root.

Heapsort

MAX-HEAPIFY

In order to maintain the max-heap property, we call the procedure MAX-HEAPIFY. Its inputs are an array A and an index i into the array. When it is called, MAX-HEAPIFY assumes that the binary trees rooted at $LEFT(i)$ and $RIGHT(i)$ are max-heaps, but that $A[i]$ might be smaller than its children, thus violating the max-heap property. MAX-HEAPIFY lets the value at $A[i]$ "float down" in the max-heap so that the subtree rooted at index i obeys the max-heap property.

Heapsort

Example

Conceptually using heapsort sort the following list:

$$10, 5, 8, 4, 12, 2, 6, 11, 3, 9, 7, 1$$

Heapsort

Exercise

Imegine that you have a max heap A and you wanna support the following operations of:

- $Increase - Key(A, i, newkey)$
- $Insert(A, i)$
- $Extract - Max(A)$

How would you do it?

Heapsort

Exercise 6.2-6 from Cormen's book

Show that the worst-case running time of MAX-HEAPIFY on a heap of size n is $\Omega(\log n)$.

- (Hint: For a heap with n nodes, give node values that cause MAX-HEAPIFY to be called recursively at every node on a simple path from the root down to a leaf.)