Explore speed of convergence and computational complexity

i. Introduction

Option pricing is important for finance. Many trader, portfolio managers, or risk analysts find different models to value this derivative securities. One of the simplest and most common methods is the tree-based model. This model shows how the price of an asset changes in steps over time. The binomial tree model is a well-known example. The model is easy to understand and quick to calculate. But the binomial model only allows two price movements in each step. This can make it less accurate, especially for more complex price changes.

The trinomial tree model improves on this. It lets the price go up, down, or stay the same at each step. This third choice makes the results more accurate. It also helps the model reach stable results. Trinomial models are better at showing complex price behaviors. They are more detailed, which makes them useful for some types of financial analysis. In this paper, I look at how the trinomial tree model works. I use it to price both European and American options. We test it in cases with and without dividends. We also compare it to the binomial model. We look at how fast each model gives steady results and how hard they are to compute when the number of steps increases.

ii. Trinomial Tree Model

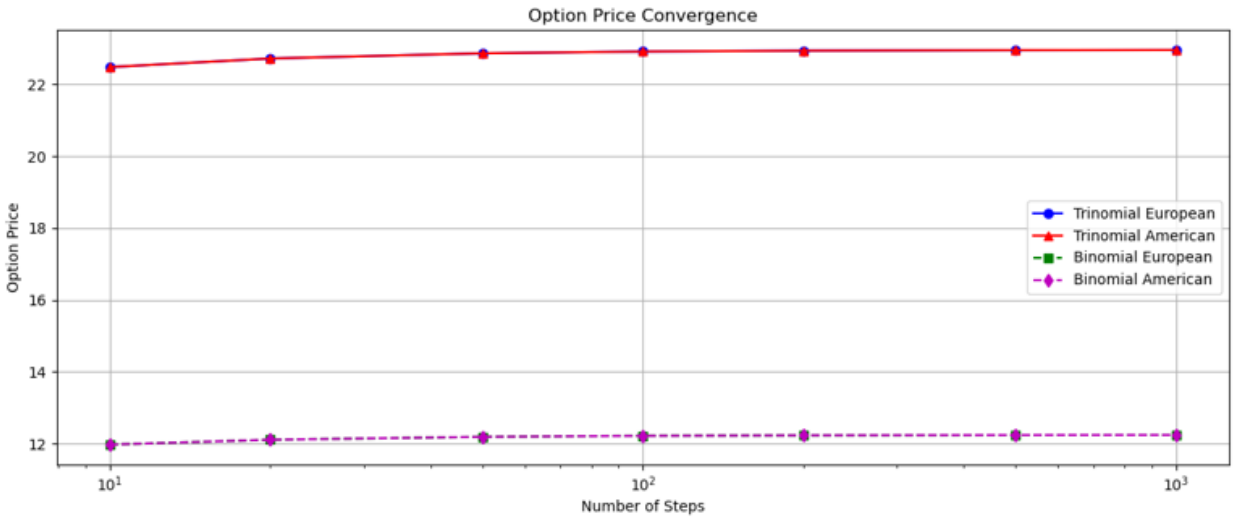
Initial price: S0, total time to maturity: T, time step: n, length: $\Delta t = T/n$, up factor: u, down factor: d, stay same factor m:1 , and the formula follows:

$u = e^{\sigma\sqrt{3\,\Delta t}}, \quad d = \frac{1}{u} = e^{-\sigma\sqrt{3\,\Delta t}}$ probability: $p_u = \frac{e^{(r-\delta)\frac{\Delta t}{2}} - e^{-\sigma\sqrt{\frac{\Delta t}{3}}}}{e^{\sigma\sqrt{\frac{\Delta t}{3}}} - e^{-\sigma\sqrt{\frac{\Delta t}{3}}}}, \quad p_d = \frac{e^{\sigma\sqrt{\frac{\Delta t}{3}}} - e^{(r-\delta)\frac{\Delta t}{2}}}{e^{\sigma\sqrt{\frac{\Delta t}{3}}} - e^{-\sigma\sqrt{\frac{\Delta t}{3}}}}, \quad p_m = 1 - p_u - p_d$

iii. Test Case

There are two test cases: First case: Theoretical Example: S0 = 100, K = 100, rfr = 5%, sd = 20%, dividend = 2%. Second case: I use yfinance to get data from AAPL. Fetched AAPL data: Price=$224.23, Volatility=24.63%, Dividend Yield=0.45%

iv. Price Convergence



With the test, I found out that the trinomial has better price convergence then binomial model in both European and American model. Because Trinomial model gives it more flexibility and better modeling of the underlying asset's behavior. The binomial model only has up and down, which makes it slightly more limited.

v. Speed of convergence

The result of two test cases:

Steps	Trinomial European	Trinomial American	Binomial European	Binomial American
10	0.000000	0.000000	0.000000	0.000000
20	0.000999	0.000000	0.000000	0.000967
50	0.000998	0.004016	0.000971	0.001994
100	0.011968	0.021942	0.005984	0.011968
200	0.024933	0.056848	0.009974	0.025932
500	0.144613	0.413893	0.077793	0.170082
1000	0.616582	1.641472	0.226399	0.702537

Steps(AAPL)	Trinomial European	Trinomial American	Binomial European	Binomial American
10	0.000000	0.000000	0.000000	0.000000
20	0.000997	0.000000	0.000000	0.000998
50	0.003989	0.005984	0.000998	0.001995
100	0.004986	0.014998	0.002981	0.005984
200	0.025931	0.068806	0.008000	0.025919
500	0.144614	0.381186	0.053828	0.179226
1000	0.571903	1.544160	0.242357	0.674368

Both of the result shows that Trinomial model will take more time to process than binomial model for both Trinomial and Binomial option. The trinomial method uses more branches at each step. Instead of just two possible price movements (up or down), it allows three (up, down, or stay the same). Because of this, the trinomial model needs more calculations at every step. So, the test result can help me conclude that takes more time to finish compared to the binomial model when the number of steps is the same. American options are more complex than European options. This is because American options can be exercised early, so the model has to check at each step if early exercise is a better choice. These extra checks make the American option pricing slower than European, no matter which model (binomial or trinomial) is used.

vi. Computational Complexity

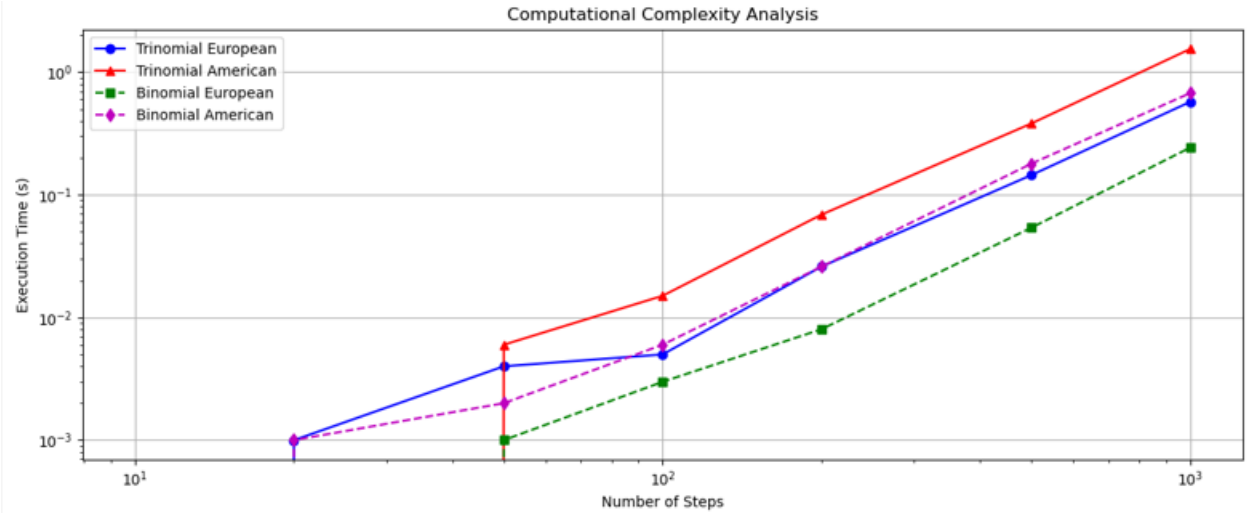


Figure above illustrates the computation time required for each of the four discrete-time models—Trinomial European, Trinomial American, Binomial European, and Binomial American. As the number of steps gets bigger, the model needs to do more work. The amount of work grows quickly. The time still looks like it increases in a straight line.

From the chart we can see trinomial model has a better performance in price prediction but a relatively bad performance in computation time. If the goal is to price many options quickly, like in a big portfolio, the binomial model might be a better choice. It's faster and still gives good results. But if the goal is to need very accurate results, or if you are working with special kinds of options, then trinomial might be better.

In the end, it's about trade off between speed and accuracy. For smaller problems, both models work fine. But for big problems with many steps, especially with American options, I will recommend you think about whether the better accuracy is worth the

extra time.