CSE 360 - Controller to Follow an Elliptical Trajectory

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The objective of this exercise is to find a control policy *u* that makes the robot move in an ellipse shape, where the major axis (aligned with the x-axis of the world frame) is equal to 4m and the minor axis (aligned with the y-axis of the world frame) is equal to 2m. The center of the ellipse is in the point $[3,\ 2]$

The parametric form for an ellipse is

$$F(t) = (x(t),\ y(t)) \tag{1}$$

If the ellipse is centered on the origin, the equations are

$$x(t) = a\cos t \tag{2}$$

$$y(t) = b\sin t \tag{3}$$

where *a* is the radius along the x-axis and *b* is the radius along the y-axis.

However, since the ellipse in Exercise 1 is not centered at origin, we need to add offsets to the x and y terms to translate (or ”move”) the ellipse to the correct location. So the full form of the equations are

$$x(t) = a\cos t + h \tag{4}$$

$$y(t) = b\sin t + k \tag{5}$$

After we substitute the true value for *a* and *b* and plug in the value for the center of the ellipse $[3,\ 2]$, we get:

$$x(t) = 4\cos t + 3 \tag{6}$$

$$y(t) = 2\sin t + 2 \tag{7}$$

If we set *t* equals to 0, we will then get the initial position of the ellipse

$$x(0) = 4\cos 0 + 3 = 7 \tag{8}$$

$$y(0) = 2\sin 0 + 2 = 2 \tag{9}$$

where the coordinate is $[7,\ 2]$.

We then take the derivative of *x* and *y* to get the velocity

$$\dot{x} = -a\sin t \tag{10}$$

$$\dot{y} = b\cos t \tag{11}$$

and this is the equation of the function *u(t)*.

After we run the simulation for 10 seconds, we get the elliptical trajectory shown below.