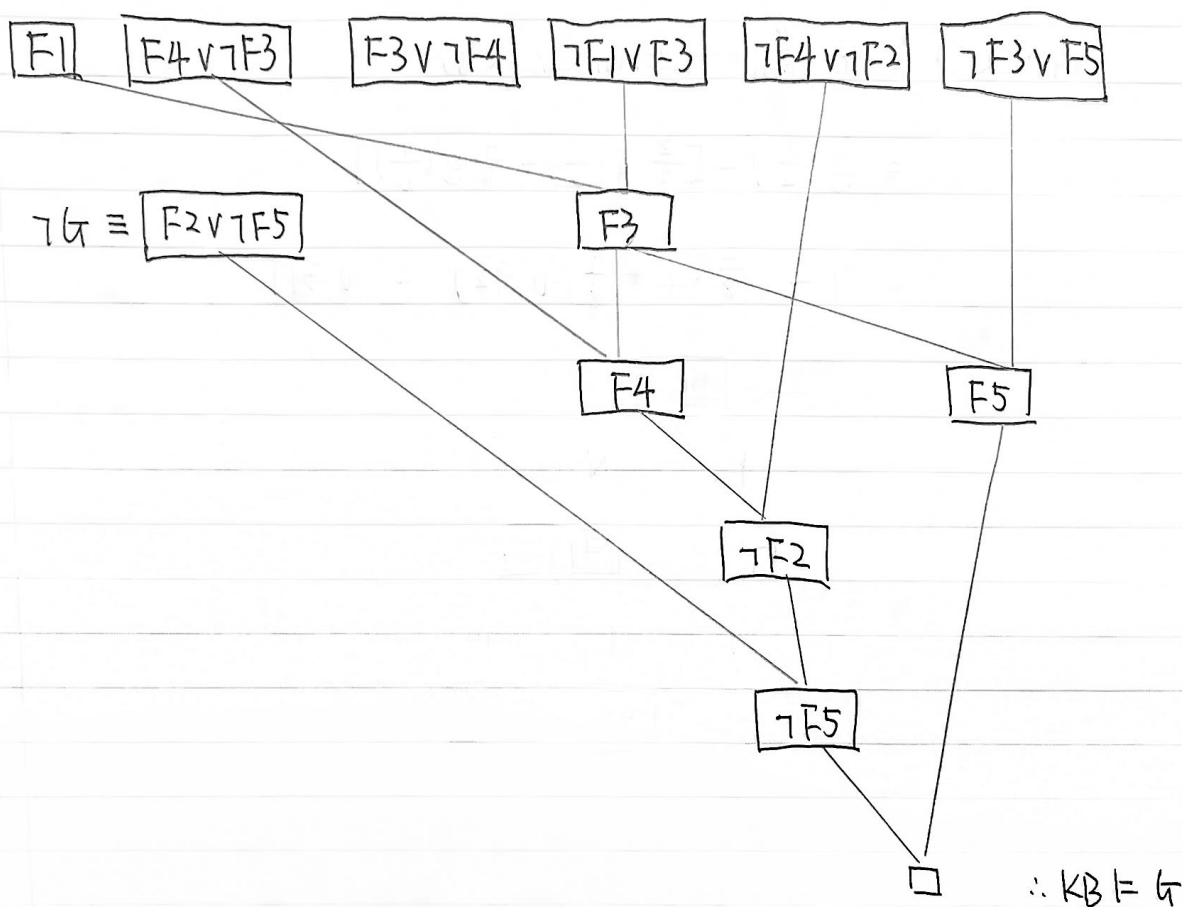


## Homework 4

1) Proof by truth table enumeration

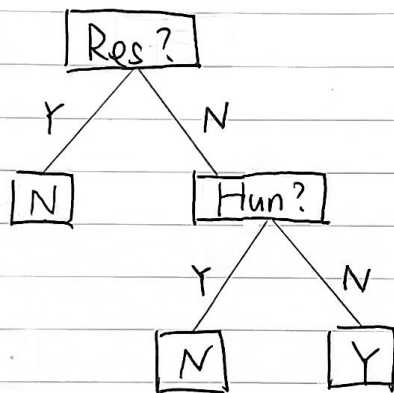
Proof by resolution

$$\begin{aligned} 2) (\neg F4 \Leftrightarrow \neg F3) &\equiv (\neg F4 \Rightarrow \neg F3) \wedge (\neg F3 \Rightarrow \neg F4) \\ &\equiv (F4 \vee \neg F3) \wedge (F3 \vee \neg F4) \end{aligned}$$



$$\begin{aligned}
 3) \text{ Gain (Hun)} &= B\left(\frac{P}{p+n}\right) - \text{Remainder (Hun)} \\
 &= B\left(\frac{4}{8}\right) - \left[\frac{3}{8} B\left(\frac{1}{3}\right) + \frac{5}{8} B\left(\frac{3}{5}\right)\right] \\
 &\approx 1 - \left(\frac{3}{8} \cdot 0.92 + \frac{5}{8} \cdot 0.97\right) \approx 0.049
 \end{aligned}$$

$$\begin{aligned}
 \text{Gain (Res)} &= B\left(\frac{P}{p+n}\right) - \text{Remainder (Res)} \\
 &= B\left(\frac{1}{2}\right) - \left[\frac{2}{8} B\left(\frac{0}{2}\right) + \frac{6}{8} B\left(\frac{4}{6}\right)\right] \\
 &\approx 1 - \left(\frac{2}{8} \cdot 0 + \frac{6}{8} \cdot 0.92\right) \approx 0.311
 \end{aligned}$$



$$\begin{aligned}
 4) \quad a_5 &= \text{ReLU}(0.1 \cdot a_1 + 0.1 a_2) = \text{ReLU}(0.25) = 0.25 \\
 a_6 &= \text{ReLU}(0.1 a_3 + 0.1 a_4) = \text{ReLU}(-0.45) = 0 \\
 a_7 &= S(0.2 a_5 + 0.2 a_6) = S(0.05) \approx 0.512
 \end{aligned}$$

$$5) \text{Err}_7 = 1.0 - 0.512 = 0.488$$

$$\Delta_7 = \text{Err}_7 \cdot g'(\text{in}_7) = 0.488 \cdot g'(0.05) = 0.488 \cdot 8(0.05)(1 - 8(0.05)) \\ \approx 0.122$$

$$W_{57} + \alpha \cdot \Delta_7 \cdot a_5 = 0.2 + 0.1 \cdot 0.122 \cdot 0.25 = 0.20305$$

$$W_{67} + \alpha \cdot \Delta_7 \cdot a_6 = 0.2 + 0.1 \cdot 0.122 \cdot 0 = 0.2$$

$$\Delta_5 = g'(\text{in}_5) (W_{57} \cdot \Delta_7) = \text{ReLU}'(0.25) \cdot 0.2 \cdot 0.122 \\ = 1 \cdot 0.2 \cdot 0.122 = 0.0244$$

$$\Delta_6 = g'(\text{in}_6) (W_{67} \cdot \Delta_7) = \text{ReLU}'(-0.45) \cdot 0.2 \cdot 0.122 = 0$$

$$W_{15} + \alpha \cdot \Delta_5 \cdot a_1 = 0.1 + 0.1 \cdot 0.0244 \cdot 1.0 = 0.10244$$

$$W_{25} + \alpha \cdot \Delta_5 \cdot a_2 = 0.1 + 0.1 \cdot 0.0244 \cdot 1.5 = 0.10366$$

$$W_{36} + \alpha \cdot \Delta_6 \cdot a_3 = 0.1 + 0 = 0.1$$

$$W_{46} + \alpha \cdot \Delta_6 \cdot a_4 = 0.1 + 0 = 0.1$$

$$\text{New Weights: } W_{15} = 0.10244$$

$$W_{25} = 0.10366$$

$$W_{36} = 0.1$$

$$W_{46} = 0.1$$

$$W_{57} = 0.20305$$

$$W_{67} = 0.2$$

$$6) (a) 5^2 = 25$$

$$(b) 124^2 \cdot 25 = 384400$$

$$(c) 60 \times 60$$

$$(d) 32^2 \cdot 32^2 = 1048576$$