

Homework 7

S520, Fall 2019

Due at the beginning of class, Wednesday October 23rd . Please upload your file to Canvas no later than 4pm on the due date. Late submission will be accepted (but penalized) before the solutions are posted.

Trosset question numbers refer to the hardcover textbook. Show all work.

1. (6 points) Trosset exercise 8.4.4.
2. (15 points) Let X be a discrete random variable with probability mass function

$$P(X = x) = \begin{cases} 0.3 & x = -2 \\ 0.6 & x = -1 \\ 0.1 & x = 12 \\ 0 & \text{otherwise.} \end{cases}$$

Let X_1, \dots, X_n be an iid sequence of random variables with the same distribution as X . Let \bar{X} be the sample mean (of X_1, \dots, X_n .)

- (a) Find $E(X)$.
- (b) Find $\text{Var}(X)$.
- (c) What is the expected value of \bar{X} ?
- (d) What is the variance of \bar{X} ? (Note: This will depend on n .)
- (e) Suppose $n = 100$. Use the R function `pnorm()` to find the approximate probability that \bar{X} is greater than 0.5.

(Question 3 is on the next page.)

3. (12 points) I want to find out the average number of people per household in the U.S. I survey a simple random sample of U.S. households and obtain the results displayed in the following table.

Household size	Number of households
1	27
2	34
3	16
4	13
5	6
6	3
7	1

- (a) Lacking any other information, our best estimate for the population mean household size is the sample mean. What is the sample mean of our data?
- (b) What is our estimate for the standard deviation of household sizes?
- (c) What is the estimated standard error of the sample mean? (That is, based on our answer to (b), what is our estimate for the standard deviation of the distribution of the sample mean?)
- (d) Our error is the difference between the sample mean and the population mean. Using the normal distribution, find the approximate probability that the absolute value of the error in a survey of this form and size is less than 0.5.