Homework 4

S520, Fall 2019

Due at the beginning of class, Monday September 23rd. Please upload your file to Canvas no later than 4pm on the due date. Late submission will be accepted (but penalized) before the solutions are posted.

Trosset question numbers refer to the hardcover textbook. Show all work and include the graphs you are asked to draw.

1. Trosset exercise 4.5.4 (6 points)

2. (9 points) A large bowl contains 50 chips (2 Red, 12 Green, 12 Blue, 12 Yellow, and 12 White). For each of the following situations, identify the distribution of Y and the parameters:

   (a) Y is the number of blue chips you get if you randomly select 5 chips from the bowl without replacement.

   (b) Y is the number of red chips you get if you randomly select 15 chips from the bowl with replacement.

   (c) Keep selecting the chips with replacement. Y is the number of chips selected before getting the first red one.

3. (6 points) Wildlife researchers are concerned with the population sizes of endangered species, such as tigers. It is rarely feasible (due to time and cost constraints) to locate and record each and every tiger in a particular region. A popular technique for estimating N, the tiger population size in a particular region, is the so-called capture-recapture method. This method consists of two steps:

   I. The researchers go through the region of interest, capture and mark (without harm) m tigers and release them back into the wild.

   II. After a period of time, k tigers are captured and the number of marked tigers out of k is recorded.

   (a) Assume that $N = 30$, $m = 10$ and $k = 15$. Find the probability that 6 marked tigers will be observed in step II.

   (b) Realistically, the researcher will not know the value of N. Suppose that, after steps I and II, the researcher observes 6 marked tigers. How can he/she use this information to estimate N, the total number of tigers in this particular region? Here we assume that all tigers have the same probability of being captured in the second sample, regardless of whether they were previously captured in the first sample.

4. (9 points) My friend Jim arrives at a bus stop at a time Uniformly distributed between 8:15AM and 8:40AM. Suppose the bus always arrives at 8:45AM.

   (a) What is the expected value and standard deviation of Jim's waiting time for the bus?

   (b) What is the probability he waits more than 10 minutes?

   (c) Given that Jim has not arrived at the bus stop by 8:30AM, what is the probability he gets there by 8:35AM?

5. Trosset exercise 5.6.2 (9 points)

6. (9 points) Let X be a random variable with probability density function

$$f(x) = \begin{cases} 2k & 0 \leq x < 3 \\ 3k & 3 \leq x < 5 \\ 0 & \text{otherwise.} \end{cases}$$

   where k is a constant.

   (a) Find k.

   (b) Find $F(4)$, the cumulative distribution function at $x = 4$.

   (c) Find the expected value of X.

7. (6 points) Let X be a random variable with PDF

$$f(x) = \begin{cases} \frac{1}{30} & 0 \leq x < 20 \\ \frac{1}{60} & 20 \leq x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the CDF of X, $F(y)$, for all y.

   (b) Find y such that $F(y) = 0.5$. Is this larger than, smaller than, or the same as $E(X)$?