Machine Vision: Homework 2

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Problem1

According to the original paper, an approach called orientation assignment is used to achieve rotation invariance. In this approach, each keypoint is assigned a consistent orientation based on local image properties, so the keypoint descriptor can be represented relative to this orientation and therefore achieve invariance to image rotation.

Problem2

$$\frac{\partial L}{\partial y_{gt}} = \frac{dL}{dp_{gt}}\frac{\partial p_{gt}}{\partial y_{gt}} = \frac{-1}{p_{gt}}\frac{exp(y_{gt})(\sum_{i=1}^{n} exp(y_i) - exp(y_{gt}))}{(\sum_{i=1}^{n} exp(y_i))^2} = \frac{-p_{gt}(1-p_{gt})}{p_{gt}} = p_{gt} - 1$$
$$\frac{\partial L}{\partial y_i} = \frac{dL}{dp_{gt}}\frac{\partial p_{gt}}{\partial y_i} = \frac{1}{p_{gt}}\frac{exp(y_{gt})}{(\sum_{i=1}^{n} exp(y_i))^2}exp(y_i) = \frac{exp(y_i)}{\sum_{i=1}^{n} exp(y_i)} = p_i, i \neq gt$$

Therefore, we have

$$\frac{\partial L}{\partial y_i} = \begin{cases} p_{gt} - 1, & i = gt \\ p_i, & i \neq gt \end{cases}$$

Problem3

We have already known $\frac{\partial L}{\partial \mathbf{y}} \in \mathbf{R}^{1\times O}$. Then we can derive:
$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial \mathbf{y}}^T \frac{\partial \mathbf{y}}{\partial \mathbf{W}} = \frac{\partial L}{\partial \mathbf{y}}^T \mathbf{x}^T \in \mathbf{R}^{O\times L}$
$\frac{\partial L}{\partial \mathbf{b}} = \frac{\partial L}{\partial \mathbf{y}} \in \mathbf{R}^{1\times O}$
$\frac{\partial L}{\partial \mathbf{x}} = \frac{\partial L}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \frac{\partial L}{\partial \mathbf{y}}\mathbf{W} \in \mathbf{R}^{1\times L}$

Problem4

Given

$$y(k,i,j) = \sum_{t=0}^{T-1}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x(t, i*s+m, j*s+n)W_k(t,m,n) + b_k \qquad (1)$$

Suppose input feature map I with size H and W, weight kernel with dimension M and N, with stride s, should produce output size $HH = (H-M)/s$ and $WW = (W-N)/s$.
By applying chain rule, the gradient for $W_k(t,m,n)$ is:

$$\frac{\partial L}{\partial W_k(t,m,n)} = \sum_{i=0}^{HH-1}\sum_{j=0}^{WW-1}\frac{\partial L}{\partial y(k,i,j)}\frac{\partial y(k,i,j)}{\partial W_k(t,m,n)}$$

Given equation (1), we have:

$$\frac{\partial y(k,i,j)}{\partial W_k(t,m,n)} = x(t, i*s+m, j*s+n)$$

Therefore, we get:

$$\frac{\partial L}{\partial W_k(t,m,n)} = \sum_{i=0}^{HH-1}\sum_{j=0}^{WW-1}\frac{\partial L}{\partial y(k,i,j)}x(t, i*s+m, j*s+n)$$

Then for each input pixel $x(t, i', j')$, it can affect output ranging from $((i' - M + 1)/s, (j' - N + 1)/s)$ to $(i'/s, j'/s)$, so $i \in [(i' - M + 1)/s, i'/s]$, $j \in [(j' - N + 1), j'/s]$

$$\frac{\partial L}{\partial x(t, i', j')} = \sum_{k=0}^{K-1} \sum_i \sum_j \frac{\partial L}{\partial y(k, i, j)} \frac{\partial y(k, i, j)}{\partial x(t, i', j')}$$

Given equation (1), for $i \in [(i' - M + 1)/s, i'/s], j \in [(j' - N + 1), j'/s]$ we get:

$$\frac{\partial L}{\partial x(t, i', j')} = \sum_{k=0}^{K-1} \sum_i \sum_j \frac{\partial L}{\partial y(k, i, j)} W_k(t, i' - i * s, j' - j * s)$$

As for $\frac{\partial L}{\partial b_k}$, it is easy to obtain:

$$\frac{\partial L}{\partial b_k} = \sum_{i=0}^{HH-1} \sum_{j=0}^{WW-1} \frac{\partial L}{\partial y(k, i, j)}$$

In sum, we obtain:

$$\frac{\partial L}{\partial W_k(t, m, n)} = \sum_{i=0}^{HH-1} \sum_{j=0}^{WW-1} \frac{\partial L}{\partial y(k, i, j)} x(t, i * s + m, j * s + n)$$

$$\frac{\partial L}{\partial b_k} = \sum_{i=0}^{HH-1} \sum_{j=0}^{WW-1} \frac{\partial L}{\partial y(k, i, j)}$$

for $i \in [(i' - M + 1)/s, i'/s], j \in [(j' - N + 1), j'/s]$

$$\frac{\partial L}{\partial x(t, i', j')} = \sum_{k=0}^{K-1} \sum_i \sum_j \frac{\partial L}{\partial y(k, i, j)} W_k(t, i' - i * s, j' - j * s)$$

Problem5

Given

$$y(c, i, j) = \max_{m=0,1,\ldots M-1} \max_{n=0,1,\ldots N-1} x(c, i * s + m, j * s + n)$$

for $i \in [(i' - M + 1)/s, i'/s], j \in [(j' - N + 1), j'/s]$,
$x(c, i', j')$ only affects when it is the maximum in kernel size, in other words, when $x(c, i', j') == y(c, i, j)$, therefore, it is easy to get:

$$\frac{\partial L}{\partial x(c, i', j')} = \sum_i \sum_j \frac{\partial L}{\partial y(k, i, j)} \mathbf{I}(x(c, i', j'), y(c, i, j))$$

$$\mathbf{I}(x, y) = \begin{cases} 1, & x = y \\ 0, & x \neq y \end{cases}$$