Case Study # 1: 1D Transient Heat Diffusion

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1 Problem Description

The problem of 1D unsteady heat diffusion in a slab of unit length with a zero initial temperature and both ends maintained at a unit temperature can be described by:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \text{ subject to } \begin{cases} T(x,0^-) = 0 & \text{for } 0 \leq x \leq 1 \\ T(0,t) = T(1,t) = 1 & \text{for } t > 0 \end{cases} \tag{1}$$

and has a well-known analytical solution:

$$T^*(x,t) = 1 - \sum_{k=1}^{\infty} \frac{4}{(2k-1)\pi} \sin[(2k-1)\pi x] \star \exp[-(2k-1)^2 \pi^2 t]. \tag{2}$$

2 Solution Algorithms

The Taylor-series (TS) method can be used on this equation to derive a finite difference approximation to the PDE.

From the definition of the derivative,

$$f'(x) \approx \frac{f(x+\varepsilon) - f(x)}{\varepsilon} \tag{3}$$

and applying this to Eqn. (1) yields:

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{T(x,t+\Delta t) - T(x,t)}{\Delta t} \\ &= \frac{T_i^{k+1} - T_i^k}{\Delta t} \end{aligned} \tag{4}$$

From the definition of the Taylor series,

$$f(x+\varepsilon) = f(x) + \varepsilon f'(x) + \frac{\varepsilon^2}{2} f''(x) + \ldots \tag{5}$$

Which, when applied to T_i^{k+1} and T_i^k gives:

$$T_{i+1} = T_i + \Delta x \frac{\partial T_i}{\partial x} + \frac{\Delta x^2}{2} \frac{\partial^2 T_i}{\partial x^2} + O(\Delta x^3) \tag{6}$$

Table 1. Figure and table captions do not end with a period

Example	Time	Cost
1	12.5	$1,000
2	24	$2,000

and

$$T_{i-1} = T_i - \Delta x \frac{\partial T_i}{\partial x} + \frac{\Delta x^2}{2} \frac{\partial^2 T_i}{\partial x^2} - O(\Delta x^3) \tag{7}$$

which, when combined, yields:

$$T_{i+1} + T_{i-1} = 2T_i + \Delta x^2 \frac{\partial^2 T_i}{\partial x^2} + O(\Delta x^4) \tag{8}$$

This problem can be solved numerically using both Forward-Time, Centered-Space (FTCS) explicit and implicit schemes.

3 Results

A Python script was used to obtain results for a 21 point mesh (N=21), and the Root Mean Square error,

$$RMS = dicks \tag{9}$$

was obtained for $s(= \Delta t / \Delta x^2)$ = 1/6, 0.25, 0.5, and 0.75, at t = 0.03, 0.06, and 0.09.

All tables should be numbered consecutively and centered above the table as shown in Table 1. The body of the table should be no smaller than 7 pt. There should be a minimum two line spaces between tables and text.

4 Discussions

Shankle chicken tail, fatback short ribs meatball pancetta ball tip sirloin short loin. Pork tongue pork belly pork loin beef ribs. Shank turkey pork belly pork loin ham hock ball tip leberkas meatloaf chuck ground round filet mignon kielbasa sirloin turducken tri-tip. Pancetta brisket sirloin beef ribs spare ribs, swine bacon ham hock. Ham kielbasa corned beef turkey turducken. Kevin biltong pork, tenderloin chuck pig ball tip filet mignon.