

```

1 import numpy as np
2 import matplotlib.pyplot as plt
3 import os
4
5 # Configure figures for production
6 WIDTH = 495.0 # the number latex spits out
7 FACTOR = 1.0 # the fraction of the width the figure should occupy
8 fig_width_pt = WIDTH * FACTOR
9
10 inches_per_pt = 1.0 / 72.27
11 golden_ratio = (np.sqrt(5) - 1.0) / 2.0 # because it looks good
12 fig_width_in = fig_width_pt * inches_per_pt # figure width in inches
13 fig_height_in = fig_width_in * golden_ratio # figure height in inches
14 fig_dims = [fig_width_in, fig_height_in] # fig dims as a list
15
16
17 def Solver(s, t_end, show_plot=False):
18     # Problem Parameters
19     L = 1. # Domain length [n.d.]
20     T0 = 0. # Initial temperature [n.d.]
21     T1 = 1. # Boundary temperature [n.d.]
22     N = 21
23
24     # Set-up Mesh
25     x = np.linspace(0, L, N)
26     dx = x[1] - x[0]
27
28     # Calculate time-step
29     dt = s * dx ** 2.0
30
31     # Initial Condition with boundary conditions
32     T_initial = [T0] * N
33     T_initial[0] = T1
34     T_initial[N - 1] = T1
35
36     # Explicit Numerical Solution
37     T_explicit = Explicit(list(T_initial), t_end, dt, s)
38
39     # Implicit Numerical Solution
40     T_implicit = Implicit(list(T_initial), t_end, dt, s)
41
42     # Analytical Solution
43     T_analytic = list(T_initial)
44     for i in range(0, N):
45         T_analytic[i] = Analytic(x[i], t_end)
46
47     # Find the RMS
48     RMS = RootMeanSquare(T_implicit, T_analytic)
49     ExplicitRMS = RootMeanSquare(T_explicit, T_analytic)
50
51     # Format our plots
52     plt.figure(figsize=fig_dims)
53     # plt.axis([0, L, T0, T1])
54     plt.xlabel('Length [nd]')
55     plt.ylabel('Temperature [nd]')
56     plt.title('s = ' + str(s)[:5] + ', t = ' + str(t_end)[:4])
57
58     # ...and finally plot
59     plt.plot(x, T_explicit, 'xr', markersize=9, label='Explicit Solution')
60     plt.plot(x, T_implicit, '+g', markersize=9, label='Implicit Solution')
61     plt.plot(x, T_analytic, 'ob', markersize=9, mfc='none', label='Analytic Solution')
62     plt.legend(loc='lower right')
63
64     # Save plots
65     save_name = 'proj_1_s_' + str(s)[:5] + '_t_' + str(t_end) + '.pdf'
66     try:

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67         os.mkdir('figures')
68     except Exception:
69         pass
70
71     plt.savefig('figures/' + save_name, bbox_inches='tight')
72     if show_plot:
73         plt.show()
74     plt.clf()
75
76     return RMS, ExplicitRMS
77
78
79 def Explicit(Told, t_end, dt, s):
80     """
81     This function computes the Forward-Time, Centered-Space (FTCS) explicit
82     scheme for the 1D unsteady heat diffusion problem.
83     """
84     N = len(Told)
85     time = 0.
86     Tnew = list(Told)
87
88     while time <= t_end:
89         for i in range(1, N - 1):
90             Tnew[i] = s * Told[i + 1] + (1 - 2.0 * s) * Told[i] + s * Told[i - 1]
91
92         Told = list(Tnew)
93         time += dt
94
95     return Told
96
97
98 def Implicit(Told, t_end, dt, s):
99     """
100     This function computes the Forward-Time, Centered-Space (FTCS) implicit
101     scheme for the 1D unsteady heat diffusion problem.
102     """
103     N = len(Told)
104     time = 0.
105
106     # Build our 'A' matrix
107     a = [-s] * N
108     a[0], a[-1] = 0, 0
109     b = [1 + 2 * s] * N
110     b[0], b[-1] = 1, 1 # hold boundary
111     c = a
112
113     while time <= t_end:
114         Tnew = TDMASolver(a, b, c, Told)
115
116         Told = list(Tnew)
117         time += dt
118
119     return Told
120
121
122 def RootMeanSquare(a, b):
123     """
124     This function will return the RMS between two lists (but does no checking
125     to confirm that the lists are the same length).
126     """
127     N = len(a)
128
129     RMS = 0.
130     for i in range(0, N):
131         RMS += (a[i] - b[i]) ** 2.
132
133     RMS = RMS ** (1. / 2.)

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134     RMS /= N**(1./2.)
135
136     return RMS
137
138
139 def TDMA solver(a, b, c, d):
140     """
141     Tridiagonal Matrix Algorithm (a.k.a Thomas algorithm).
142     """
143     N = len(a)
144     Tnew = list(d)
145
146     # Initialize arrays
147     gamma = np.zeros(N)
148     xi = np.zeros(N)
149
150     # Step 1
151     gamma[0] = c[0] / b[0]
152     xi[0] = d[0] / b[0]
153
154     for i in range(1, N):
155         gamma[i] = c[i] / (b[i] - a[i] * gamma[i - 1])
156         xi[i] = (d[i] - a[i] * xi[i - 1]) / (b[i] - a[i] * gamma[i - 1])
157
158     # Step 2
159     Tnew[N - 1] = xi[N - 1]
160
161     for i in range(N - 2, -1, -1):
162         Tnew[i] = xi[i] - gamma[i] * Tnew[i + 1]
163
164     return Tnew
165
166
167 def Analytic(x, t):
168     """
169     The analytic answer is 1 - Sum(terms). Though there are an infinite
170     number of terms, only the first few matter when we compute the answer.
171     """
172     result = 1
173     large_number = 1E6
174
175     for k in range(1, int(large_number) + 1):
176         term = ((4. / ((2. * k - 1.) * np.pi)) *
177                 np.sin((2. * k - 1.) * np.pi * x) *
178                 np.exp(-(2. * k - 1.) ** 2. * np.pi ** 2. * t))
179
180         # If subtracting the term from the result doesn't change the result
181         # then we've hit the computational limit, else we continue.
182         # print '{0} {1}, {2:.15f}'.format(k, term, result)
183         if result - term == result:
184             return result
185         else:
186             result -= term
187
188
189 def main():
190     """
191     Main function to call solver over assigned values and create some plots to
192     look at the trends in RMS compared to s and t.
193     """
194     # Loop over requested values for s and t
195     s = [1. / 6., .25, .5, .75]
196     t = [0.03, 0.06, 0.09]
197
198     RMS = []
199     with open('results.dat', 'w+') as f:
200         for i, s_ in enumerate(s):

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201         sRMS = [0] * len(t)
202         for j, t_ in enumerate(t):
203             sRMS[j], ExplicitRMS = Solver(s_, t_, False)
204             f.write('{0:.3f} {1:.2f} {2:.2e} {3:.2e} \n'.format(s_, t_, sRMS[j], ExplicitRMS))
205             # print i, j, sRMS[j]
206         RMS.append(sRMS)
207
208         # Convert to np array to make this easier...
209         RMS = np.array(RMS)
210
211         # Check for trends in RMS vs t
212         plt.figure(figsize=fig_dims)
213         plt.plot(t, RMS[0], '.r', label='s = 1/6')
214         plt.plot(t, RMS[1], '.g', label='s = .25')
215         plt.plot(t, RMS[2], '.b', label='s = .50')
216         plt.plot(t, RMS[3], '.k', label='s = .75')
217         plt.xlabel('t')
218         plt.ylabel('RMS')
219         plt.title('RMS vs t')
220         plt.legend(loc='best')
221
222         save_name = 'proj_1_rms_vs_t.pdf'
223         plt.savefig('figures/' + save_name, bbox_inches='tight')
224         plt.clf()
225
226         # Check for trends in RMS vs s
227         plt.figure(figsize=fig_dims)
228         plt.plot(s, RMS[:, 0], '.r', label='t = 0.03')
229         plt.plot(s, RMS[:, 1], '.g', label='t = 0.06')
230         plt.plot(s, RMS[:, 2], '.b', label='t = 0.09')
231         plt.xlabel('s')
232         plt.ylabel('RMS')
233         plt.title('RMS vs s')
234         plt.legend(loc='best')
235
236         save_name = 'proj_1_rms_vs_s.pdf'
237         plt.savefig('figures/' + save_name, bbox_inches='tight')
238         plt.clf()
239
240     if __name__ == "__main__":
241         main()

```