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1 import numpy as np
2 import matplotlib.pyplot as plt
3 import os
4
5 # Configure figures for production
6 WIDTH = 495.0 # the number latex spits out
7 FACTOR = 1.0 # the fraction of the width you'd like the figure to occupy
8 fig_width_pt = WIDTH * FACTOR
9
10 inches_per_pt = 1.0 / 72.27
11 golden_ratio = (np.sqrt(5) - 1.0) / 2.0 # because it looks good
12
13 fig_width_in = fig_width_pt * inches_per_pt # figure width in inches
14 fig_height_in = fig_width_in * golden_ratio # figure height in inches
15 fig_dims = [fig_width_in, fig_height_in] # fig dims as a list
16
17
18 def Solver(s, t_end, show_plot=False):
19     # Problem Parameters
20     L = 1. # Domain lengthth [n.d.]
21     T0 = 0. # Initial temperature [n.d.]
22     T1 = 1. # Boundary temperature [n.d.]
23     N = 21
24
25     # Set-up Mesh
26     x = np.linspace(0, L, N)
27     dx = x[1] - x[0]
28
29     # Calculate time-step
30     dt = s * dx ** 2.0
31
32     # Initial Condition with boundary conditions
33     T_initial = [T0] * N
34     T_initial[0] = T1
35     T_initial[N - 1] = T1
36
37     # Explicit Numerical Solution
38     T_explicit = Explicit(np.array(T_initial).copy(), t_end, dt, s)
39
40     # Implicit Numerical Solution
41     T_implicit = Implicit(np.array(T_initial).copy(), t_end, dt, s)
42
43     # Analytical Solution
44     T_analytic = np.array(T_initial).copy()
45     for i in range(0, N):
46         T_analytic[i] = Analytic(x[i], t_end)
47
48     # Find the RMS
49     RMS = RootMeanSquare(T_implicit, T_analytic)
50     ExplicitRMS = RootMeanSquare(T_explicit, T_analytic)
51
52     # Format our plots
53     plt.figure(figsize=fig_dims)
54     plt.axis([0, L, T0, T1])
55     plt.xlabel('Length [nd]')
56     plt.ylabel('Temperature [nd]')
57     plt.title('s = ' + str(s)[:5] + ', t = ' + str(t_end)[:4])
58
59     # ...and finally plot
60     plt.plot(x, T_explicit, 'xr', markersize=9, label='Explicit Solution')
61     plt.plot(x, T_implicit, '+g', markersize=9, label='Implicit Solution')
62     plt.plot(x, T_analytic, 'ob', markersize=9, mfc='none', label='Analytic Solution')
63     plt.legend(loc='lower right')
64
65     # Save plots
66     save_name = 'proj_1_s_' + str(s)[:5] + '_t_' + str(t_end) + '.pdf'

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67     try:
68         os.mkdir('figures')
69     except Exception:
70         pass
71
72     plt.savefig('figures/' + save_name, bbox_inches='tight')
73     if show_plot:
74         plt.show()
75     plt.clf()
76
77     return RMS, ExplicitRMS
78
79
80 def Explicit(Told, t_end, dt, s):
81     """
82     This function computes the Forward-Time, Centered-Space (FTCS) explicit
83     scheme for the 1D unsteady heat diffusion problem.
84     """
85     N = len(Told)
86     time = 0.
87     Tnew = Told
88
89     while time <= t_end:
90         for i in range(1, N - 1):
91             Tnew[i] = s * Told[i + 1] + (1 - 2.0 * s) * Told[i] + s * Told[i - 1]
92
93         Told = Tnew
94         time += dt
95
96     return Told
97
98
99 def Implicit(Told, t_end, dt, s):
100     """
101     This function computes the Forward-Time, Centered-Space (FTCS) implicit
102     scheme for the 1D unsteady heat diffusion problem.
103     """
104     N = len(Told)
105     time = 0.
106
107     # Build our 'A' matrix
108     a = [-s] * N
109     a[0], a[-1] = 0, 0
110     b = [1 + 2 * s] * N
111     b[0], b[-1] = 1, 1 # hold boundary
112     c = a
113
114     while time <= t_end:
115         Tnew = TDMASolver(a, b, c, Told)
116
117         Told = Tnew
118         time += dt
119
120     return Told
121
122
123 def RootMeanSquare(a, b):
124     """
125     This function will return the RMS between two lists (but does no checking
126     to confirm that the lists are the same length).
127     """
128     N = len(a)
129
130     RMS = 0.
131     for i in range(0, N):
132         RMS += (a[i] - b[i]) ** 2.
133

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134     RMS = RMS ** (1. / 2.)
135     RMS /= N
136
137     return RMS
138
139
140 def TDMA_solver(a, b, c, d):
141     """
142     Tridiagonal Matrix Algorithm (a.k.a Thomas algorithm).
143     """
144     N = len(a)
145     Tnew = d
146
147     # Initialize arrays
148     gamma = np.zeros(N)
149     xi = np.zeros(N)
150
151     # Step 1
152     gamma[0] = c[0] / b[0]
153     xi[0] = d[0] / b[0]
154
155     for i in range(1, N):
156         gamma[i] = c[i] / (b[i] - a[i] * gamma[i - 1])
157         xi[i] = (d[i] - a[i] * xi[i - 1]) / (b[i] - a[i] * gamma[i - 1])
158
159     # Step 2
160     Tnew[N - 1] = xi[N - 1]
161
162     for i in range(N - 2, -1, -1):
163         Tnew[i] = xi[i] - gamma[i] * Tnew[i + 1]
164
165     return Tnew
166
167
168 def Analytic(x, t):
169     """
170     The analytic answer is 1 - Sum(terms). Though there are an infinite
171     number of terms, only the first few matter when we compute the answer.
172     """
173     result = 1
174     large_number = 1E6
175
176     for k in range(1, int(large_number) + 1):
177         term = ((4. / ((2. * k - 1.) * np.pi)) *
178                 np.sin((2. * k - 1.) * np.pi * x) *
179                 np.exp(-(2. * k - 1.) ** 2. * np.pi ** 2. * t))
180
181         # If subtracting the term from the result doesn't change the result
182         # then we've hit the computational limit, else we continue.
183         # print '{0} {1}, {2:.15f}'.format(k, term, result)
184         if result - term == result:
185             return result
186         else:
187             result -= term
188
189
190 def main():
191     """
192     Main function to call solver over assigned values and create some plots to
193     look at the trends in RMS compared to s and t.
194     """
195     # Loop over requested values for s and t
196     s = [1. / 6., .25, .5, .75]
197     t = [0.03, 0.06, 0.09]
198
199     RMS = []
200     with open('results.dat', 'w+') as f:

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201     for i, s_ in enumerate(s):
202         sRMS = [0] * len(t)
203         for j, t_ in enumerate(t):
204             sRMS[j], ExplicitRMS = Solver(s_, t_, False)
205             f.write('{0:.3f} {1:.2f} {2:.2e} {3:.2e} \n'.format(s_, t_, sRMS[j], ExplicitRMS))
206             # print i, j, sRMS[j]
207         RMS.append(sRMS)
208
209     # Convert to np array to make this easier...
210     RMS = np.array(RMS)
211
212     # Check for trends in RMS vs t
213     plt.figure(figsize=fig_dims)
214     plt.plot(t, RMS[0], '.r', label='s = 1/6')
215     plt.plot(t, RMS[1], '.g', label='s = .25')
216     plt.plot(t, RMS[2], '.b', label='s = .50')
217     plt.plot(t, RMS[3], '.k', label='s = .75')
218     plt.xlabel('t')
219     plt.ylabel('RMS')
220     plt.title('RMS vs t')
221     plt.legend(loc='best')
222
223     save_name = 'proj_1_rms_vs_t.pdf'
224     plt.savefig('figures/' + save_name, bbox_inches='tight')
225     plt.clf()
226
227     # Check for trends in RMS vs s
228     plt.figure(figsize=fig_dims)
229     plt.plot(s, RMS[:, 0], '.r', label='t = 0.03')
230     plt.plot(s, RMS[:, 1], '.g', label='t = 0.06')
231     plt.plot(s, RMS[:, 2], '.b', label='t = 0.09')
232     plt.xlabel('s')
233     plt.ylabel('RMS')
234     plt.title('RMS vs s')
235     plt.legend(loc='best')
236
237     save_name = 'proj_1_rms_vs_s.pdf'
238     plt.savefig('figures/' + save_name, bbox_inches='tight')
239     plt.clf()
240
241 if __name__ == "__main__":
242     main()

```