Eq. da advecção-difusão

- Alterar os elementos da matriz -**done**
- Efeitos da cond. contorno na matriz -**done**
- Alterar a f -**done**

$$\frac{\partial(\rho u\phi)}{\partial x} + \frac{\partial(\rho v\phi)}{\partial y} = \frac{\partial}{\partial x}(k\frac{\partial\phi}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial\phi}{\partial y}) + f(x,y)$$

$$\rho u = \rho v = k = 1$$

$$f = -\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial y^2} + \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y}$$

Para uma dimensão, temos:

$$f = -\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial x}$$

Discretizando:

$$f = -\frac{(\phi_{i+1} - 2\phi_i + \phi_{i-1})}{\Delta x^2} + \frac{(\phi_{i+1} - \phi_{i-1})}{2\Delta x}$$

Desta forma temos que os coeficientes da matriz A1 são dados por:

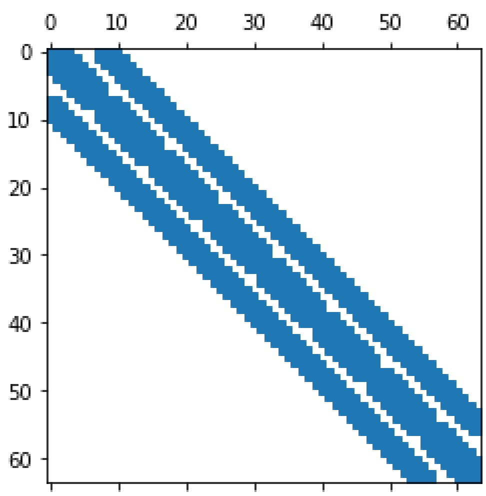
$$-\Delta x^2 f = \phi_{i+1}(1 + \Delta x/2) - 2\phi_i + \phi_{i-1}(1 - \Delta x/2) + O(\Delta x^2)$$

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.sparse import diags, csr_matrix
from scipy.sparse.linalg import spsolve

def phi(x,y):
    return np.sin(2*np.pi*x)*np.sin(2*np.pi*y)

def f(x,y):
    return 8*(np.pi**2)*np.sin(2*np.pi*x)*np.sin(2*np.pi*y) + 2*np.pi*(np.cos(2*np.pi*x)*n

def main():
    N = 8
    a = 0
    b = 1
    dx = (b-a)/N

    k = [np.ones(N-1)*(1+dx/2),-2*np.ones(N),np.ones(N-1)*(1-dx/2)]
    offset = [-1,0,1]
    A1 = diags(k,offset).toarray()

    A1[0][0] = A1[0][0] -1
    A1[N-1][N-1] = A1[N-1][N-1] -1

    I = np.eye(N)

    A = csr_matrix(np.kron(A1,I) + np.kron(I,A1))

    plt.figure(1)
    plt.spy(A)
```

```
    phiex = np.zeros(N**2)
    rhs = np.zeros(N**2)

    x = np.zeros(N)
    y= np.zeros(N)
    for i in range(N):
        x[i] = a + (i+0.5)*dx
        y[i] = a + (i+0.5)*dx

    for j in range(N):
        for i in range(N):
            phiex[i+j*N] = phi(x[i],y[j])
            rhs[i+j*N] = -1*(dx**2)*f(x[i],y[j])

    for i in range(N):
        rhs[i] = rhs[i] + 2*phi(x[i],a)
        rhs[N**2 - 1 - i] = rhs[N**2 - 1 - i] + 2*phi(x[N-1-i], b)
    for j in range(N):
        rhs[j*N] = rhs[j*N] + 2*phi(a,y[j])
        rhs[(N-1)+j*N] = rhs[(N-1)+j*N] + 2*phi(b,y[j])

    phiaprox = spsolve(A,rhs)
    #print(x)
    err = np.zeros(N**2)
    for i in range(N**2):
        err[i] = np.abs(phiex[i] - phiaprox[i])
    #print("N =", N, "| err ->", np.linalg.norm(err, ord = np.inf))
    print(A1)
    print(err.max())

main()
```

```
[[-3.      0.9375  0.      0.      0.      0.      0.      0.    ]
 [ 1.0625 -2.      0.9375  0.      0.      0.      0.      0.    ]
 [ 0.      1.0625 -2.      0.9375  0.      0.      0.      0.    ]
 [ 0.      0.      1.0625 -2.      0.9375  0.      0.      0.    ]
 [ 0.      0.      0.      1.0625 -2.      0.9375  0.      0.    ]
 [ 0.      0.      0.      0.      1.0625 -2.      0.9375  0.    ]
 [ 0.      0.      0.      0.      0.      1.0625 -2.      0.9375]
 [ 0.      0.      0.      0.      0.      0.      1.0625 -3.    ]]
0.053914064318804744
```



In [ ]: