

# Introductory Computational Finance

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### Group C: Monte Carlo Method

a) Studied the source code and imported the necessary files for the program to run successfully.

b) We ran the MC program with Batches 1 and 2 and experimented with various step values and simulation counts. Here is the results table for them:

EXACT CALL: 2.13337      EXACT PUT: 5.84628

		BATCH 1					
		NSIM					
		10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>
NT	10	Call Price: 2.38525 Call SD: 5.68943 Call SE: 0.568943	Call Price: 2.08001 Call SD: 4.50887 Call SE: 0.142583	Call Price: 2.12297 Call SD: 4.48641 Call SE: 0.0448641	Call Price: 2.11117 Call SD: 4.43419 Call SE: 0.0140221	Call Price: 2.11674 Call SD: 4.44983 Call SE: 0.00444983	Call Price: 2.11928 Call SD: 4.45082 Call SE: 0.00140747
		Put Price: 6.0245 Put SD: 6.34635 Put SE: 0.634635	Put Price: 6.02809 Put SD: 6.2552 Put SE: 0.197807	Put Price: 5.84177 Put SD: 6.07773 Put SE: 0.0607773	Put Price: 5.8456 Put SD: 6.08728 Put SE: 0.0192497	Put Price: 5.84074 Put SD: 6.08132 Put SE: 0.00608132	Put Price: 5.83512 Put SD: 6.08127 Put SE: 0.00192307
	100	Call Price: 2.094 Call SD: 4.35982 Call SE: 0.435982	Call Price: 2.0741 Call SD: 4.48768 Call SE: 0.141913	Call Price: 2.1378 Call SD: 4.54234 Call SE: 0.0454234	Call Price: 2.13043 Call SD: 4.51298 Call SE: 0.0142713	Call Price: 2.13271 Call SD: 4.51624 Call SE: 0.00451624	Call Price: 2.13223 Call SD: 4.50954 Call SE: 0.00142604
		Put Price: 6.58781 Put SD: 6.52263 Put SE: 0.652263	Put Price: 5.84851 Put SD: 5.89612 Put SE: 0.186452	Put Price: 5.90807 Put SD: 6.0547 Put SE: 0.060547	Put Price: 5.87321 Put SD: 6.05775 Put SE: 0.0191563	Put Price: 5.85125 Put SD: 6.04898 Put SE: 0.00604898	Put Price: 5.84183 Put SD: 6.04901 Put SE: 0.00191286
	500	Call Price: 2.17558 Call SD: 4.41928 Call SE: 0.441928	Call Price: 2.02328 Call SD: 4.41728 Call SE: 0.139687	Call Price: 2.12707 Call SD: 4.60808 Call SE: 0.0460808	Call Price: 2.14763 Call SD: 4.55354 Call SE: 0.0143996	Call Price: 2.13071 Call SD: 4.51286 Call SE: 0.00451286	Call Price: 2.13391 Call SD: 4.51504 Call SE: 0.00142778
		Put Price: 5.95189 Put SD: 6.1765 Put SE: 0.61765	Put Price: 6.03298 Put SD: 6.06848 Put SE: 0.191902	Put Price: 5.93754 Put SD: 6.09454 Put SE: 0.0609454	Put Price: 5.83818 Put SD: 6.05203 Put SE: 0.0191382	Put Price: 5.84125 Put SD: 6.04743 Put SE: 0.00604743	Put Price: 5.84580 Put SD: 6.04719 Put SE: 0.00191
	1000	Call Price: 2.36385 Call SD: 4.36268 Call SE: 0.43627	Call Price: 2.10416 Call SD: 4.54809 Call SE: 0.14382	Call Price: 2.12827 Call SD: 5.93043 Call SE: 0.04556	Call Price: 2.13874 Call SD: 4.55622 Call SE: 0.01441	Call Price: 2.13658 Call SD: 4.51429 Call SE: 0.00451	Call Price: 2.13320 Call SD: 4.51605 Call SE: 0.00143
		Put Price: 6.14841 Put SD: 6.09495 Put SE: 0.60949	Put Price: 6.02480 Put SD: 5.92219 Put SE: 0.18728	Put Price: 5.93043 Put SD: 6.08910 Put SE: 0.06089	Put Price: 5.86757 Put SD: 6.05476 Put SE: 0.01915	Put Price: 5.83817 Put SD: 6.04834 Put SE: 0.00605	Put Price: 5.84573 Put SD: 6.04705 Put SE: 0.00191

Figure B.1

One of the first things I noticed from testing various combinations of NSIM and NT values is that the time it took to run simulations grew as NT and NSIM values increased. We can see that small simulation steps (10<sup>2</sup>) have no values converging to our exact values no matter what NT value was used for our computations.

On the other hand, most computations with NT of at least 100 and NSIM of  $10^7$  converged with our exact value. From our observations, we can conclude that the growth of NT and NSIM results in higher accuracies for put and call calculations.

EXACT CALL: 7.96557

EXACT PUT: 7.96557

		BATCH 2					
		NSIM					
		$10^2$	$10^3$	$10^4$	$10^5$	$10^6$	$10^7$
NT	10	Call Price: 8.4227 Call SD: 15.8341 Call SE: 1.58341	Call Price: 7.86977 Call SD: 13.1167 Call SE: 0.414788	Call Price: 7.98069 Call SD: 13.09 Call SE: 0.1309	Call Price: 7.9613 Call SD: 12.9674 Call SE: 0.0410067	Call Price: 7.96922 Call SD: 13.0034 Call SE: 0.0130034	Call Price: 7.97938 Call SD: 13.0058 Call SE: 0.0041128
		Put Price: 8.10554 Put SD: 11.2165 Put SE: 1.12165	Put Price: 8.36038 Put SD: 10.881 Put SE: 0.344088	Put Price: 7.98379 Put SD: 10.5308 Put SE: 0.105308	Put Price: 8.0067 Put SD: 10.5416 Put SE: 0.0333355	Put Price: 7.99098 Put SD: 10.5321 Put SE: 0.0105321	Put Price: 7.98315 Put SD: 10.5304 Put SE: 0.00333001
		Call Price: 7.77513 Call SD: 12.8422 Call SE: 1.28422	Call Price: 7.73557 Call SD: 13.0707 Call SE: 0.413333	Call Price: 7.94097 Call SD: 13.212 Call SE: 0.13212	Call Price: 7.94362 Call SD: 13.1477 Call SE: 0.0415767	Call Price: 7.9625 Call SD: 13.1517 Call SE: 0.0131517	Call Price: 7.96824 Call SD: 13.1354 Call SE: 0.00415378
	100	Put Price: 9.44836 Put SD: 11.3238 Put SE: 1.13238	Put Price: 7.88494 Put SD: 10.1262 Put SE: 0.320219	Put Price: 8.06336 Put SD: 10.435 Put SE: 0.10435	Put Price: 8.0079 Put SD: 10.4359 Put SE: 0.0330012	Put Price: 7.97439 Put SD: 10.4143 Put SE: 0.0104143	Put Price: 7.96088 Put SD: 10.4125 Put SE: 0.00329271
		Call Price: 7.98253 Call SD: 12.9557 Call SE: 1.29557	Call Price: 7.62472 Call SD: 12.8813 Call SE: 0.407342	Call Price: 7.93258 Call SD: 13.3434 Call SE: 0.133434	Call Price: 8.01256 Call SD: 13.2345 Call SE: 0.0418511	Call Price: 7.96142 Call SD: 13.1421 Call SE: 0.0131421	Call Price: 7.96866 Call SD: 13.1481 Call SE: 0.004158
		Put Price: 8.10974 Put SD: 10.7909 Put SE: 1.07909	Put Price: 8.28685 Put SD: 10.4581 Put SE: 0.330715	Put Price: 8.13742 Put SD: 10.5022 Put SE: 0.105022	Put Price: 7.95795 Put SD: 10.4107 Put SE: 0.0329215	Put Price: 7.95663 Put SD: 10.4052 Put SE: 0.0104052	Put Price: 7.96539 Put SD: 10.4071 Put SE: 0.00329101
	500	Call Price: 8.86798 Call SD: 13.5847 Call SE: 1.35847	Call Price: 7.87164 Call SD: 13.3407 Call SE: 0.421871	Call Price: 7.92226 Call SD: 13.1747 Call SE: 0.131747	Call Price: 7.98184 Call SD: 13.2354 Call SE: 0.0418541	Call Price: 7.97529 Call SD: 13.1393 Call SE: 0.0131393	Call Price: 7.96646 Call SD: 13.1506 Call SE: 0.00415859
		Put Price: 8.88433 Put SD: 10.7928 Put SE: 1.07928	Put Price: 8.28691 Put SD: 10.3063 Put SE: 0.325913	Put Price: 8.12311 Put SD: 10.476 Put SE: 0.10476	Put Price: 8.00674 Put SD: 10.4307 Put SE: 0.0329847	Put Price: 7.94982 Put SD: 10.407 Put SE: 0.010407	Put Price: 7.96533 Put SD: 10.4042 Put SE: 0.00329008
	1000						

Figure B.2

This is the table for our batch 2 test. The values yielded by this test follow a similar pattern where more simulations and steps resulted in better accuracy at matching our data. Moreover, we can see that our accuracy of two places behind the decimal point is achieved at NSIM =  $10^5$  and NT = 10. Generally, the highest NSIM value of  $10^7$  yields the most accurate results.

c) This is for stress testing the Monte Carlo method using batch 4:

EXACT CALL: 92.1747      EXACT PUT: 1.24651

=== CALLS ===

NT = 100: NSIM: 100000: CALL = 90.1398

NT = 100: NSIM: 500000: CALL = 90.2954

NT = 100: NSIM 1000000: CALL = 89.7989

NT = 100: NSIM: 5000000: CALL = 89.4013

NT = 100: NSIM: 10000000: CALL = 89.3694

NT = 500: NSIM: 100000: CALL = 93.02111

NT = 500: NSIM: 500000: CALL = 91.8002

NT = 500: NSIM 1000000: CALL = 91.9223

NT = 500: NSIM: 5000000: CALL = 91.8342

NT = 500: NSIM: 10000000: CALL = 91.7806

NT = 1000: NSIM: 100000: CALL = 92.3474

NT = 1000: NSIM: 500000: CALL = 91.9592

NT = 1000: NSIM 1000000: CALL = 91.9568

NT = 1000: NSIM: 5000000: CALL = 91.9540

NT = 1000: NSIM: 10000000: CALL = 91.8598

=== PUTS ===

NT = 100: NSIM: 100000: PUT = 1.29850

NT = 100: NSIM: 500000: PUT = 1.29123

NT = 100: NSIM 1000000: PUT = 1.29534

NT = 100: NSIM: 5000000: PUT = 1.29017

NT = 100: NSIM: 10000000: PUT = 1.28956

NT = 500: NSIM: 100000: PUT = 1.25552

NT = 500: NSIM: 500000: PUT = 1.25897

NT = 500: NSIM 1000000: PUT = 1.25419

NT = 500: NSIM: 5000000: PUT = 1.25379

NT = 500: NSIM: 10000000: PUT = 1.25527

NT = 1000: NSIM: 100000: PUT = 1.25722

NT = 1000: NSIM: 500000: PUT = 1.24840

NT = 1000: NSIM 1000000: PUT = 1.24880

NT = 1000: NSIM: 5000000: PUT = 1.25103

NT = 1000: NSIM: 10000000: PUT = 1.25071

From the results of Batch 4, we can see there is a recurring pattern for higher NSIM/NT values resulting in higher accuracy but it requires more computational power than Batches 1 and 4. We tried to use the Monte Carlo simulation to get an accuracy of two places after the decimal but an NT of 100 and NSIM of  $10^7$  did not suffice. To get that degree of accuracy, it may be safe to assume we need at least a time step of 1000 and an even higher NSIM value.

### Group D: Advanced Monte Carlo

a) I created generic functions to compute the standard deviation and standard error using the formula from the PDF document. These functions were put in SDandSE.hpp files and included in our main so we can use them

b) Now we will analyze some patterns from SD and SE calculations:

		BATCH 1					
		NSIM					
		10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>
NT	10	Call Price: 2.38525	Call Price: 2.08001	Call Price: 2.12297	Call Price: 2.11117	Call Price: 2.11674	Call Price: 2.11928
		Call SD: 5.68943	Call SD: 4.50887	Call SD: 4.48641	Call SD: 4.43419	Call SD: 4.44983	Call SD: 4.45082
		Call SE: 0.568943	Call SE: 0.142583	Call SE: 0.0448641	Call SE: 0.0140221	Call SE: 0.00444983	Call SE: 0.00140747
		Put Price: 6.0245	Put Price: 6.02809	Put Price: 5.84177	Put Price: 5.8456	Put Price: 5.84074	Put Price: 5.83512
		Put SD: 6.34635	Put SD: 6.2552	Put SD: 6.07773	Put SD: 6.08728	Put SD: 6.08132	Put SD: 6.08127
		Put SE: 0.634635	Put SE: 0.197807	Put SE: 0.0607773	Put SE: 0.0192497	Put SE: 0.00608132	Put SE: 0.00192307
	100	Call Price: 2.094	Call Price: 2.0741	Call Price: 2.1378	Call Price: 2.13043	Call Price: 2.13271	Call Price: 2.13223
		Call SD: 4.35982	Call SD: 4.48768	Call SD: 4.54234	Call SD: 4.51298	Call SD: 4.51624	Call SD: 4.50954
		Call SE: 0.435982	Call SE: 0.141913	Call SE: 0.0454234	Call SE: 0.0142713	Call SE: 0.00451624	Call SE: 0.00142604
		Put Price: 6.58781	Put Price: 5.84851	Put Price: 5.90807	Put Price: 5.87321	Put Price: 5.85125	Put Price: 5.84183
		Put SD: 6.52263	Put SD: 5.89612	Put SD: 6.0547	Put SD: 6.05775	Put SD: 6.04898	Put SD: 6.04901
		Put SE: 0.652263	Put SE: 0.186452	Put SE: 0.060547	Put SE: 0.0191563	Put SE: 0.00604898	Put SE: 0.00191286
	500	Call Price: 2.17558	Call Price: 2.02328	Call Price: 2.12707	Call Price: 2.14763	Call Price: 2.13071	Call Price: 2.13391
		Call SD: 4.41928	Call SD: 4.41728	Call SD: 4.60808	Call SD: 4.55354	Call SD: 4.51286	Call SD: 4.51504
		Call SE: 0.441928	Call SE: 0.139687	Call SE: 0.0460808	Call SE: 0.0143996	Call SE: 0.00451286	Call SE: 0.00142778
		Put Price: 5.95189	Put Price: 6.03298	Put Price: 5.93754	Put Price: 5.83818	Put Price: 5.84125	Put Price: 5.84580
		Put SD: 6.1765	Put SD: 6.06848	Put SD: 6.09454	Put SD: 6.05203	Put SD: 6.04743	Put SD: 6.04719
		Put SE: 0.61765	Put SE: 0.191902	Put SE: 0.0609454	Put SE: 0.0191382	Put SE: 0.00604743	Put SE: 0.00191
	1000	Call Price: 2.36385	Call Price: 2.10416	Call Price: 2.12827	Call Price: 2.13874	Call Price: 2.13658	Call Price: 2.13320
		Call SD: 4.36268	Call SD: 4.54809	Call SD: 5.93043	Call SD: 4.55622	Call SD: 4.51429	Call SD: 4.51605
		Call SE: 0.43627	Call SE: 0.14382	Call SE: 0.04556	Call SE: 0.01441	Call SE: 0.00451	Call SE: 0.00143
		Put Price: 6.14841	Put Price: 6.02480	Put Price: 5.93043	Put Price: 5.86757	Put Price: 5.83817	Put Price: 5.84573
		Put SD: 6.09495	Put SD: 5.92219	Put SD: 6.08910	Put SD: 6.05476	Put SD: 6.04834	Put SD: 6.04705
		Put SE: 0.60949	Put SE: 0.18728	Put SE: 0.06089	Put SE: 0.01915	Put SE: 0.00605	Put SE: 0.00191

We can see that standard deviation and standard error seem to decrease as NSIM approaches infinity. Standard error seems to have more of a correlation with NSIM approaching infinity as it is decreasing at a much faster rate. This supports our claim that more data and tests results in better accuracy.

		BATCH 2					
		NSIM					
		10^2	10^3	10^4	10^5	10^6	10^7
NT	10	Call Price: 8.4227 Call SD: 15.8341 Call SE: 1.58341	Call Price: 7.86977 Call SD: 13.1167 Call SE: 0.414788	Call Price: 7.98069 Call SD: 13.09 Call SE: 0.1309	Call Price: 7.9613 Call SD: 12.9674 Call SE: 0.0410067	Call Price: 7.96922 Call SD: 13.0034 Call SE: 0.0130034	Call Price: 7.97938 Call SD: 13.0058 Call SE: 0.0041128
		Put Price: 8.10554 Put SD: 11.2165 Put SE: 1.12165	Put Price: 8.36038 Put SD: 10.881 Put SE: 0.344088	Put Price: 7.98379 Put SD: 10.5308 Put SE: 0.105308	Put Price: 8.0067 Put SD: 10.5416 Put SE: 0.0333355	Put Price: 7.99098 Put SD: 10.5321 Put SE: 0.0105321	Put Price: 7.98315 Put SD: 10.5304 Put SE: 0.00333001
		Call Price: 7.77513 Call SD: 12.8422 Call SE: 1.28422	Call Price: 7.73557 Call SD: 13.0707 Call SE: 0.413333	Call Price: 7.94097 Call SD: 13.212 Call SE: 0.13212	Call Price: 7.94362 Call SD: 13.1477 Call SE: 0.0415767	Call Price: 7.9625 Call SD: 13.1517 Call SE: 0.0131517	Call Price: 7.96824 Call SD: 13.1354 Call SE: 0.00415378
	100	Put Price: 9.44836 Put SD: 11.3238 Put SE: 1.13238	Put Price: 7.88494 Put SD: 10.1262 Put SE: 0.320219	Put Price: 8.06336 Put SD: 10.435 Put SE: 0.10435	Put Price: 8.0079 Put SD: 10.4359 Put SE: 0.0330012	Put Price: 7.97439 Put SD: 10.4143 Put SE: 0.0104143	Put Price: 7.96088 Put SD: 10.4125 Put SE: 0.00329271
		Call Price: 7.98253 Call SD: 12.9557 Call SE: 1.29557	Call Price: 7.62472 Call SD: 12.8813 Call SE: 0.407342	Call Price: 7.93258 Call SD: 13.3434 Call SE: 0.133434	Call Price: 8.01256 Call SD: 13.2345 Call SE: 0.0418511	Call Price: 7.96142 Call SD: 13.1421 Call SE: 0.0131421	Call Price: 7.96866 Call SD: 13.1481 Call SE: 0.004158
		Put Price: 8.10974 Put SD: 10.7909 Put SE: 1.07909	Put Price: 8.28685 Put SD: 10.4581 Put SE: 0.330715	Put Price: 8.13742 Put SD: 10.5022 Put SE: 0.105022	Put Price: 7.96795 Put SD: 10.4107 Put SE: 0.0329215	Put Price: 7.95663 Put SD: 10.4052 Put SE: 0.0104052	Put Price: 7.96539 Put SD: 10.4071 Put SE: 0.00329101
	500	Call Price: 8.86798 Call SD: 13.5847 Call SE: 1.35847	Call Price: 7.87164 Call SD: 13.3407 Call SE: 0.421871	Call Price: 7.92226 Call SD: 13.1747 Call SE: 0.131747	Call Price: 7.98184 Call SD: 13.2354 Call SE: 0.0418541	Call Price: 7.97529 Call SD: 13.1393 Call SE: 0.0131393	Call Price: 7.96646 Call SD: 13.1506 Call SE: 0.00415859
		Put Price: 8.88433 Put SD: 10.7928 Put SE: 1.07928	Put Price: 8.28691 Put SD: 10.3063 Put SE: 0.325913	Put Price: 8.12311 Put SD: 10.476 Put SE: 0.10476	Put Price: 8.00674 Put SD: 10.4307 Put SE: 0.0329847	Put Price: 7.94982 Put SD: 10.407 Put SE: 0.010407	Put Price: 7.96533 Put SD: 10.4042 Put SE: 0.00329008

A similar pattern also is shown in Batch 2 test. An accuracy of two places behind the decimal is met for PUT when NT=100 and NSIM = 10<sup>6</sup> with standard error being quite low around 0.01. Call SD seems to approach 13.1 as Put SD approaches 10.4 as NSIM and NT increases.