

# Numerical Methods (Building Simulator for Real World)

## Source

- YouTube
  - Numerical Methods Course [https://www.youtube.com/playlist?list=PLDea8VeK4MUTOBXLpvx\\_WKtVrMkojEh52](https://www.youtube.com/playlist?list=PLDea8VeK4MUTOBXLpvx_WKtVrMkojEh52)
- Mathematic
  - MIT Calculus Course
  - 3 Blue 1 Brown
- Related to
  - Machine Learning

## Keywords

approximate solution

“Function is everything”

Mathematical Modeling

How can we use numerical methods to solve real-world problems?

only to find a solution that is close enough to the true answer.

“Creating a simulator” which can model complex systems and predict their behavior over time.

## Review

review in Freeman

## Calculus

- derivatives
- zero point theorem

$f \in C[a, b]$ ,  $f(a)f(b) < 0$ , then there exists at least one point  $c \in (a, b)$  such that  $f(c) = 0$ .

what can it use for?

- intermediate value theorem

$f \in C[a, b]$ ,

reflect what?

existence of the continuous point

- mean value theorem

reflect what? the relation between instantaneous rate of change and average rate of change.  
How?

```
#include<iostream>
using namespace std;
int main(){
```

```
}
```

- Rolle's theorem

if  $f \in C[a, b]$  and  $f'(x)$  exists for all  $x \in (a, b)$ , and  $f(a) = f(b)$ , then there exists at least one point  $c \in (a, b)$  such that  $f'(c) = 0$ .

- Lagrange's mean value theorem

if  $f \in C[a, b]$  and  $f'(x)$  exists for all  $x \in (a, b)$ , then there exists at least one point  $c \in (a, b)$  such that  $f'(c) = \frac{f(b)-f(a)}{b-a}$ .

- Cauchy's mean value theorem

$y=f(x)$  let  $y=g(t)$   $x=w(t)$

Parameterisation of Lagrange's mean value theorem

fit the function in real world scenarios

## Sequences and Series

Taylor series

using polynomial approximation to approximate a function, and find out the "lost part/lost function/remainder"

using Lagrange's mean value theorem to estimate the remainder and ensure the lost function is disappear.

## Binary Review

「乘二取整，順序排列」

「除二取餘，逆序排列」

why?

float point number can not represent all the real number.

- number theory

IEEE 754 <> what is it?

how to trans real number to double point number?

「差之毫釐，謬以千里」

Error analysis

- Roundoff error
- Truncation error

can not eliminate these errors.

"Perfect is impossible, good enough is enough."

## week 1

### error analysis

We hope the number itself can reflect its precision.

make it to significant digit.

$\frac{1}{2}$  the  $n$  digit. where  $n$  is the  $n$ th of  $\neq 0$  digit.

$\frac{1}{2} 10^{-n}$

$|p - p^*| / |p| < \frac{10^{1-d}}{2}(a_1 + 1)$

why? below is the proof.

$$a_1 * 10^{m-1} \leq |p *| < (a_1 + 1) * 10^{m-1}$$

1) assume  $p *$  got  $n$  significant digit.

2) assume  $r(p *) < \frac{1}{2}(a_1 + 1)$ , where  $r(p *)$  is the relative error of  $p *$ ,

making error reasonable

how to use it  $>> ?$

Taylor expansion  $O()$

in computer

error is reasonable ???

## Numerical Stability

question

### Ill-posed Problem

adding a small perturbation to the input and observing whether it causes a large change in the output.

transform Subtraction (-) into Addition (+)

GNU

forget to hw

匯編不會編

practical solution

- avoid

### summary

error analysis

- absolute error
  - we need to make the value close to  $\frac{1}{2}$  n digit, where nth number  $\neq 0$
  - here is the methods
- relative error

## nonlinear equation

transcendence function

using some iterative method to find the answer.

### bisection method

### regula-falsi , position method

enhance speed

replace mid point

secant

[https://en.wikipedia.org/wiki/Regula\\_falsi](https://en.wikipedia.org/wiki/Regula_falsi)

using a coefficient to solve the problem.

womier

what if we use linear transform in false position method? crazy mind

### **Fixed-Point Iteration**

[https://en.wikipedia.org/wiki/Banach\\_fixed-point\\_theorem](https://en.wikipedia.org/wiki/Banach_fixed-point_theorem)

should under what consequence?

if  $|g'(x)| < 1$  sometime it works

else if  $|g'(x)| \geq 1$  it fails

when  $|g'(x)| = 1$  can't judge

-> theorem of banach contraction mapping

question what is lipschitz and how to speed-up

Lagrange mean value theorem to prove it.

### **Newton's method**

Newton's method [https://en.wikipedia.org/wiki/Newton%27s\\_method](https://en.wikipedia.org/wiki/Newton%27s_method)

BFGS(Broyden-Fletcher-Goldfarb-Shanno algorithm) L-BFGS(Limited memory Broyden-Fletcher-Goldfarb-Shanno algorithm)

why use it? don't need to calculate the Hessian Matrix

### **Addition**