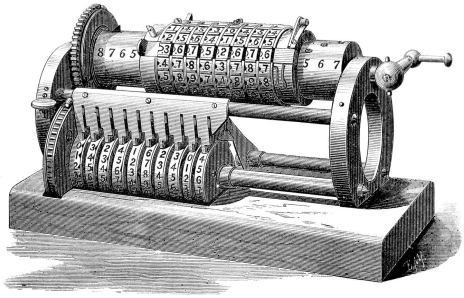
Numerical Methods (Building Simulator for Real World)



Source

- YouTube
  - Numerical Methods Course https://www.youtube.com/playlist?list=PLDea8VeK4MUTOBXLpvx_WKtVrMkojEh52
- Mathematic
  - MIT Calculus Course
  - 3 Blue 1 Brown
- Related to
  - Machine Learning

Keywords

approximate solution

“Function is everything”

Mathematical Modeling

How can we use numerical methods to solve real-world problems?

only to find a solution that is close enough to the true answer.

“Creating a simulator” which can model complex systems and predict their behavior over time.

Review

review in Freeman

Calculus

- derivatives
- zero point theorem

$f \in C[a,b]$, $f(a)f(b) < 0$, then there exists at least one point $c \in (a,b)$ such that $f(c) = 0$.

what can it use for?

- intermediate value theorem

$f \in C[a,b]$,

reflect what?

existence of the continuous point

- mean value theorem

reflect what? the relation between instantaneous rate of change and average rate of change. How?

```
#include<iostream>
using namespace std;
int main(){

}
```

- Rolle's theorem

if $f \in C[a,b]$ and $f'(x)$ exists for all $x \in (a,b)$, and $f(a) = f(b)$, then there exists at least one point $c \in (a,b)$ such that $f'(c) = 0$.

- Lagarange's mean value theorem

if $f \in C[a,b]$ and $f'(x)$ exists for all $x \in (a,b)$, then there exists at least one point $c \in (a,b)$ such that $f'(c)$ =frac{$f(b) - f(a)$}{$b - a$}.

- Cauchy's mean value theorem

y=f(x) let y=g(t) x=w(t)

Parameterisation of Lagarange's mean value theorem

/…. fit the function in real world scenarios

Sequences and Series

Taylor series

using polynomial approximation to approximate a function, and find out the "lost part/lost function/remainder"

|

using Lagarange's mean value theorem to estimate the remainder.and ensure the lost function is disappear.

Binary Review

「乘二取整，順序排列」

「除二取餘，逆序排列」

why?

float point number can not represent all the real number.

fenmu

- number theory

IEEE 754 <> what is it?

how to trans real number to double point number?

「差之毫釐，謬以千里」

Error analysis

- Roudoff error
- Truncation error

can not eliminate these errors.

“Perfect is impossible, good enough is enough.”

week 1

error analysis
We hope the number itself can reflect its precision.

make it to significant digit.

$\frac{1}{2}$ the n digit. where n is the nth of !=0 digit.

$\frac{1}{2}\ 10^{-}n$

$|p - p*| \ / \ |p| < \frac{10^{\{1-d\}}}{2}(a_1 + 1)$

why? below is the proof.

$a_1 * 10^{\{m-1\}} \leq |p*| < (a_1 + 1) * 10^{\{m-1\}}$

1) assume $p*$ got n significant digit.

2) assume $r(p*) < \frac{1}{2}(a_1 + 1)$, where $r(p*)$ is the relative error of $p*$,

making error resonable

how to use it >> ?

Taylor expansion O()

in computer

| | error is reasonable ???

Numerical Stability
question

Ill-posed Problem
adding a small perturbation to the input and observing whether it causes a large change in the output.

transform Subtraction (-) into Addition (+)

GNU

forget to hw
匯編不會編

pratical solution

- avoid

summary

error analysis

- absolute error
  - we need to make the value close to $\frac{1}{2}$ n digit, where nth number !=0
  - here is the methods
- relative error

nonlinear equation

transcendence function

using some interative method to find the answer.

bisection method

regula-falsi , position method

enhace speed

replace mid point

secant

https://en.wikipedia.org/wiki/Regula_falsi

using a coefficient to solve the problem.

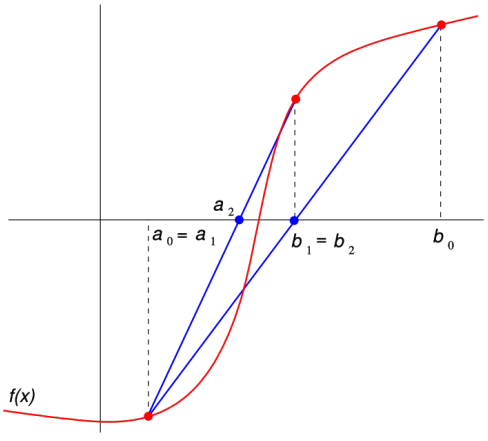
womier

what if we use linear transform in false position method? crazy mind

```
// bisection method find the solution of one-dimantional equation
// when in real world problem, we first draw the plot to figure out the area of the
solution.

#include <iostream>
#include <cstdio>
using namespace std;

double a,b,c,d;
```



```
double f(double x)
{
    return a*x*x*x + b*x*x + c*x + d;
}

double search (double l, double r)
{
    double mid;
    bool flat = true;
    for (int i=0; i<=500; i++)
    {
        mid = (l+r)/2.0;
        if (f(l)*f(mid)<=0)
            r=mid;
        else if (f(r)*f(mid)<=0)
            l=mid;
        else
        {
            flat = false;
            break;
        }
    }
    if (flat)
        return mid;

    return -101;
}

int main ()
{

    cin >> a >> b >> c >> d;
    double x[4]={0};
    double solution;
    int count = 0;
    for (int i = -100; i<= 100; i++)
    {
        solution = search (i,i+1);
        if (solution != -101 && x[1]!=solution && x[2]!= solution && x[3]!=solution)
        {
            count ++;
            x[count]=solution;
        }
    }
    printf("%.2lf %.2lf %.2lf",x[1],x[2],x[3]);

    return 0;
}
```

Fixed-Point Iteration

https://en.wikipedia.org/wiki/Banach_fixed-point_theorem

should under what consequence?

if $|g'(x)| < 1$ sometime it works

else if $|g'(x)| \geq 1$ it fails

when $|g'(x)| = 1$ can't judge

-> theorem of banach contraction mapping

question what is lipchitz and how to speed-up

Lagarange mean value theorem to prove it.

Newton's method

Newton's method https://en.wikipedia.org/wiki/Newton%27s_method

BFGS(Broyden-Fletcher-Goldfarb-Shanno algorithm) L-BFGS(Limited memory Broyden-Fletcher-Goldfarb-Shanno algorithm)

why use it? don't need to claculate the Hessian Matrix

Addition