Probability and statistics

AI/ML → involve the use of large amounts of data → To understand and analyze the data, we need probability and statistics

Statistics

FREQUENTIST

- probabilities are evaluated based on data collected regarding a certain hypothesis or event
- no importance is given to other events which may or may not influence the event under consideration

BAYESIAN

- probabilities are evaluated from two perspectives – prior to knowing a piece of evidence and posterior to it.
- based on the celebrated Bayes' theorem in probability

Frequentist Statistics

Given an event A, the probability of the event occurring, P(A), is determined by repeatedly conducting a relevant experiment, and noting down its outcome in each case.

$$P(A) = \frac{n(A)}{n(\Omega)}$$

$n(A)$ → # of times the event A occurred

$n(\Omega)$ → the # of times the experiment was conducted

⇓

size of the sample space.

Ω → sample space

↓

set of possible outcomes

Eg: A coin is tossed 50 times and a head is obtained 24 times.

$$P(\text{Heads}) = \frac{24}{50} = \frac{12}{25}$$

Ideal $P(\text{Head}) = \frac{n(H)}{n(\Omega)} = \frac{1}{n(\{H,T\})} = \frac{1}{2}$

Bayesian Statistics

This form of statistics is based on the concept of conditional probability, which provides a way to account for probabilities before (prior to) and after (posterior to) knowing a piece of evidence

Given two events A and B, the conditional probability of event A occurring, provided event B has occurred is

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Bayes' theorem

$P(A)$: PRIOR PROBABILITY, i.e. the probability of event A occurring, prior to knowing whether event B has occurred or not. Event A is treated as a proposition, i.e. a hypothesis whose likelihood is being estimated, given some EVIDENCE represented by event B

$P(B)$: EVIDENTIAL PROBABILITY, i.e. the probability of the evidential event B.

$P(A|B)$: POSTERIOR PROBABILITY, i.e. the probability of the hypothesis

$P(B|A)$: LIKELIHOOD FUNCTION, ie how likely the evidence B is, given that proposition A has occurred.

likelihood function

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Posterior probability

prior probability

evidential probability

$$\boxed{\text{POSTERIOR} = \frac{\text{LIKELIHOOD} \times \text{PRIOR}}{\text{EVIDENCE}}}$$

$$P(A \cap B) = P(B \cap A)$$

$$P(A) \cdot P(B|A) = P(B) \cdot P(A|B) \Rightarrow P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}.$$

Application of Bayes' theorem to multiple events:

If there are several mutually exclusive and exhaustive events A_i that can occur following the evidential event B, then

$$P(B) = \sum_i P(A_i \cap B) = \sum_i P(A_i) \cdot P(B|A_i)$$

$$\underbrace{P(A_i|B) = \frac{P(A_i) \cdot P(B|A_i)}{P(B)}}_{\text{Bayes' theorem}} = \frac{P(A_i) \cdot P(B|A_i)}{\sum_j P(A_j) \cdot P(B|A_j)}$$