Receiver operating characteristic (ROC)

→ one of the ways to understand how well the binary classifier is performing

Others include:

- Confusion
- Balanced score.

In the ROC, we plot the true positive rate (TPR) versus the false positive rate (FPR), for various values of the classification threshold probability (p^*).

$$\hat{y} = \begin{cases} 1 \; ; & p \geq p^* \\ 0 \; ; & p < p^* \end{cases}$$

$\hat{y} = 1$: Class II

$\hat{y} = 0$: class I

p: logistic function

$$TPR = \left(\frac{TP}{TP + FN} \right)$$

$$FPR = \left(\frac{FP}{FP + TN} \right)$$

Varying p^* from 1 to 0

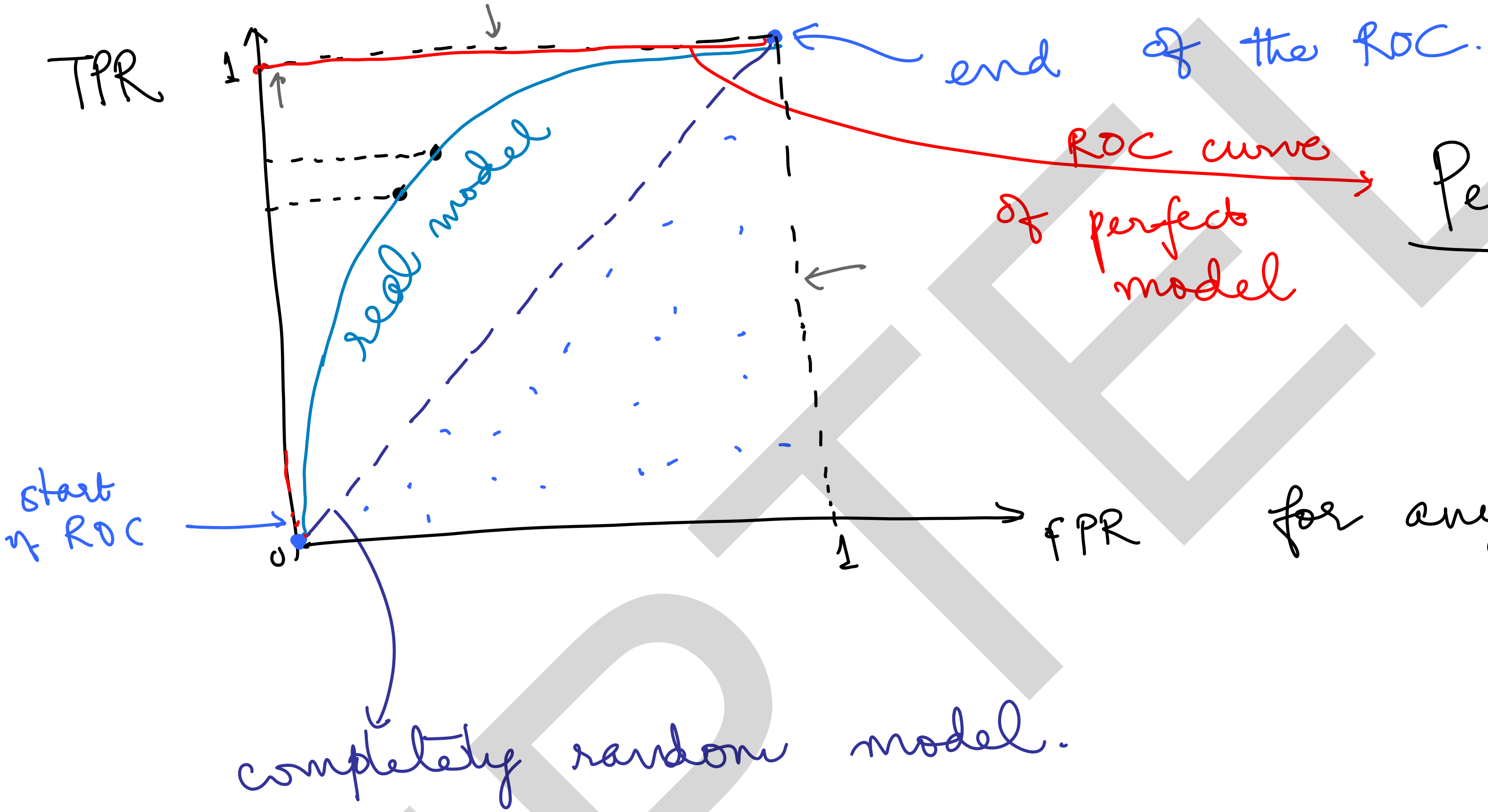
plotting TPR vs. FPR

$\Downarrow$

Receiver operating characteristic (ROC).

* When $\boxed{p^* = 1}$: TPR = 0 , FPR = 0. since no point will be classified as positive (both TP = FP = 0).

* When $p^* = 0$: TPR = 1 , FPR = 1, since all points will be classified as positive (both TN = FN = 0).



Perfect model:

$TPR = 1$

$FPR = 0$ for any threshold p^*

→ **For a real model:** The ROC can be used to choose p^* based on the tradeoff between TPR & FPR.

Area under the curve (AUC) for a receiver operating characteristic (ROC) is a threshold-independent metric to assess the quality of a binary classification model.

AUC-ROC for a perfect model = 1.

AUC-ROC for a completely random model = 0.5

Typically, a real model will have AUC ROC between 0.5 to 1, and the closer it is to 1, the better it is.

Multi-class classification $\longrightarrow$ Softmax regression or multinomial logistic regression.

Probabilities of the classes are evaluated using the softmax function:

$$\longrightarrow P(y=k \mid x) = \frac{e^{\beta_{0k} + \beta_k^T x}}{\sum_{j=1}^{K} e^{(\beta_{0k} + \beta_k^T x)}}$$

$0 \leq p \leq 1$

index for the classes

total number of classes

parameters of the softmax regression ML model.

The predicted value of $\hat{y}$ corresponds to the largest predicted $P(y=k|x)$, i.e.

$$\boxed{\hat{y}(x) = \underset{k}{\mathrm{argmax}}\ P(y=k|x)}$$

Here, the loss function used is the categorical cross-entropy loss:

$$\mathcal{L} = \sum_{i=1}^{n} \sum_{k=1}^{K} -\mathbb{I}(y_i=k) \log(P(y_i=k|x_i))$$

$\mathbb{I}(y_i=k)$ is 1 if $y_i=k$ and 0 otherwise.

Some example applications:

Mechanical engineering
- fault diagnosis in machinery
- failure detection in a component.

Civil engineering
- soil type classification
- structural health monitoring

Aerospace engineering
- Aircraft fault detection
- landing success classification

Chemical engineering
- Process anomaly detection
- Catalyst deactivation prediction

Industrial engineering
- Quality control check in a production line
- Predictive maintenance

Environmental engineering
- Water potability detection ←
- Air pollutant detection ←