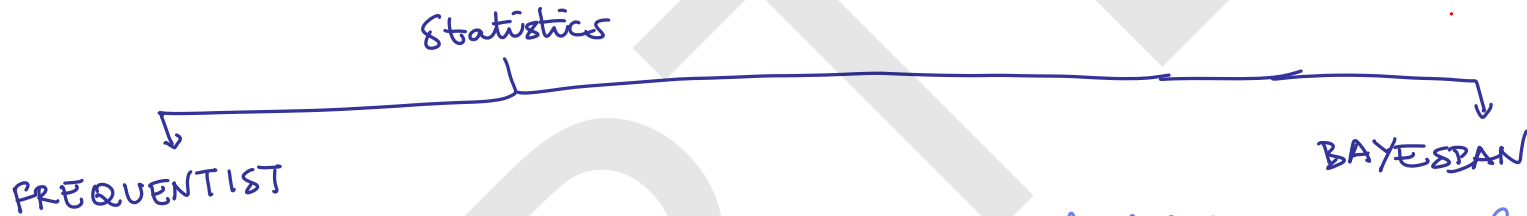
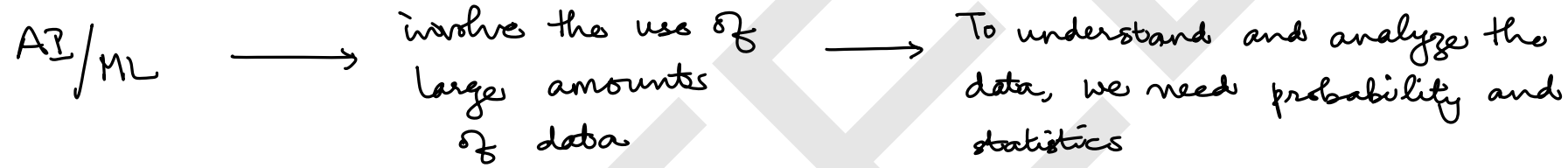


Probability and Statistics



- probabilities are evaluated based on data collected regarding a certain hypothesis or event
- no importance is given to other events which may or may not influence the event under consideration

- probabilities are evaluated from two perspectives - prior to knowing a piece of evidence and posterior to it.
- based on the celebrated Bayes' theorem in probability

Frequentist Statistics

Given an event A , the probability of the event occurring, $P(A)$, is determined by repeatedly conducting a relevant experiment, and noting down its outcome in each case.

$$P(A) = \frac{n(A)}{n(\Omega)}$$

\rightarrow # of times the event A occurred

\rightarrow the # of times the experiment was conducted

sample space
 \downarrow
set of possible outcomes

\Downarrow
size of the sample space.

Eg: A coin is tossed 50 times and a head is obtained 24 times.

$$P(\text{Heads}) = \frac{24}{50} = \frac{12}{25}$$

$$\text{Ideal } P(\text{Head}) = \frac{n(H)}{n(\Omega)} = \frac{1}{n(\{H, T\})} = \frac{1}{2}$$

Bayesian statistics

This form of statistics is based on the concept of conditional probability, which provides a way to account for probabilities before (prior to) and after (posterior to) knowing a piece of evidence

Given two events A and B, the conditional probability of event A occurring, provided event B has occurred is

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

Bayes' theorem

$P(A)$: PRIOR PROBABILITY, i.e. the probability of event A occurring, prior to knowing whether event B has occurred or not. Event A is treated as a proposition, i.e. a hypothesis whose likelihood is being estimated, given some EVIDENCE represented by event B.

$P(B)$: EVIDENTIAL PROBABILITY, i.e. the probability of the evidential event B.

$P(A|B)$: POSTERIOR PROBABILITY, i.e. the probability of the hypothesis

$P(B|A)$: LIKELIHOOD FUNCTION, ie how likely the evidence B is, given that proposition A has occurred.

$$\underset{\substack{\text{Posterior} \\ \text{probability}}}{P(A|B)} = \frac{\overset{\substack{\text{likelihood function}}}{P(B|A)} \cdot \overset{\substack{\text{prior probability}}}{P(A)}}{\underset{\substack{\text{evidential probability}}}{P(B)}}$$

$$\boxed{\text{POSTERIOR} = \frac{\text{LIKELIHOOD} \times \text{PRIOR}}{\text{EVIDENCE}}}$$

$$\begin{aligned} P(A \cap B) &= P(B \cap A) \\ P(A) \cdot P(B|A) &= P(B) \cdot P(A|B) \Rightarrow P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}. \end{aligned}$$

Application of Bayes' theorem to multiple events:

If there are several mutually exclusive and exhaustive events A_i that can occur following the evidential events B , then

$$P(B) = \sum_i P(A_i \cap B) = \sum_i P(A_i) \cdot P(B|A_i)$$

$$P(A_i|B) = \frac{P(A_i) \cdot P(B|A_i)}{P(B)} = \frac{P(A_i) \cdot P(B|A_i)}{\sum_i P(A_i) \cdot P(B|A_i)}$$

Bayes' theorem