Interim Report

On

Image Sharpening and De-noising using Inverse Diffusion Process

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by

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Chapter 1

Introduction

In this proposal we shall discuss about Anisotropic Inverse diffusion process for enhancement of images in context of sharpening and denoising the image. Before proceeding onto the topic let us first try to know, why we are going to use only this PDE based Inverse diffusion process, where ample number of methods are available for solving this proposed problem.

Image processing is one of the vast areas in which many problems regarding images, such as image enhancement, sharpening, denoising from deblurred images are found out using simple techniques from mathematics and more subjects. Image enhancement in the presence of noise using PDE methods is considered in the scale-space as one of the finest method. In order to smooth an image and to simultaneously enhance important features such as edges, we apply an anisotropic diffusion process (filter) whose diffusivity is given by derivatives of the non-linear diffusion equation (evolving image). These filters act locally like a backward (Inverse) diffusion process and results in enhancement of transients and singularities in the one-dimensional case and of edges in images, while locally denoising smoother segments of the image. In order to carry out the dissertation we shall require few basic concepts regarding the image processing which use PDE methods.

Chapter 2

Problem Definition

To find an image sharpening(enhancing image objects) method based on Diffusion model(i.e spread of matter from high concentation level to low concentration level), which considers local properties of the image, removes the noise in the image and thereby improving the visibility of the image objects leading to a most stable enhanced image.

Chapter 3

Progress So far in the Dissertation

3.0.0.0.1 The following material describes the way I had carried out the dissertation work so far in the semester. Firstly I tried knowing what the diffusion is all about and how the concept of diffusion can be related for image processing.Secondly, read about what a digital image is and what constitutes image processing.Thirdly, read about the various image noise.Then, read about the various De-noising techniques which are useful for image processing.Then, read about the Linear diffusion process, Scale-space concepts and Non-linear diffusion process.

3.1 Diffusion Process:

The process where by a concentration difference occur among high and low concentration area.The flow of concentration is called flux *'J.'* (concentration change in the direction of flow per unit volume) flux is given by

$$J = -D(\frac{dc}{dx}) \tag{3.1}$$

where
$\frac{dc}{dx}$:is concentration gradient per unit length.

D :Diffusion constant, if J is parallel to the gradient change- D is scalar

and known as isotropic coefficient(i.e uniform over the space) and if J is non parallel to the gradient change then D is space dependent and known as anisotropic coefficient.
Then net change in flux as a function of time is given by the equation,

$$\frac{\partial c}{\partial t} = \frac{[J(x) - J(x + \delta x)]}{dx} = -\frac{\partial J}{\partial x} \tag{3.2}$$

Thus Diffusion equation is given by,

$$\frac{\partial u}{\partial t} = D\Delta u \tag{3.3}$$

whereΔ: is Laplacian($\frac{d^2}{dx^2}$)

In image processing we deal with 'u' as gray scale value or intensity value at a point of the image. Remark:Differentiation is basically analogous to discretizations and in image processing many algorithms have been worked out for discretizing an image from a continous domain, so diffusion process which deals with differentiation comes in handy for carrying out various image related operations like enhancing,denoising or sharpening.

3.2 Digital image and its properties

An image can be defined as two-dimensional function,*f(x,y)* , where x,y are spatial coordinates and f is the amplitude(i.e brightness) of the image at the real coordinate position(x,y).An image consists of sub-images regerred to as regions of interest(ROI).The amplitude of a given image will be of integer numbers, so range of the amplitude runs from 0-255 which represent various shades of colours between white and black(e.g: 0-black,1-white) in computer representation.A digital image a[m,n] in discrete space is analogous to f(x,y) in a 2D continous space and a[m,n] is derived using Sampling process. The 2D continuous image a(x,y) is divided into N rows and M columns. The intersection of a row and a column is termed a pixel. The value assigned to the integer coordinates [m,n] with {m=0,1,2,,M-1} and {n=0,1,2,,N-1} is a[m,n].Digital image is composed of a finite number of elements,each of which has a particular position and value.Each of these elements are commonly referred to as *pixels*.

3.2.1 Types of Digital images:

Binary: In binary image the value of each pixel is either black or white. The image have only two possible values for each pixel either 0 or 1, we need one bit per pixel.
Grayscale: In grayscale image each pixel is shade of gray, which have value normally 0 [black] to 255 [white]. This means that each pixel in this image can be shown by eight bits, that is exactly of one byte. Other grayscale ranges can be used, but usually they are also power of 2. rue Color or RGB: Each pixel in the RGB image has a particular color; that color in the image is described by the quantity of red, green and blue value in image. If each of the components has a range from 0255, this means that this gives a total of 2563 different possible colors values. That means such an image is stack of three matrices; that represent the red, green and blue values in the image for each pixel. This way we can say that for every pixel in the RGB image there are corresponding 3 values.

3.2.2 Image Operations:

There are variety of ways to characterize image operations.This reason for this characterization depends on what type of results we might want to achieve with a given type of operation. The type of operations that can be applied to digital images to transform image a[m,n] into an output image b[m,n] are:
• Point operation: The output value at a specific coordinate is dependent only on the input value at the same coordinate.
• Local operation: The output the output value at a specific coordinate is dependent on the input values in the neighborhood of that same coordinate.
• Global operation: The output value at a specific coordinate is dependent on all the values in the input image.

3.2.3 Types of neighbourhoods:

Neighborhood operations play a key role in modern digital image processing. It is therefore important to understand how images can be sampled and

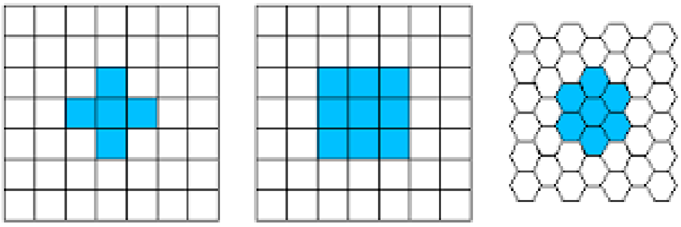

Figure 3.1: various image neighbourhood

how that relates to the various neighborhoods that can be used to process an image.

• Rectangular sampling: In most cases, images are sampled by laying a rectangular grid over an image.

• Hexagonal sampling : An alternative sampling scheme is shown in Figure 3c and is termed hexagonal sampling.

3.3 Image Noise:

In image processing, noise reduction and restoration of image is expected to improve the qualitative inspection of an image and the performance criteria of quantitative image analysis techniques Digital image is inclined to a variety of noise which affects the quality of image. The main purpose of de-noising the image is to restore the detail of original image as much as possible. The criteria of the noise removal problem depends on the noise type by which the image is corrupting. The main source of noise in digital images arises during image acquisition (digitization) or during image transmission.

3.3.1 Types of Noises:

- **Uniform Noise:** The uniform noise cause by quantizing the pixels of image to a number of distinct levels is known as quantization noise.In the uniform noise the level of the gray values of the noise are uniformly distributed across a specified range. Uniform noise can be used to generate any different type of noise distribution. This noise is often used to degrade images for the evaluation of image restoration algorithms.

This noise provides the most neutral or unbiased noise.

- **Gaussian Noise or Amplifier Noise:** This noise has a probability density function [pdf] of the normal distribution. It is also known as Gaussian distribution. It is a major part of the read noise of an image sensor that is of the constant level of noise in the dark areas of the image.

- Salt and Pepper Noise: The salt-and-pepper noise are also called shot noise, impulse noise or spike noise that is usually caused by faulty memory locations malfunctioning pixel elements in the camera sensors, or there can be timing errors in the process of digitization.This noise has only two values a and b, each has probability of 0.2.

- Spekle Noise: speckle noise is commonly found in synthetic aperture radar images, satellite images and medical images.Mostly referred to as noise in texture of the image.Speckle noise is a random and deterministic in an image. Speckle has negative impact on ultrasound imaging, Radical reduction in contrast resolution may be responsible for the poor effective resolution of ultrasound as compared to MRI.In case of medical literatures,speckle noise is modelled as

$$g(n, m) = f(n, m) * u(n, m) + \epsilon(n, m) \tag{3.4}$$

In the field of reducing the image noise several type of linear and non linear filtering techniques have been proposed.Different approaches for reduction of noise and image enhancement have been considered, each of which has their own limitation and advantages.

This brings us to different de-noising techniques:

3.4 De-noising techniques:

The fundamental operations for image processing are divided into four categories:1.Image Histogram 2.Simple mathematics 3.Convolution 4.Mathematical Morphology

Image histogram: These are an important class of point operations.These operations include contrast stretching(i.e manipulations of brightness values.) and equalization(i.e correspond to a brightness distribution where all values are equally probable.)

Mathematics-based operations: These include simple basic mathematical operations like Not, Or, And, Xor, Sub and Add. The gray-value point operations that form the basis for image processing are based on ordinary mathematics and include: Log, Mul, Div, Exp, Sqrt, Trig, invert(i.e $c = (2^B - 1) - a$)

Convolution-based operations: Convolution filtering is used to modify spatial frequency characteristics of an image.It works by determining the valued of central pixel by adding the weighted values of all its neighbours together, giving a new modified filtered image.It is basically an averaging filter.The basic idea is that a window of some finite size and shapethe supportis scanned across the image. The output pixel value is the weighted sum of the input pixels within the window where the weights are the values of the filter assigned to every pixel of the window itself. The window with its weights is called the convolution kernel.

Smoothing operations: These are applied to reduce noise and prepare images for futher processing such as segmentation.

Derivative-based operations: Smoothing is a fundamental operation in image processing so is the ability to take one or more spatial derivatives of the image.The fundamental problem is that, according to mathematicval definition of a derivative, this cannot be done.A digitized image is not a continous function a(x,y) of the spatial variables but rather a discrete function a[m,n] of the integer spatial coordinates, as discussed in the beginning of introduction.In this case we can only find the approximations to the spatial derivatives of the original spatial continuous image.e.g:First order derivatives and Second order derivatives. In later part of interim report we shall see how we use derivative based operations for carrying out the proposed work

Morphology-based operations: Image can be alternatively defined based on the notion that image consists of a set of either continuous or discrete coordinates.i.e the set corresponds to the points or pixels that belong to the objects in the image.These operations include *union, intersection* $\bigcup, \bigcap$, *complement and translation.*

3.5 Denoising and Sharpening methods in detail:

In this topic I would like to describe why Partial differential equations are considered for image processing and what are the methods which I used for carrying out the process.
In any image denoising algorithm it is very important that the denoising process has no blurring effect on the image, and makes no changes or relocation to image edges. Before going on to the discussion of methods, i would like to tell about most important topic of image processing *Scale spaces.*

3.5.0.0.2 Scale-space: Scale-space theory is a framework of multiscale image/signal representation, in other words it is a representation of multiple scales simultaneously. Scale-space is a process of representing image as a one-parameter family of smoothed images.The parameter t is referred to as the scale parameter.The notion of scale is an essential part of early visual processing, where the main task is to separate the image into relevant and irrelevant parts. main type of scale space is the linear(Gaussian) scale space, which can be used as a basis for expressing a large class of visual opearations which are invariant for visual information. This processes the qualitative description of image in terms of its local extrema and its derivatives and intervals bounded by extrema.The main reason for scale space is the difficulty of finding the image information which change enormously with size and extent of image, so with every value of scale parameter we get a different information of the image.And computing descriptions at multiple scales just leads to the problem of increasing the volume of data. So Scale space is found out to represent the image information in an organized manner i.e in a way relating one scale to another.

Definition:

For a given image $f(x, y)$, its linear (Gaussian) scale-space representation is a family of derived signals $L(x, y; t)$ defined by the convolution of $f(x, y)$ with the two-dimensional Gaussian kernel

$$g(x, y, t) = \frac{\exp^{-}((x^2)/2t)}{2\pi t} \tag{3.5}$$

such that $L(x, y; t)$=$g(x, y; t) * f(x, y)$

The scale parameter t=σ^2 is the variance of the Gaussian filter and as a limit for $t = 0$ the filter g becomes an impulse funciton such that $L(x, y; 0) = f(x, y)$.As t incresed, L is the result of smoothing f with a larger filter with more details of the image getting removed. The gaussian convolution of a signal depends both on space parameter and on the σ standard deviation.At any value of σ the extrema in that particular resolution is given by $L_{xx} =$ 0,(inflection point) a double derivative in 1-D case. The multiscale images must satisfy:

- Casuality: No unwanted image details should be created while passing from finer to coarser scales.
- Localization: At each resolution, the region boundaries should be sharp and coincide with the semantically meaningful boundaries at that resolution.
- Piecewise Smoothing:At all scales, intraregion smoothing should occur preferentially over interregion smoothing.

So in order to overcome localization problem(which includes distortion and dislocation of image details.) we must track coarse extrema to their fine-scale locations, a coarse scale may be used to identify extrema, and a fine scale, to localize them.Therefore reduces to an (z, σ) pair, specifying its fine-scale location on the z-axis, and the coarsest scale at which the contour appears.
Now we shall reduce the scale space image to a simple tree describing the qualitative structure of the signal over all scales.So, for carrying out this process we consider an interval I which is bounded by the extrema of the smoothed signal(obtained by the inflection points), each interval is divided into sub intervals based on the singular point of the signal between the two extremas. Each interval corresponds to a node in the tree and parent node represents the larger interval from which I and offspring represent the smaller intervals into which I subdivides. Each interval also defines a rectangle in scale-space, denoting its location and extent on the scale dimension.
Thus we obtained a stable one-parameter structure for representing images which vary consistently on varying the scale parameter.
Now its time to venture into the De-noising methods.

3.6 Linear diffusion filtering:

The linear diffusion (heat) equation is the best investigated PDE method in image processing. Let $f(x)$ denote a grayscale(noisy) input image and $u(x, t)$ be initialized with $u(x, 0) = f(x)$ Then, the linear diffusion process can be defined by the equation

$$\frac{\partial u}{\partial t} = \nabla \cdot (\nabla u) = \Delta u \tag{3.6}$$

where $\nabla \cdot$ denote the divergence operator(amount of change in concentration per unit volume).Thus the equation turns out to be of form,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \tag{3.7}$$

in two dimensional case.The diffusion process can be seen as an evolution process with an artificial time variable t denoting the diffusion time where the input image is smoothed at a constant rate in all directions. Starting from the initial image $u^0(x)$, the evolving images $u(x,t)$ under the governed equation represent the successively smoothed versions of the initial input image $f(x)$, and thus create a scale space representation of the given image f, with $t > 0$ being the scale. The solution to the linear equation is given by,

$$u(x,T) = (G_(\sqrt{2}T) * f)(x) \tag{3.8}$$

with

$$G_\sigma = \frac{\exp(-\frac{|x|^2}{2\sigma^2})}{2\pi\sigma^2} \tag{3.9}$$

3.6.1 Numerical implementaiton:

Since we involve digital images, we discretize the problem in both spatial and time coordinates.Using Finite differences methods we shall represent the spatial derivatives in the following manner.

$$\frac{d^2u_{i,j}}{dx^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_x}, \frac{d^2u_{i,j}}{dy^2} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h_y} \tag{3.10}$$

where $u_{i,j}$ denotes the gray value or the brightness of the evolving image at pixel location(i,j) values of h_x and h_y can be set to 1.
Then to solve (3.7), we shall use the formula

$$\frac{u_{i,j}^{k+1} - u_{i,j}^k}{\Delta t} = u_{i+1,j}^k + u_{i-1,j}^k + u_{i,j+1}^k + u_{i,j-1}^k - 4u_{i,j}^k \tag{3.11}$$

where Δt is the time step, and u^k represents the restored image u at iteration k. And Stable condition for the discrete scheme requires Δt ≤ 0.25

3.6.2 Relation Between Variational Regularization and Diffusion Equations

Now we have found the approximation of the de-noised image.We shall check the a closer approximation to the original image.i.e $f(x) = u(x) + n(x)$ where $n(x)$ is the additive noise. The simple regularization is given by the Tikhonov energy funtional which is,

$$E(u) = \int ((u - f)^2 + \alpha \mod (\nabla u)^2) dx \tag{3.12}$$

where

- $\Omega \subset R^2$ is connected, bounded, open subset representing the image domain,
- f is an image defined on Ω,
- u is the smooth approximation of f ,
- $\alpha > 0$ is the scale parameter.

The first term in $E(u)$ penalizes the deviations between u(initial image) and f , and thus forces the restored image to be close to the original image. The second term is called the *regularization* or smoothness term which penalizes the high gradients, and gives preference to smooth approximations.
The main drawback of linear diffusion filtering is that
• the smoothing process does not consider information regarding important image features such as edges. It applies same amount of smoothing at every image location(irrespective of the image features). As a result, the diffusion process does smooth not only noise, but also image edges and thus causing difficulty in finding the edges.
• Linear diffusion filtering dislocates edges when moving from finer to coarser scales.So structures which are identified at a coarse scale do not give the right location and have to be traced back to the original image.So in this case we require the use of scale-space.

So this brings us to the main important de-noising method *Non-Linear Diffusion.*

3.7 Non-Linear Diffusion filter

We shall now study the smoothing methods which depend on the local properties of the image(commonly known as Adaptive method).The Pde formulation of this Non-linear diffusion method is given by the Perona and Malik which uses a diffusion coefficient dependent on image space rather than a scalar diffusivity. Consider the anisotropic diffusion equation,

$$I_t = \nabla(c(x,y,t)\nabla I) = c(x,y,t)\Delta I + \nabla c \cdot \nabla I \tag{3.13}$$

If c is a constant (3.13) reduces to isotropic diffusion equaiton. For a simple case, Perona and Malik suggested that we would want to invovle smoothing within a region in preference to smoothing across the boundaries.This can be achieved by setting the conduction coefficient to be 1 in the interior of each region and 0 at he boundaries. Then blurring would take place separately in each region with no interactions between regions.Since we does not know the exact locations of edges in various smoothing levels, we shall find an estimate of a vector valued function($E(x,y,t)$) defined on the image with values: $E(x,y,t)=\begin{cases} 0 & \text{interior of each region} \\ \text{Ke(x,y,t)} & \text{at each edge point} \end{cases}$

For an estimate $E(x,y,t)$ the conduction coefficient can be chosen as c=$g(\parallel E \parallel)$ such that $g(0) = 1$.In this way diffusion process takes place in the interior regions, and will not affect the region boundaries where the gradient change is large.Thus best choice of the conduction coefficient is chosen to be

$$c = g(\parallel \nabla I(x,y,t) \parallel) \tag{3.14}$$

So the non-linear diffusion equation turns out to be,

$$\partial_t I = \nabla \cdot ((g|\nabla I|^2)\nabla I) \tag{3.15}$$

it use diffusivities such as:

$$g(\nabla I) = \begin{cases} \exp -(\frac{\|\nabla I\|}{K^2}) \\ \frac{1}{1+(\frac{\|\nabla I\|}{K})^2} \end{cases} \tag{3.16}$$

Since this non-linear diffusion is ill posed problem, we shall discretize the problem similar to linear diffusion process.Then (3.15) can be discretized on a square lattice, with brightness values associated to the vertices, and conduction coefficients to the arcs.A 4-nearest neighbours discretization of the Laplacian operator can be used:

$$I_{i,j}^{t+1} = I_{i,j}^{t} + \lambda[c_N \cdot \bigtriangledown_N I + c_S \cdot \bigtriangledown_S I + c_W \cdot \bigtriangledown_W I + c_E \cdot \bigtriangledown_E I] \tag{3.17}$$

where $0<\lambda<0.25$ for the problem to be stable, and the subscripts are the surrounding neighbours of a central pixel, $\bigtriangledown$ operator represents the nearest-neighbour differences. Such as if we consider a pixel at (i, j) location, then the differences are given to be:

$$\bigtriangledown_N I_{i,j} = I_{i-1,j} - I_{i,j}, \tag{3.18}$$

$$\bigtriangledown_S I_{i,j} = I_{i+1,j} - I_{i,j}, \tag{3.18}$$

$$\bigtriangledown_E I_{i,j} = I_{i,j+1} - I_{i,j}, \tag{3.18}$$

$$\bigtriangledown_W I_{i,j} = I_{i,j-1} - I_{i,j}, \tag{3.18}$$

(3.18)

The conduction coefficients are calculated using equation (3.14) and (3.16) respectively. And using the functions defined in (16) gives scale spaces with privileges of high contrast edges over low contrast ones, wide regions over smaller regions.The above mentioned Non-linear problem is semi-discretized problem having scale-parameter and time as continous parameters, whereas discrete in space parameters like coordinates axis.This Non-linear diffusion model is also known as *Forward and Backward diffusion process.* depending on the"K' parameter in the flux function i.e

$$\phi(\nabla I)=g(\nabla I)\cdot\nabla I;\ "K"\ \text{a parameter of } g(\nabla I) \text{ in } (3.16)$$

The flux function satisfies $\phi\prime(\nabla I) \geq 0$ for $| \nabla$ I$| \leq$ K for this inequality the PM filter is forward parabolic type i.e filter encourages diffusion process which is forward process in the regions of small gradient less than K. And the filter is backward(i.e discourages diffusion process in the regions of high gradient value i.e grater than K.) "K" is the contrast parameter which has to dependent on the problem, usually set default by the problem solver. Thus a non-linear diffusion process which keeps region boundaries at each resolution sharp and coincides with semantically meaningful boundaries at that resolution, and as well as promotes intra-region smoothing over inter-region smoothing.
The regularized model for PM diffusion problem is given by Catte, Lions, Morel and Coll, they replaced the diffusivity g($| \nabla$ I $|$) of the PeronaMalik model by a Gaussian-smoothed version g($| \nabla I_\sigma |$) where I_σ:=$G_\sigma*$I. Though we use a special diffusivity coefficient it is still a scalar valued diffusivity and not a diffusion tensor(component of scalar and direction(vectors)). So, we shall know about the anisotropic diffusion equation of PM method.

3.7.1 Anisotropic regularization of the Perona-Malik process:

In the interior of a segment the nonlinear isotropic diffusion equation (3.15) behaves almost like the linear diffusion filter, but at edges diffusion is discouraged. Therefore, noise at edges cannot be eliminated successfully by this process. To overcome this problem, a desirable method should prefer diffusion along edges to diffusion perpendicular to them.Anisotropic models do not only take into account the modulus of the edge detector ∇I_σ , but also its direction(orientation). To this end, we construct the orthonormal system of eigenvectors $v1 \parallel$,and $v2 \perp$ to ∇I_σ of the diffusion tensor D such that they reflect the estimated edge structure.

In order to prefer smoothing along the edge *Weickert* proposed to choose the corresponding eigenvalues λ_1 and λ_2 as

$$\lambda_1(\nabla I_\sigma) = g(|\nabla I_\sigma|^2), \tag{3.19}$$
$$\lambda_2(\nabla I_\sigma) = 1. \tag{3.19}$$

(3.19)

3.7.2 Inverse diffusion:

This linear backward (inverse) diffusion is analogous to a Gaussian deconvolution, where the noise amplification explodes with frequency. Problems associated with backward diffusion process are explosive instability, noise amplification and oscillations.

To overcome these shortcomings, first is to reduce explosion, for this we need to diminish the value of inverse diffusion coefficient at high gradients. To reduce amplification of noise, we need to use low gradients. Finally to reduce oscillations, we should combine a forward diffusion force, that smoothes low gradients. These can be solved just by using the PM conductance coefficient which was explained in the section (3.7.1),

$$c_{P-M}(s) = 1/(1 + (s/k)^2) \tag{3.20}$$

So, we have found the basic inverse problem which is a solution to the problem defined in the chapter (2).

Chapter 4

Literature Review

The basic definitions of diffusion and its problems relating to the diffusion equation are read from the Anisotropic Diffusion in Image Processing by Joachim Weickert. The author gives an overview of the requirement of PDE-based methods for image enhancement and smoothing, and discusses of the various types of diffusion equations, such as of linear diffusion process, requirement of Gaussian scale-space and its advantages in overcoming the ill-posedness of the non-linear anisotropic diffusion equation. He as put emphasis on the unified description of the underlying ideas, theoretical results, numerical approximations, generalizations and applications. And he also presented an indepth treatment of an interesting class of parabolic equations which bridge the gap between scale-space and restorations ideas: such as nonlinear diffusion filters by Perona Malik models, have been discussed thoroughly.[1]

The introduction to basic concepts and methodologies for digital image processing are well presented in [2].This main text in its first chapter describes the basic definitions of image, their generation, its properties and current areas that use digital image processing.In the second chapter the author describes the process of sampling(convertion of continous images into discrete image)

, aliasing, image zooming and shrinking.Third chapter presents the image transforms, spatial methods for image processing(eg convolution) and frequency domain, usage of first derivatives for enhancement-The gradient.[2]

Digital image is inclined to a variety of noise which affects the quality of image. The main purpose of de-noising the image is to restore the detail of original image as much as possible. The criteria of the noise removal problem depends on the noise type by which the image is corrupting.The various types of image noise are described in this material.[3][4]

The linear diffusion method for image processing is the fundamental topic for the study of image processing based on partial differential equations.The author described the various advantages, disadvantages of linear diffusion and other regularization methods[5]

A fundamental problem in computing such descriptions is scale: a derivative must be taken over some neighborhood, but there is seldom a principled basis for choosing its size. Scale-space filtering is a method that describes signals qualitatively, managing the ambiguity of scale in an organized and natural way. The signal is first expanded by convolution with gaussian masks over a continuum of sizes. This "scale-space" image is then collapsed, using its qualitative structure, into a tree providing a concise but complete qualitative description covering all scales of observation.[6]

The work regarding the Perona-Malik equation in detail, and its essential role in image processing is described. The first part gives a survey of results on existance, uniqueness and stability of solutions, the second part introduces discretisations of equation and deals with an analysis of discrete problem.[7][8]

The basic concepts of convolution and its detailed working are briefly explained in this text[9.]

This paper devoloped and analyzed a robust and fast image segmentation algorithm which is pervasive , robust to large amplitude noise. And discussed its application in synthetic apeture radar segmentation for removal of speckle noise which is a well known problem that has defeated many algorithms.[10]

Reducing noise from the medical images, a satellite image etc. is a challenge for the researchers in digital image processing. Several approaches are there for noise reduction. Generally speckle noise is commonly found in synthetic aperture radar images, satellite images and medical images. This paper proposes filtering techniques for the removal of speckle noise from the digital images. Quantitative measures are done by using signal to noise ration and noise level is measured by the standard deviation.[11]

4.1 References

1 Anisotropic Diffusion in Image Processing-Joachim Weickert

2 Digital Image Processing - Rafael C.Gonzalez, Richard E. Woods

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8 Perona-Malik equation and its numerical properties -Maciej Wielgus

9 Image Convolution - Jamie Ludwig, Satellite Digital Image Analysis, 581 Portland State University.

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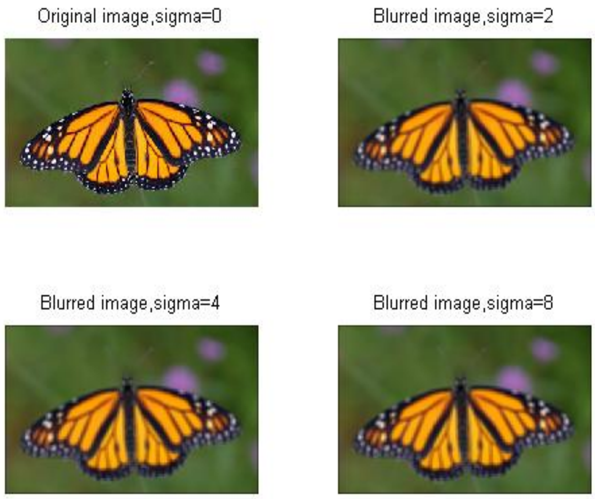
11 Inverse Problems In Image Processing-Ramesh Neelamani

Chapter 5

Implementation results

5.1 Linear Diffusion process

In this section the details of the simple linear diffusion method applied to the image in matlab is described.
Firstly I converted the image into a discretized image of [m,n] matrix of size equal to the image.Let us consider a 2-D structure. Then the discretization in x coordinate of image I can be written as $I_{xx} = [I(:, 2 : n, :)I(:, n, :)] - 2 * I + [I(:, 1, :)I(:, 1 : n - 1, :)]$ similar to the differential equation of double derivative. And the discretization with respect to y coordinate is $I_{yy} = [I(2 : m, :, :); I(m, :, :)] - 2 * I + [I(1, :, :); I(1 : m - 1, :, :)]$
And then using the equation (3.7) we use the I_{xx} and I_{yy} to compute the diffused image $\hat{I}$=I + $dt \cdot (I_{xx} + I_{yy})$ where dt is the time-steps(i.e order of 0.2 or 0.5 etc)
The results are as follows:



5.2 Planned Methodology

To enhance the PM filter and work more on the backward(inverse) diffusion method for images and implementing the methods described for carrying out this inverse diffusion problem.