

$$(N2) \quad g \sim N(a, b^2)$$

$$f_g(x) = \frac{1}{\sqrt{2\pi}b^2} \exp \left\{ -\frac{(x-a)^2}{2b^2} \right\}$$

$$\mathbb{E}g = a$$

$$\mathbb{E}g^2 = \int_{-\infty}^{+\infty} t^2 \frac{1}{\sqrt{2\pi}b^2} \exp \left\{ -\frac{(t-a)^2}{2b^2} \right\} dt =$$

$$= \frac{1}{\sqrt{2\pi}b^2} \int_{-\infty}^{+\infty} (t-a)^2 dt$$

$$= \frac{1}{\sqrt{2\pi}b^2} \left(\int_{-\infty}^{+\infty} (t-a)^2 \exp \left\{ -\frac{(t-a)^2}{2b^2} \right\} dt \right)$$

$$+ 2a \int_{-\infty}^{+\infty} t \exp \left\{ -\frac{(t-a)^2}{2b^2} \right\} dt - a^2 \int_{-\infty}^{+\infty} \exp \left\{ -\frac{(t-a)^2}{2b^2} \right\} dt$$

$$= \left\{ \begin{array}{l} y = \frac{t-a}{b} \\ dy = \frac{dt}{b} \end{array} \quad t = by + a \right\} = \frac{1}{\sqrt{2\pi}b^2} \left(\int_{-\infty}^{+\infty} b^3 y^2 e^{-\frac{y^2}{2}} dy + \right.$$

$$+ 2a^2 b - a^2 b \int_{-\infty}^{+\infty} e^{-\frac{y^2}{2}} dy \Big) = \frac{b^2}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} y^2 e^{-\frac{y^2}{2}} dy +$$

$$+ 2a^2 - a^2 = \frac{b^2}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} y^2 e^{-\frac{y^2}{2}} dy =$$

$$= \frac{b^2}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} y^2 e^{-\frac{y^2}{2}} dy + a^2 = \frac{b^2}{\sqrt{2\pi}} \left(\frac{\sqrt{\pi}}{\sqrt{2}} \phi \left(\frac{y}{\sqrt{2}} \right) - \right.$$

$$\left. - y e^{-\frac{y^2}{2}} \right) \Big|_{-\infty}^{+\infty} + a^2 = \frac{b^2}{\sqrt{2\pi}} \left(\frac{\sqrt{\pi}}{\sqrt{2}} \phi(+\infty) - \frac{\sqrt{\pi}}{\sqrt{2}} \phi(-\infty) \right) + a^2$$

$$= a^2 = \frac{b^2}{\sqrt{2\pi}} \cdot \sqrt{2\pi} + a^2 = b^2 + a^2$$

$$\mathbb{D}g = \mathbb{E}g^2 - (\mathbb{E}g)^2 = b^2$$

Answer: $\mathbb{D}g = b^2$