№6 $\xi \sim N(a, \sigma^2)$

$$f_{\xi_1}(x) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma_1^2}} \qquad f_{\xi_2}(x) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{(x-b)^2}{2\sigma_2^2}}$$

$$\cancel{f_{\xi_1+\xi_2}(x) = \int_{-\infty}^{\infty} f_\xi(x-u) f_\xi(u)\,du = \frac{1}{2\pi\sigma^2}\int_{-\infty}^{\infty} e^{-\frac{(x-u-a)^2}{2\sigma^2}} \cdot e^{-\frac{(u-a)^2}{2\sigma^2}}\,du = \frac{1}{2\pi\sigma^2}\int_{-\infty}^{\infty} e^{-\frac{x^2-2ux+u^2-2xa+2ua+a^2}{2\sigma^2} - \frac{u^2-2ua+a^2}{2\sigma^2}}\,du = \frac{1}{2\pi\sigma^2}\int_{-\infty}^{\infty} e^{-\frac{x^2-2ux-2xa+4ua+2u^2+2a^2}{2\sigma^2}}\,du}$$

$$f_{\xi_1+\xi_2}(x) = \frac{\cancel{\sigma_2+\sigma_1}\ 1}{\sigma_1\sigma_2\sqrt{2\pi}^2}\int_{-\infty}^{\infty} e^{-\frac{(x-u-a)^2}{2\sigma_1}} e^{-\frac{(u-b)^2}{2\sigma_2}}\,du =$$

$$= \underbrace{\frac{\cancel{\sigma_1+\sigma_2}\ 1}{\sigma_1\sigma_2\sqrt{2\pi}^2}}_{C}\int_{-\infty}^{\infty} e^{\left(\sqrt{-\frac{1}{2\sigma_2^2}-\frac{1}{2\sigma_1^2}}\,u + \frac{\frac{x-a}{\sigma_1^2}+\frac{b}{\sigma_2^2}}{2\sqrt{-\frac{1}{2\sigma_2^2}-\frac{1}{2\sigma_1^2}}}\right)^2 - \frac{\left(\frac{x-a}{\sigma_1^2}+\frac{b}{\sigma_2^2}\right)^2}{4\left(-\frac{1}{2\sigma_2^2}-\frac{1}{2\sigma_1^2}\right)} - \frac{(x-a)^2}{2\sigma_1^2} - \frac{b^2}{2\sigma_2^2}}\,du = \left\{ y = \frac{\sigma_1^2\left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_1^2}\right)\sigma_2^2 u + \sigma_2^2(a-x) - b\sigma_1^2}{\sqrt{2}\,\sigma_1^2\sigma_2^2\sqrt{\frac{1}{\sigma_2^2}+\frac{1}{\sigma_1^2}}} \right\} =$$

$$= \cancel{\sqrt{\pi}\sigma_1\sigma_2}\, C \cdot \frac{\sqrt{\pi}\,\sigma_1\sigma_2}{\sqrt{2}\sqrt{\sigma_2^2+\sigma_1^2}}\, e^{\frac{\left(\frac{x-a}{\sigma_1^2}+\frac{b}{\sigma_2^2}\right)^2}{2\left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_1^2}\right)} - \frac{(x-a)^2}{2\sigma_1^2} - \frac{b^2}{2\sigma_2^2}} \cdot \int_{-\infty}^{\infty} \frac{2e^{-y^2}}{\sqrt{\pi}}\,dy = \frac{\cancel{\sigma_1+\sigma_2}\ 1}{\sigma_1\sigma_2\sqrt{2\pi}^2}\,\frac{\sqrt{2\pi}}{\sqrt{\sigma_2^2+\sigma_1^2}} \cdot \sigma_1\sigma_2 \cdot e^{-\frac{(x-a-b)^2}{2\sigma_1^2+2\sigma_2^2}} =$$

$$\Leftrightarrow \frac{1}{\sqrt{2\pi}\sqrt{\sigma_2^2+\sigma_1^2}} e^{-\frac{(x-a-b)^2}{2\sigma_1^2+2\sigma_2^2}}$$

$$\Downarrow$$

f_g $\quad g_1+g_2 \sim \mathcal{N}(a+b,\ \sigma_1^2+\sigma_2^2)$