

(NT) $\text{Bi}(n, p)$

$$P(Y=k) = C_n^k p^k (1-p)^{n-k}$$

$$\begin{aligned} EY &= \sum_{k=0}^n k P(Y=k) = \sum_{k=0}^n k C_n^k p^k (1-p)^{n-k} = \\ &= \sum_{k=0}^n \frac{n!}{k!(n-k)!} \cdot k \cdot p^k (1-p)^{n-k} = \sum_{k=0}^n \frac{n!}{(k-1)!(n-k)!} \\ &\cdot p^k (1-p)^{n-k} = \sum_{k=0}^n \frac{n(n-1)!}{(k-1)!(n-k)!} p^k (1-p)^{n-k} = \\ &= \sum_{k=1}^n n C_{n-1}^{k-1} p \cdot p^{k-1} (1-p)^{n-k} = np \underbrace{\sum_{k=1}^n C_{n-1}^{k-1} p^{k-1} (1-p)^{n-k}}_{=1} = \\ &= np \end{aligned}$$

$$EY = np$$

$$EY^2 = \sum_{k=0}^n k^2 C_n^k p^k (1-p)^{n-k}$$

$$EY^2 = EY(Y-1) + EY$$

$$\begin{aligned} EY(Y-1) &= \sum_{k=1}^n k(k-1) C_n^k p^k (1-p)^{n-k} = \\ &= \sum_{k=1}^n \frac{n! k(k-1)}{k!(n-k)!} \cdot p^k (1-p)^{n-k} = \sum_{k=1}^n \frac{n! p^k (1-p)^{n-k}}{(k-2)!(n-k)!} = \\ &= \sum_{k=2}^n n \cdot p^2 \frac{(n-2)!}{(k-2)!(n-k)!} p^{k-2} (1-p)^{n-k} = n^2 p^2 - np^2 \end{aligned}$$

$$\begin{aligned} DY &= EY^2 - (EY)^2 = EY(Y-1) + EY - (EY)^2 = \\ &= n^2 p^2 - np^2 + np - n^2 p^2 = np(1-p) \end{aligned}$$

Answer: $EY = np$

$$DY = np(1-p)$$