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3D Point Clouds

Lecture 8 – Feature Description

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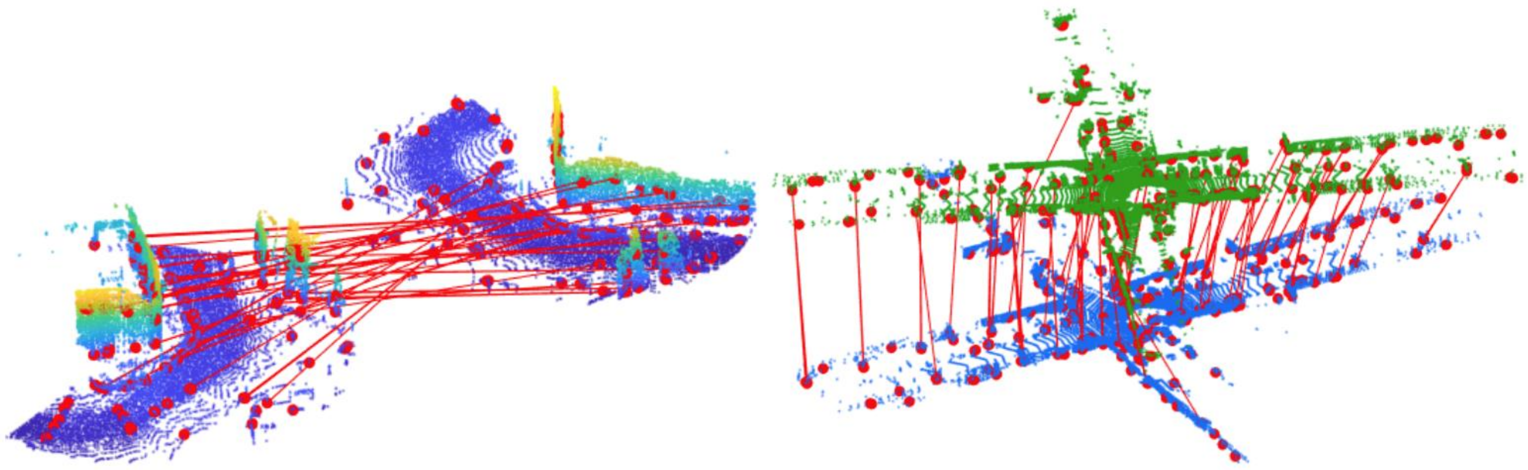
1. Introduction

2. Classical – PFH, FPFH, SHOT

3. Modern – Deep Learning Methods

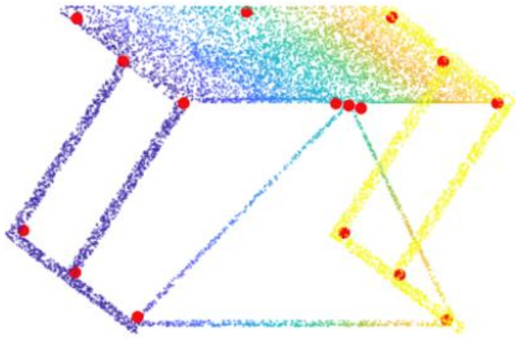
Features

- **Detection**: Identity the keypoints
- **Description**: Extract a vector around the keypoint to descript it.
- **Matching**: Determine correspondence between descriptors

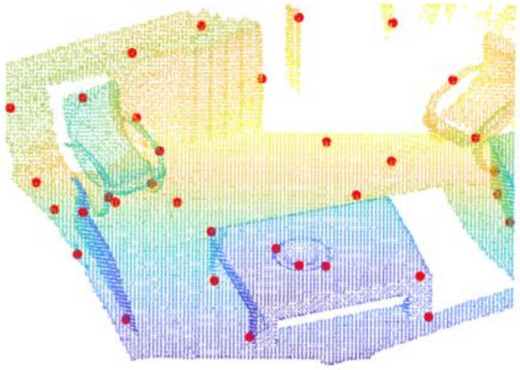


Point Cloud Keypoints Summary

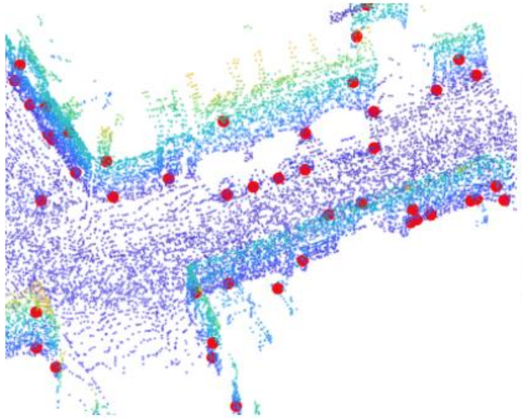
- Handcrafted
  - Harris family
    - Harris 3D with/without intensity
    - Harris 6D with intensity
  - ISS
- Deep learning
  - USIP



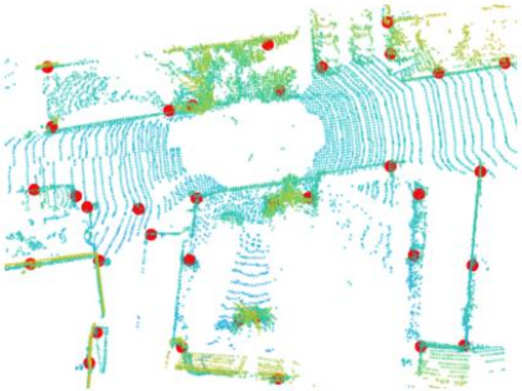
CAD Model



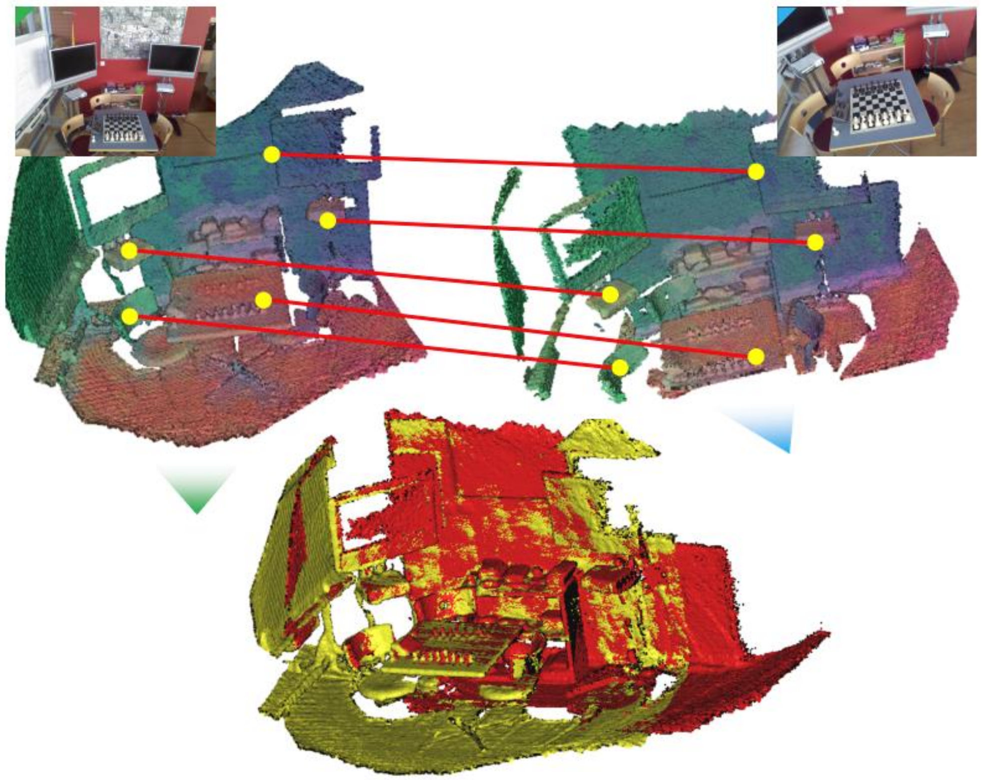
RGBD



Oxford Robotcar



KITTI

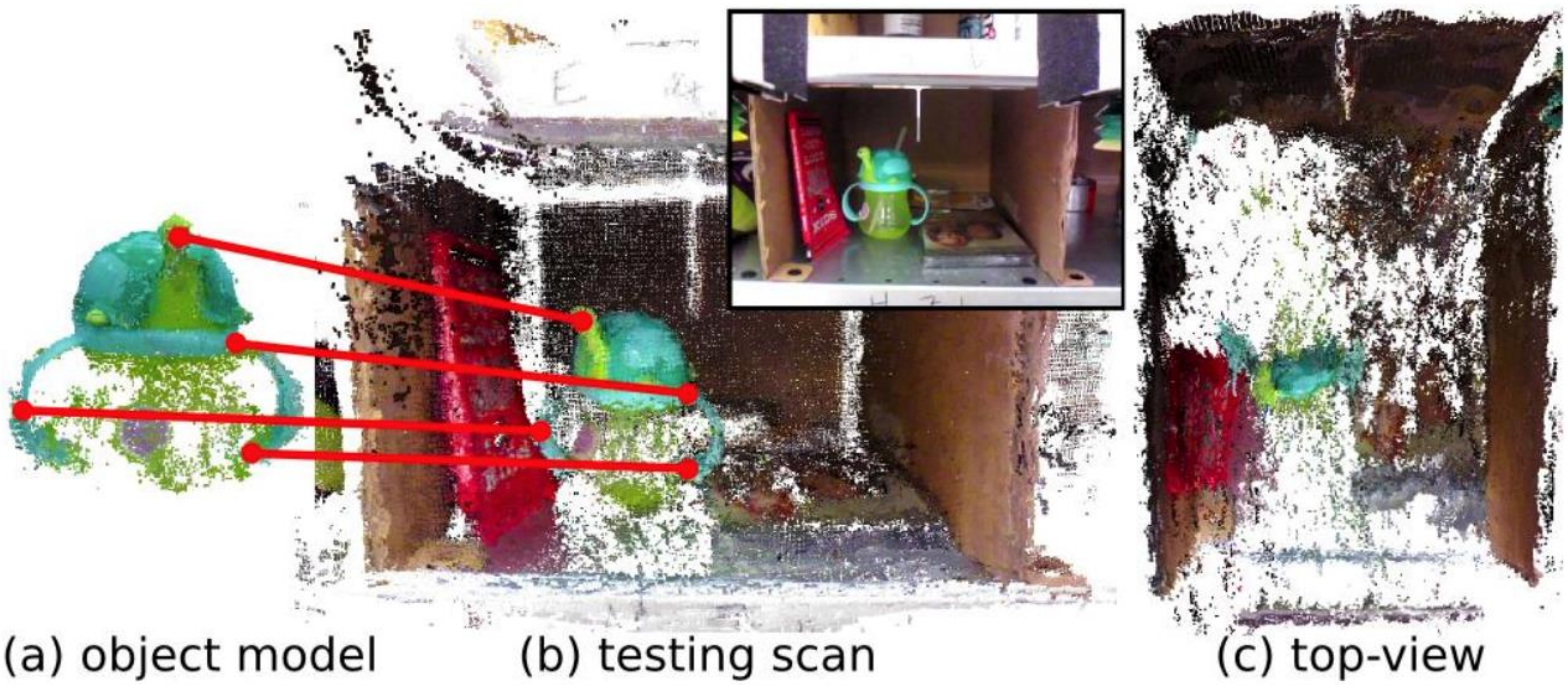


Registration

- Find a transform to align two point clouds
- A transform consists of
  - Rotation R
  - Translation t
- Method 1 - Iterative Closest Point (ICP)?
  - ICP requires proper initial guess
  - Low overlapping ratio
- Method 2 – Detect and match features
  - No initialization required
  - Works for low overlapping ratio

Source: PPFNet: Global Context Aware Local Features for Robust 3D Point Matching

Point Cloud Features – Object 6D Pose

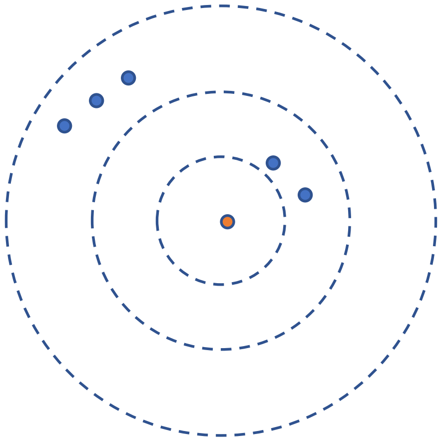


- In fact, it is still registration

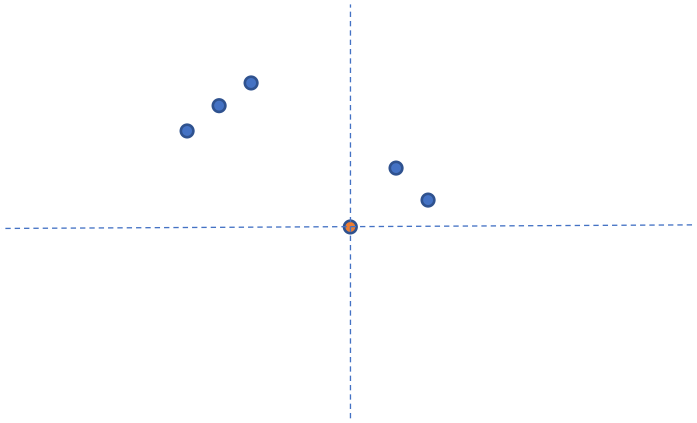
Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions

Point Cloud Feature Descriptor - Handcrafted

- Histogram based
  - Encode local geometric variations, put them into a histogram
  - Methods:
    - Point Feature Histogram (PFH)
    - Fast Point Feature Histogram (FPFH)
- Signature based
  - Compute geometric measures based on local reference frame
  - Methods
    - Structure Indexing
    - Signature of Histograms of OrienTations (SHOT)



0	2	3



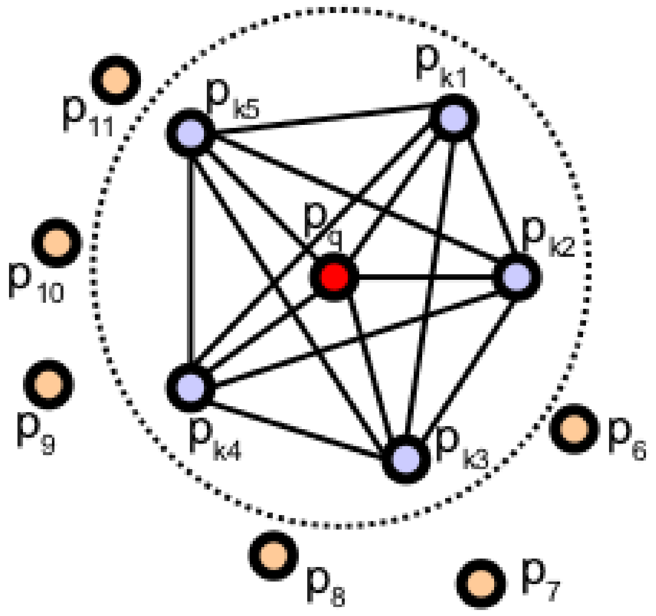
Avg(Quadrant 1)	Avg(Quadrant 2)	Avg(Quadrant 3)	Avg(Quadrant 4)

PFH – Point Feature Histogram

- Input
  - Point coordinates of the point cloud
  - Surface normal vectors of the point cloud
  - A query point
- Output
  - Array of length B^3
    - B is the number of bins on each histogram, e.g., $B = 5$
    - 3 - there are 3 histograms
- Ideas
  - 6D-Pose independent
  - Captures surface variations in a neighborhood

PFH – Neighborhood

- Captures surface variations in a neighborhood
- Given point p_q, find its neighbors within radius r
- PFH considers each pair within this ball/circle

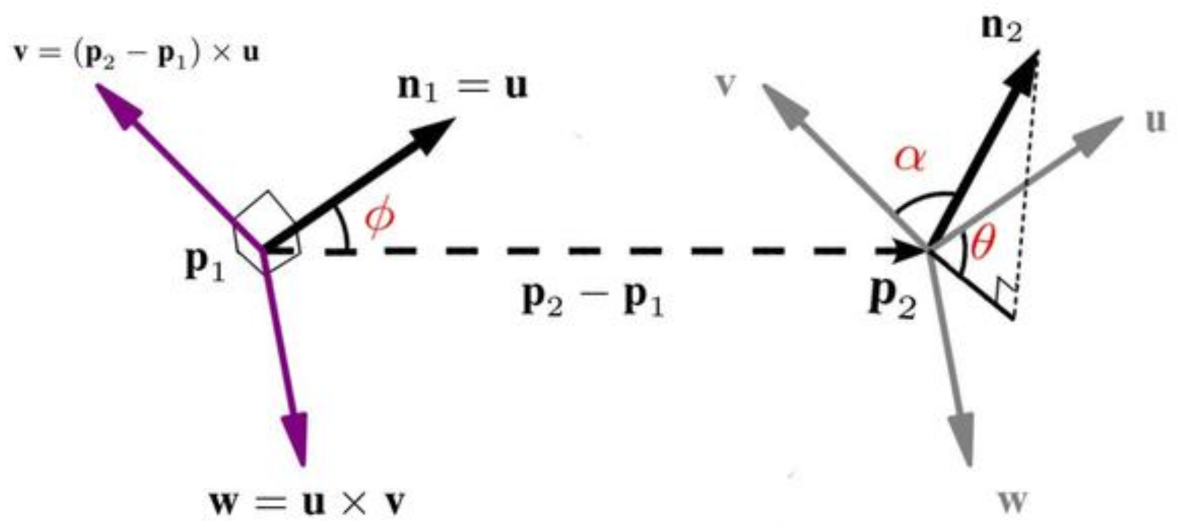


Source: PFH PCL, http://pointclouds.org/documentation/tutorials/pfh_estimation.php

- Define a local reference frame that is independent to 6D pose.

n_1: surface normal at p_1
n_2: surface normal at p_2

$$u = n_1$$
$$v = u \times \frac{p_2 - p_1}{\|p_2 - p_1\|_2}$$
$$w = u \times v$$



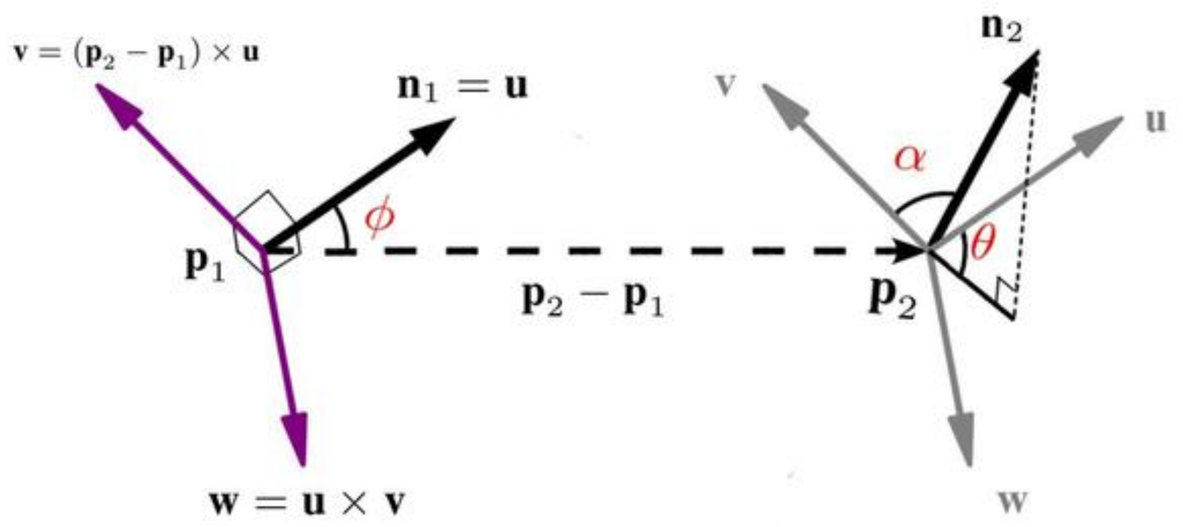
Source: PFH PCL, http://pointclouds.org/documentation/tutorials/pfh_estimation.php

- Goal: difference between surface normal $n_1, n_2 \rightarrow [\alpha, \phi, \theta, d]$

$$d = \|p_2 - p_1\|_2$$
$$\alpha = v \cdot n_2$$
$$\phi = u \cdot \frac{p_2 - p_1}{\|p_2 - p_1\|_2}$$
$$\theta = \arctan(w \cdot n_2, u \cdot n_2)$$

n_2 projection on w: $\frac{n_2 \cdot w}{\|w\|_2} = w \cdot n_2$

n_2 projection on u



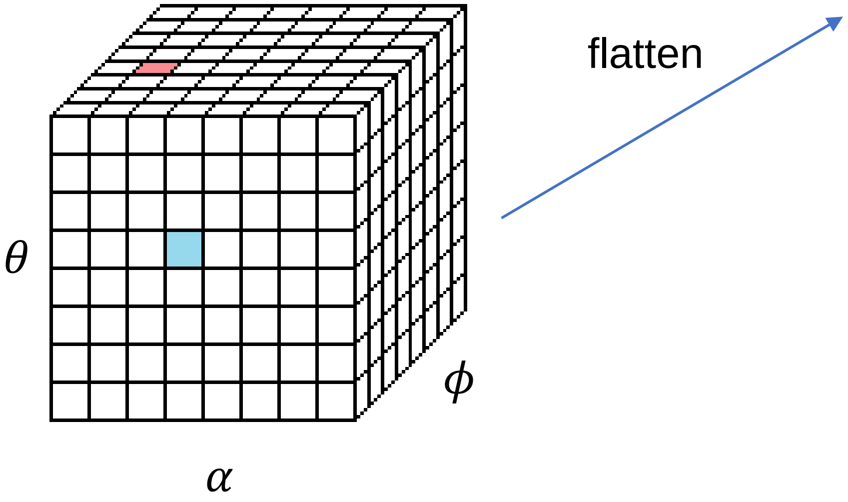
Source: PFH PCL, http://pointclouds.org/documentation/tutorials/pfh_estimation.php

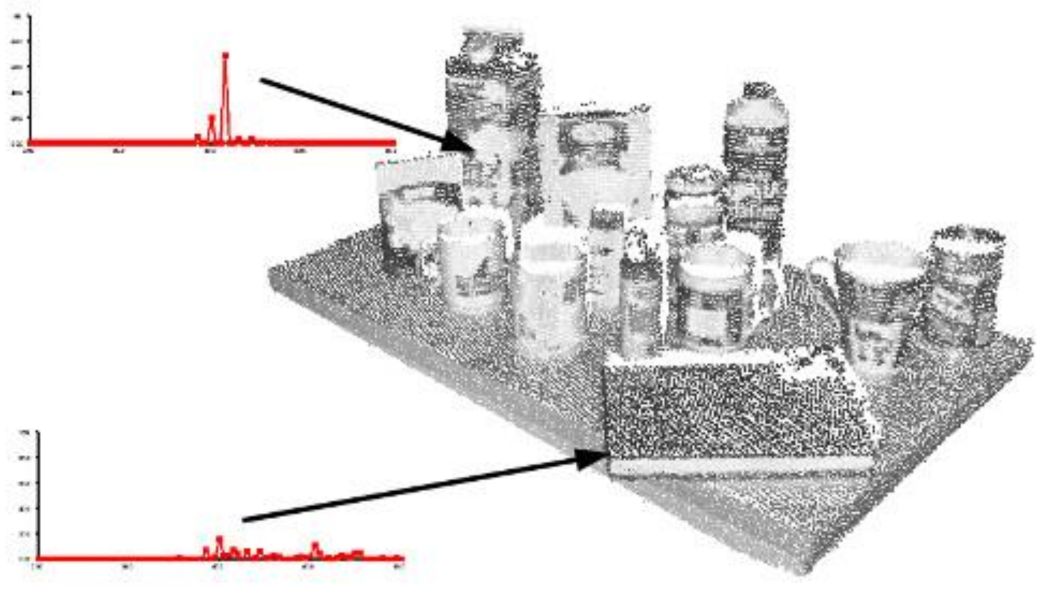
PFH – Histogram

- Each pair of points gives a quadruplet $[\alpha, \phi, \theta, d]$
  - Usually d is ignored because
    - Usually point densities changes according to viewpoint
    - We don't want the descriptors depends on viewpoint
- If there are k points in the neighborhood, there are k^2 quadruplets/triplets
- Put the k^2 triplets $[\alpha, \phi, \theta]$ into histograms
  - Treat each triplet as a 3D data point
  - Each dimension has B bins
  - Put each triplet into this 3D "voxel grid"
  - The flatten "voxel grid" is a B^3 array - PFH feature vector
  - Normalize, e.g., sum/norm equals to some value.

PFH Example – Different regions has different histograms

Put the k^2 triplets $[\alpha, \phi, \theta]$ into histograms





Source: PFH PCL, http://pointclouds.org/documentation/tutorials/pfh_estimation.php

PFH Example – Semantic Segmentation

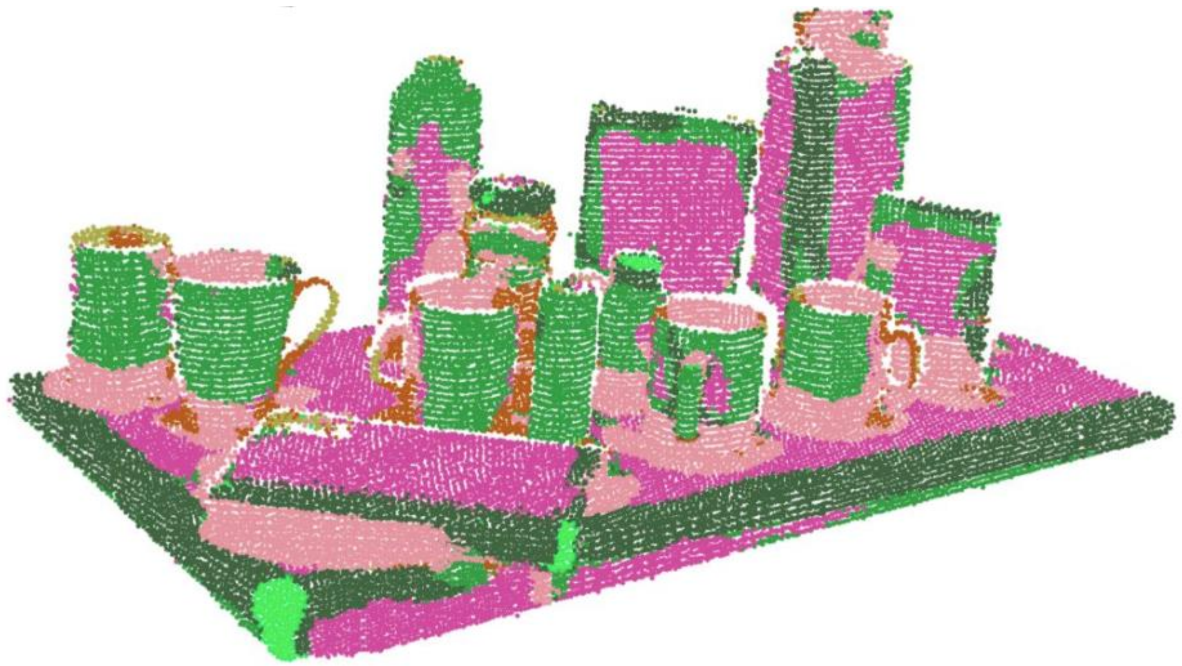
Semantic segmentation

↓

Per-point classification

↓

Per-point PFH + SVM

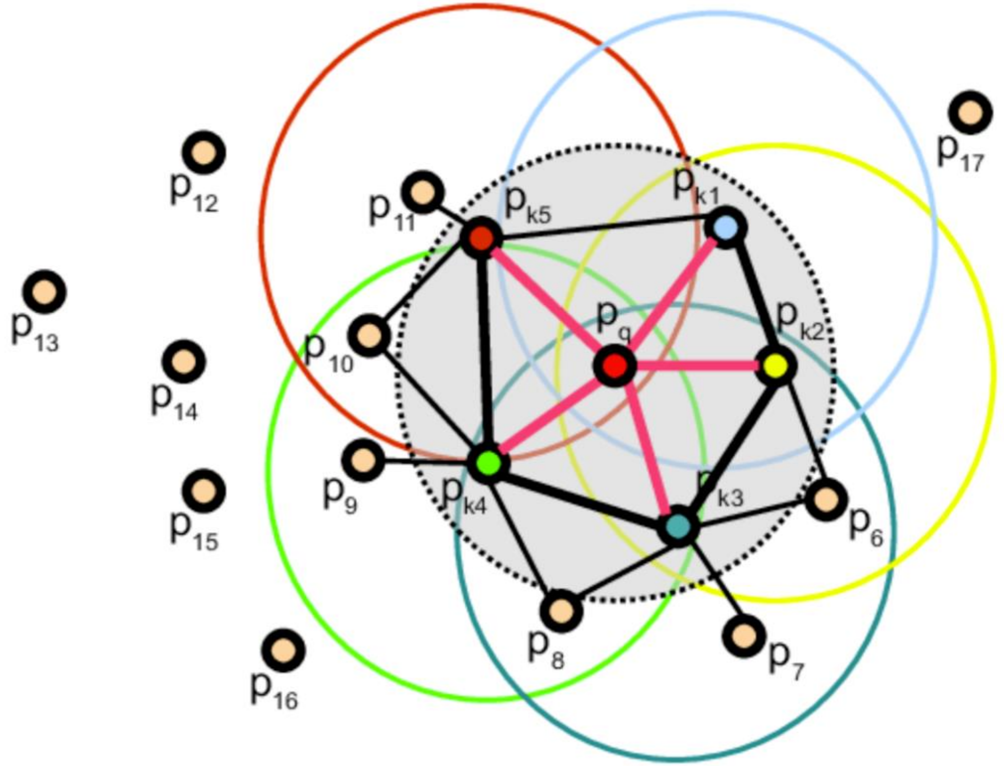


PFH Summary

- Idea:
  - 6D-Pose independent
    - Create a local reference frame based on surface normal.
  - Captures surface variations in a neighborhood
    - Compute the surface normal difference for each pair of points
- Complexity:
  - For each keypoint p_q with k points in the neighborhood: $O(k^2)$
  - n points: $O(nk^2)$
  - Can be improved to be $O(k)/O(nk)$ - FPFH
- Characteristics
  - Simple and effective
  - Sensitive to surface normal estimation

FPFH – Fast Point Feature Histogram

- Simplified Point Feature Histogram (SPFH)
  - Compute triplet $[\alpha, \phi, \theta]$ between query point and its neighbors within r
  - PFH – compute triplet between every pair within r
- Output is 3 histograms (each has B bins) by binning triplet $[\alpha, \phi, \theta]$



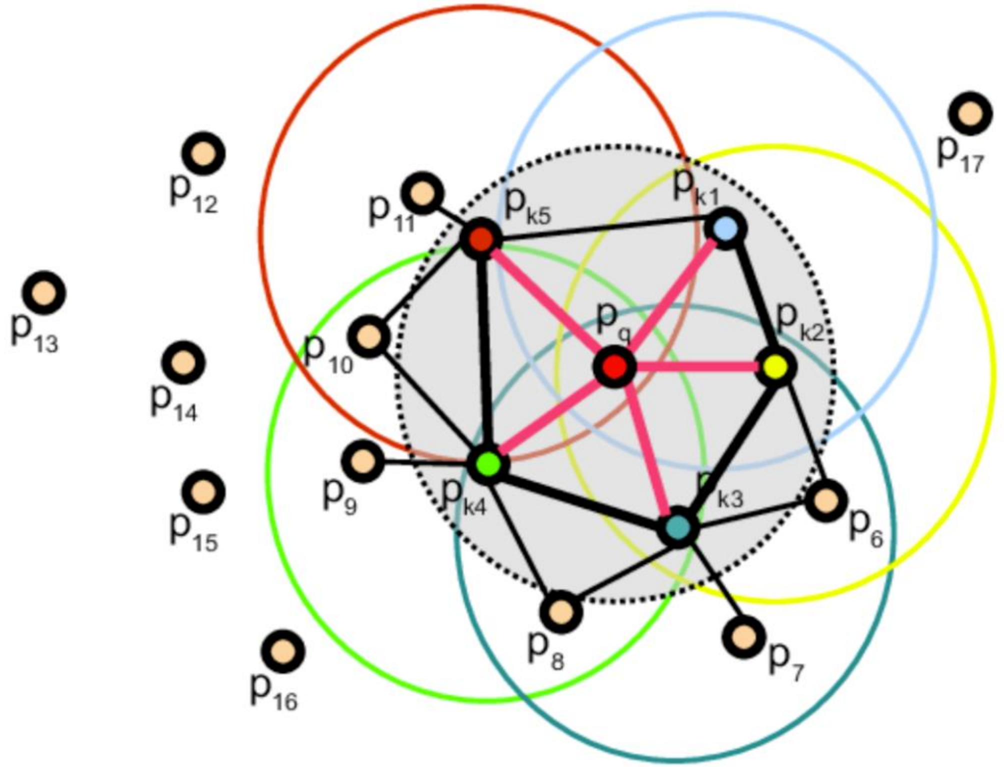
Source: FPFH PCL, http://pointclouds.org/documentation/tutorials/fpfh_estimation.php

- FPFH is the weighted sum of neighboring SPFH

1. Compute SPFH of query point
2. Compute SPFH of neighbor points
3. FPFH = weighted sum of (1)(2)

$$FPFH(p_q) = SPFH(p_q) + \frac{1}{k}\sum_{i=1}^{k} w_k \cdot SPFH(p_k)$$

$$w_k = \frac{1}{\left\|p_q - p_k\right\|_2}$$



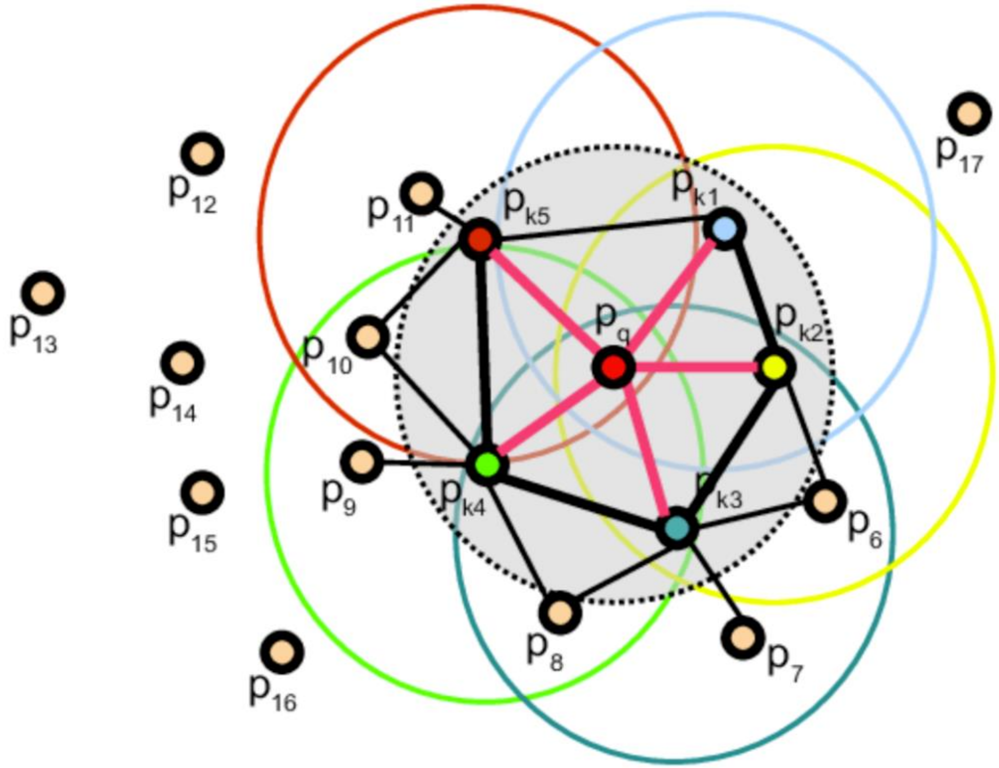
Source: FPFH PCL, http://pointclouds.org/documentation/tutorials/fpfh_estimation.php

FPFH – Fast Point Feature Histogram

- FPFH is the weighted sum of neighboring SPFH
- Some edges are counted twice (thick edges)
- 3 Histograms are concatenated, not “voxel grid”.

$$FPFH(p_q) = SPFH(p_q) + \frac{1}{k}\sum_{i=1}^{k} w_k \cdot SPFH(p_k)$$

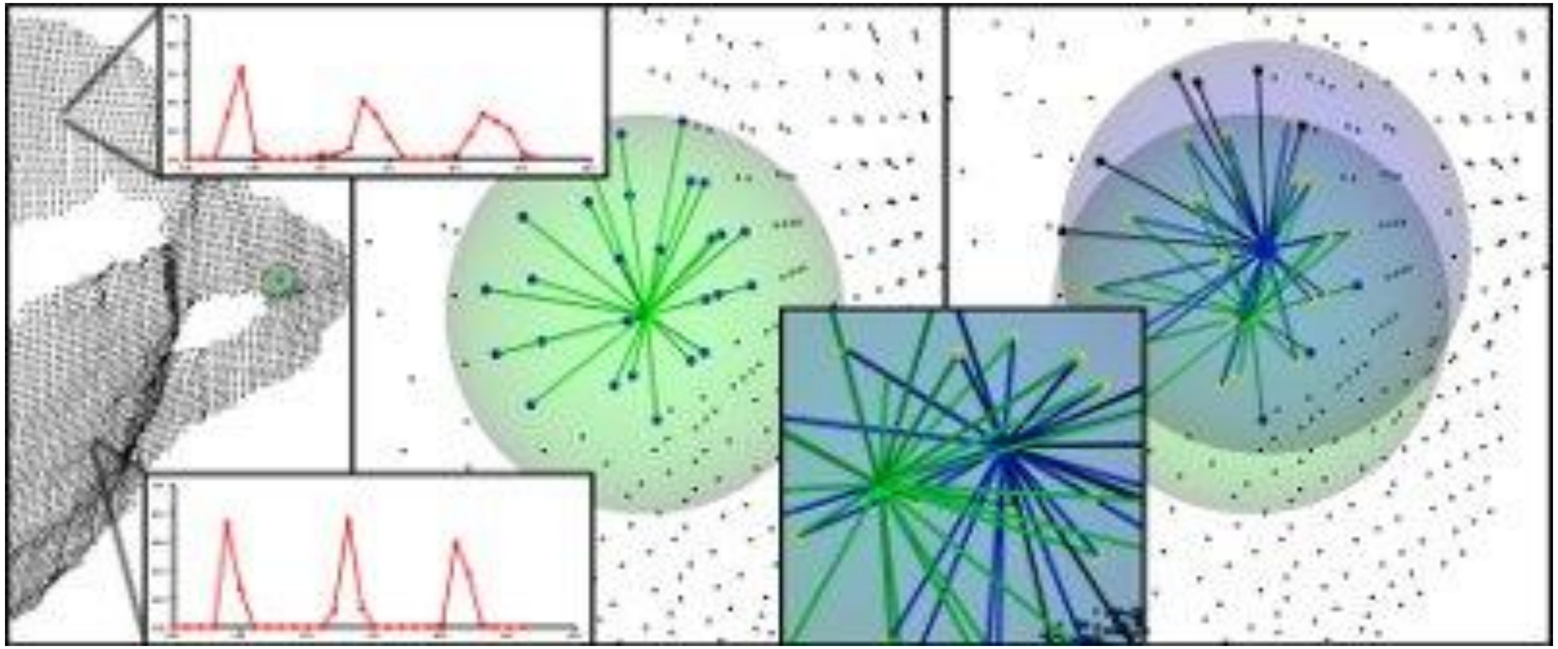
$$w_k = \frac{1}{\left\|p_q - p_k\right\|_2}$$



Source: FPFH PCL, http://pointclouds.org/documentation/tutorials/fpfh_estimation.php

FPFH Example

- In this example, there are 3 peaks in the FPFH histogram
- Because it is concatenation of 3 histograms. Each histogram has a peak in this example.



Source: FPFH PCL, http://pointclouds.org/documentation/tutorials/fpfh_estimation.php

FPFH

- Partial connected neighbors
- Neighborhood of range $[r, 2r]$
- Some edge are counted twice
- $O(nk)$
- Histogram size $3B$

PFH

- Fully connected neighbors
- Neighborhood of range r
- Each edge is counted once
- $O(nk^2)$
- Histogram size B^3

- PFH / FPFH encodes pair-wise information with α, ϕ, θ
  - 6D-Pose independent
  - Captures surface variations in a neighborhood
  - Neighbor positions are not directly recorded

- Why don't we encode neighborhood position information?
  - E.g., 5 points on 6 o'clock direction, 8 points on 9 o'clock direction, etc.
  - But local coordinate has to be 6D-Pose independent.
  - Solution: build a canonical pose of the local neighborhood
    - Local Reference Frame (LRF)

Unique Signatures of Histograms for Local Surface Description. Federico Tombari et.al.

SHOT LRF

PCA? There is the positive / negative ambiguity – each principle vector has two directions

1. Weighted covariance matrix within radius R

$$\mathbf{M} = \frac{1}{\sum_{i:d_i \le R} (R - d_i)} \sum_{i:d_i \le R} (R - d_i)(\mathbf{p}_i - \mathbf{p})(\mathbf{p}_i - \mathbf{p})^T \qquad d_i = \|\mathbf{p}_i - \mathbf{p}\|_2$$

2. Compute the three eigenvectors in decreasing eigenvalue order
   - Denoted as $\mathrm{x}^+, \mathrm{y}^+, \mathrm{z}^+$
   - Opposite direction denoted as $\mathrm{x}^-, \mathrm{y}^-, \mathrm{z}^-$

3. Determine x

$$S_x^+ \doteq \{i : d_i \le R \wedge (\mathbf{p}_i - \mathbf{p}) \cdot \mathbf{x}^+ \ge 0\}$$

Number of points on the half space of x^+

$$S_x^- \doteq \{i : d_i \le R \wedge (\mathbf{p}_i - \mathbf{p}) \cdot \mathbf{x}^- > 0\}$$

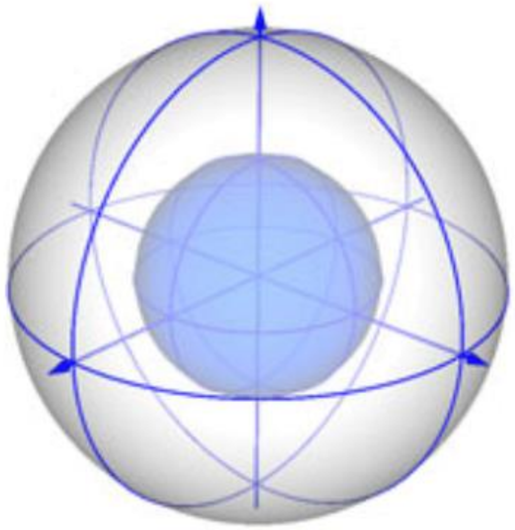
Number of points on the half space of x^-

$$\mathbf{x} = \begin{cases} \mathbf{x}^+, & |S_x^+| \ge |S_x^-| \\ \mathbf{x}^-, & \text{otherwise} \end{cases}$$

4. Determine z similarly.

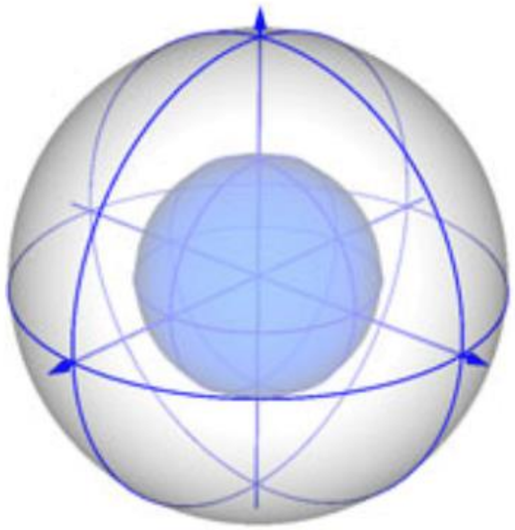
5. $\mathrm{y} = \mathrm{z} \times \mathrm{x}$

SHOT - Signatures of Histogram



- With the LRF, we can
  1. Divide the space into several small volumes
  2. Compute local histogram of each volume
  3. Concatenate local histograms into a ”signature”
     - With LRF, the signature is 6D pose invariant
  4. Normalize the “signature” into sum=1. This is the descriptor

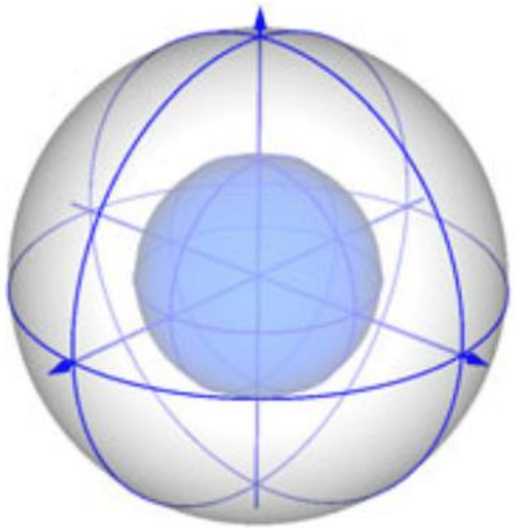
SHOT - Signatures of Histogram



1. Division into 32 volumes
   - 8 azimuth divisions
     - Figure on the right shows only 4 azimuth divisions for clarity
   - 2 elevation divisions
   - 2 radial divisions
2. Build a histogram of $cos\theta_i$ for each volume
   - Surface normal of a point in that volume, n_{v_i}
   - Surface normal of the keypoint, n_u
   - $\cos\theta_i = n_u \cdot n_{v_i}$
   - Increase the corresponding bin of $cos\theta_i$ for that volume
   - For example, *pcl::SHOT352* builds histogram of length 11 for each volume. $11 \times 32 = 352$

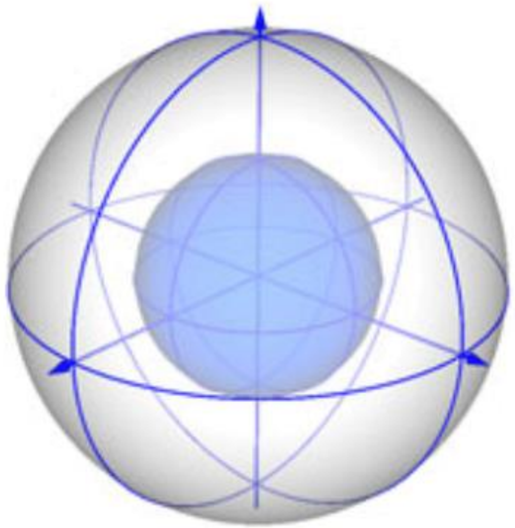
- Boundary Effect!
  - Points at the edge of each volume should contribute to neighboring volume as well
  - Small perturbation of LRF changes all the local histograms

SHOT – Boundary Effect



- For a point $\mathrm{x_i} = (\rho_i, \alpha_i, \beta_i)$ with computed $\cos\theta_i$
  - ρ_i is distance to keypoint. Division resolution r_ρ
  - α_i is azimuth angle in LRF. Division resolution r_α
  - β_i is elevation angle in LRF. Division resolution r_β
  - $\cos\theta_i$ is the surface normal dot product. Division resolution r_θ
  - It contributes to $2^3 = 8$ volumes
    - $\left(\left\lfloor\frac{\rho_i}{r_\rho}\right\rfloor and \left\lceil\frac{\rho_i}{r_\rho}\right\rceil, \left\lfloor\frac{\alpha_i}{r_\alpha}\right\rfloor and \left\lceil\frac{\alpha_i}{r_\alpha}\right\rceil, \left\lfloor\frac{\beta_i}{r_\beta}\right\rfloor and \left\lceil\frac{\beta_i}{r_\beta}\right\rceil\right)$
  - In each volume, it contributes to 2 bins in the histogram
    - $\left\lfloor\frac{\cos\theta_i}{r_\theta}\right\rfloor and \left\lceil\frac{\cos\theta_i}{r_\theta}\right\rceil$
  - It is quadrilinear interpolation (interpolation that involves 4 dimensions $\rho, \alpha, \beta, \cos\theta$)

SHOT – Boundary Effect



- Let's look at the contribution into
  - volume $\left\lceil \frac{\rho_i}{r_\rho} \right\rceil, \left\lfloor \frac{\alpha_i}{r_\alpha} \right\rfloor, \left\lfloor \frac{\beta_i}{r_\beta} \right\rfloor$
  - Histogram bin $\left\lfloor \frac{\cos\theta_i}{r_\theta} \right\rfloor$
- Weighting in each dimension
  - $w_\rho = 1 - \frac{\left\lceil \frac{\rho_i}{r_\rho} \right\rceil r_\rho - \rho_i}{r_\rho}, w_\alpha = 1 - \frac{\alpha_i - \left\lfloor \frac{\alpha_i}{r_\alpha} \right\rfloor r_\alpha}{r_\alpha}, w_\beta = 1 - \frac{\beta_i - \left\lfloor \frac{\beta_i}{r_\beta} \right\rfloor r_\beta}{r_\beta}, w_\theta = 1 - \frac{\cos\theta_i - \left\lfloor \frac{\cos\theta_i}{r_\theta} \right\rfloor r_\theta}{r_\theta}$
- The contribution to that bin at that volume is $w = w_\rho w_\alpha w_\beta w_\theta$

Registration Recall on 3DMatch Dataset

	Spin Image [16]	SHOT [35]	FPFH [34]	3DMatch [51]	CGF [18]	PPFNet [8]	FoldNet [48]	Ours	Ours-5K
Kitchen	0.1937	0.1779	0.3063	0.5751	0.4605	**0.8972**	0.5949	0.7352	0.7866
Home 1	0.3974	0.3718	0.5833	0.7372	0.6154	0.5577	0.7179	0.7564	**0.7628**
Home 2	0.3654	0.3365	0.4663	**0.7067**	0.5625	0.5913	0.6058	0.6250	0.6154
Hotel 1	0.1814	0.2080	0.2611	0.5708	0.4469	0.5796	0.6549	0.6593	**0.6814**
Hotel 2	0.2019	0.2212	0.3269	0.4423	0.3846	0.5769	0.4231	0.6058	**0.7115**
Hotel 3	0.3148	0.3889	0.5000	0.6296	0.5926	0.6111	0.6111	0.8889	**0.9444**
Study	0.0548	0.0719	0.1541	0.5616	0.4075	0.5342	**0.7123**	0.5753	0.6199
MIT Lab	0.1039	0.1299	0.2727	0.5455	0.3506	**0.6364**	0.5844	0.5974	0.6234
Average	0.2267	0.2382	0.3589	0.5961	0.4776	0.6231	0.6130	0.6804	**0.7182**

Registration Recall on Rotated 3DMatch Dataset

	Spin Image [16]	SHOT [35]	FPFH [34]	3DMatch [51]	CGF [18]	PPFNet [8]	FoldNet [48]	Ours	Ours-5K
Kitchen	0.1779	0.1779	0.2905	0.0040	0.4466	0.0020	0.0178	0.7352	**0.7885**
Home 1	0.4487	0.3526	0.5897	0.0128	0.6667	0.0000	0.0321	0.7692	**0.7821**
Home 2	0.3413	0.3365	0.4712	0.0337	0.5288	0.0144	0.0337	0.6202	**0.6442**
Hotel 1	0.1814	0.2168	0.3009	0.0044	0.4425	0.0044	0.0133	0.6637	**0.6770**
Hotel 2	0.1731	0.2404	0.2981	0.0000	0.4423	0.0000	0.0096	0.6058	**0.6923**
Hotel 3	0.3148	0.3333	0.5185	0.0096	0.6296	0.0000	0.0370	0.9259	**0.9630**
Study	0.0582	0.0822	0.1575	0.0000	0.4178	0.0000	0.0171	0.5616	**0.6267**
MIT Lab	0.1169	0.1299	0.2857	0.0260	0.4156	0.0000	0.0260	0.6104	**0.6753**
Average	0.2265	0.2337	0.3640	0.0113	0.4987	0.0026	0.0233	0.6865	**0.7311**

Source: PPF-FoldNet: Unsupervised Learning of Rotation Invariant 3D Local Descriptors, Haowen Deng, et.al.

Summary for PFH, FPFH, SHOT

FPFH

- Partial connected neighbors
- Neighborhood of range $[r, 2r]$
- Pairwise Reference Frame
- $O(nk)$
- Histogram size $3B$

PFH

- Fully connected neighbors
- Neighborhood of range r
- Pairwise Reference Frame
- $O(nk^2)$
- Histogram size B^3

Similar to PPFNet / PPF-FoldeNet

SHOT

- Only connects a keypoint with its neighbors
- Neighborhood of range r
- Local Reference Frame
- $O(nk)$
- Descriptor size $32 \times histogram\_size$

Similar to LRF in PerfectMatch

Why do we need Deep Learning Descriptor

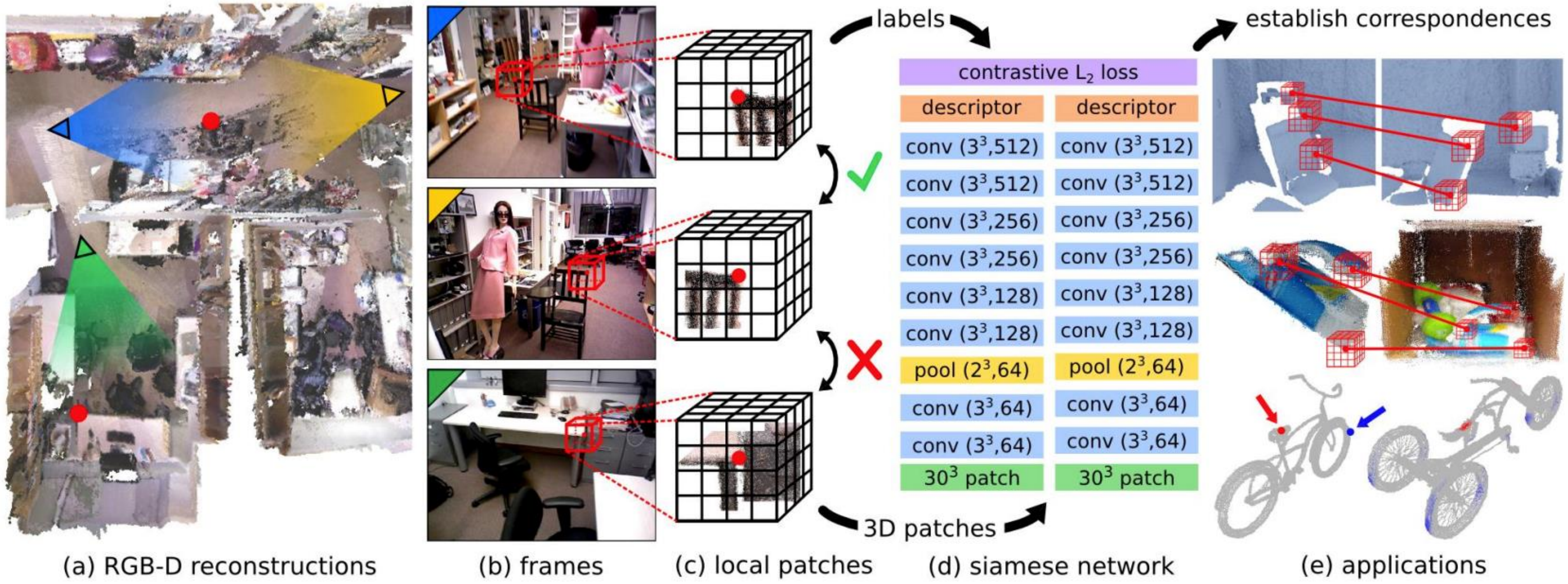
- All handcrafted descriptors are based on geometry, e.g.,
  - Surface normal variations around the keypoint.
  - Point distributions around the keypoint
- They are not reliable in case of
  - Noise
  - Occlusion / incomplete shape
  - Sparsity
- Deep learning
  - Includes semantic information
  - Smarter way to encode geometry
  - Robust to noise

Point Cloud Feature Descriptor – Deep Learning

- Lots of approaches
- Usually better performance
- Methods:
  - 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions
  - The Perfect Match: 3D Point Cloud Matching with Smoothed Densities
  - PPFNet: Global Context Aware Local Features for Robust 3D Point Matching
  - PPF-FoldNet: Unsupervised Learning of Rotation Invariant 3D Local Descriptors
  - CGF: Learning Compact Geometric Features
  - 3DFeat-Net: Weakly Supervised Local 3D Features for Point Cloud Registration
  - USIP: Unsupervised Stable Interest Point Detection from 3D Point Clouds
  - … …

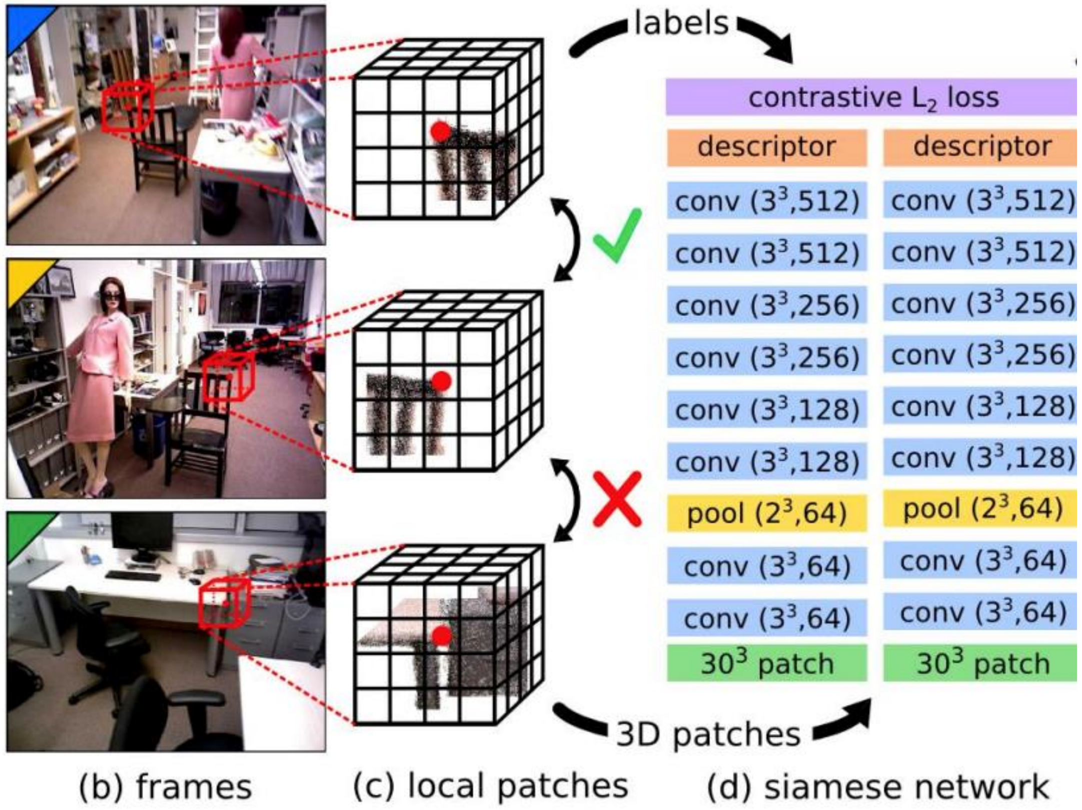
3DMatch - Inference

- Inference
  - Input: 3D patch, voxel grid of $30 \times 30 \times 30$.
    - **Truncated Distance Function (TDF)** / Binary Voxel Grid / Probabilistic Voxel Grid, etc.
  - Output: Descriptor for that patch, vector of 512



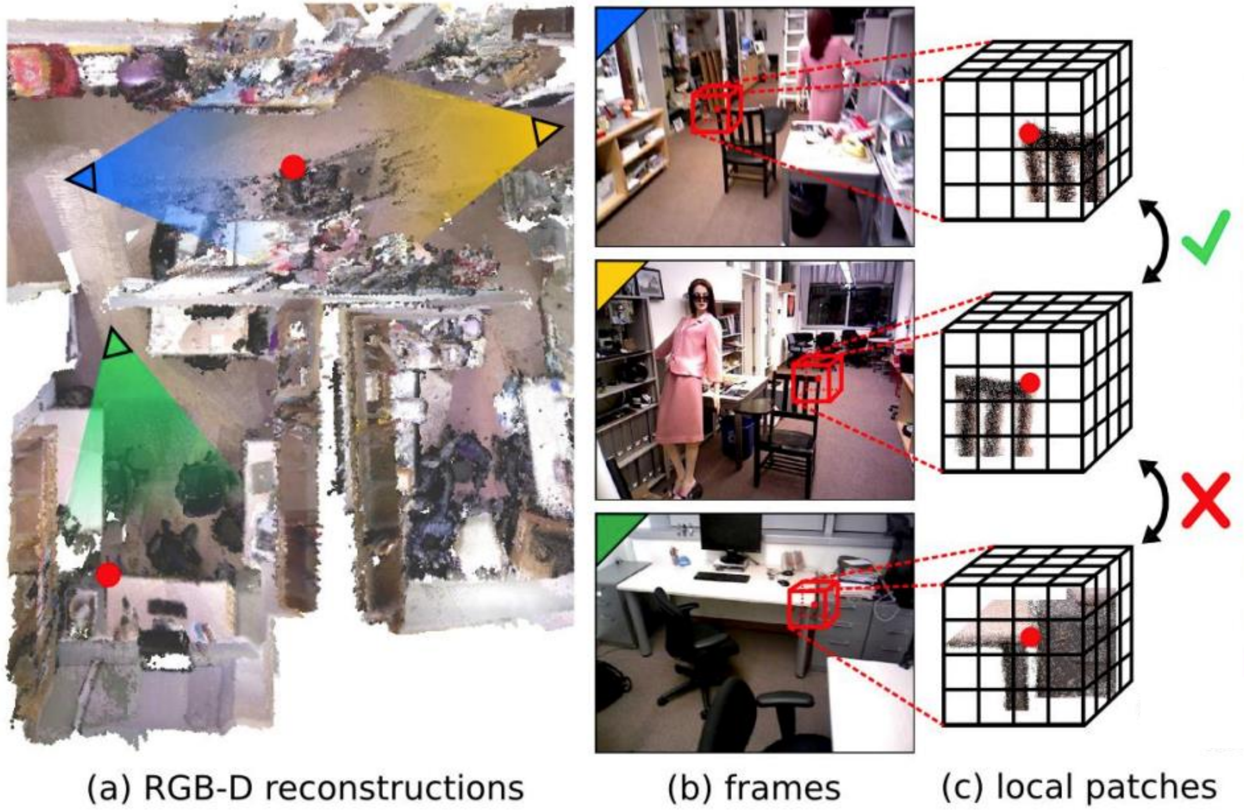
Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions, Andy Zeng, et.al.

- Training:
  - *Input*: two patches
  - *Output*: two descriptors
  - If the two patches are from the same location:
    - Make them similar
  - Else:
    - Make them different.
- Two problems to be solved:
  - How to build dataset
  - How to define “similar” / “different”



Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions, Andy Zeng, et.al.

- Utilize RGB-D reconstructions
  - Reconstructed from multiple RGB-D frames.
  - We know the position of each frame.
- Positive pairs of patches:
  - Same physical location (e.g. <0.05m)
  - Different frames that are captures at least 1m apart.
- Negative pairs:
  - At least 0.1m apart.



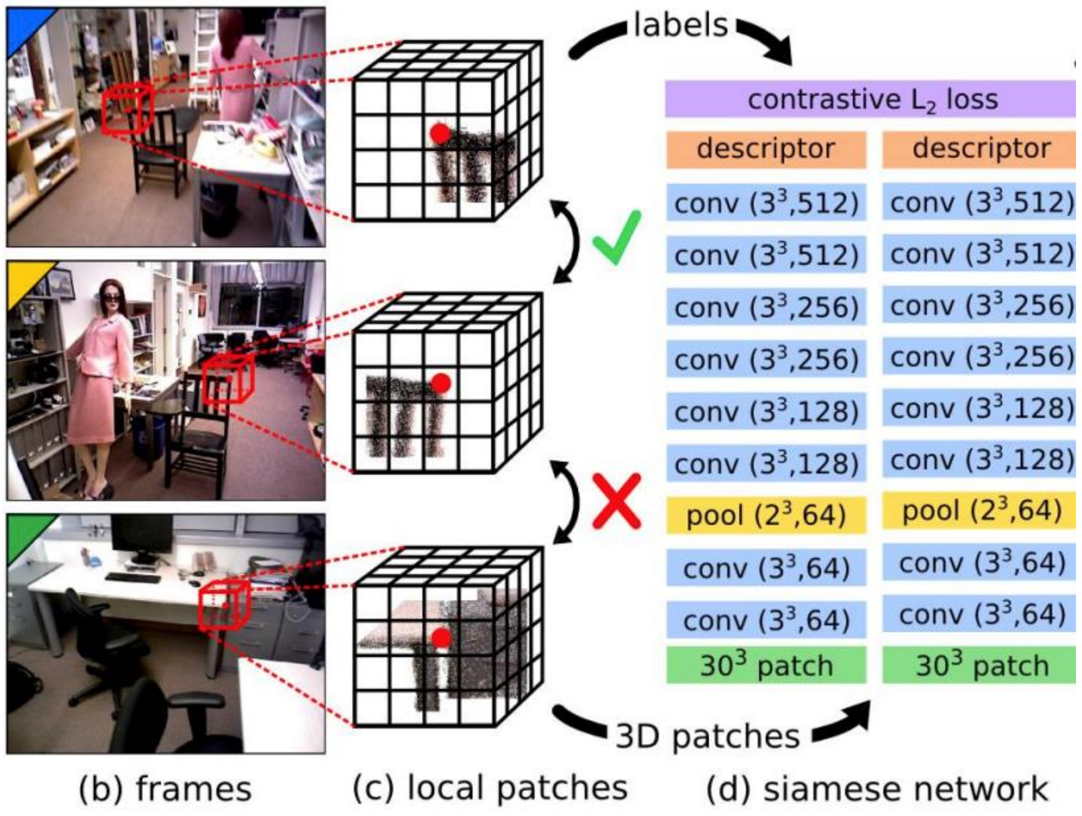
Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions, Andy Zeng, et.al.

- There is ground truth label y_{ij}
  - Positive pairs: $y_{ij} = 1$
  - Negative pairs: $y_{ij} = 0$
- Contrastive Loss

$$L = \frac{1}{N}\sum_{n=1}^{N} y_{ij} d_{ij}^2 + (1 - y_{ij}) \max(\tau - d_{ij}, 0)^2$$

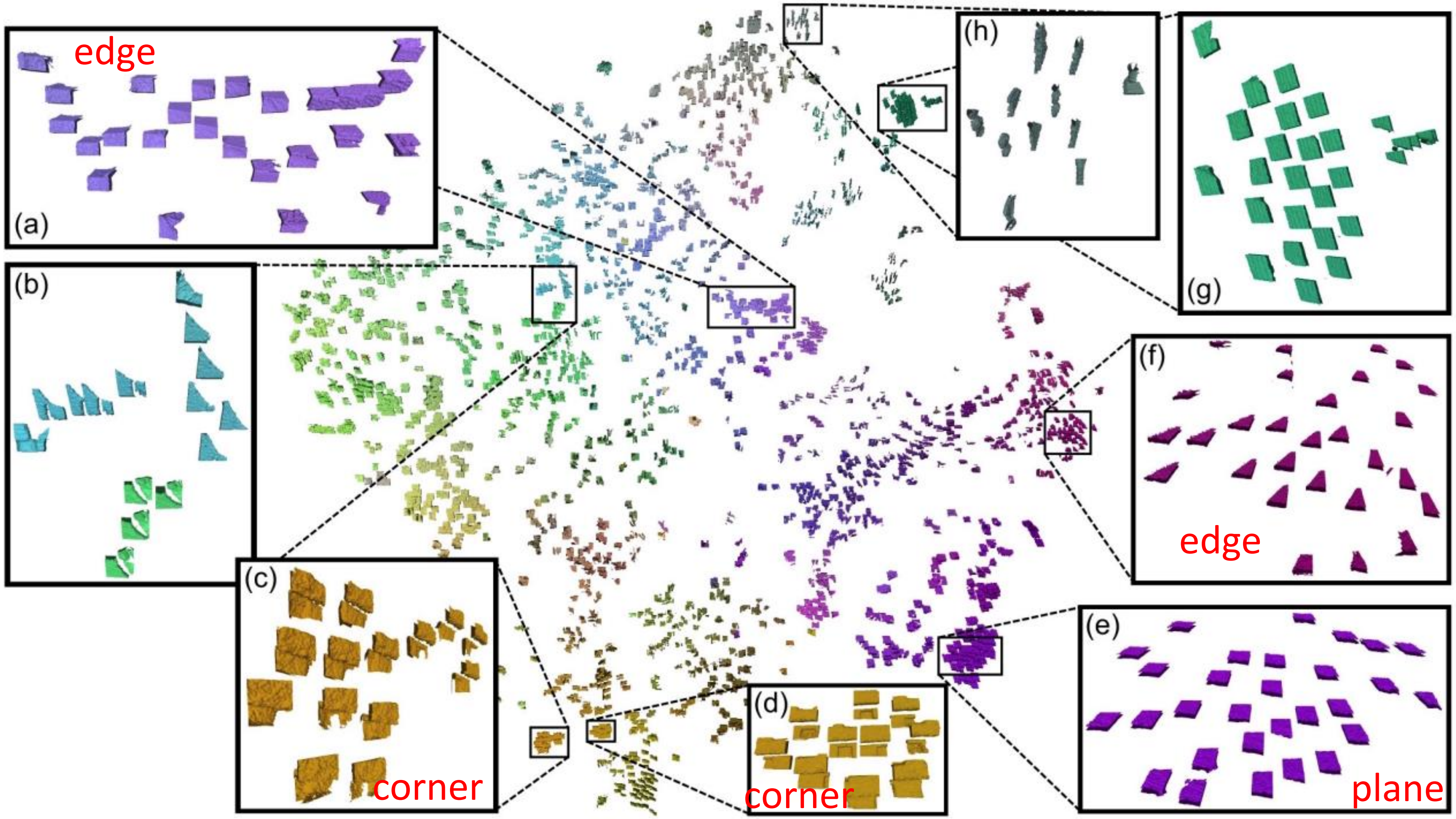
$$d_{ij} = \| f(x_i) - f(x_j) \|_2$$

- Positive: Pull them together
- Negative: Push them to τ away.
  - τ is the margin. If the negative pair is already τ away, ignore it.



Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions, Andy Zeng, et.al.

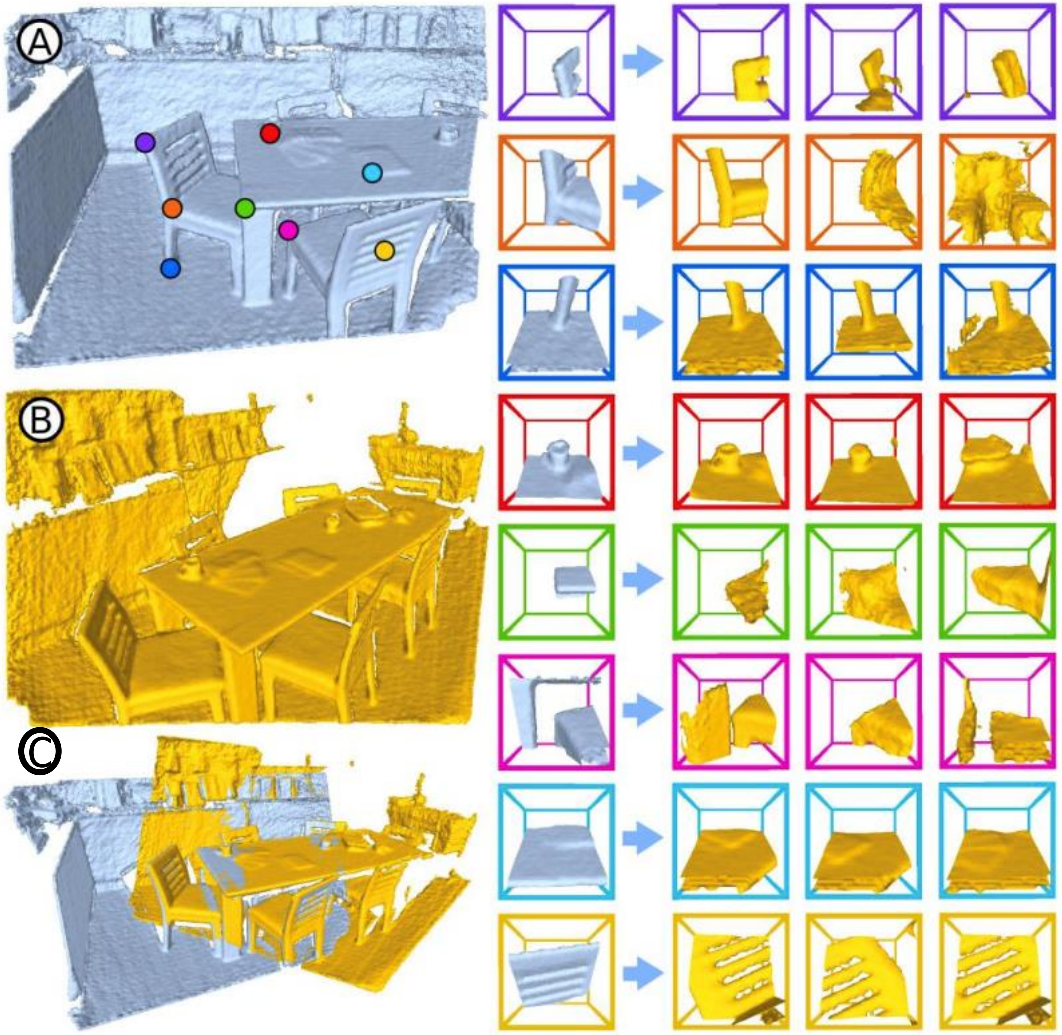
3DMatch – t-SNE embedding of descriptors



Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions, Andy Zeng, et.al.

3DMatch – Results

- A, B: RGBD scans from different view point
- C: Registered with 3DMatch + RANSAC (covered in Lecture 9)
- The 3 columns on the right: 3NN search of A’s patches in B, based on the 3DMatch descriptors.



Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions, Andy Zeng, et.al.

Registration Recall on 3DMatch Dataset

	Spin Image [16]	SHOT [35]	FPFH [34]	3DMatch [51]	CGF [18]	PPFNet [8]	FoldNet [48]	Ours	Ours-5K
Kitchen	0.1937	0.1779	0.3063	0.5751	0.4605	**0.8972**	0.5949	0.7352	0.7866
Home 1	0.3974	0.3718	0.5833	0.7372	0.6154	0.5577	0.7179	0.7564	**0.7628**
Home 2	0.3654	0.3365	0.4663	**0.7067**	0.5625	0.5913	0.6058	0.6250	0.6154
Hotel 1	0.1814	0.2080	0.2611	0.5708	0.4469	0.5796	0.6549	0.6593	**0.6814**
Hotel 2	0.2019	0.2212	0.3269	0.4423	0.3846	0.5769	0.4231	0.6058	**0.7115**
Hotel 3	0.3148	0.3889	0.5000	0.6296	0.5926	0.6111	0.6111	0.8889	**0.9444**
Study	0.0548	0.0719	0.1541	0.5616	0.4075	0.5342	**0.7123**	0.5753	0.6199
MIT Lab	0.1039	0.1299	0.2727	0.5455	0.3506	**0.6364**	0.5844	0.5974	0.6234
Average	0.2267	0.2382	0.3589	0.5961	0.4776	0.6231	0.6130	0.6804	**0.7182**

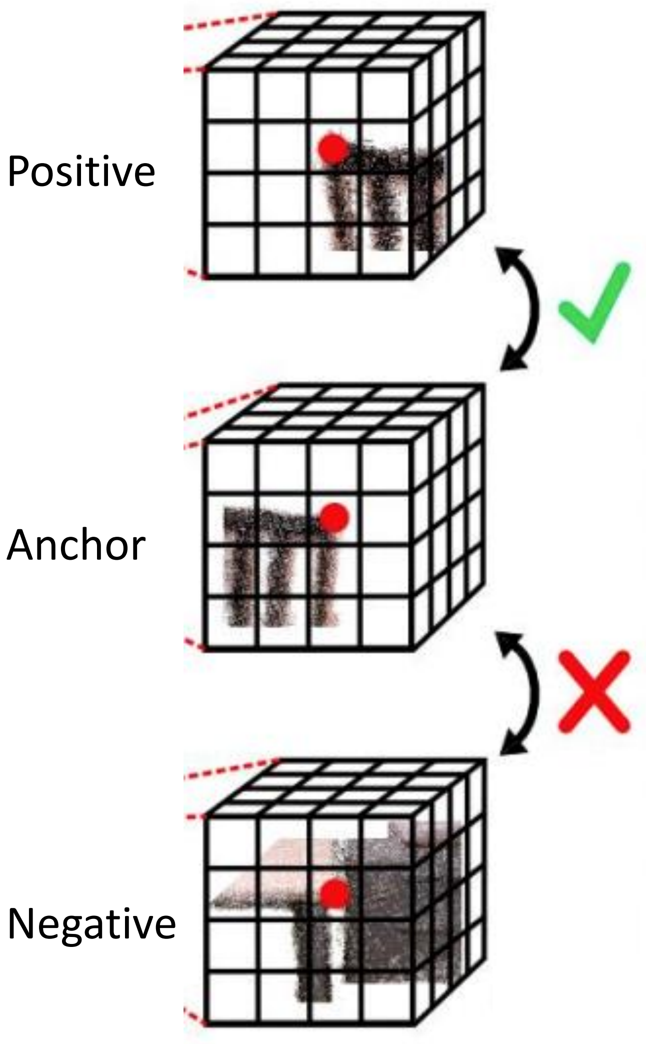
Registration Recall on Rotated 3DMatch Dataset

	Spin Image [16]	SHOT [35]	FPFH [34]	3DMatch [51]	CGF [18]	PPFNet [8]	FoldNet [48]	Ours	Ours-5K
Kitchen	0.1779	0.1779	0.2905	0.0040	0.4466	0.0020	0.0178	0.7352	**0.7885**
Home 1	0.4487	0.3526	0.5897	0.0128	0.6667	0.0000	0.0321	0.7692	**0.7821**
Home 2	0.3413	0.3365	0.4712	0.0337	0.5288	0.0144	0.0337	0.6202	**0.6442**
Hotel 1	0.1814	0.2168	0.3009	0.0044	0.4425	0.0044	0.0133	0.6637	**0.6770**
Hotel 2	0.1731	0.2404	0.2981	0.0000	0.4423	0.0000	0.0096	0.6058	**0.6923**
Hotel 3	0.3148	0.3333	0.5185	0.0096	0.6296	0.0000	0.0370	0.9259	**0.9630**
Study	0.0582	0.0822	0.1575	0.0000	0.4178	0.0000	0.0171	0.5616	**0.6267**
MIT Lab	0.1169	0.1299	0.2857	0.0260	0.4156	0.0000	0.0260	0.6104	**0.6753**
Average	0.2265	0.2337	0.3640	0.0113	0.4987	0.0026	0.0233	0.6865	**0.7311**

Source: PPF-FoldNet: Unsupervised Learning of Rotation Invariant 3D Local Descriptors, Haowen Deng, et.al.

How to Improve 3DMatch?

- Contrastive Loss is too "greedy".
  - $L = \frac{1}{N} \sum_{n=1}^{N} y_{ij} d_{ij}^2 + (1 - y_{ij}) \max(\tau - d_{ij}, 0)^2$
- Another representation of contrastive loss:
  - Anchor sample x_i^a
  - Positive sample x_i^p - same location
  - Negative sample x_i^n - different location
  - Assume equal number of positive and negative pairs
  - $L = \frac{1}{N} \left(\begin{array}{c} \sum_{i=1}^{N} \| f(x_i^a) - f(x_i^p) \|_2^2 + \\ \sum_{i=1}^{N} \max(\tau - \| f(x_i^a) - f(x_i^n) \|_2^2, 0) \end{array} \right)$
- It considers two samples only, either A+P or A+N



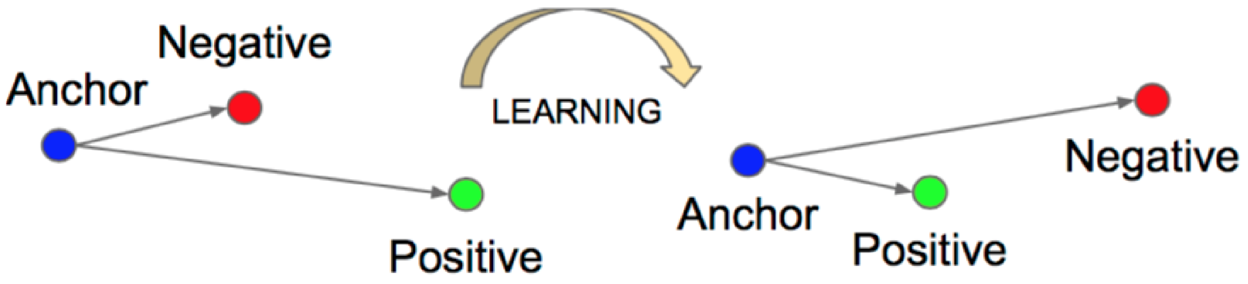
Source: 3DMatch: Learning Local Geometric Descriptors from RGB-D Reconstructions, Andy Zeng, et.al.

Triplet Loss

- ”Far” and “Close” is relative
  - It is sufficient to distinguish positive and negative when $d(a,n) \gg d(a,p)$
  - Not necessary to ensure $d(a,p) \sim 0, d(a,n) > \tau$
- Triplet Loss:
  - Pull A, P together, push A, N away

γ is the margin

$$L = \sum_{i=1}^{N} L_i = \sum_{i=1}^{N} \max(d_i(a,p) - d_i(a,n) + \boxed{\gamma}, 0)$$



FaceNet: A Unified Embedding for Face Recognition and Clustering, F Schroff, et.al.

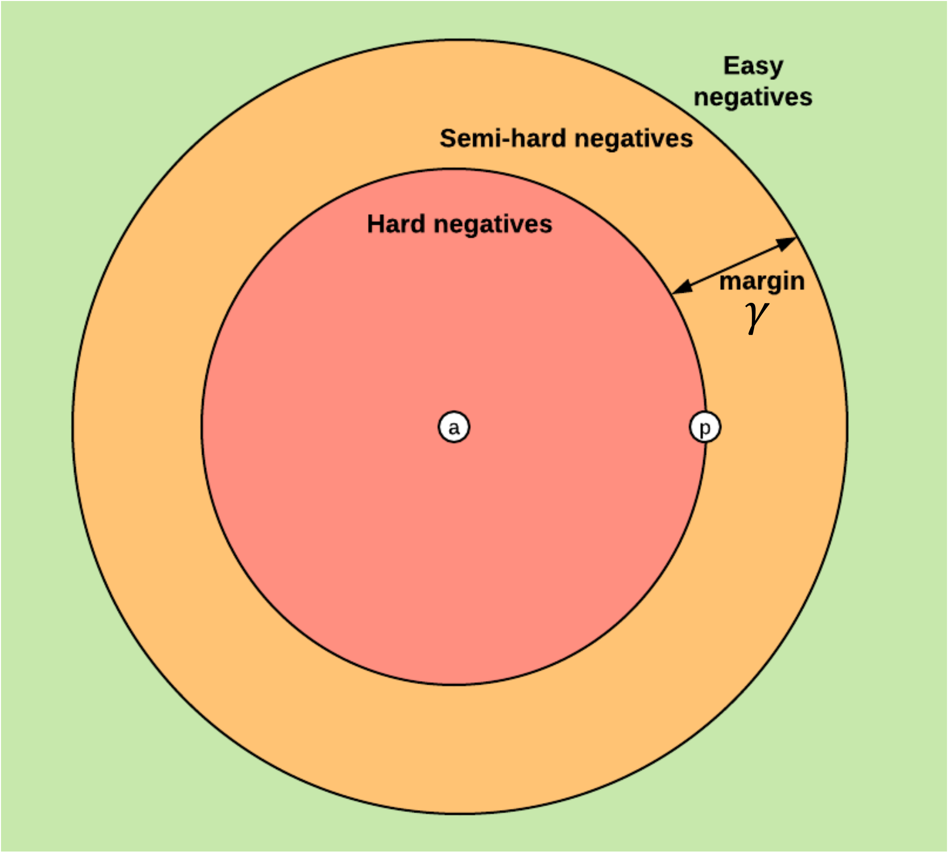
- A triplet loss involves 3 samples: anchor, positive, negative

$$L_i = \max(d_i(a,p) - d_i(a,n) + \gamma, 0)$$

- Problem 1 – $L_i \approx 0$, not supervising the network.
  - For each anchor, randomly sample a negative sample.
  - In most cases, that randomly sampled negative is very different to anchor - easy to distinguish
  - L_i is close to 0 in most cases!
- Problem 2 – $L_i > \gamma$, network not converging.
  - For each anchor, always find the most similar negative sample, i.e., $d_{i(a,n)} \approx 0$

Triplet Loss

- Easy triplet
  - Triplet loss is close to 0
  - $d(a,p) - d(a,n) + \gamma < 0$
- Hard triplet
  - The negative is closer to the anchor than the positive
  - $d(a,n) < d(a,p)$
- Semi-hard triplet
  - The positive is closer to the anchor than the negative, but not too much
  - $d(a,p) < d(a,n) < d(a,p) + \gamma$



Source: https://omoindrot.github.io/triplet-loss

- General guideline
  - Usually triplets should contain semi-hard and/or hard ones
  - Experiment with validation/test set to determine the best configuration and γ
  - *In Defense of the Triplet Loss for Person Re-Identification, Alexander Hermans et.al.*

- Offline negative mining – Not efficient
  - Pre-build a list of $[a, p, n]$
  - After each training epoch
    - Go through the dataset
    - Pick hard positive & negative samples for each anchor

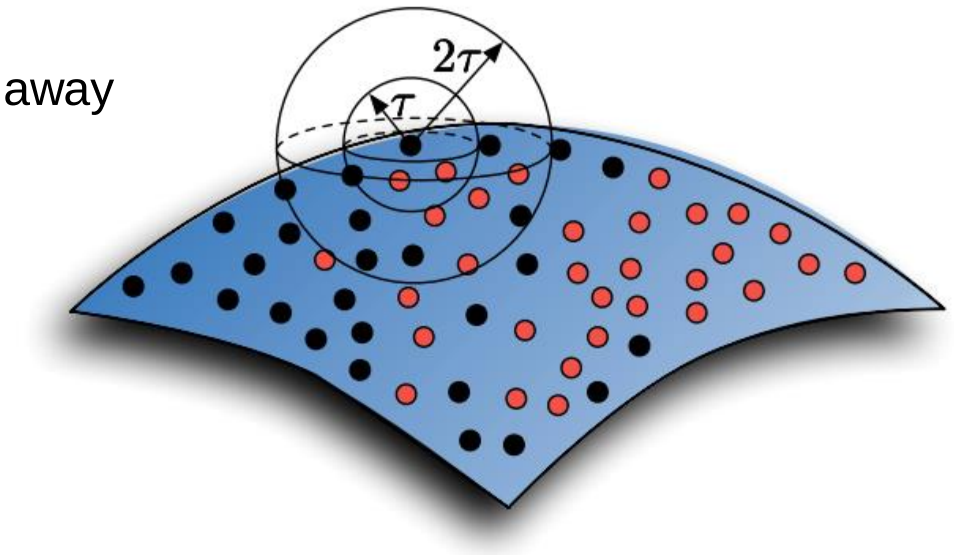
- Batch size B

1. There are B pairs of a, p
   - p is determined by
     - Getting same location from different view point
     - Randomly getting a patch from the same geometric location (within some small distance from a)
   - Optional: select p from frames that is some distance away
   - Not necessary to mine the hardest positives
2. Randomly sample B negatives n
   - Far away from a
3. Feed the 3B samples into network, get 3B descriptors
4. For each anchor a
   - Find hard negatives by getting $n^* = \text{argmin } d(a, n)$
5. Compute Triplet loss $L_i = \max(d_i(a,p) - d_i(a,n^*) + \gamma, 0)$

- Batch size B

1. There are B pairs of a, p
   - p is determined by randomly getting a patch from the same geometric location (within some small distance τ from a)
   - Optional: select p from frames that is some distance away
2. Select B negatives n
   - Distance from a is $[\tau, 2\tau]$
3. Compute Triplet loss

$$L_i = \max(d_i(a, p) - d_i(a, n) + \gamma, 0)$$



Source: Learning Compact Geometric Features, Marc Khoury, et.al.

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Source: PPF-FoldNet: Unsupervised Learning of Rotation Invariant 3D Local Descriptors, Haowen Deng, et.al.

How to Make Descriptor Robust to Rotation

- Point cloud / Voxel grid is different after rotation.

- Solution: rotate the patch to canonical representation, i.e., LRF
  - SHOT has a LRF
  - The Perfect Match: 3D Point Cloud Matching with Smoothed Densities

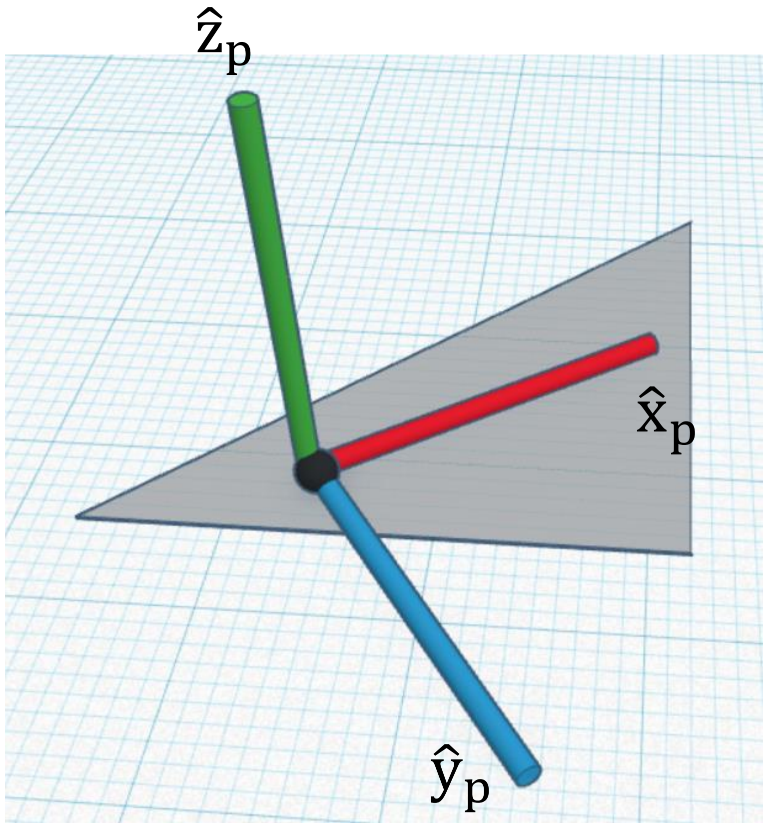
Local Reference Frame (LRF)

1. Compute covariance matrix around interest point p
   - The local support (neighborhood) is defined as $S = \{p_i : \|p_i - p\|_2 < r\}$

$$\tilde{\boldsymbol{\Sigma}}_{\mathcal{S}} = \frac{1}{|\mathcal{S}|} \sum_{\mathbf{p}_i \in \mathcal{S}} (\mathbf{p}_i - \mathbf{p})(\mathbf{p}_i - \mathbf{p})^{\mathrm{T}}$$

2. Determine $\hat{\mathrm{z}}_{\mathrm{p}}$
   - $\hat{\mathrm{n}}_{\mathrm{p}}$ is the surface normal
   - Determine direction of $\hat{\mathrm{z}}_{\mathrm{p}}$ (e.g., pointing up or down)
   - Select the direction that sum of p_{i} projection is positive.

$$\hat{\mathbf{z}}_{\mathbf{p}} = \begin{cases} \hat{\mathbf{n}}_{\mathbf{p}}, & \text{if } \sum_{\mathbf{p}_i \in \mathcal{S}} \langle \hat{\mathbf{n}}_{\mathbf{p}}, \overrightarrow{\mathbf{p}_i \mathbf{p}} \rangle \geq 0 \\ -\hat{\mathbf{n}}_{\mathbf{p}}, & \text{otherwise} \end{cases}$$



3. Determine $\hat{x}_p$

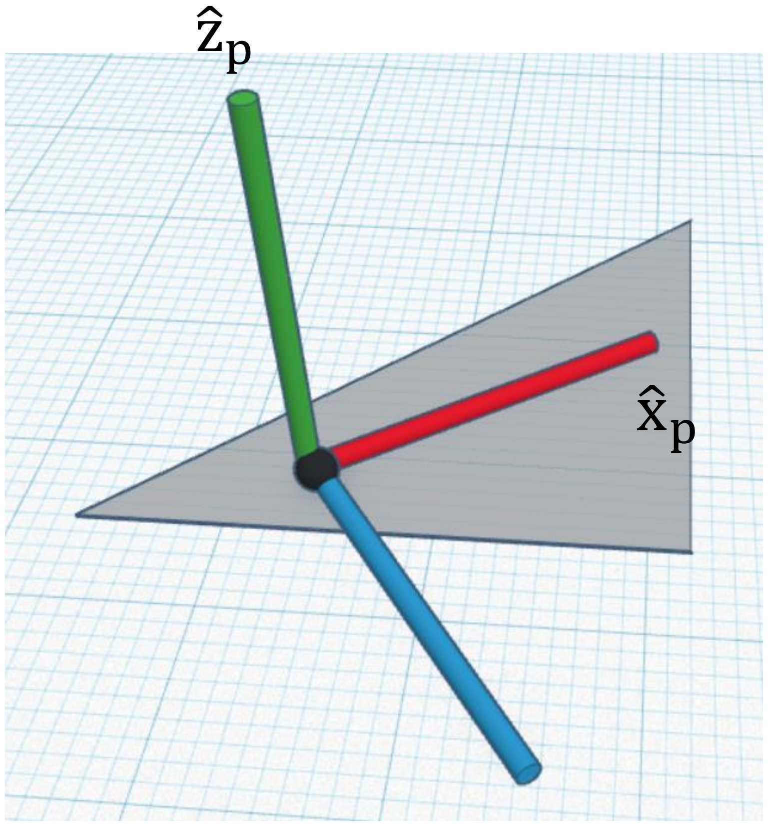
- Project p_i onto the plane, get the weighted average as $\hat{x}_p$

$$\hat{\mathbf{x}}_{\mathbf{p}} = \frac{1}{||\sum_{\mathbf{p}_i \in \mathcal{S}} \alpha_i \beta_i \mathbf{v}_i||_2} \sum_{\mathbf{p}_i \in \mathcal{S}} \alpha_i \beta_i \mathbf{v}_i$$

$$\underbrace{\mathbf{v}_i}_{\text{Projection of } p_i \text{ on to plane}} = \overrightarrow{\mathbf{pp}_i} - \underbrace{\langle \overrightarrow{\mathbf{pp}_i}, \hat{\mathbf{z}}_{\mathbf{p}} \rangle \hat{\mathbf{z}}_{\mathbf{p}}}_{\text{Projection of } p_i \text{ on to } \hat{z}_p}$$

$$\alpha_i = (r_{LRF} - ||\mathbf{p} - \mathbf{p}_i||_2)^2$$ Closer point → higher weights

$$\beta_i = \langle \overrightarrow{\mathbf{pp}_i}, \hat{\mathbf{z}}_{\mathbf{p}} \rangle^2$$ Points further away from plane → higher weights



$$\hat{y}_p = \hat{x}_p \times z_p$$

Registration recall
(higher means better descriptor matching)

	3DMatch data set			
	Original		Rotated	
	Average	STD	Average	STD
FPFH [28]	54.3	11.8	54.8	12.1
SHOT [38]	73.3	7.7	73.3	7.6
3DMatch [49][2]	57.3	7.8	3.6	1.7
CGF [17]	58.2	14.2	58.5	14.0
PPFNet [5]	62.3	11.5	0.3	0.5
PPF-FoldNet [4]	71.8	9.9	73.1	11.1
Ours (16 dim)	92.8	3.4	93.0	3.2
Ours (32 dim)	**94.7**	2.7	**94.9**	2.5

The Perfect Match

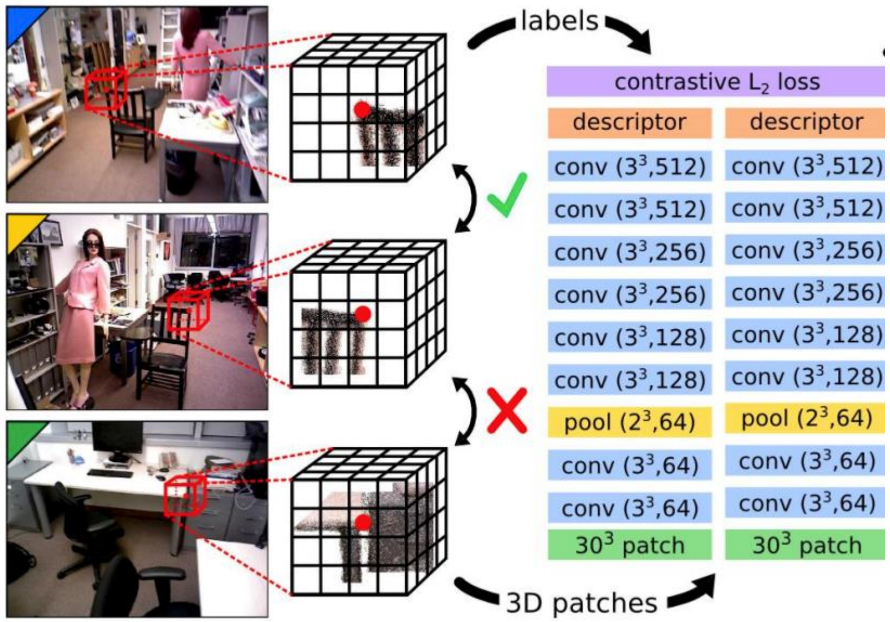
Smooth Density Value: another way to fill the voxel grid. (3DMatch utilizes Truncated Distance Function, TDF)

	3DMatch data set			
	Original		Rotated	
	$\tau_2 = 0.05$	$\tau_2 = 0.2$	$\tau_2 = 0.05$	$\tau_2 = 0.2$
All together	94.7	72.7	**94.9**	**72.8**
W/o SDV	92.5	63.5	92.5	63.6
W/o LRF	**96.3**	**81.6**	11.6	2.7
W/o SDV & LRF	95.6	78.6	9.7	2.1

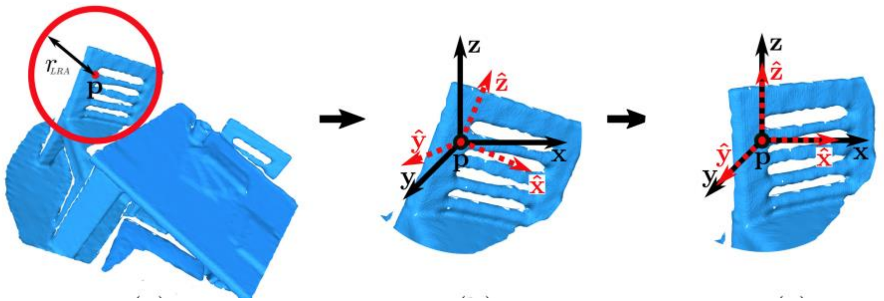
LRF is the key to make descriptors rotational robust

Source: The Perfect Match: 3D Point Cloud Matching with Smoothed Densities, Zan Gojcic, et.al.

- Extract patches around keypoints
  - Truncated Distance Function (TDF)
  - Smooth Density Value (SDV)
  - ...
- Processing
  - Local Reference Frame (LRF)
  - 3D convolutions
- Loss function
  - Contrastive loss
  - Triplet loss
    - Semi-hard / hard triplet mining
    - Online / Offline triplet mining



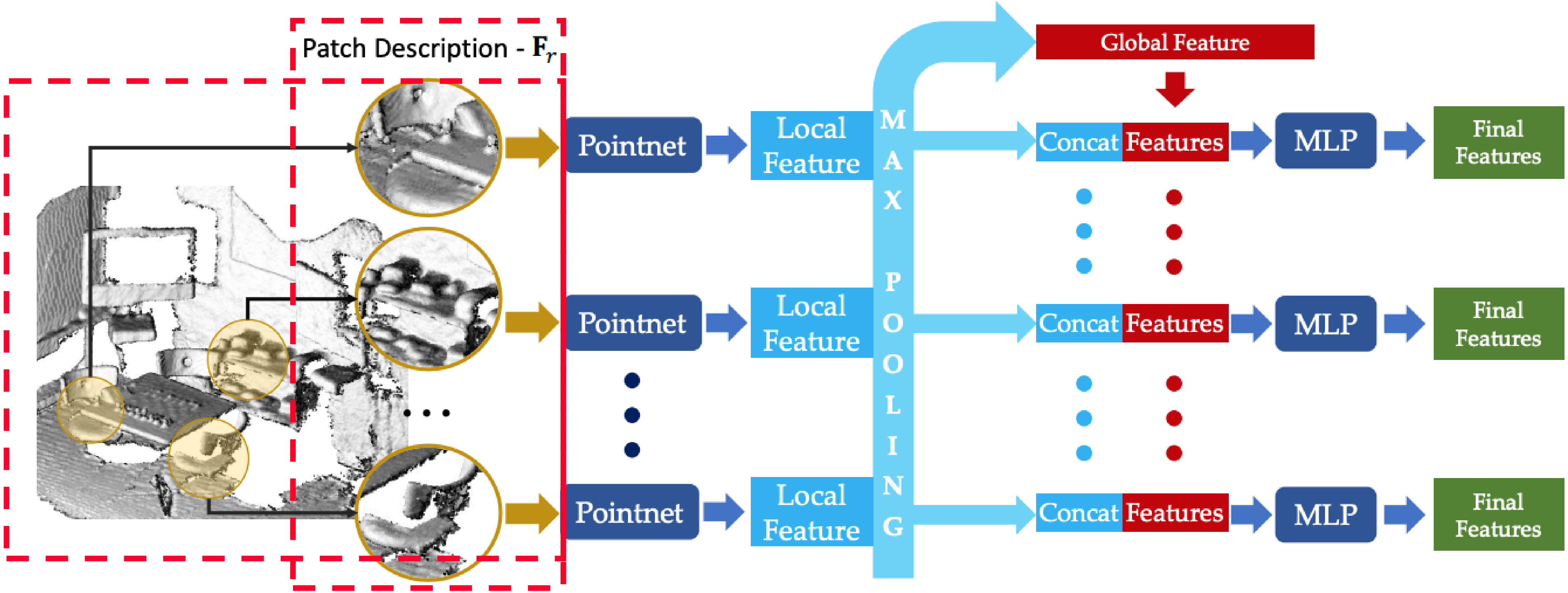
3DMatch - Architecture



The Perfect Match - Local Reference Frame

PPFNet – Point Pair Feature NETwork

How to encode a patch for PointNet



Consider all patches in one frame at the same time.

Source: PPFNet: Global Context Aware Local Features for Robust 3D Point Matching, Haowen Deng, et.al.

PPFNet – Point Pair Features (PPF)

- Simplest way
  - For query point $\mathrm{x_r} \in \mathbb{R}^3$, find its neighbors Ω by kNN/RadiusNN
  - Stack the M points into $M \times 3$ matrix

- Generally more information is better
  - Surface normal vectors improve classification/segmentation (Lecture 5)

- What else?
  - Pair wise features like the PFH?

PPFNet – Point Pair Features (PPF)

- For a pair of points $\mathrm{x}_1, \mathrm{x}_2$
- The PPF is

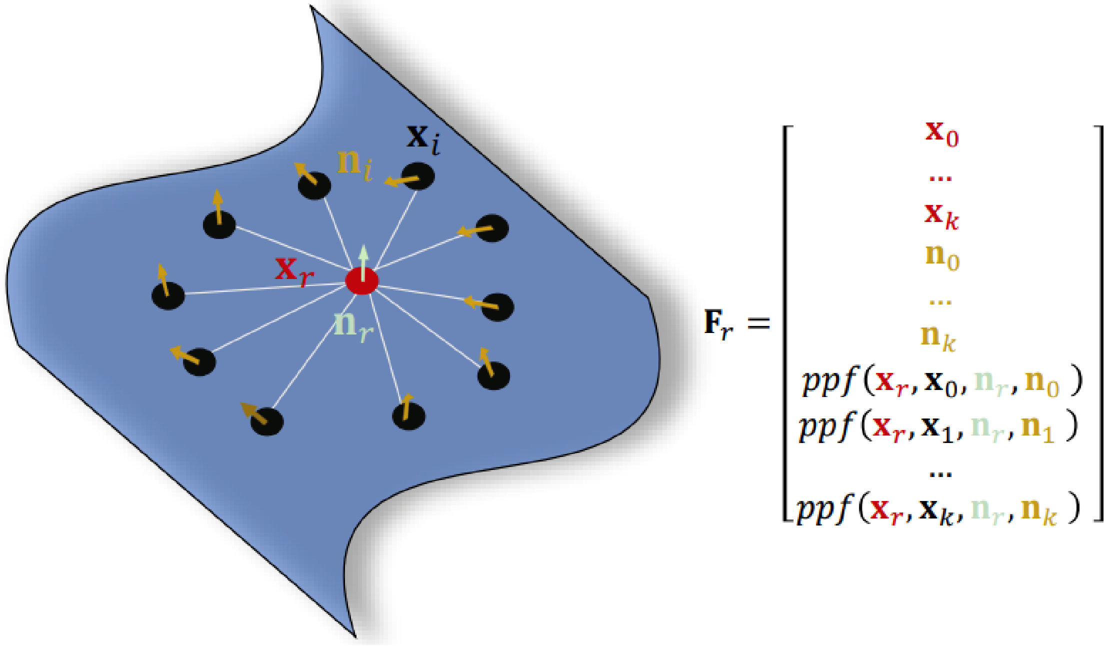
$$\psi_{12} = (\|\mathbf{d}\|_2, \angle(\mathbf{n}_1, \mathbf{d}), \angle(\mathbf{n}_2, \mathbf{d}), \angle(\mathbf{n}_1, \mathbf{n}_2))$$

$$\angle(\mathbf{v}_1, \mathbf{v}_2) = \mathrm{atan2}\left(\|\mathbf{v}_1 \times \mathbf{v}_2\|, \mathbf{v}_1 \cdot \mathbf{v}_2\right)$$

- For a patch around x_r,
    - Compute $\psi_{ri}, \forall i \in \Omega$
    - The PPF for the patch is

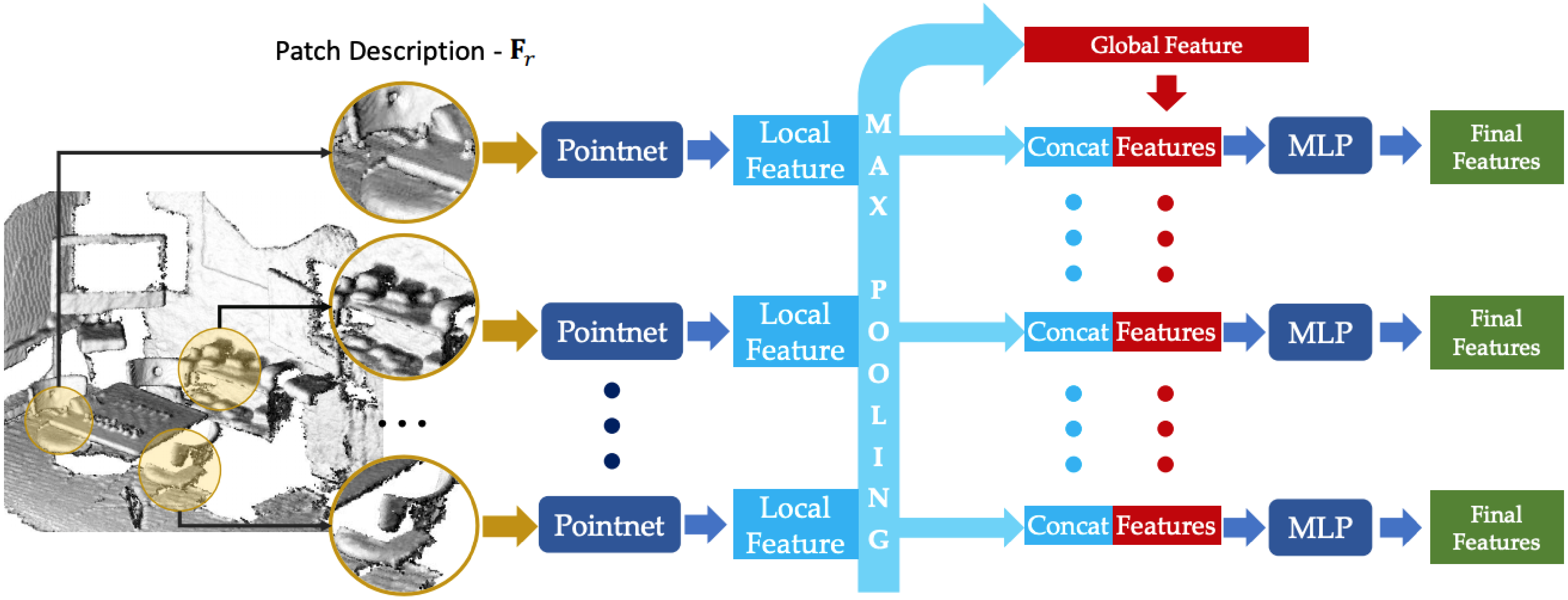
$$\mathbf{F}_r = \{\mathbf{x}_r, \mathbf{n}_r, \mathbf{x}_i, \cdots, \mathbf{n}_i, \cdots, \psi_{ri}, \cdots\}$$

Local Pathch Encoding



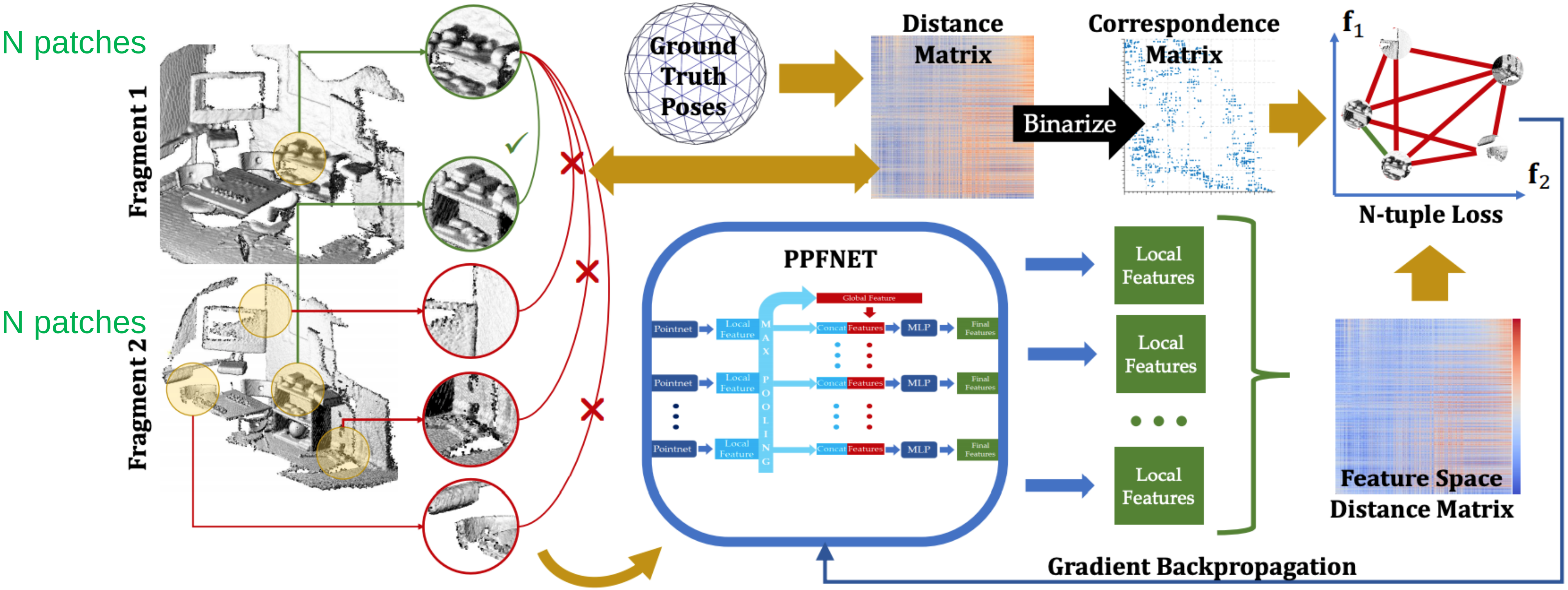
PPFNet

- Local Feature: feed F_r into a shared PointNet.
- Global Feature: maxpool of all local features in the same frame.
- Final Feature: concatenation of Local Feature and Global Feature + MLP



PPFNet – Training

- Input: A pair of frames (fragments) with known ground truth pose
- Positive pairs are trivial – Ground truth transformation is given at training
- Loss function – How to make use of “global” information?



- Contrastive Loss

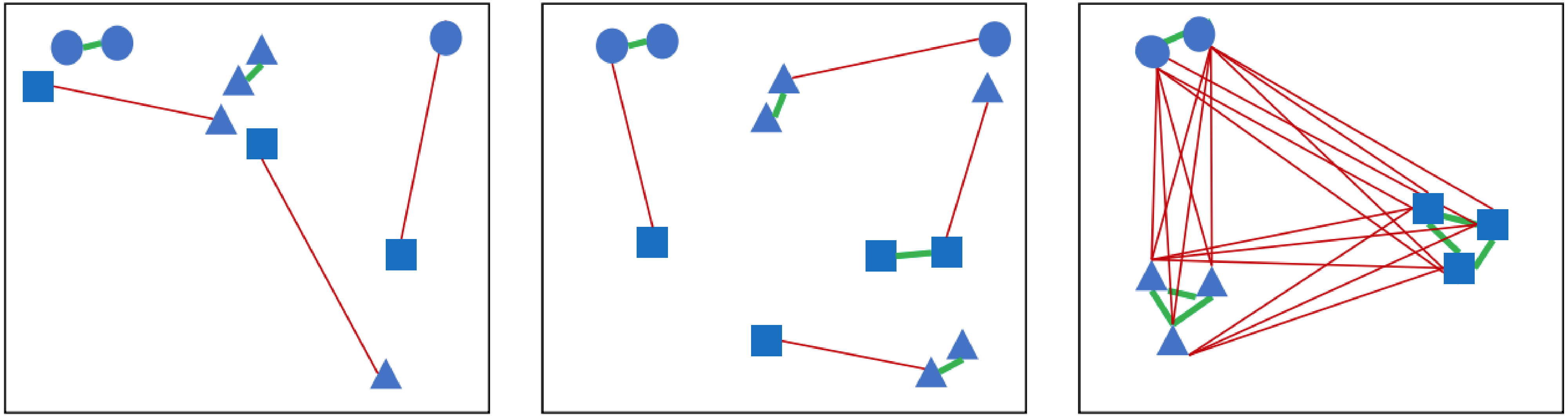
$$L = \frac{1}{N}\sum_{n=1}^{N} y_{ij}d_{ij}^2 + \left(1 - y_{ij}\right)\max\left(\tau - d_{ij}, 0\right)^2$$

  - Considers 2 samples at a time. A+N / A+P
- Triplet Loss

$$L = \sum_{i=1}^{N} L_i = \sum_{i=1}^{N} \max(d_i(a,p) - d_i(a,n) + \gamma, 0)$$

  - Considers 3 samples at a time. A + N + P
- Can we consider N samples at a time?
  - PPFNet considers two frames at a time
  - N samples at each frame

- N-tuple Loss
  - N keypoints in each of the two frames
  - N to N association → N tuples
  - Similar samples stay together, dis-similar samples stay away



(a) Pair Samples **(b)** Triplet Samples **(c)** N-Tuple Configuration

1. There are N patches in each frame
   - Frame 1: keypoint locations $\mathrm{x_i}$
   - Frame 2: keypoint locations $\mathrm{y_i}$
   - Transformation matrix between frames: T
2. Construct the Euclidean space patch distance matrix
   - Distance between N keypoints in frame1 to N keypoints in frame2
3. Keypoint distance matrix → Corresponding matrix $\mathbf{M} \in \mathbb{R}^{N \times N}$

$$m_{ij} = \mathbb{1}(\|\mathbf{x}_i - \mathbf{T}\mathbf{y}_j\|_2 < \tau)$$

Indicator function

Threshold to determine positive pairs

4. Compute feature space distance matrix $\mathbf{D} \in \mathbb{R}^{N \times N}$ between destriptors $f(\mathrm{x_i}), f(\mathrm{y_j})$. $f(\cdot)$ is the PPFNet.

$$d_{ij} = \|f(\mathbf{x}_i) - f(\mathbf{y}_j)\|_2$$

5. The N-tuple loss is given by

Negative descriptor distance

Positive descriptor distance

$$L = \sum{}^{*} \left(\frac{\boxed{\mathbf{M} \circ \mathbf{D}}}{\|\mathbf{M}\|_2^2} + \alpha \frac{max(\theta - \boxed{(1 - \mathbf{M}) \circ \mathbf{D}}, 0)}{N^2 - \|\mathbf{M}\|_2^2} \right)$$

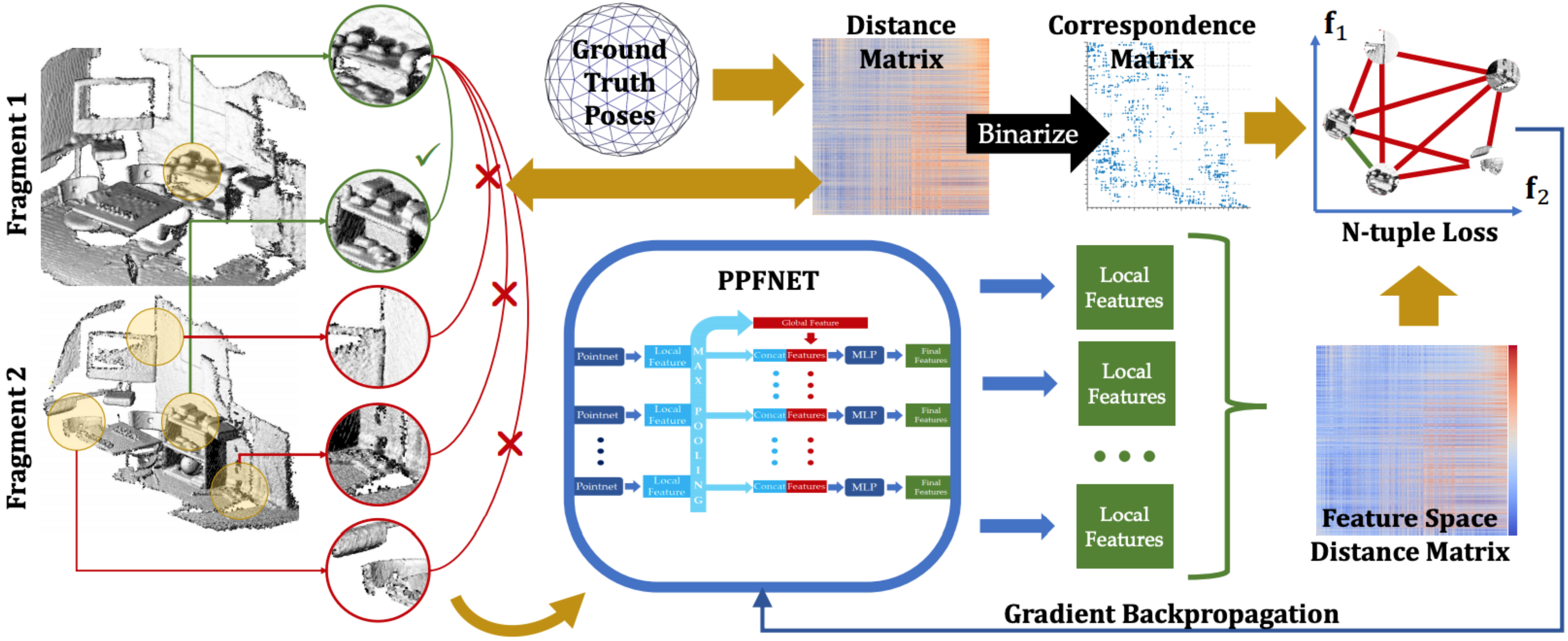
Number of 1 (# of positive matches)

Hyper-parameter that balance the weight between positive and negative

Margin of typical contrastive loss

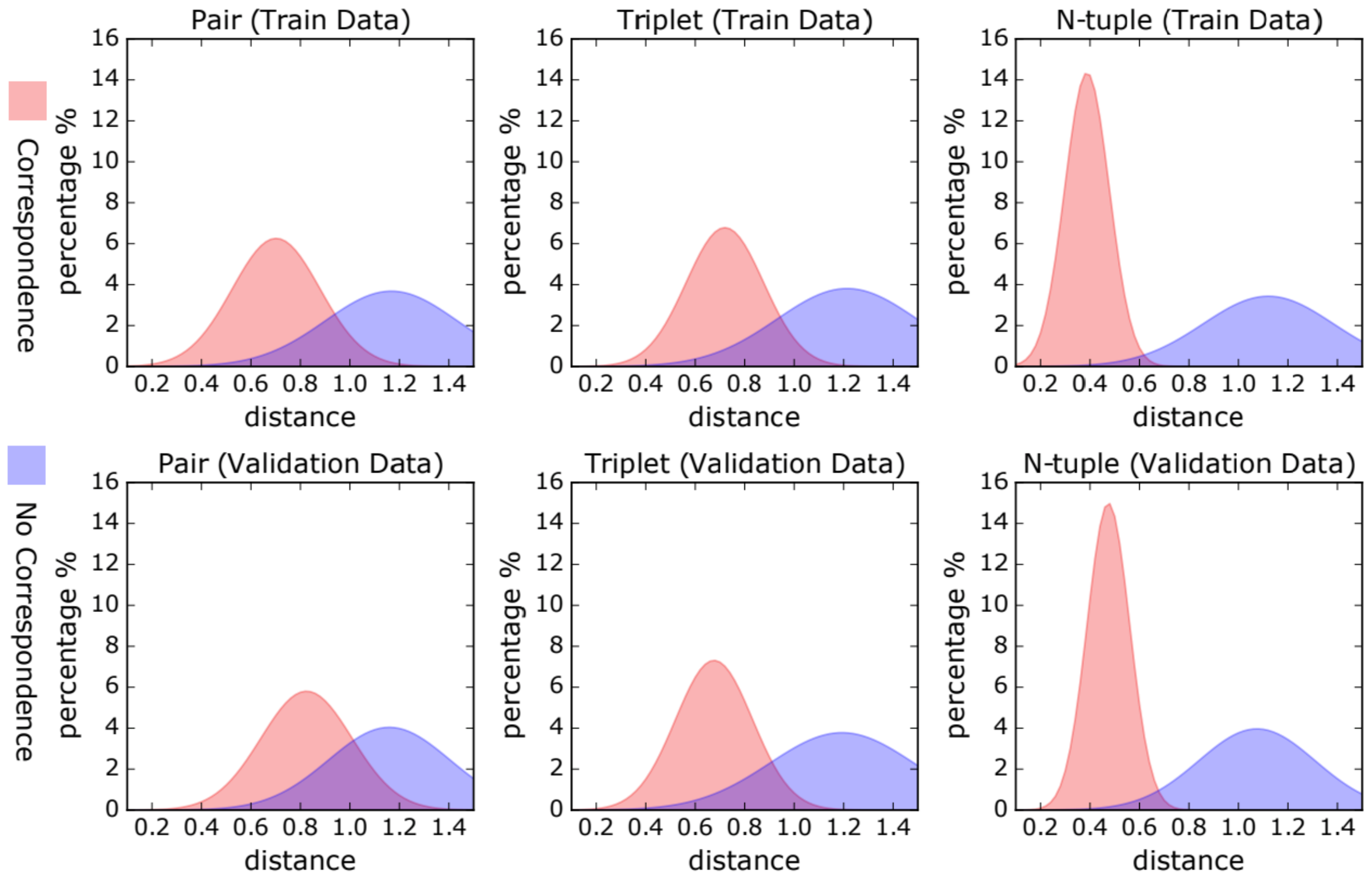
Element-wise multiplication

PPFNet – N-tuple Loss



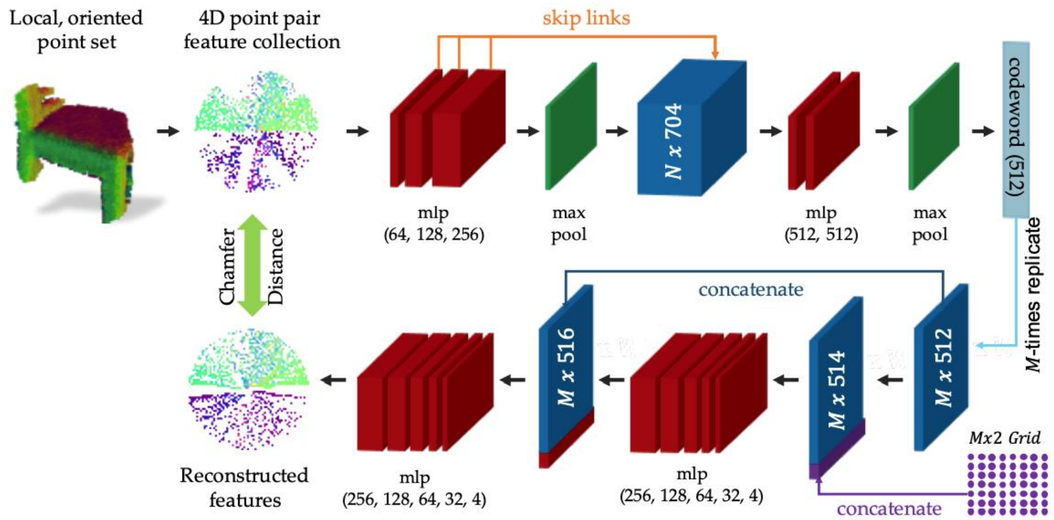
PPFNet – N-tuple Loss

- Pink: descriptor distance between positive pairs
- Purple: descriptor distance between negative pairs
- Column 1:
  Contrastive loss
- Column 2:
  Triplet loss
- Column 3:
  N-tuple loss

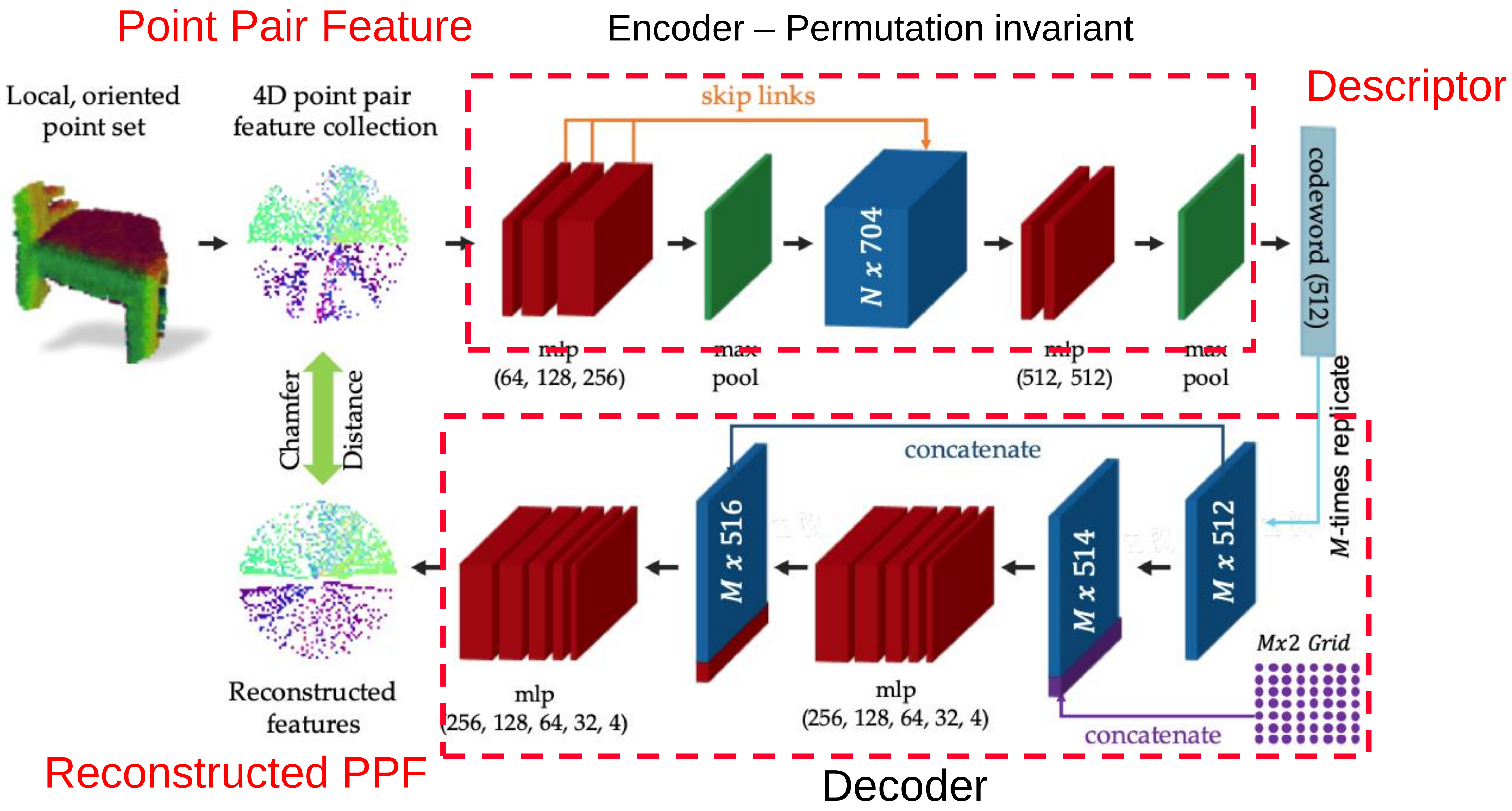


PPF-FoldNet – Descriptor with Autoencoder

- Common points about 3DMatch, PefectMatch, PPFNet
  - Relies on loss that pulls positive pairs & push negative pairs
  - Requires ground truth label of positive / negative, via ground truth pose
- No contrastive / triple loss? Unsupervised? – PPF-FoldNet



Source: PPF-FoldNet: Unsupervised Learning of Rotation Invariant 3D Local Descriptors, Haowen Deng, et.al.


Point Pair Feature
Encoder – Permutation invariant
Descriptor
Local, oriented point set
4D point pair feature collection
skip links
N x 704
codeword (512)
mlp (64, 128, 256)
max pool
mlp (512, 512)
max pool
Chamfer Distance
M-times replicate
concatenate
M x 516
M x 514
M x 512
Mx2 Grid
Reconstructed features
mlp (256, 128, 64, 32, 4)
mlp (256, 128, 64, 32, 4)
concatenate
Reconstructed PPF
Decoder

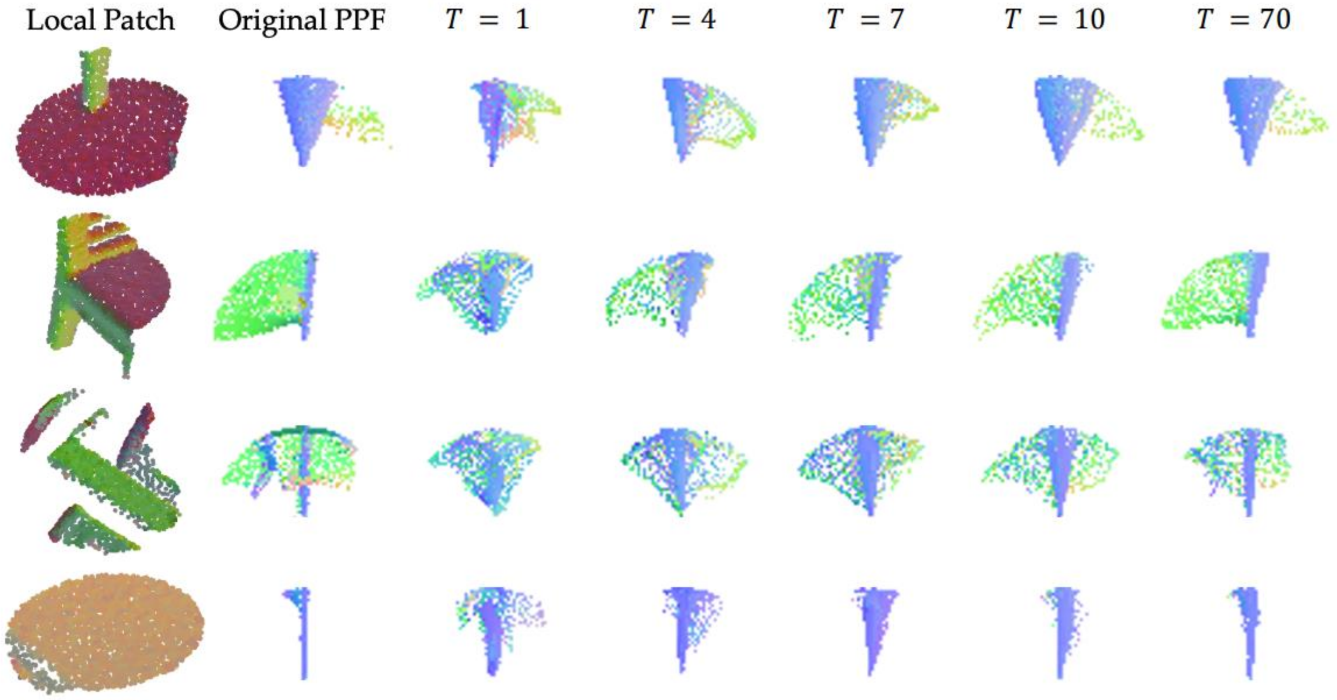
- Inference – Green. Training – Blue.

1. Convert a patch into PPF features $\mathrm{F} = \{\psi_{r1}, \cdots, \psi_{ri}, \cdots \psi_{rn}\} \in \mathbb{R}^{n \times 4}$
   $\psi_{12} = \left(\|d\|_2, \angle(n_1, d), \angle(n_2, d), \angle(n_1, n_2)\right)$ -- Rotation Invariant
2. Get the codeword (descriptor, length 512) with Encoder
3. Get the reconstructed PPF features $\hat{\mathrm{F}} \in \mathbb{R}^{n \times 4}$ with Decoder
4. Compute Chamfer distance between $\mathrm{F}, \hat{\mathrm{F}}$

Chamfer Loss

$$d(\mathbf{F}, \hat{\mathbf{F}}) = \max\left\{ \frac{1}{|\mathbf{F}|} \sum_{\mathbf{f} \in \mathbf{F}} \min_{\hat{\mathbf{f}} \in \hat{\mathbf{F}}} \|\mathbf{f} - \hat{\mathbf{f}}\|_2,\ \frac{1}{|\hat{\mathbf{F}}|} \sum_{\mathbf{f} \in \hat{\mathbf{F}}} \min_{\mathbf{f} \in \mathbf{F}} \|\mathbf{f} - \hat{\mathbf{f}}\|_2 \right\}$$

Original & reconstructed PPF in different training epochs



PPFFold-Net

Registration Recall on 3DMatch Dataset

	Spin Image [16]	SHOT [35]	FPFH [34]	3DMatch [51]	CGF [18]	PPFNet [8]	FoldNet [48]	Ours	Ours-5K
Kitchen	0.1937	0.1779	0.3063	0.5751	0.4605	**0.8972**	0.5949	0.7352	0.7866
Home 1	0.3974	0.3718	0.5833	0.7372	0.6154	0.5577	0.7179	0.7564	**0.7628**
Home 2	0.3654	0.3365	0.4663	**0.7067**	0.5625	0.5913	0.6058	0.6250	0.6154
Hotel 1	0.1814	0.2080	0.2611	0.5708	0.4469	0.5796	0.6549	0.6593	**0.6814**
Hotel 2	0.2019	0.2212	0.3269	0.4423	0.3846	0.5769	0.4231	0.6058	**0.7115**
Hotel 3	0.3148	0.3889	0.5000	0.6296	0.5926	0.6111	0.6111	0.8889	**0.9444**
Study	0.0548	0.0719	0.1541	0.5616	0.4075	0.5342	**0.7123**	0.5753	0.6199
MIT Lab	0.1039	0.1299	0.2727	0.5455	0.3506	**0.6364**	0.5844	0.5974	0.6234
Average	0.2267	0.2382	0.3589	0.5961	0.4776	0.6231	0.6130	0.6804	**0.7182**

PPFFold-Net

Registration Recall on Rotated 3DMatch Dataset

	Spin Image [16]	SHOT [35]	FPFH [34]	3DMatch [51]	CGF [18]	PPFNet [8]	FoldNet [48]	Ours	Ours-5K
Kitchen	0.1779	0.1779	0.2905	0.0040	0.4466	0.0020	0.0178	0.7352	**0.7885**
Home 1	0.4487	0.3526	0.5897	0.0128	0.6667	0.0000	0.0321	0.7692	**0.7821**
Home 2	0.3413	0.3365	0.4712	0.0337	0.5288	0.0144	0.0337	0.6202	**0.6442**
Hotel 1	0.1814	0.2168	0.3009	0.0044	0.4425	0.0044	0.0133	0.6637	**0.6770**
Hotel 2	0.1731	0.2404	0.2981	0.0000	0.4423	0.0000	0.0096	0.6058	**0.6923**
Hotel 3	0.3148	0.3333	0.5185	0.0096	0.6296	0.0000	0.0370	0.9259	**0.9630**
Study	0.0582	0.0822	0.1575	0.0000	0.4178	0.0000	0.0171	0.5616	**0.6267**
MIT Lab	0.1169	0.1299	0.2857	0.0260	0.4156	0.0000	0.0260	0.6104	**0.6753**
Average	0.2265	0.2337	0.3640	0.0113	0.4987	0.0026	0.0233	0.6865	**0.7311**

Source: PPF-FoldNet: Unsupervised Learning of Rotation Invariant 3D Local Descriptors, Haowen Deng, et.al.

Deep Learning Descriptors

	Representation	Loss Function	Rotation Handling
3DMatch	Voxel grid	Contrastive Loss	No
PerfectMatch	Voxel grid	Triplet Loss	Local Reference Frame
PPFNet	Coordinates + surface normal+ PPF	N-tuple Loss	No
PPF-FoldNet	PPF	Reconstruction Chamfer	Rotation invariant

Descriptor Summary

- Handcrafted
  - PFH, FPFH, SHOT
  - Advantages
    - Captures local surface variations
    - Invariant to 6D pose
  - Disadvantages:
    - Sensitive to surface normal estimation
    - Sensitive to noise / occlusion / sparsity.
- Deep Learning
  - 3DMatch, PerfectMatch, PPFNet, PPF-FoldNet
  - Advantages:
    - More distinctive, better performance
    - Robust to noise / occlusion / sparsity
  - Disadvantages:
    - Requires GPU
    - Requires training
    - Generalization

Homework

- Implement your own ISS keypoint detection.

- Implement feature descriptors FPFH, SHOT
  - You may call PCL library via python binding OR implement your own version.
  - Example of python binding of PCL: https://github.com/lijx10/PCLKeypoints
  - Test with ModelNet40 to ensure correctness.
    - For example, find a symmetric table, compute descriptor around the end-point of each leg. Check whether the descriptors are similar. Submit a report.
    - The correctness will be reflected in Lecture 9 homework.

- Register two point clouds by feature detection and description
  - Homework of Lecture 9.