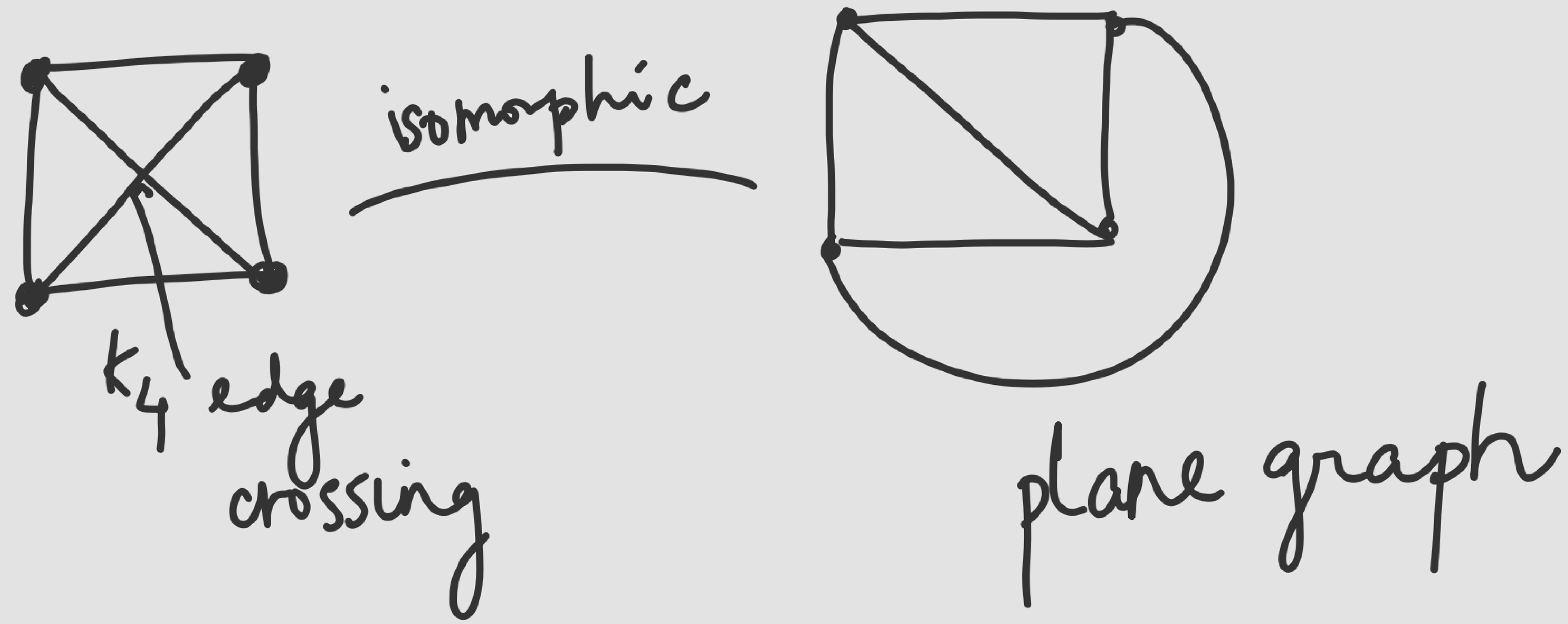
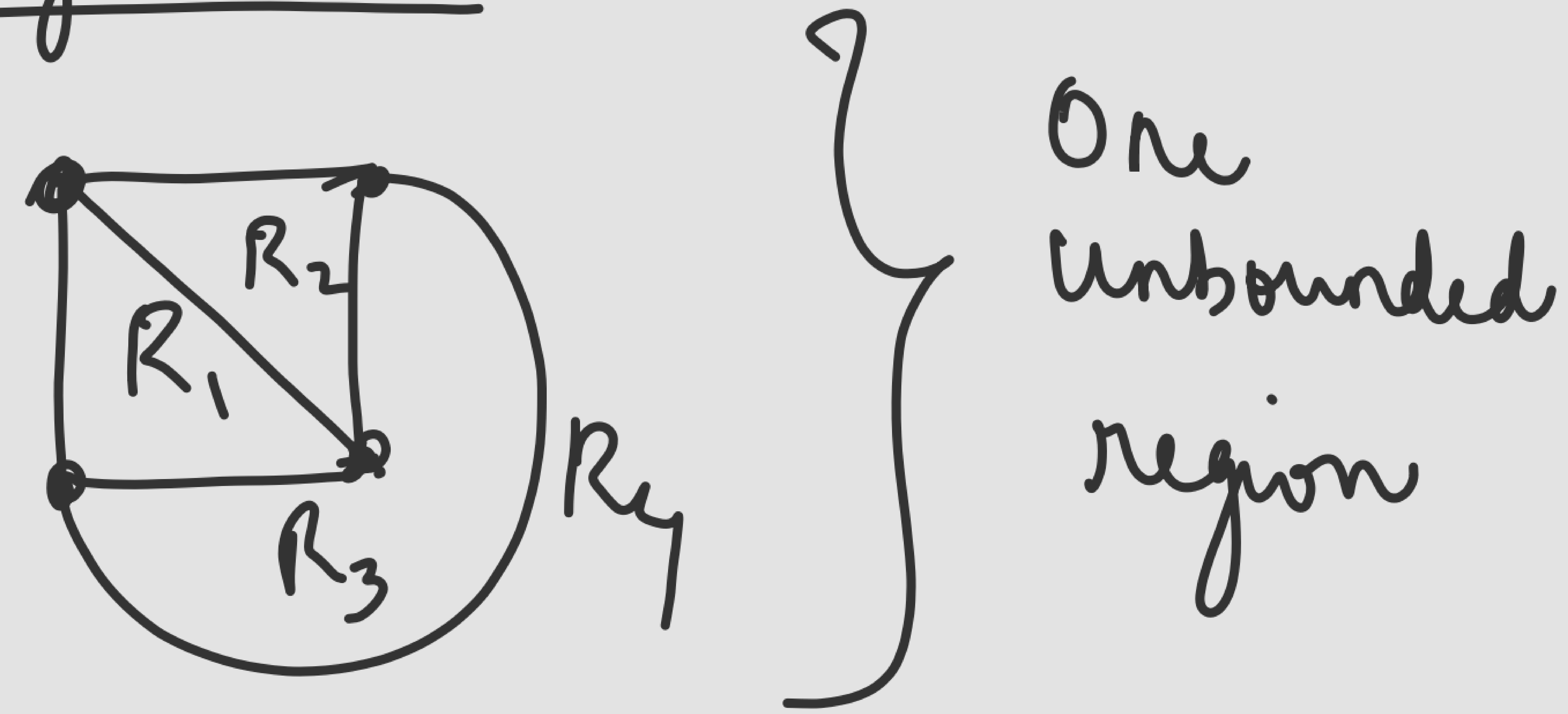
Planar Graphs

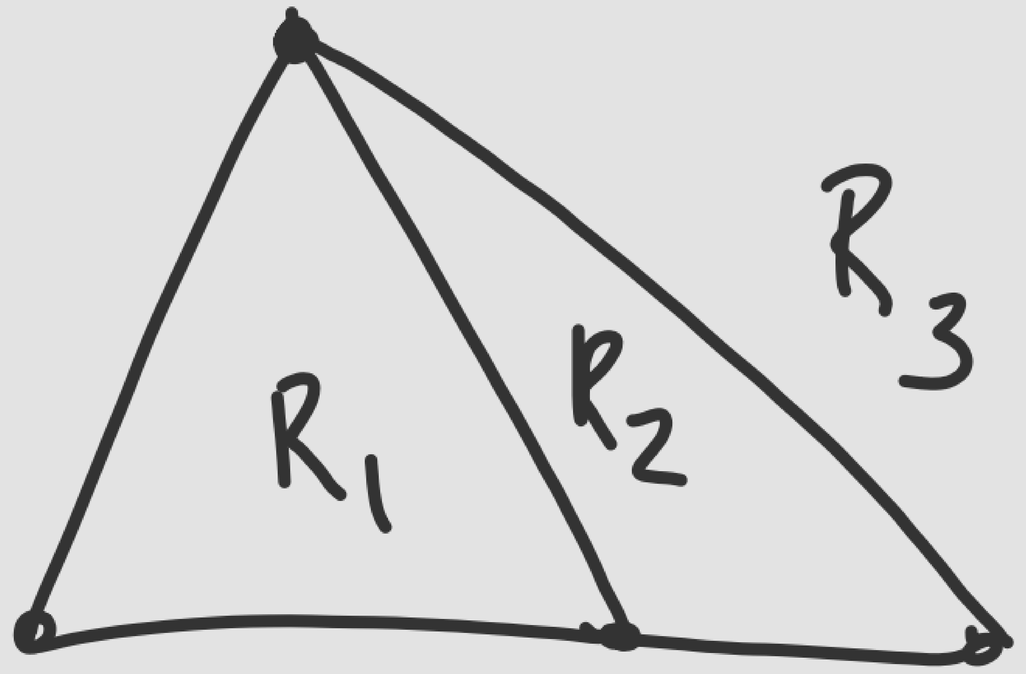


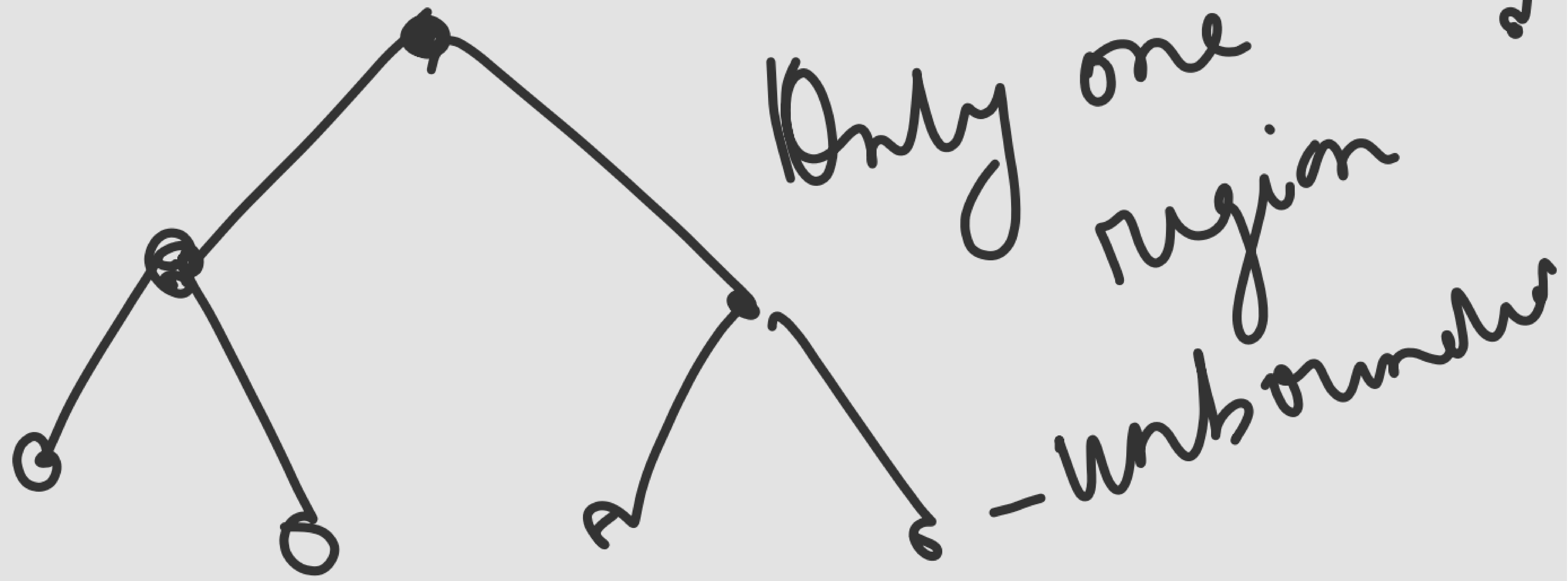
- planar graph — can be drawn on a plane without edge crossing

plane graph — Planar graph drawn in a plane without edge crossing.

regions

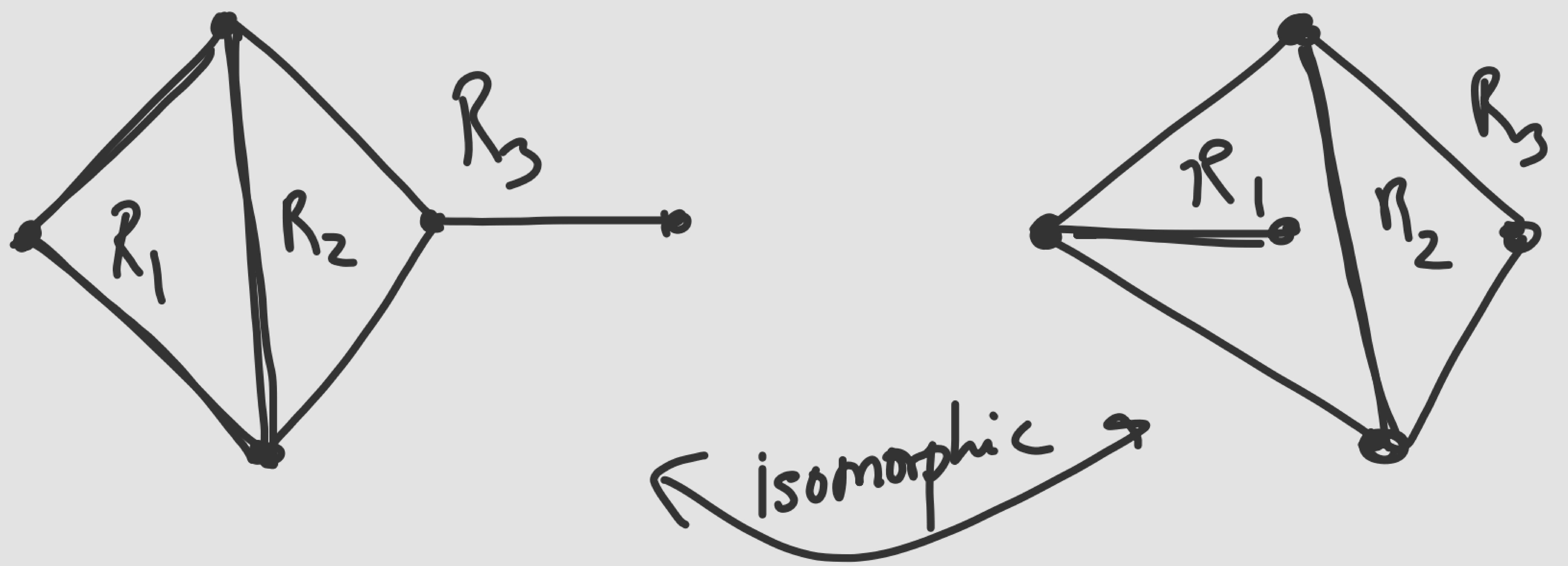




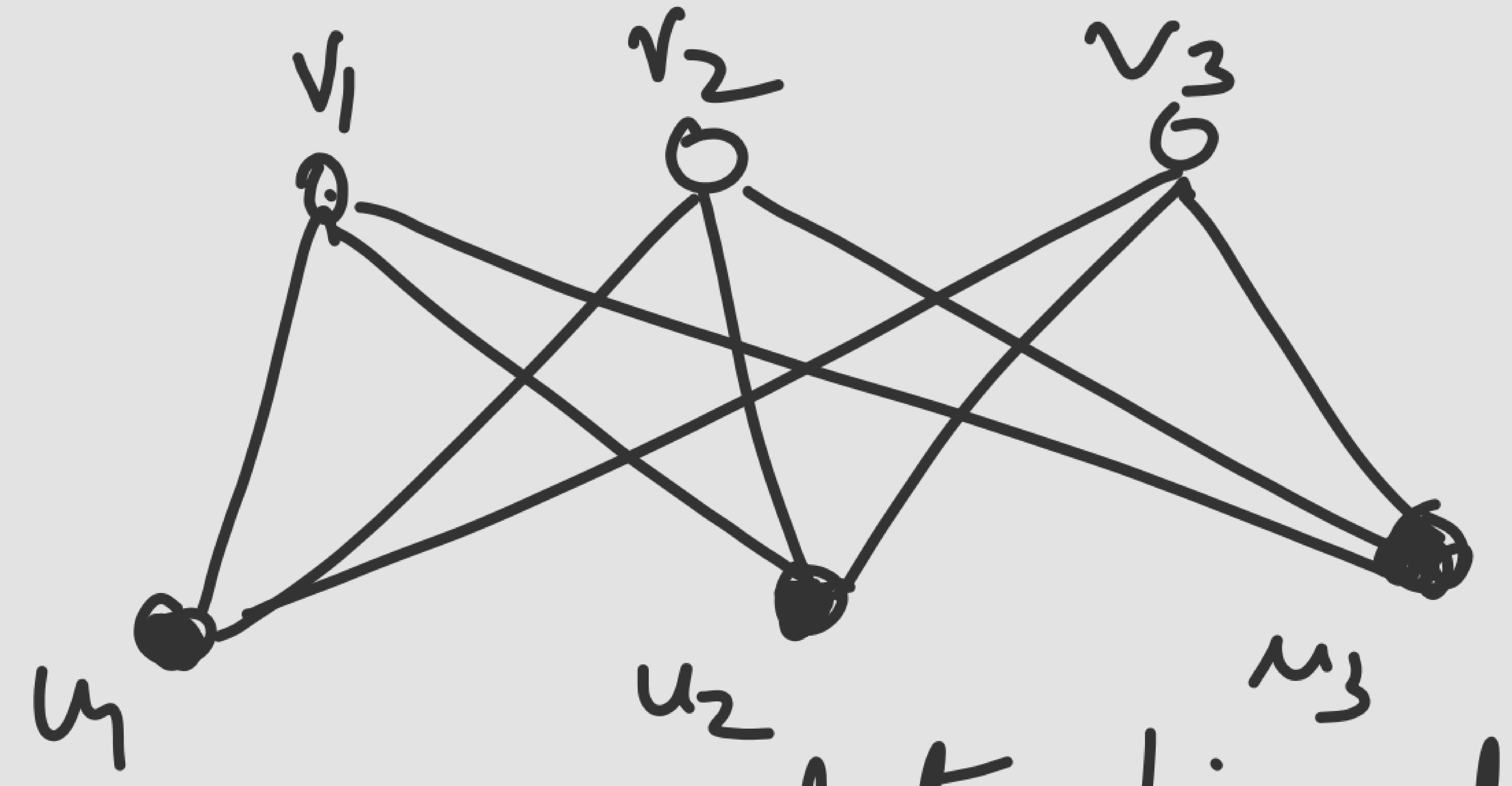


Acyclic graphs / trees
Connected

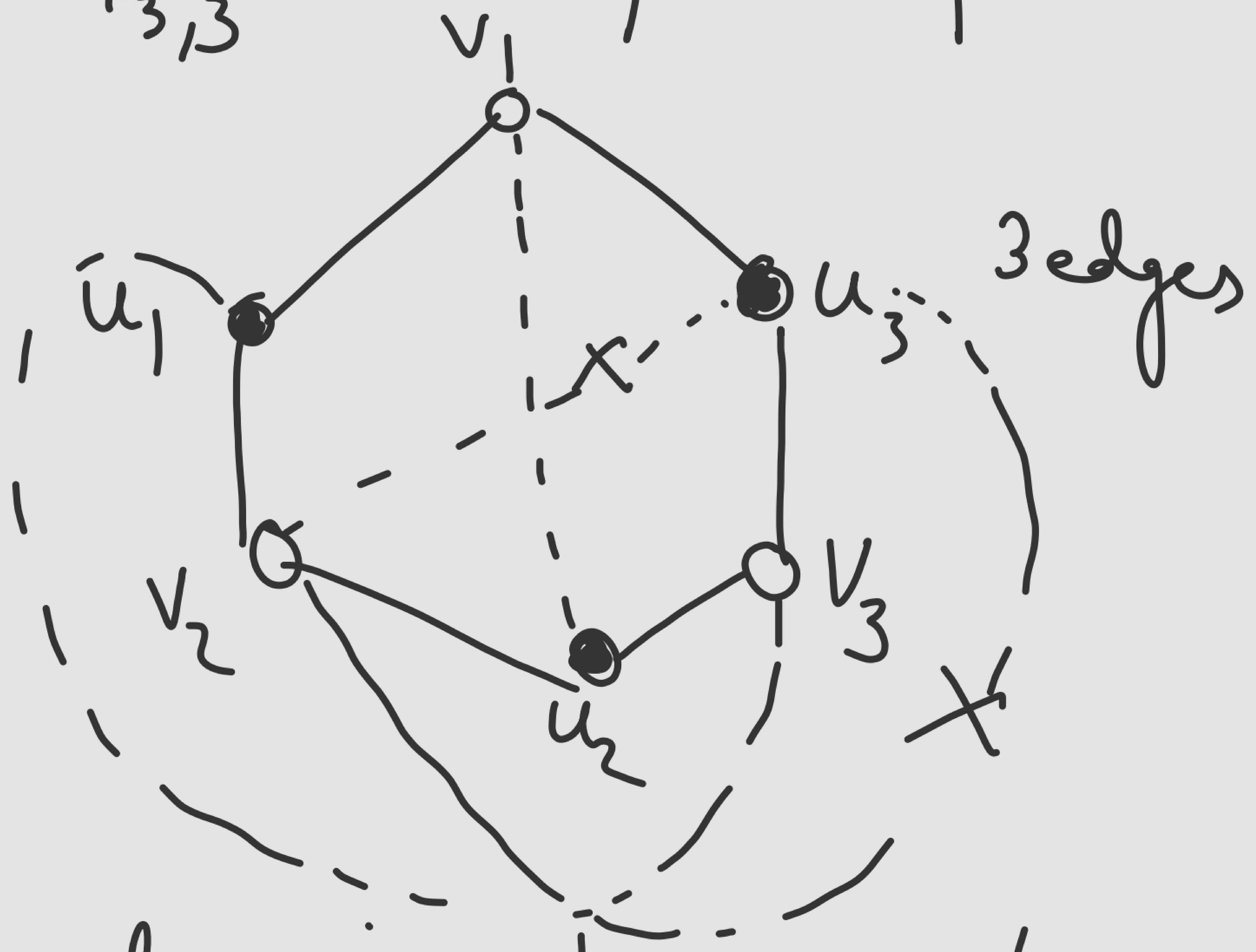
Only one region — Unbounded



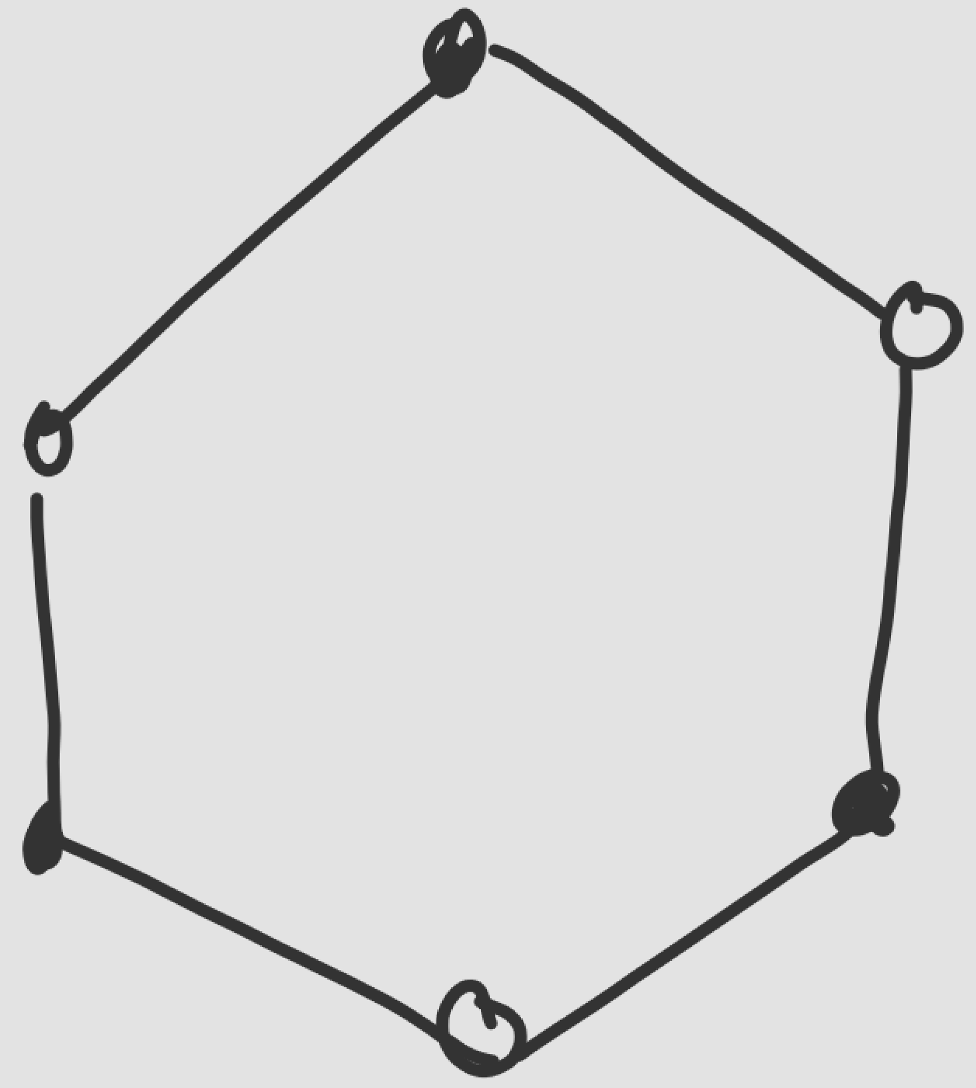
— They are not same plane graph — $\boxed{\text{boundary}}$ of R_1 is different



$K_{3,3}$ — complete bipartite



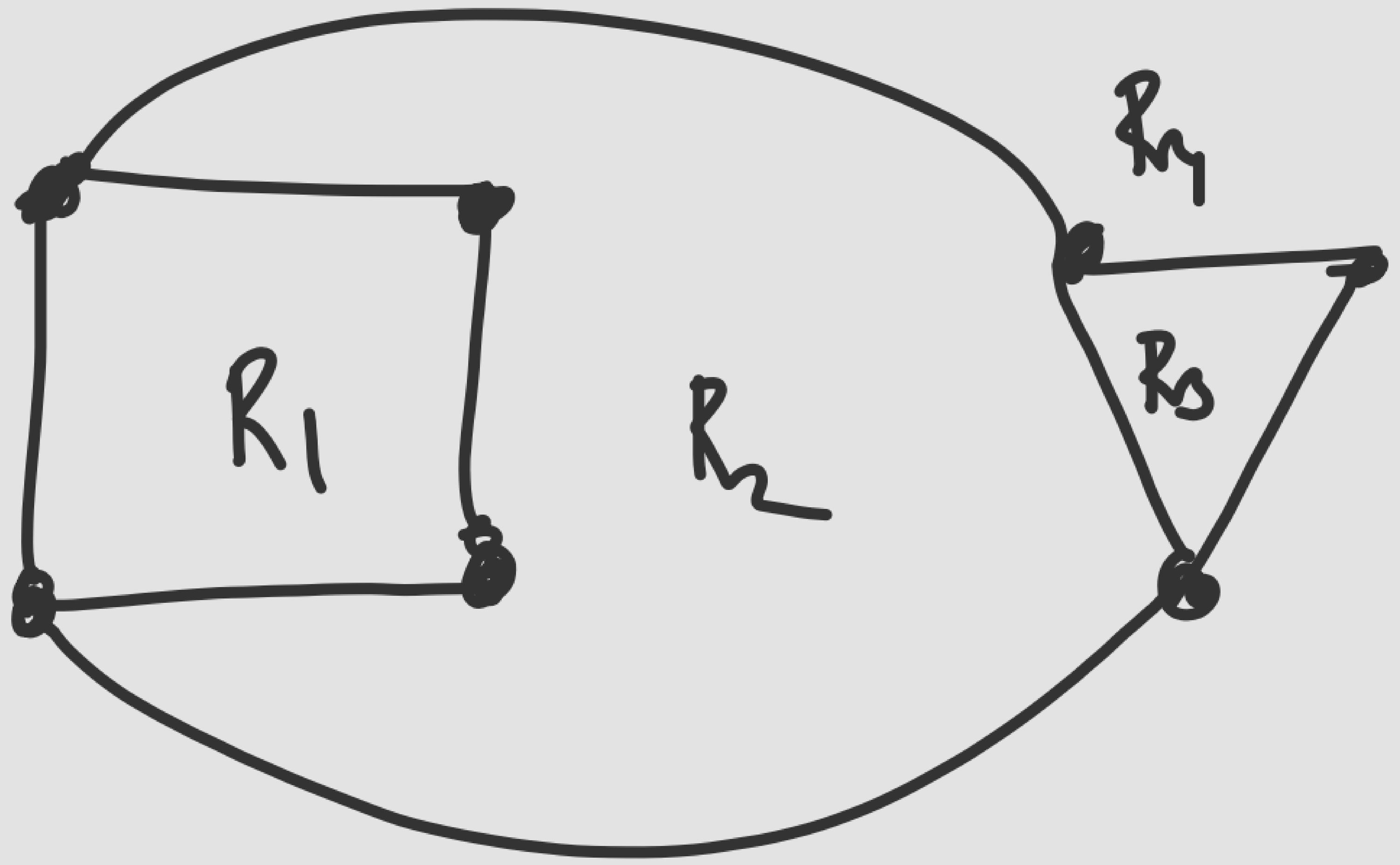
edgecrossing — $K_{3,3}$ is not planar



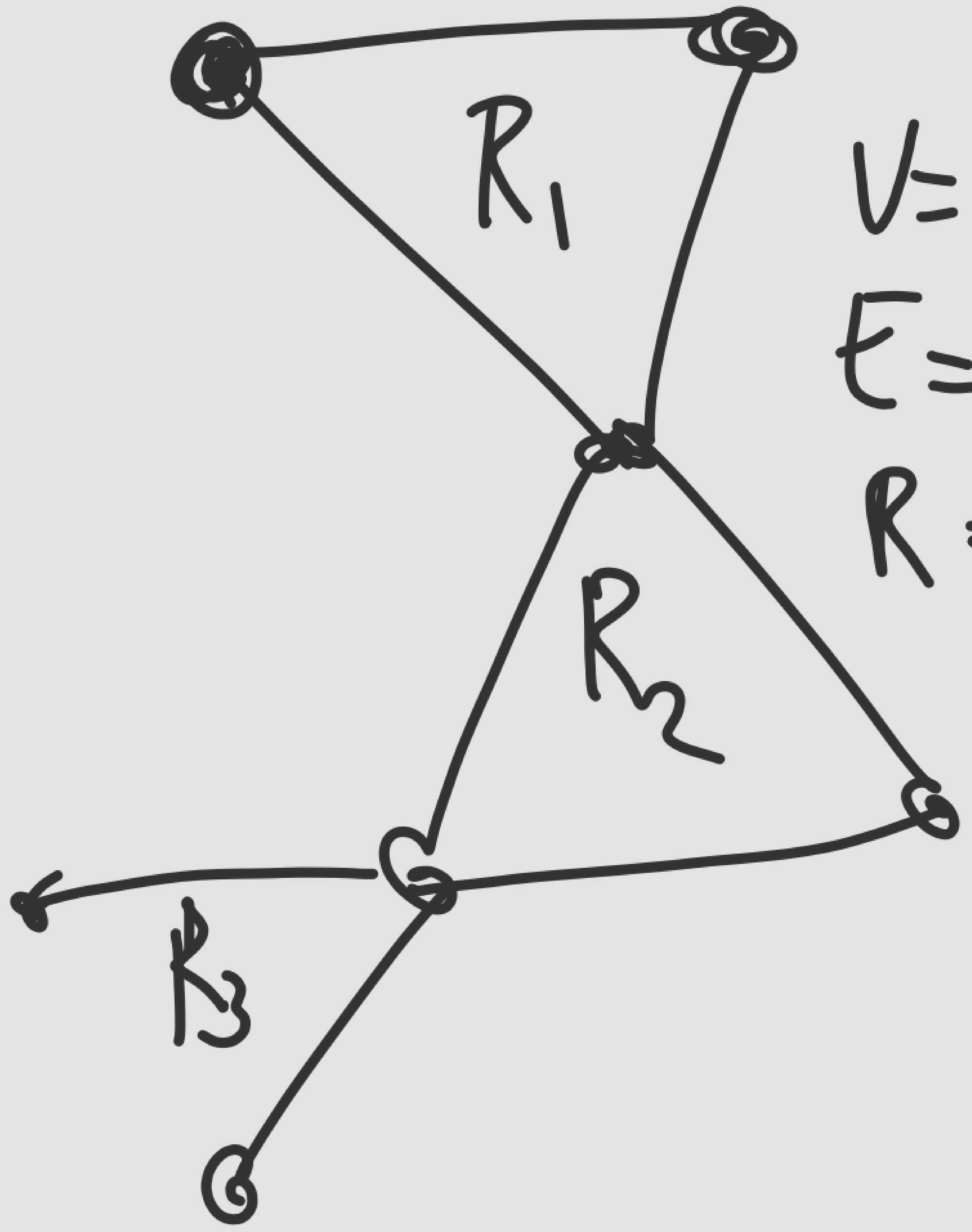
3 edges and only
2 regions
(interior, exterior)

By pigeohole principle, atleast there will be one edge crossing

Hence, $K_{3,3}$ is non-planar



$$\left.\begin{aligned} V = n &= 7 \\ E = m &= 9 \\ r &= 4 \end{aligned}\right\} \quad n - m + r = 2$$



$V = n = 7$
$E = m = 8$
$R = r = 3$

$n - m + r = 2$

Euler's Theorem/ formula for planarity

connected planar graph

Induction on no. of edges

Base case:

$m = 0$

$n = 1 \quad r = 1 \quad v_1$

isolated vertex

$n - m + r = 2$

It holds for $m = 0$.

Induction hypothesis

If $G(V,E)$ is a connected planar graph with $V = n$, $E < m$ such that $m \geq 1$ then

acyclic

$m = n - 1$, $r = 1$

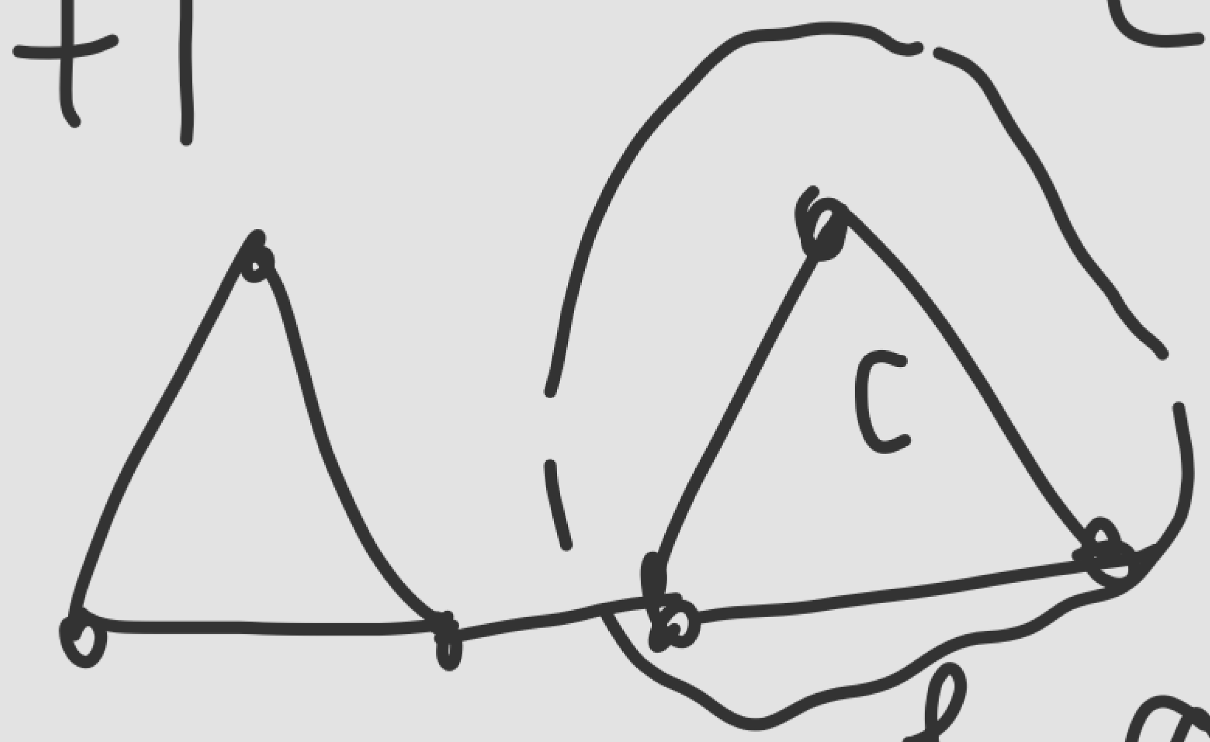
$n - m + r$

$= n - (n-1) + 1$

$= 2$

cyclic

Let e be an edge in cycle C. Hence e is not a bridge



Remove 'e' from G

(G-e)

$n' = n$

$m' = m-1$

$r' = r-1$

$n' - m' + r'$

$= n - (m-1) + (r-1)$

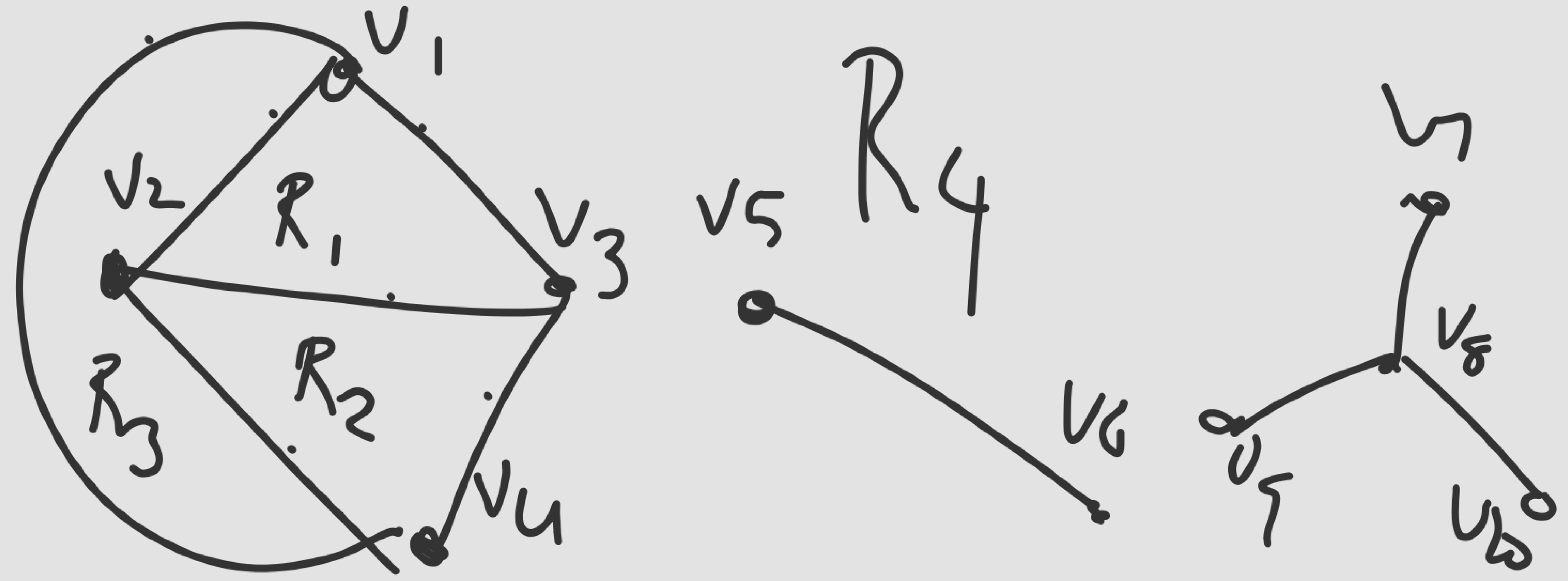
$= \underbrace{n - m + r}$

$= 2$ (Using induction hypothesis)

Hence for any connected planar graph $G(V,E)$ with $V=n$ and $E=m$

$n-m+r=2$

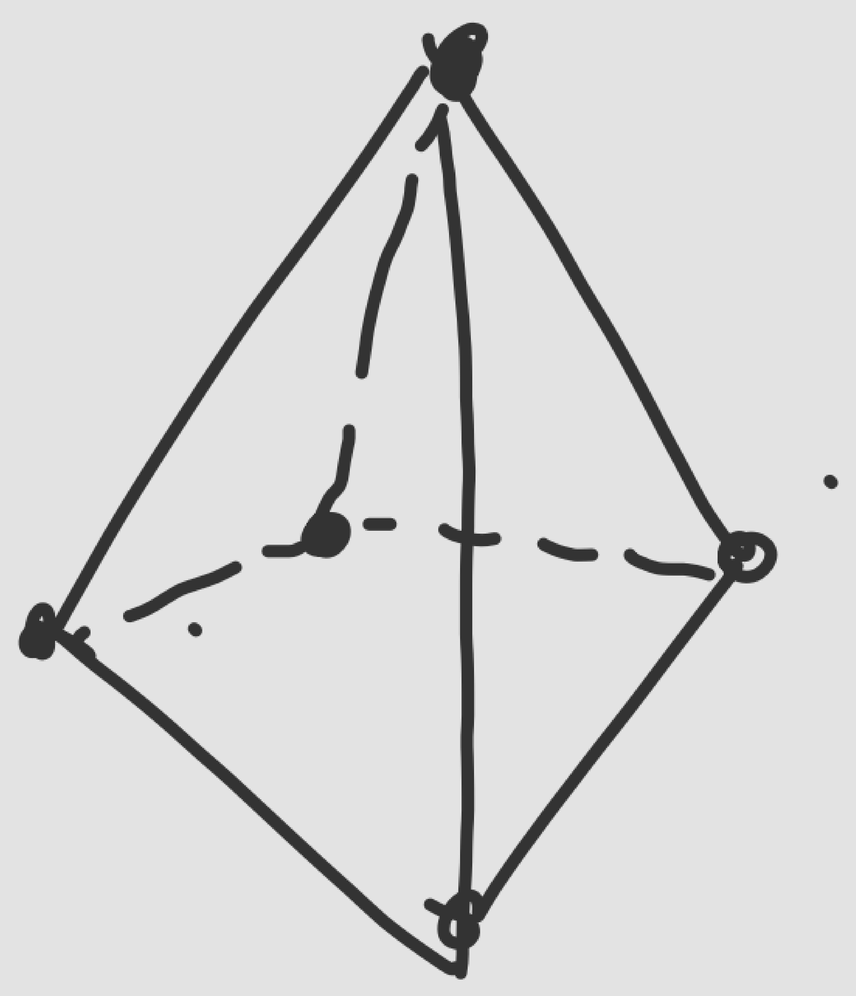
<u>Corollary</u>

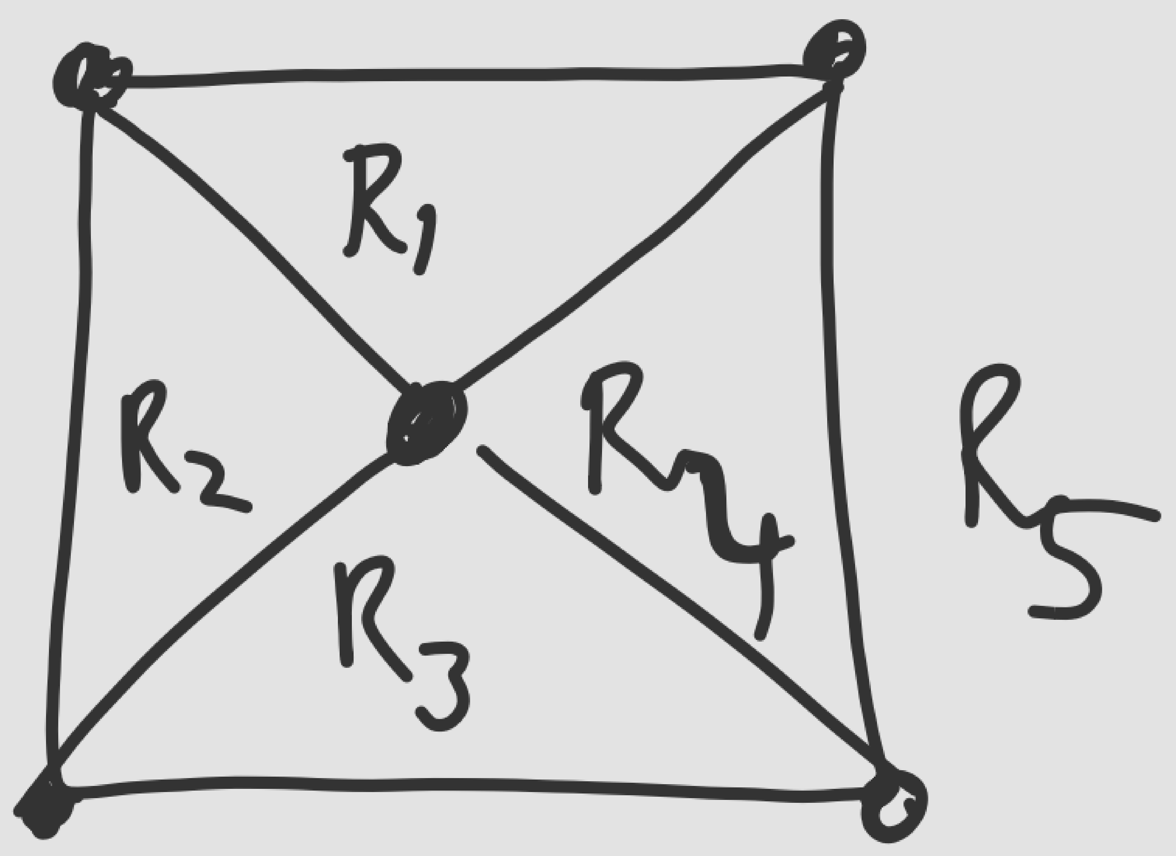


$\underbrace{n-m+r}_{=1+c(G)} = 10-10+4$

$c(G) \rightarrow$ connected component

Euler's formula for Polyhedron





$V = 5$

$E = 8$

$F = 5$

$V - E + F = 5 - 8 + 5$

$= 2$

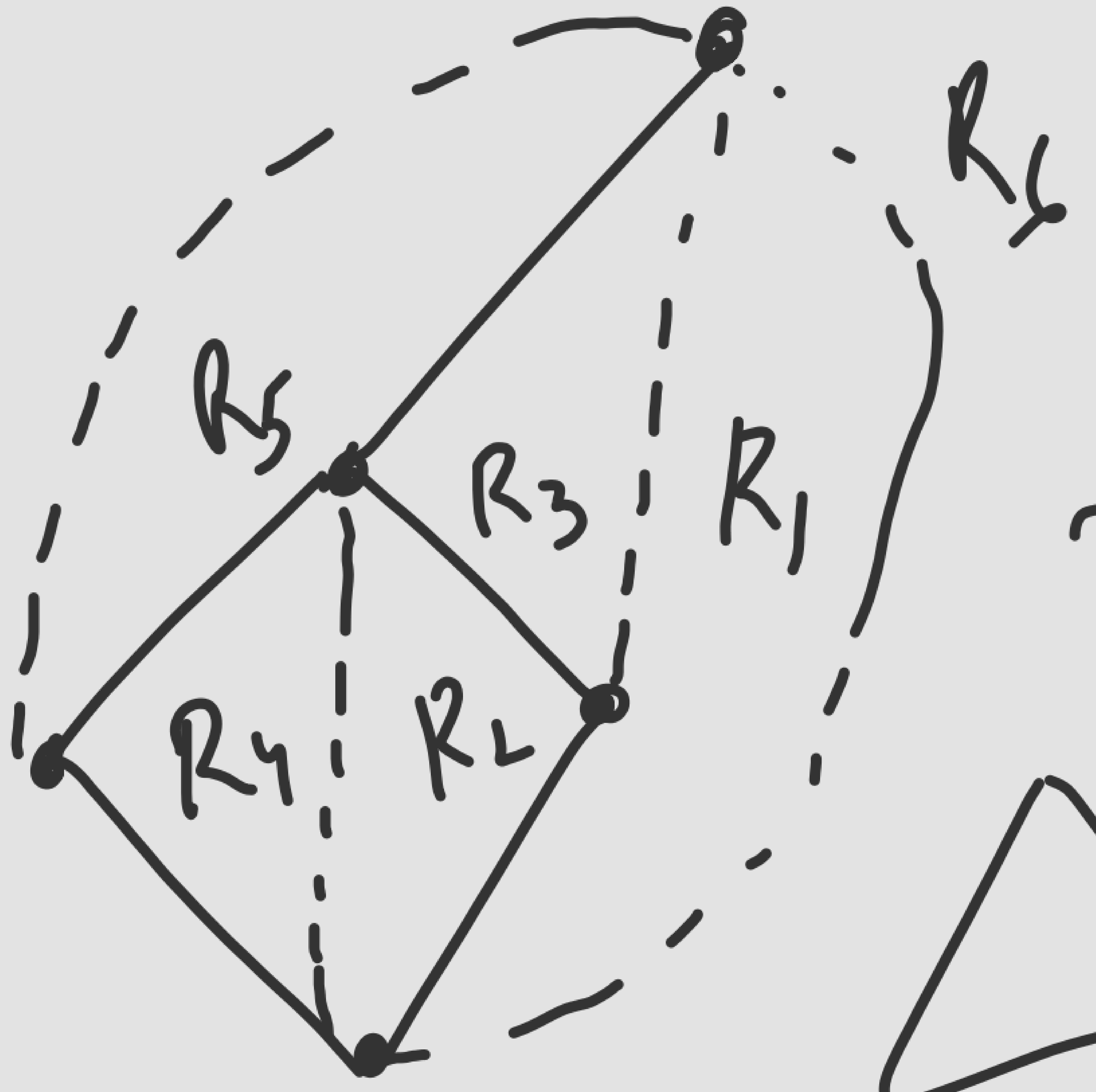
$n = 5$

$m = 8$

$r = 5$

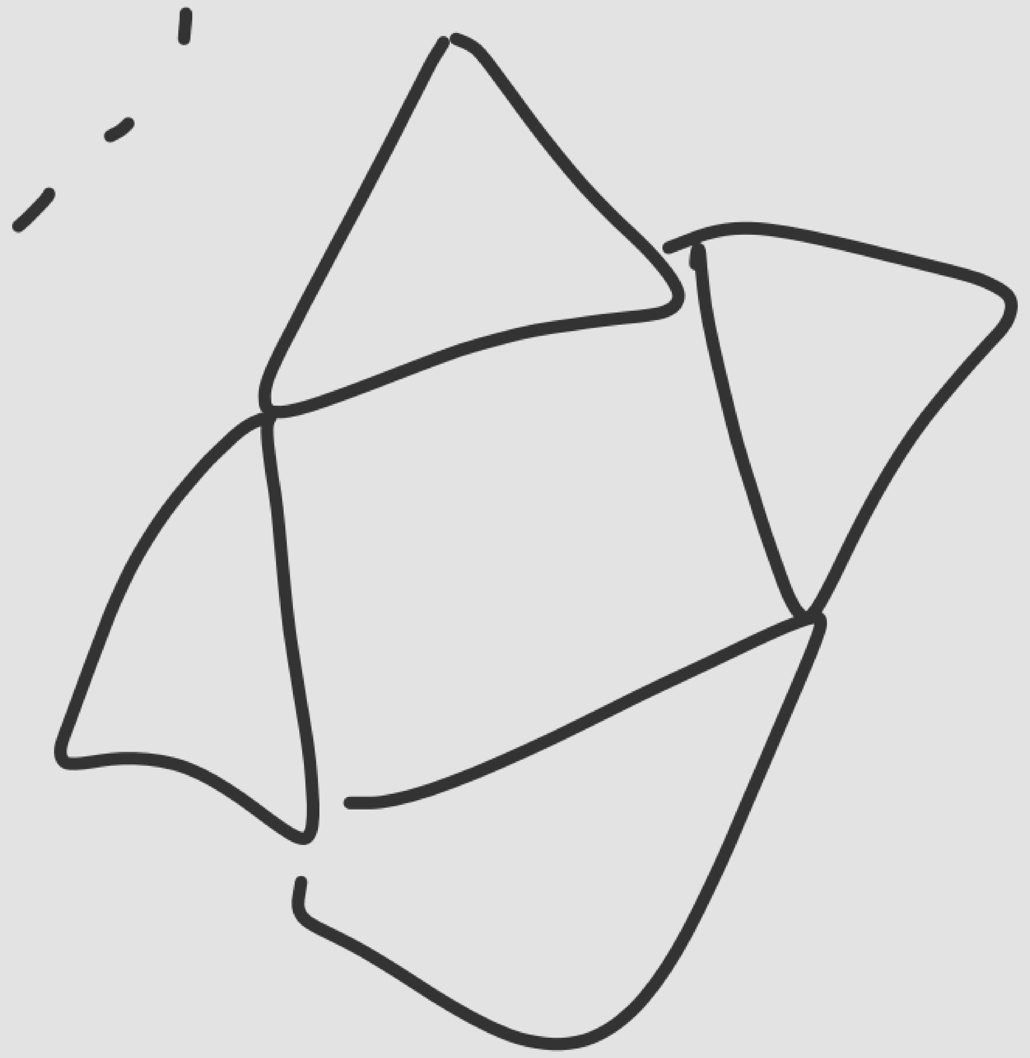
$n - m + r = 2$

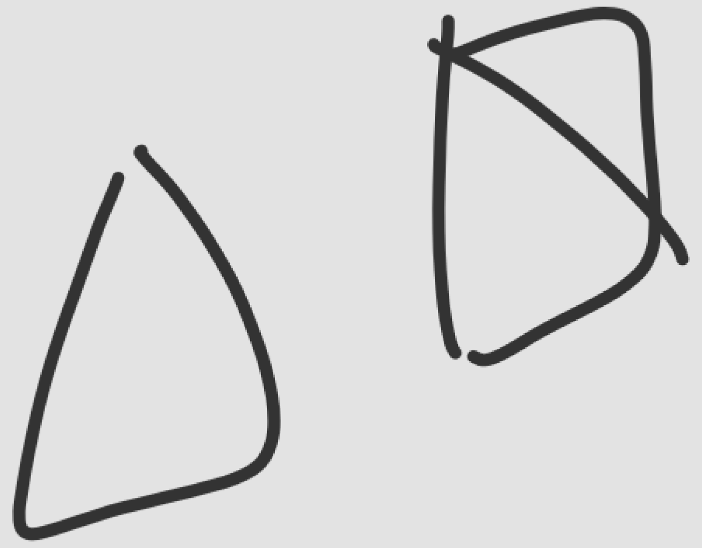
$V - E + F = 2$



→ maximal planar graph

plane/planar graph





Δgular regions

$$\sum_{\text{all edges of } G} \#\text{ edges over region } R$$

$$\boxed{2m = 3r}$$

maximal planar graphs

$$n - m + r = 2$$

$$n - m + \frac{2m}{3} = 2$$

$$\Rightarrow 3n - m = 6$$

$$m = 3n - 6$$

$\boxed{m = 3n - 6}$

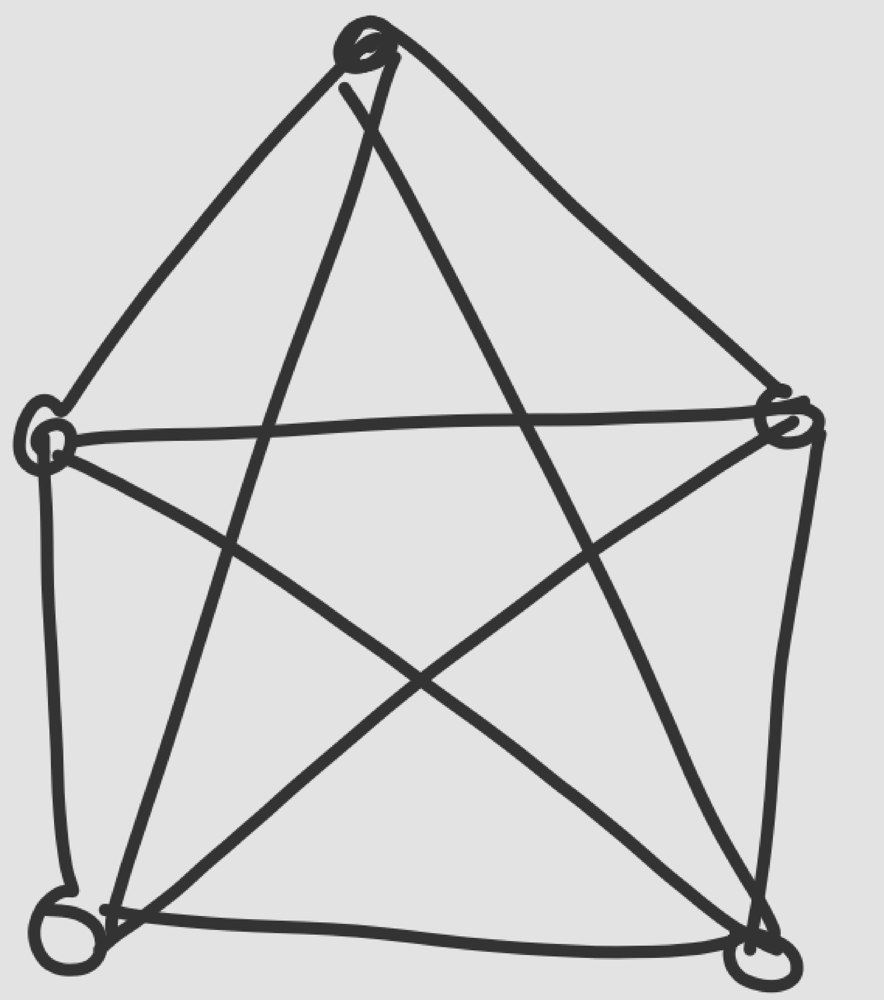
$m' \leq m$

$\boxed{m' \leq 3n - 6}$

Nonplanar

$K_{3,3}$ is non-planar

K_5



K_5

$n = 5$

$m = 10$

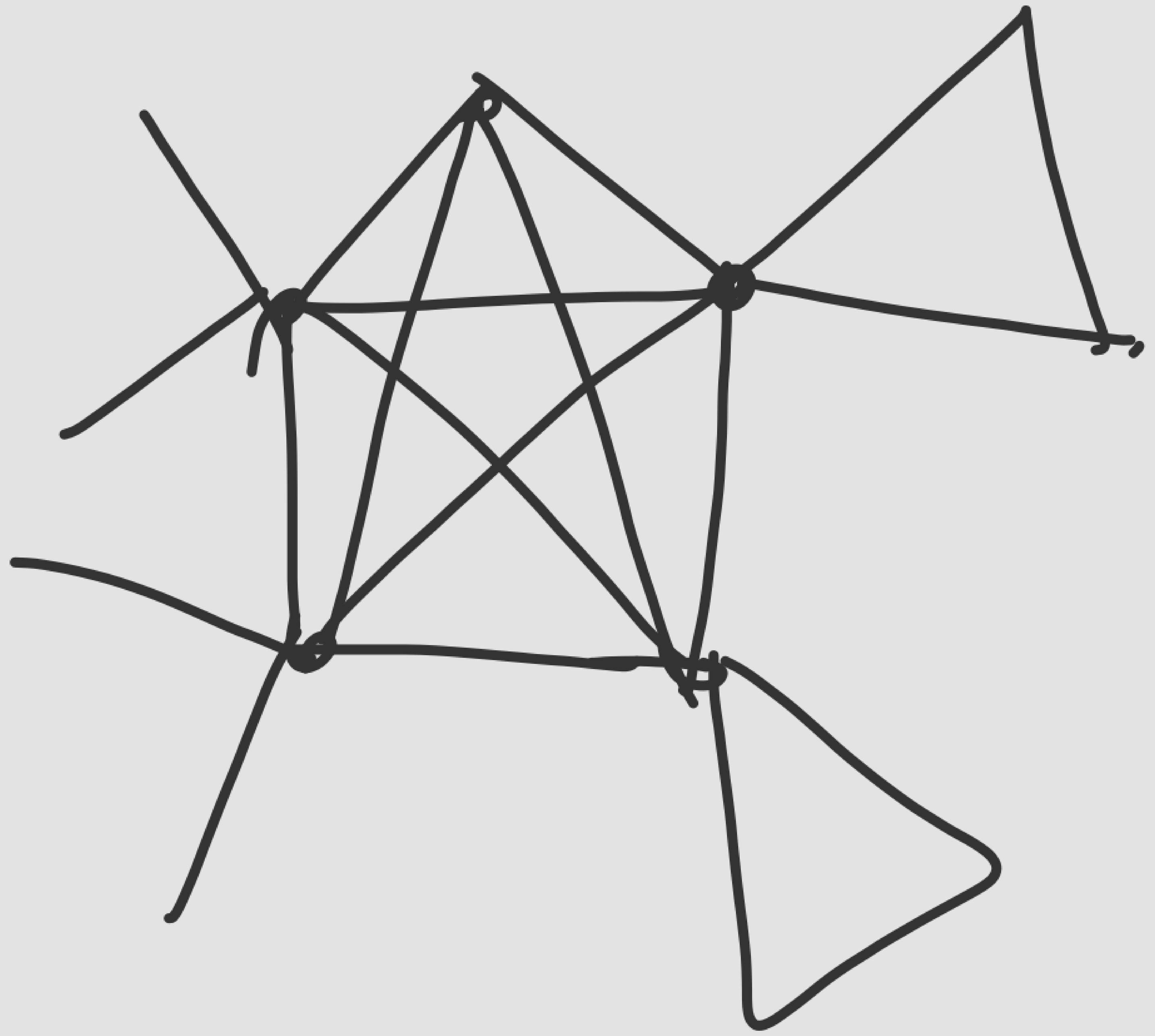
$m \leq 3n - 6$

$10 \leq 9$

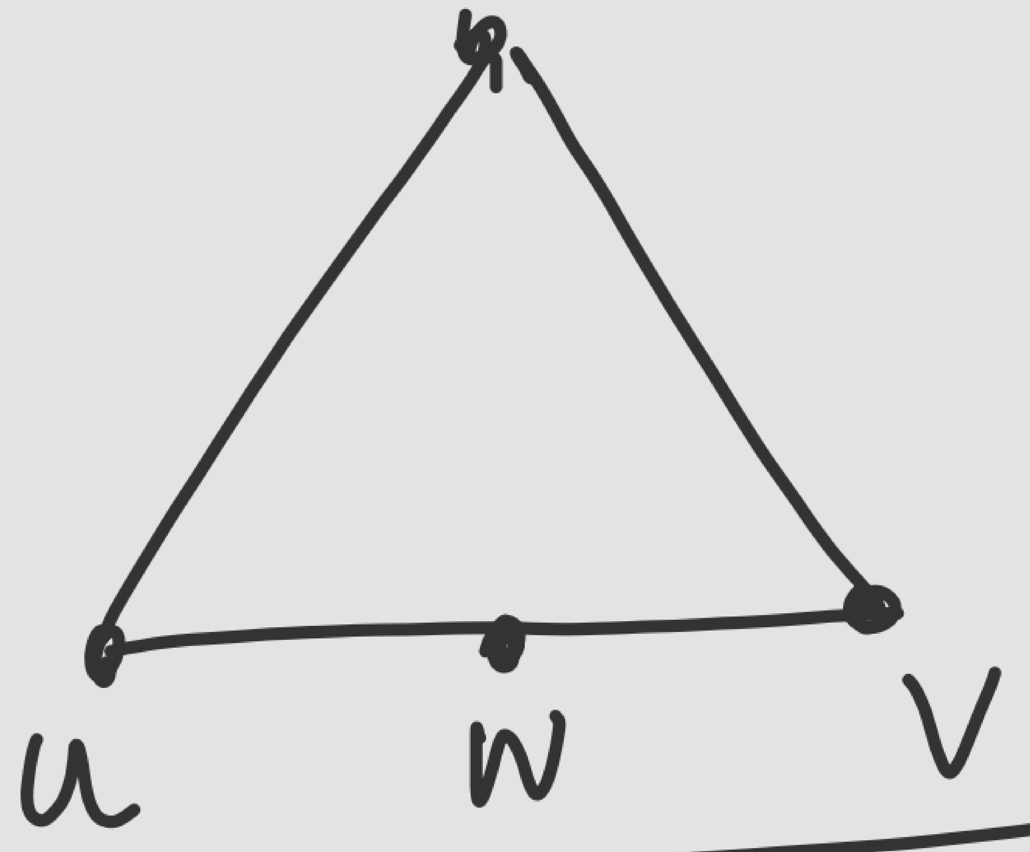
★ $K_{3,3}$ - not planar

★ K_5 - not planar

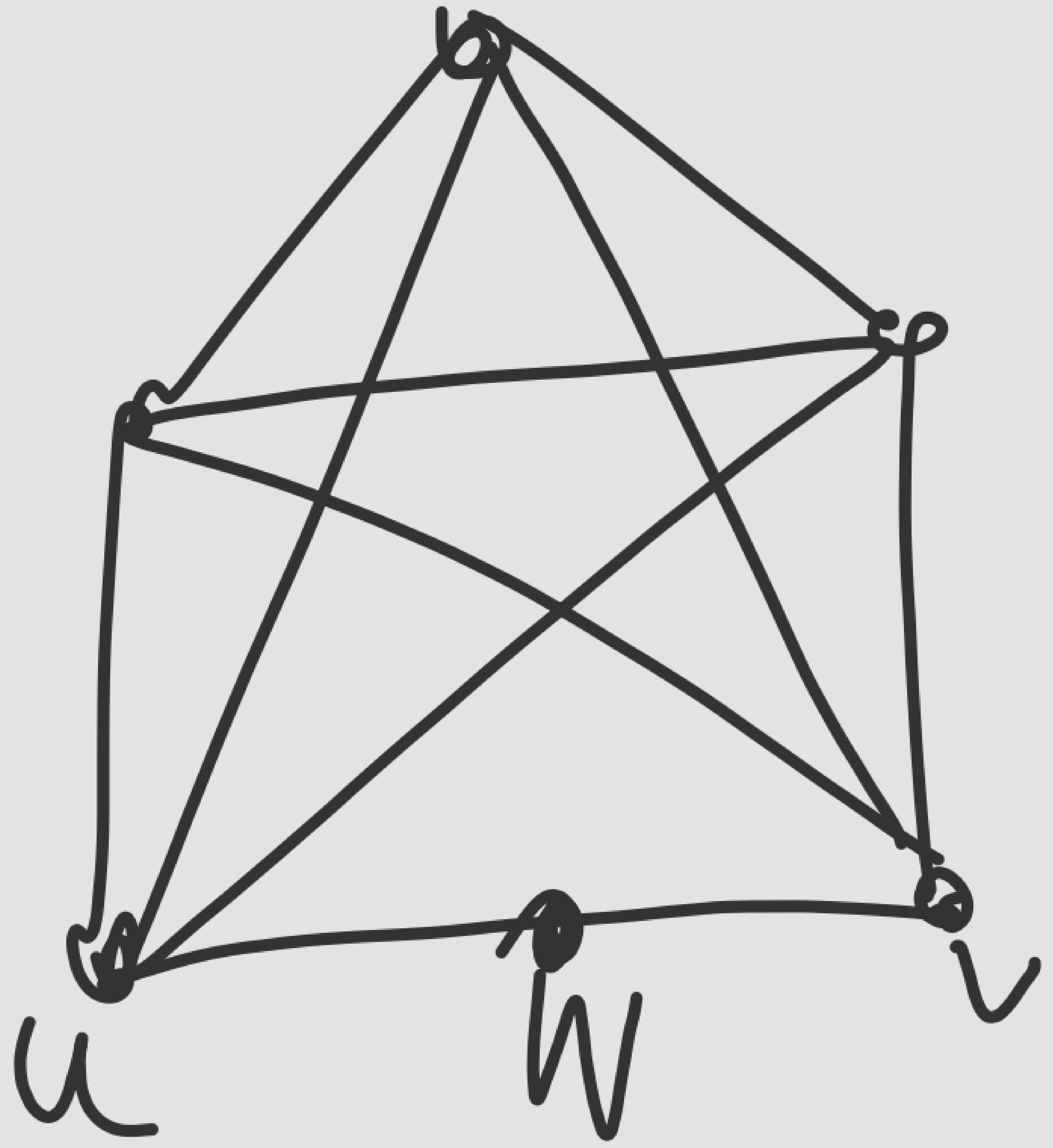
★



★ Any graph having K_5 or $K_{3,3}$ as subgraph is not planar



$\boxed{(u,w) \ \& \ (w,v)}$



Non planarity

Kurtowski

K_5 or $K_{3,3}$

planar graph w/o Δgular regions

$$\boxed{2m \le 4r}$$

$$r \geq \frac{m}{2}$$

$$n - m + r = 2$$

$$2 = n - m + r \geq n - m + \frac{m}{2}$$

$$2 \geq \frac{2n - m}{2}$$

$$\boxed{4 \geq 2n - m} \Rightarrow m$$

Ques MST