Let's Recap

* Sets

Collection of well defined distinct objects

$A = \{a, b, c\}$

Members

Set Notation

- $S = \{2, 4, 6, \ldots\ldots\}$
- $S = \{x : x \text{ is a even number}\}$

Empty Set

$\emptyset = \{\ \}$

* Can sets have sets as members?

Yes

$$\underbrace{\{a, b\}}_{2} \neq \underbrace{\{\{a, b\}\}}_{1}$$

* Cardinality

* Subset

A, B - if every element of A belongs to set B

$$A \subseteq B$$

* Proper subset

$A \subseteq B$ and $A \neq B$

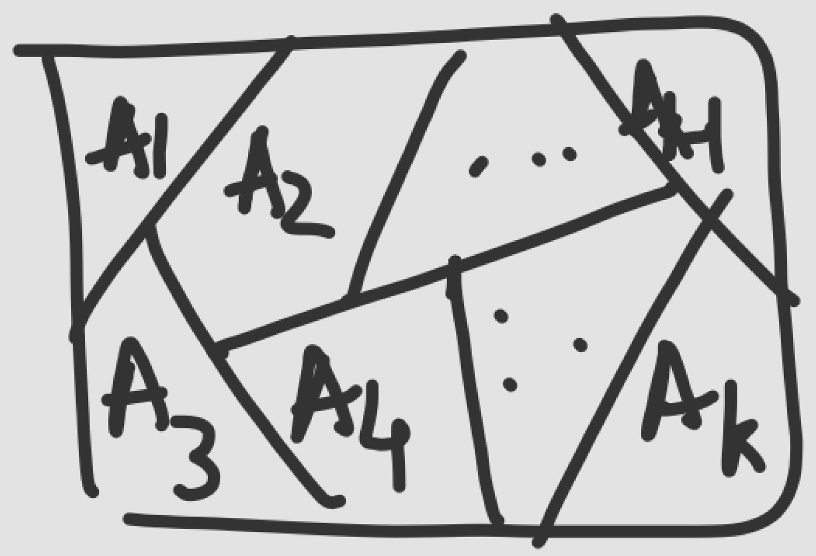
* $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

(proper subset)

* Equal sets

A, B if all elements of A are in B

* Partition



A

$A_1, \ldots, A_k$ are partitions of set A if each element of A appears in exactly one A_i $(1 \leq i \leq k)$

i) $A_i \cap A_j = \emptyset$

ii) $A_1 \cup A_2 \cup A_3 \ldots \cup A_k = A$

A_i

partite

Union $\cup$ A, B

elements that are in A or in B

Intersection $\cap$

Intersection is set of all elements that are in A and B

Complement $\bar{A}$

Not in A but in universe

set of elements under consideration

Set Difference $A - B$

all elements in A and; not in B

$$A - B = A \cap B^c$$

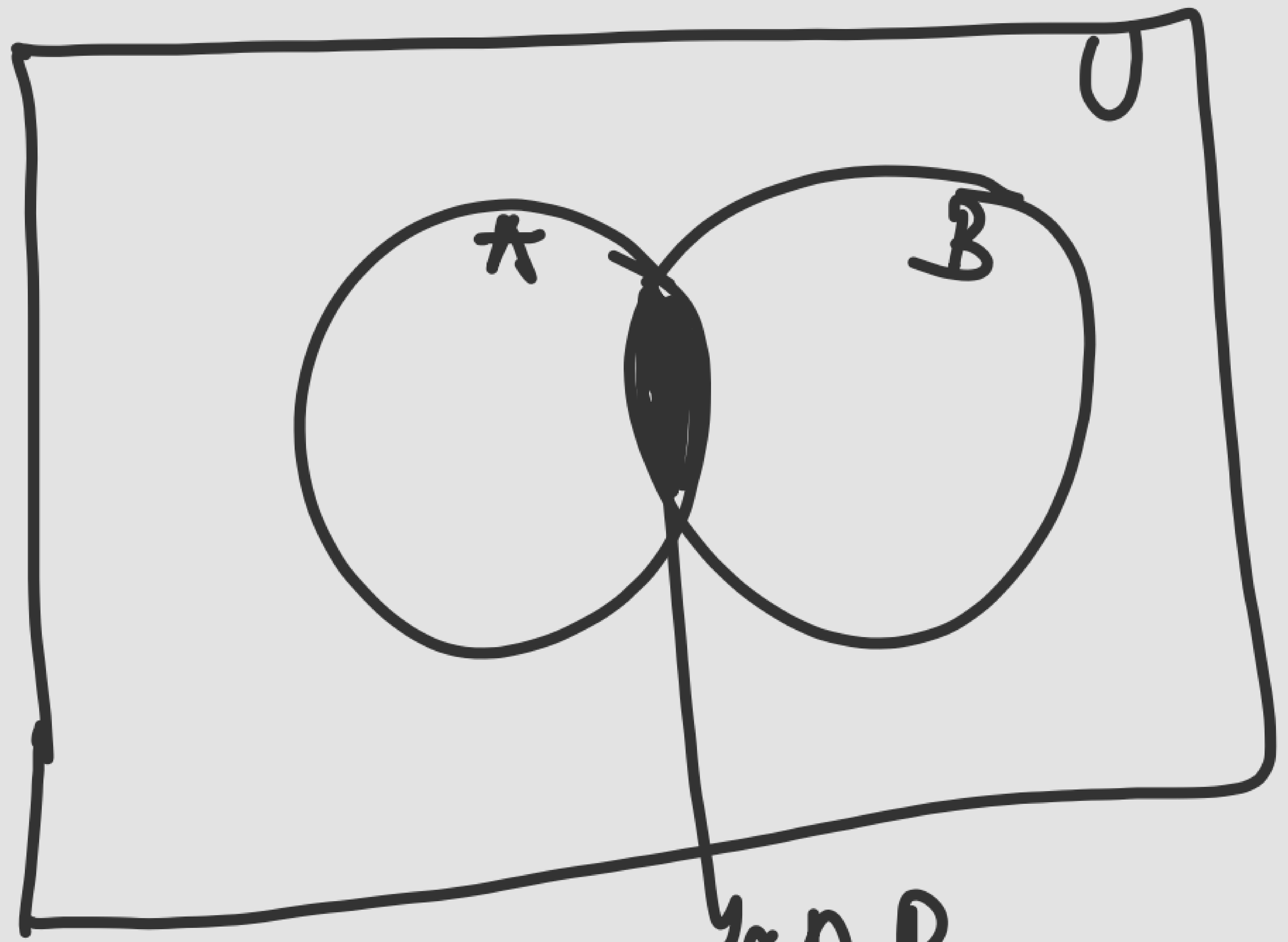
FINITE SET

A set is finite if there is one-to-one correspondance with some $n \in \mathbb{N} \cup \{0\}$

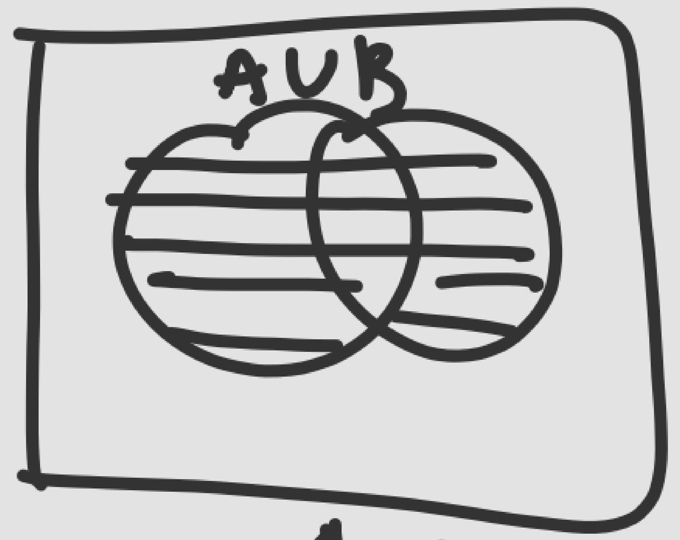
DISJOINT

$A \cap B = \emptyset$

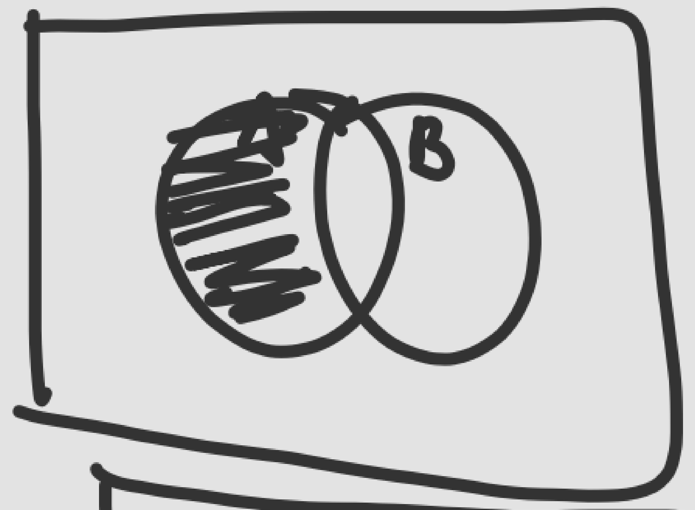
Venn Diagram

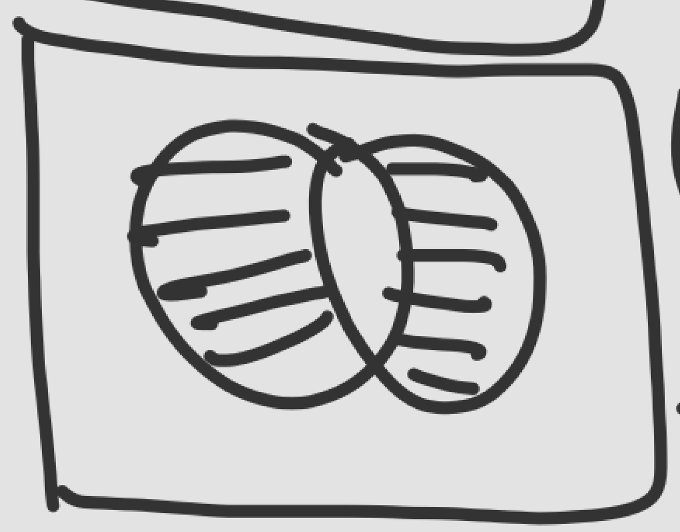


$A \cap B$



$A - B$





$(A-B) \cup (B-A)$
$= A \oplus B$

$A \oplus B$

Symmetric Difference

POWER SET $P(S)$

class of subsets for a set S

$S = \{1,2,3\}$

$P(S) = \{\emptyset \; . \; . \; . \}$

$|P(S)| = 8 \rightarrow 3$

$|S| = n$

$|P(S)| = 2^n = 2^{|S|}$

$S = \{1,2\}$

$|P(S)| = 4 \rightarrow$
2 elemen

Proof by Induction

A proposition, $P(n)$ holds if -

1) $P(1)$ is true - Basis step

2) $P(k+1)$ is true assuming $P(k)$ is true

$1 \le k \le n$

induction step

- hypothesis

COUNTING

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Cartesian Product

A and B

$A \times B = \{(a,b) : a \in A \text{ and } b \in B$ for all $a \in A$ and $b \in B\}$

Ordered pair

(a,b)

$A = \{1,2\}$

$B = \{3,4\}$

$A \times B = \{(1,3)\,(1,4)\,(2,3),(2,4)\}$

$m \quad n$

$|A \times B| = m \times n$

Relation

$R \subseteq A \times B$

(a, b)

Domain ↑ ↑ Range

R is on $A \times A$

Reflexive

A relation R is reflexive if $(a, a) \in R$ for all $a \in A$ [$aRa \ \forall a \in A$]

Symmetric

if $(a, b) \in R$ then $(b, a) \in R$

if aRb then bRa

$\forall \rightarrow$ for all

$\exists \rightarrow$ ~~some~~

Transitive

If aRb and bRc then aRc

Equivalence

Reflexive

Symmetric

Transitive