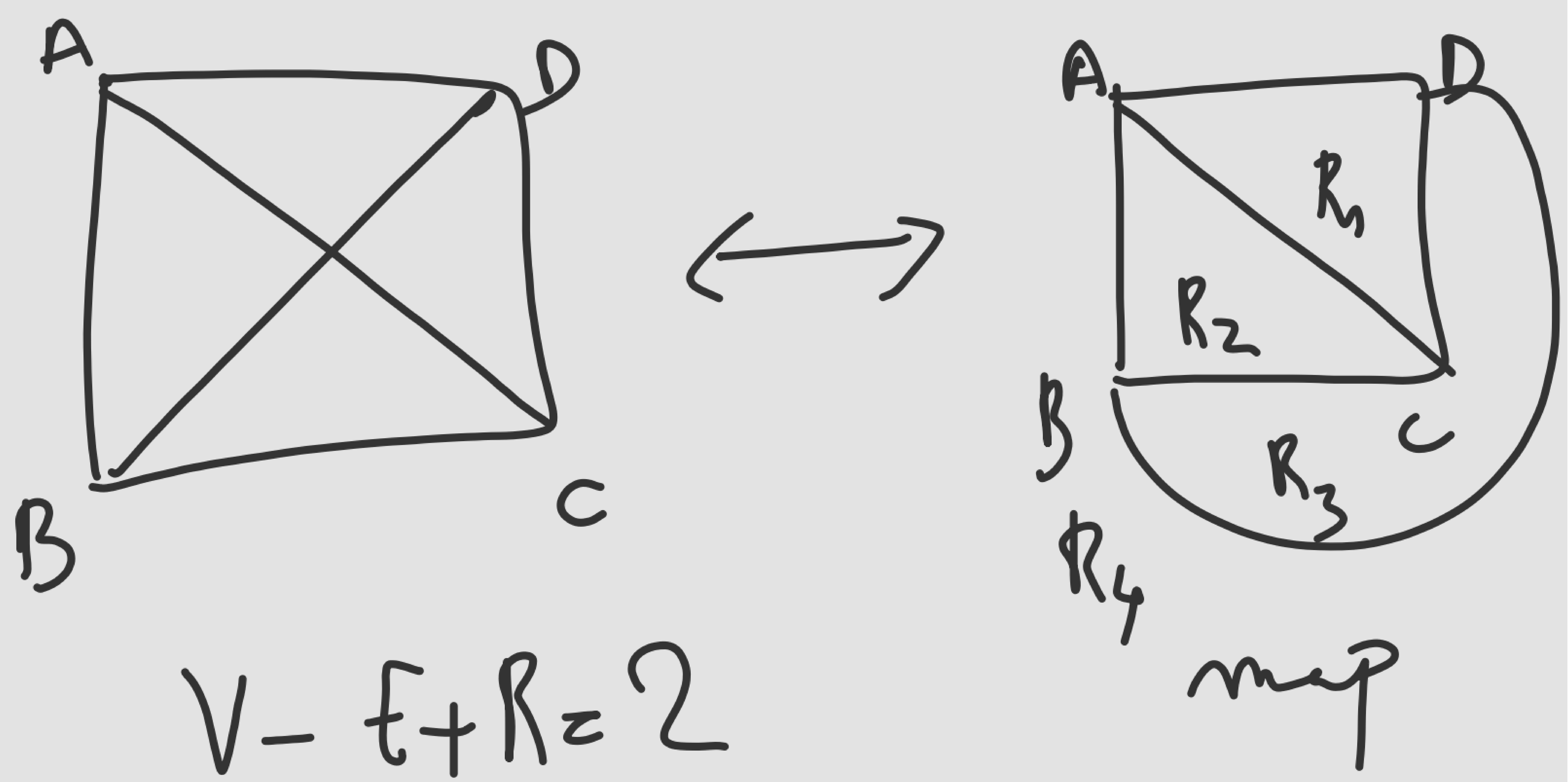
planar graphs

Chromatic number

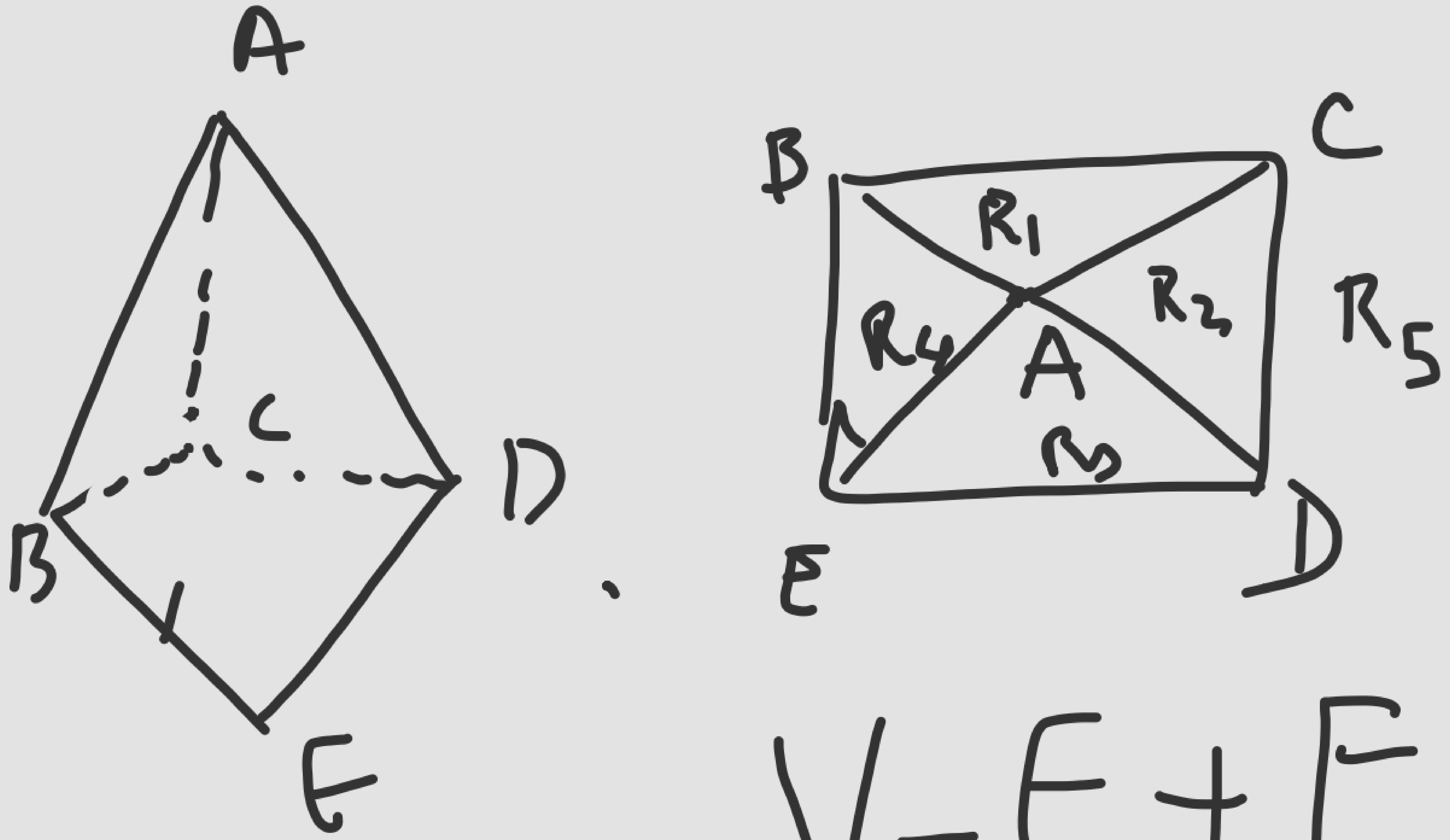
Vertex Coloring / Graph Coloring

Euler's Formula for planar

Hamiltonian cycle



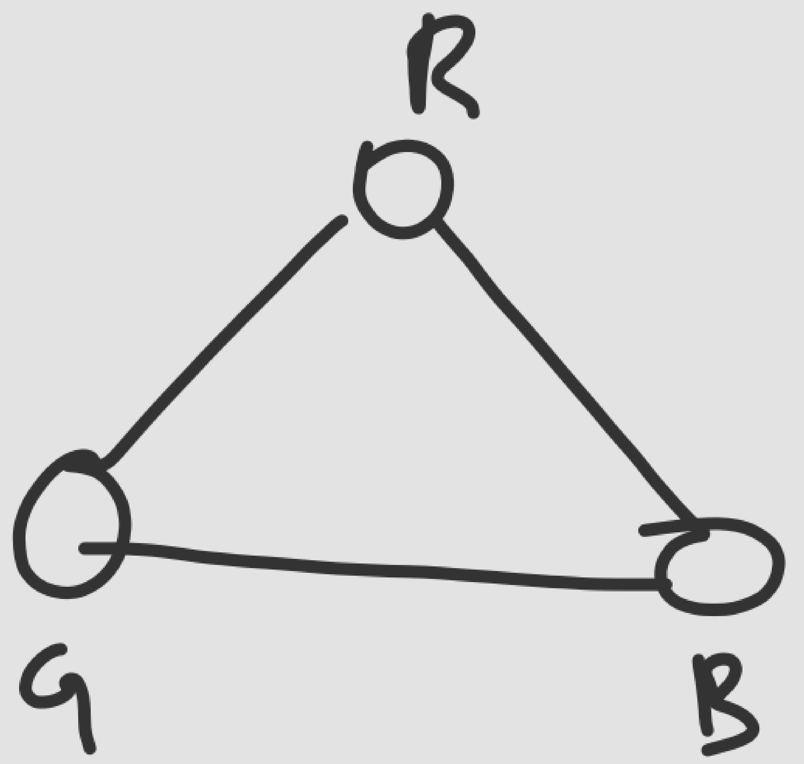
$$V - E + R = 2$$



$$V - E + F = 2$$

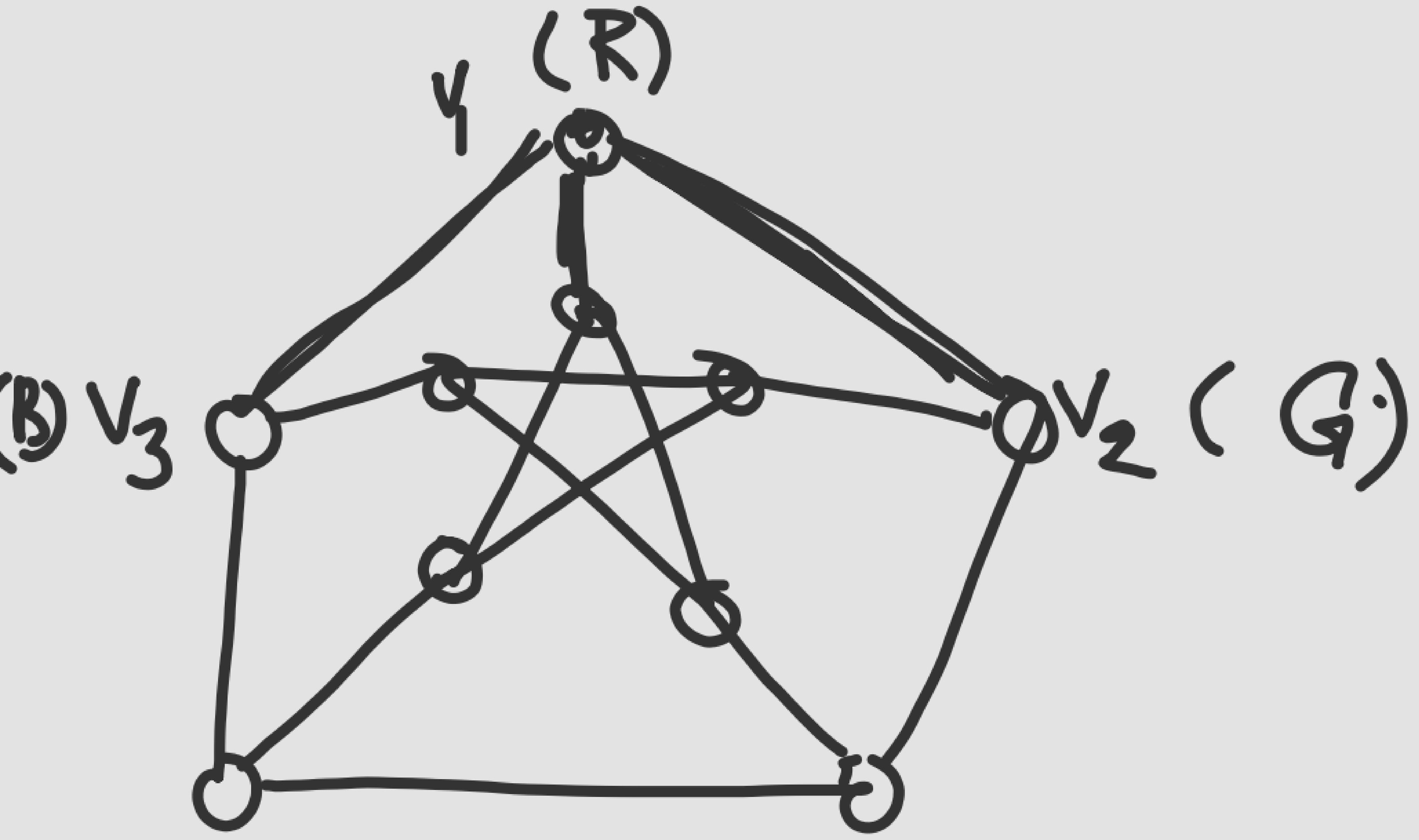
$$\underbrace{5 - 8 + 5}$$

Coloring Problem

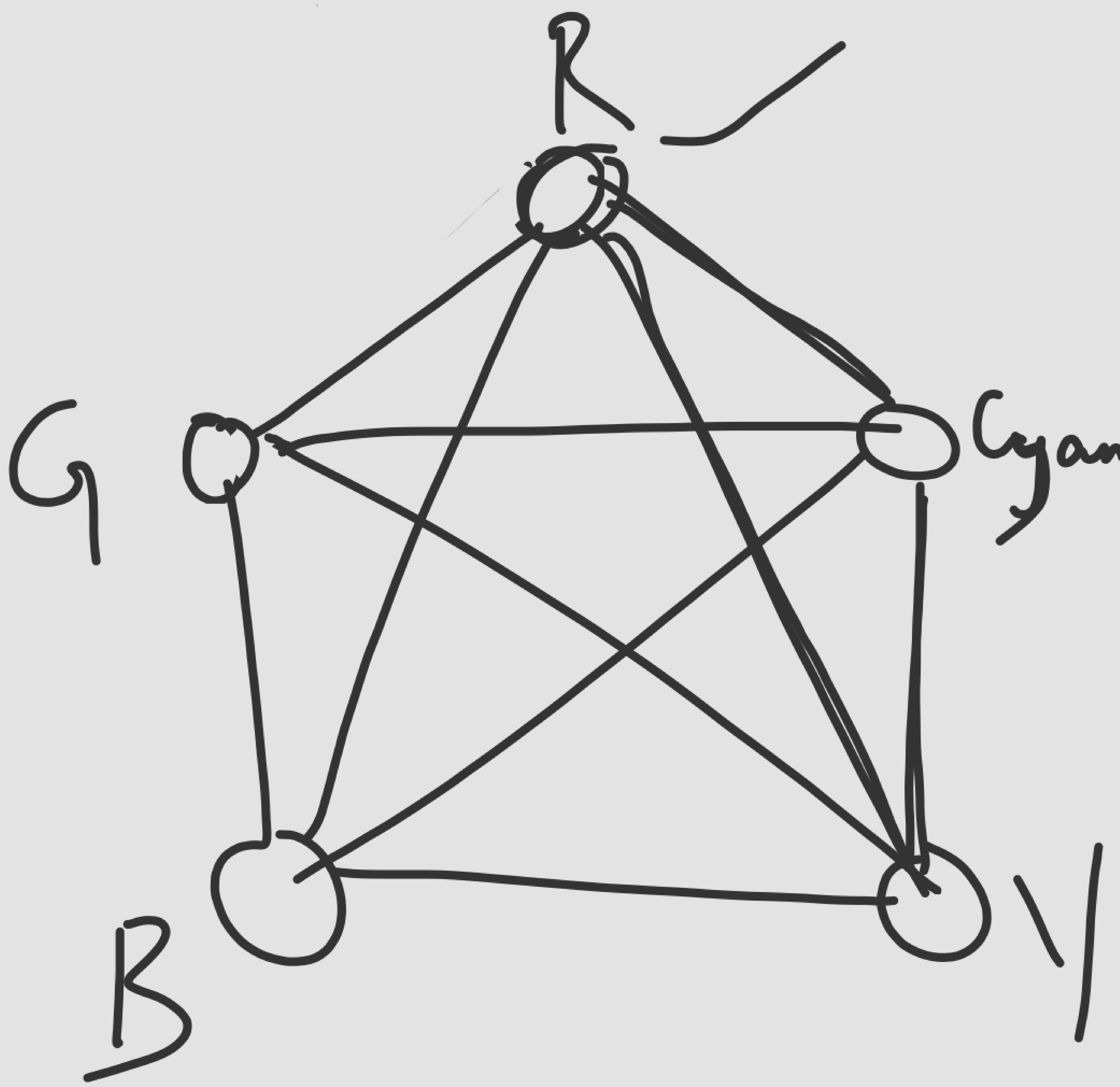


3-colorable

$\underbrace{R, G, B, Y}$

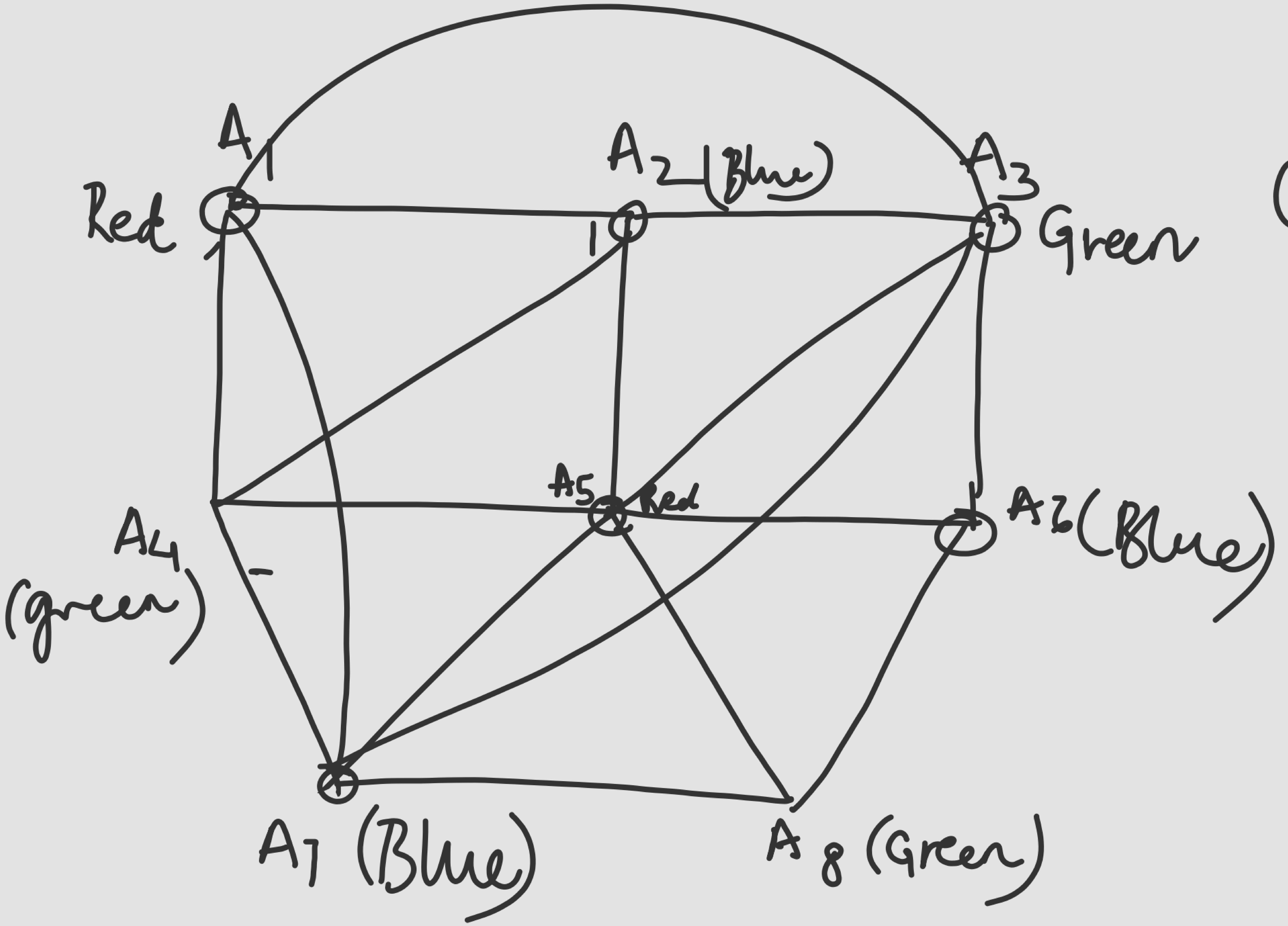


Petersen graph



n
↓
n colors

Subgraph



$C = \{ \}$

1. Order your vertices in. non-Ising order of degree
2. Assign C_1 to first vertex and assign C_1 to all non-adjacent vertices
3. Repeat by assigning C_2 to next vertex

Welch-Powell

3-colorable

$\{A_5, A_3, A_7, A_1, A_2, A_4, A_6, A_8\}$

Facts

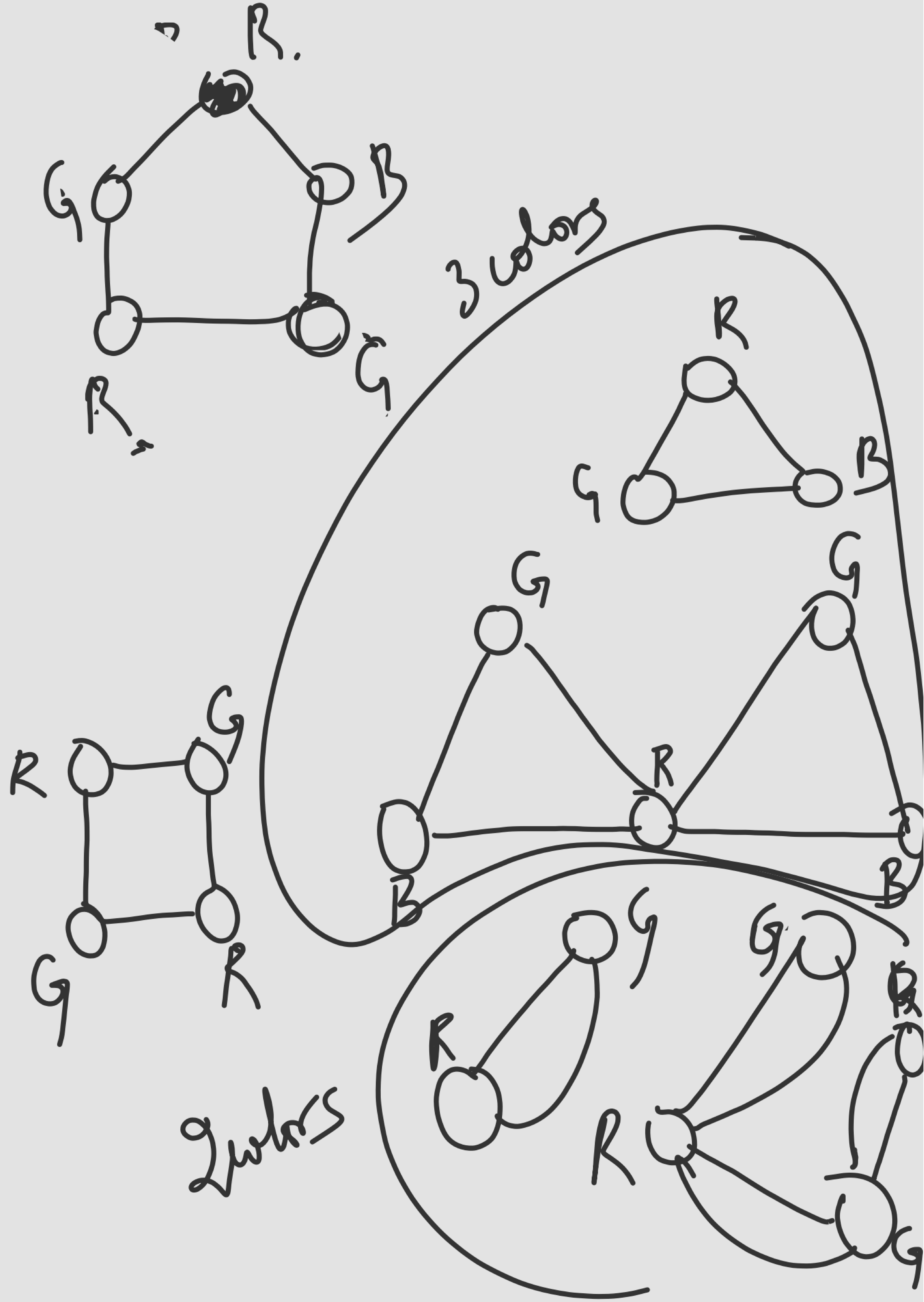
$G(V,E)$ and $|V|=n$

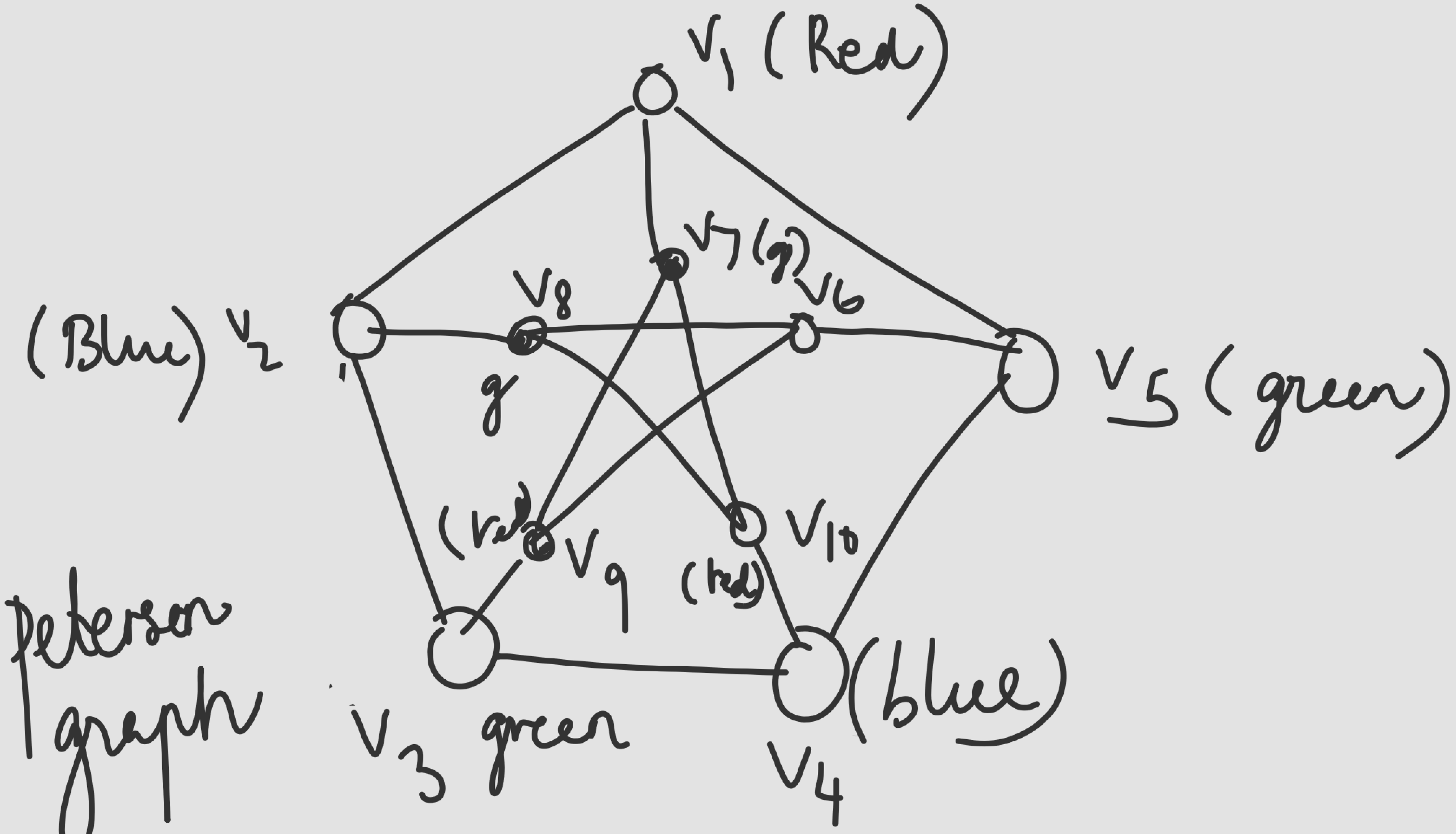
* $\chi(G) \leq n$
* $\chi(G)=1 \iff$ G has <u>no</u> edges
* $\chi(G)=n \iff$ complete graph
* $\chi(G)=2 \iff$ even cycle

$\chi(G)=3 \iff$ odd cycle

* $\chi(G) \geq \chi(H)$

$\Leftarrow$ if $\underset{(V,E)}{H} \subseteq G(V,E)$





Peterson graph

$V_1 = \{ \quad \}$

$V_2 = \{ \quad \}$

$V_3 = \{ \quad \}$

3-colorable

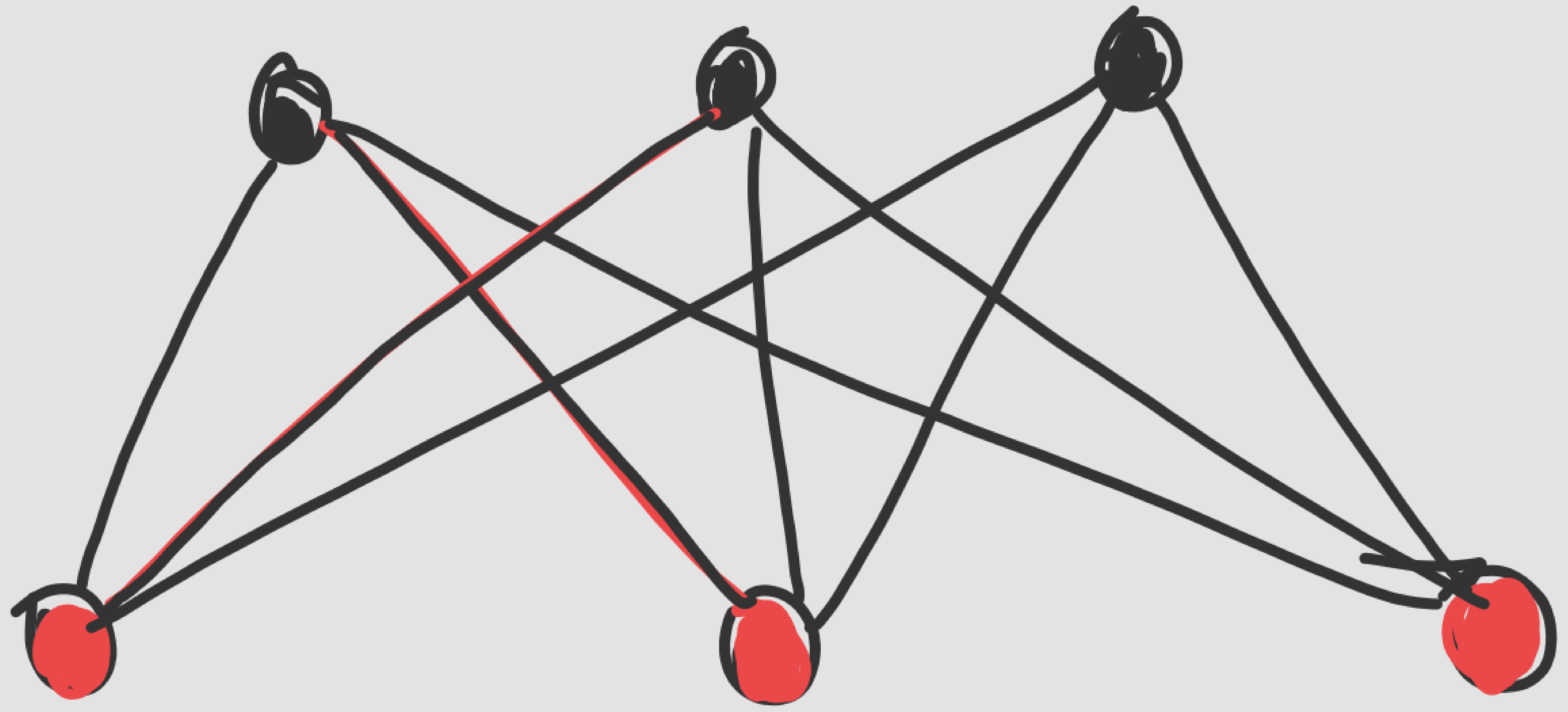
Let $G(V, E)$

A k-colouring of G partitions the vertex set V into k sets $V_1, V_2, \ldots, V_K$ where each V_i is an <u>independent</u> set (ie)

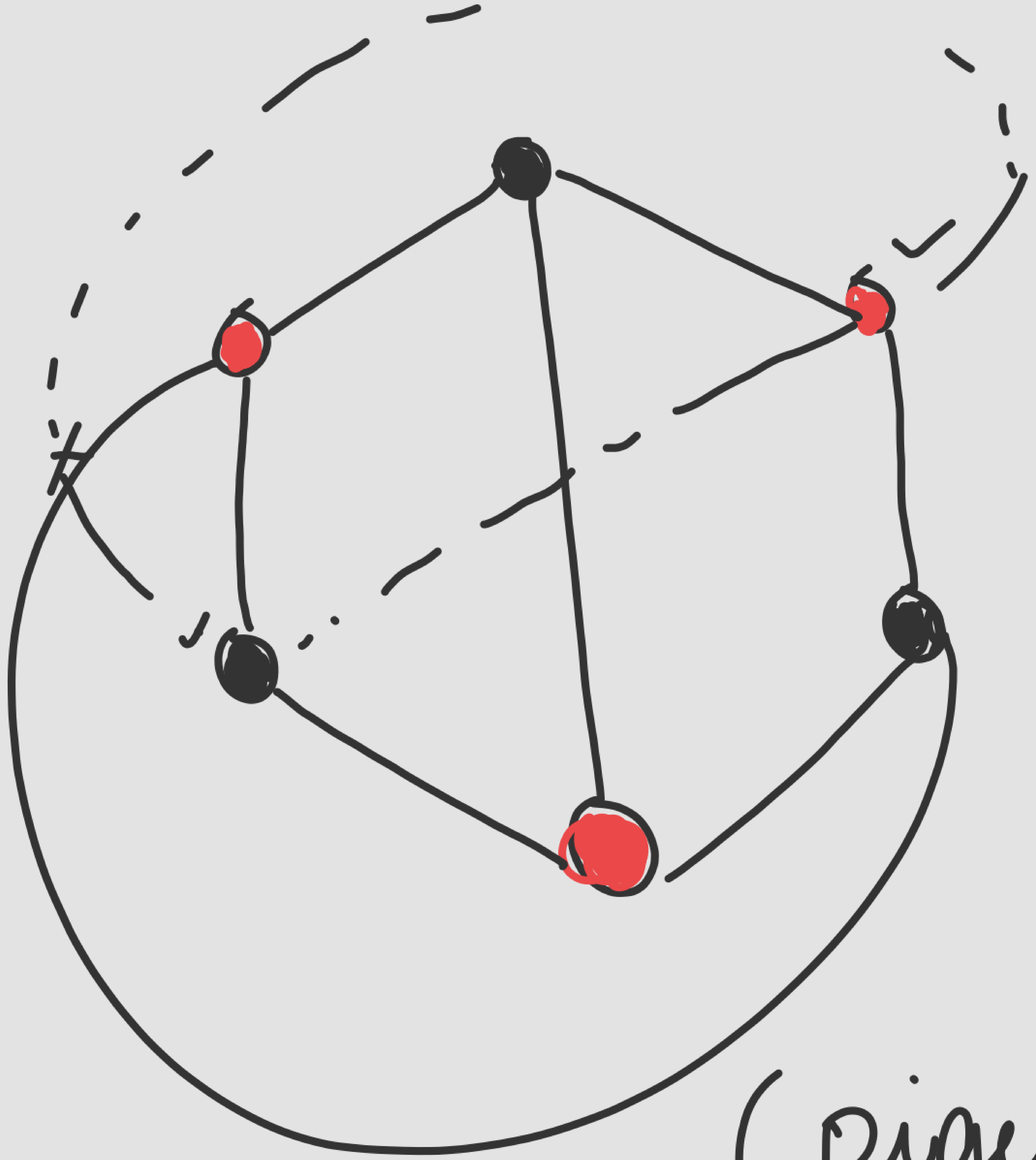
$$V_1 \cup V_2 \cup \ldots \cup V_K = V$$

$$V_i \cap V_j = \emptyset \quad \forall\, i, j$$

Each independent set is called <u>colour class</u>

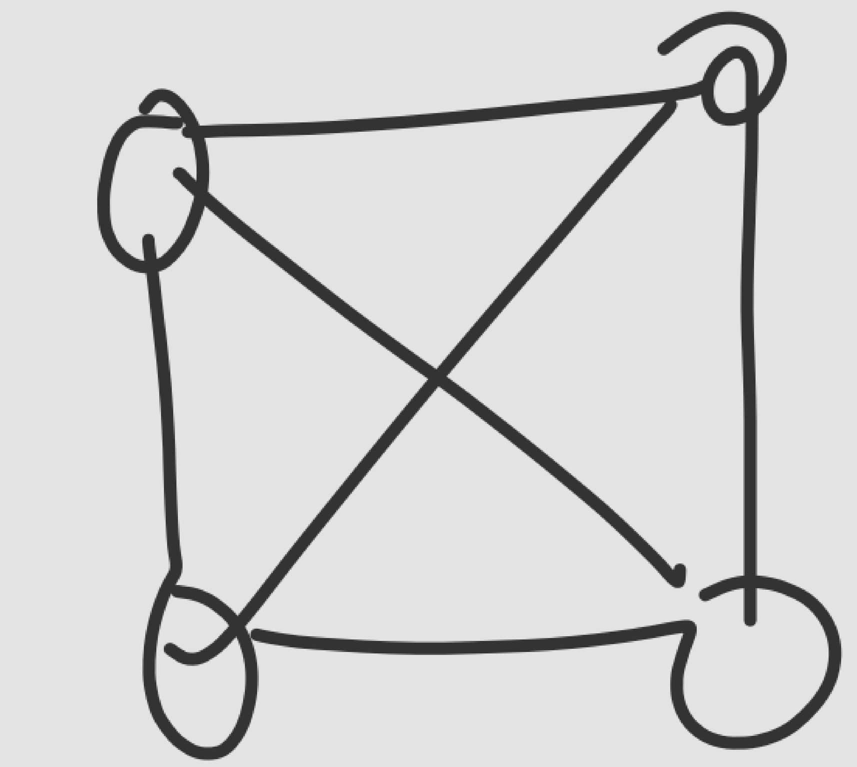


Bipartite Graph
(Complete)
$K_{3,3}$

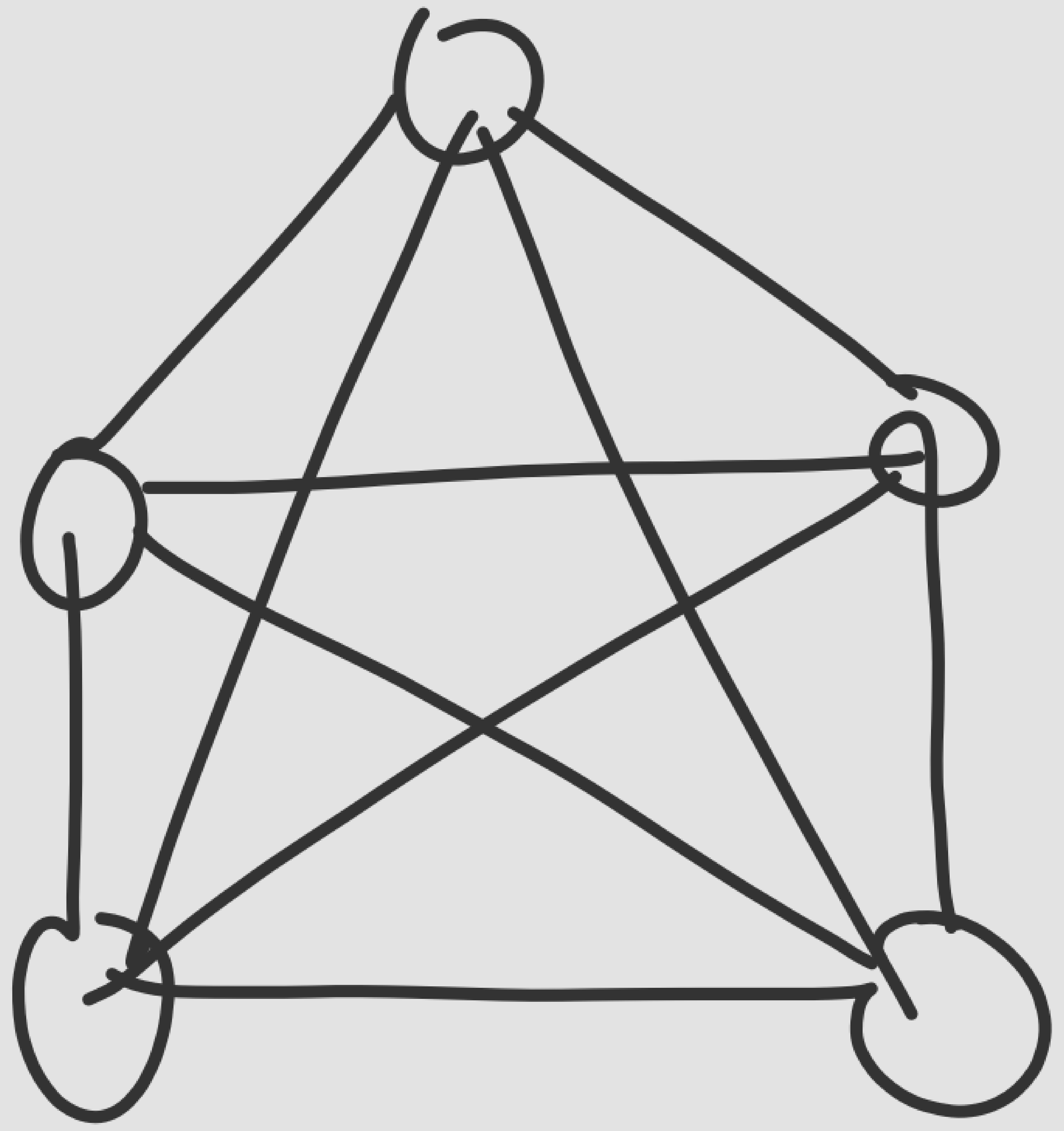


non-planar

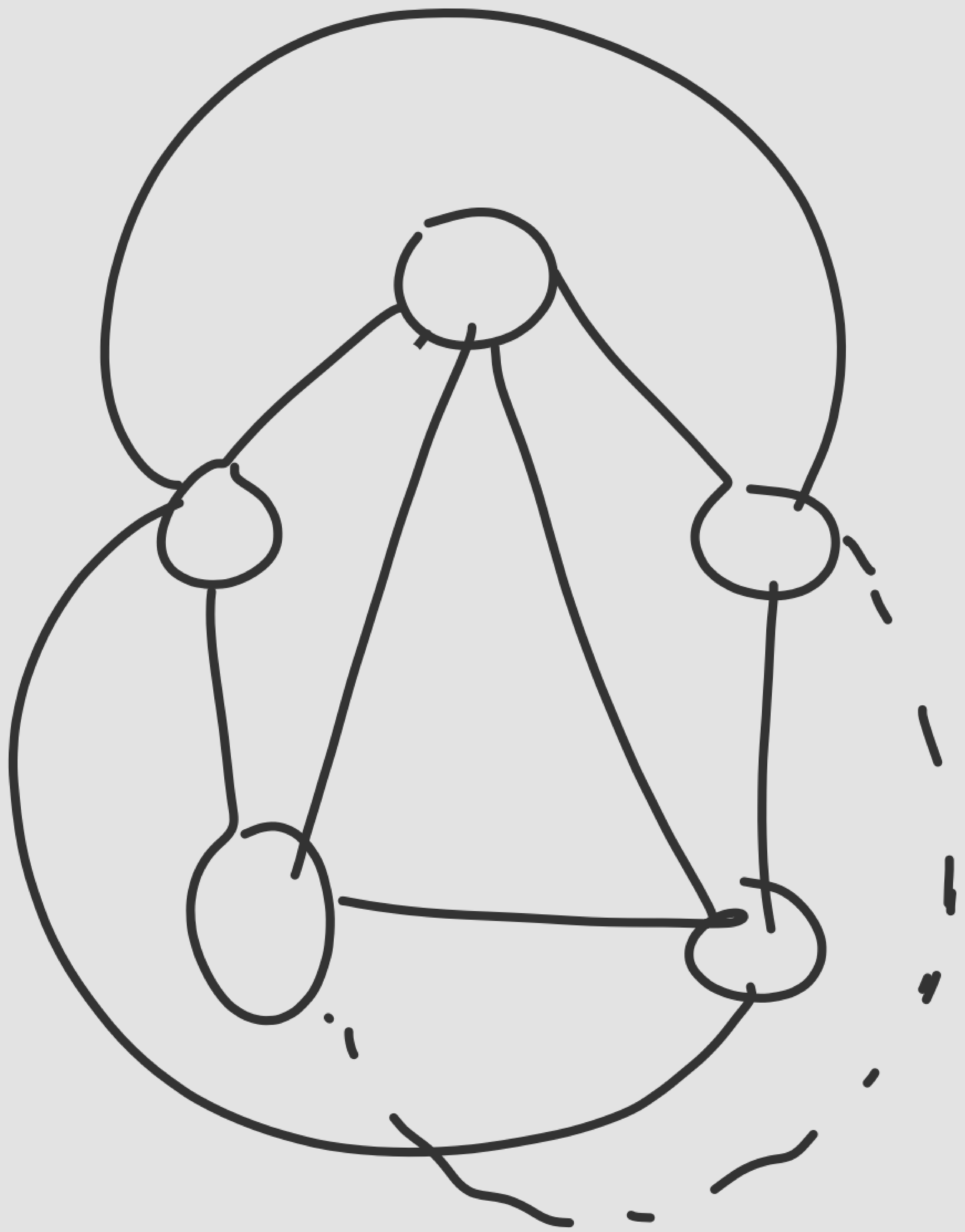
(pigeonhole principle)



K_4



K_5



any graph isomorphic to

K_5 and $K_{3,3}$ are

not planer

non-planar

Theorem

Any graph is non-planar iff it is homeomorphic to K_5 or $K_{3,3}$

Kuratowski theorem

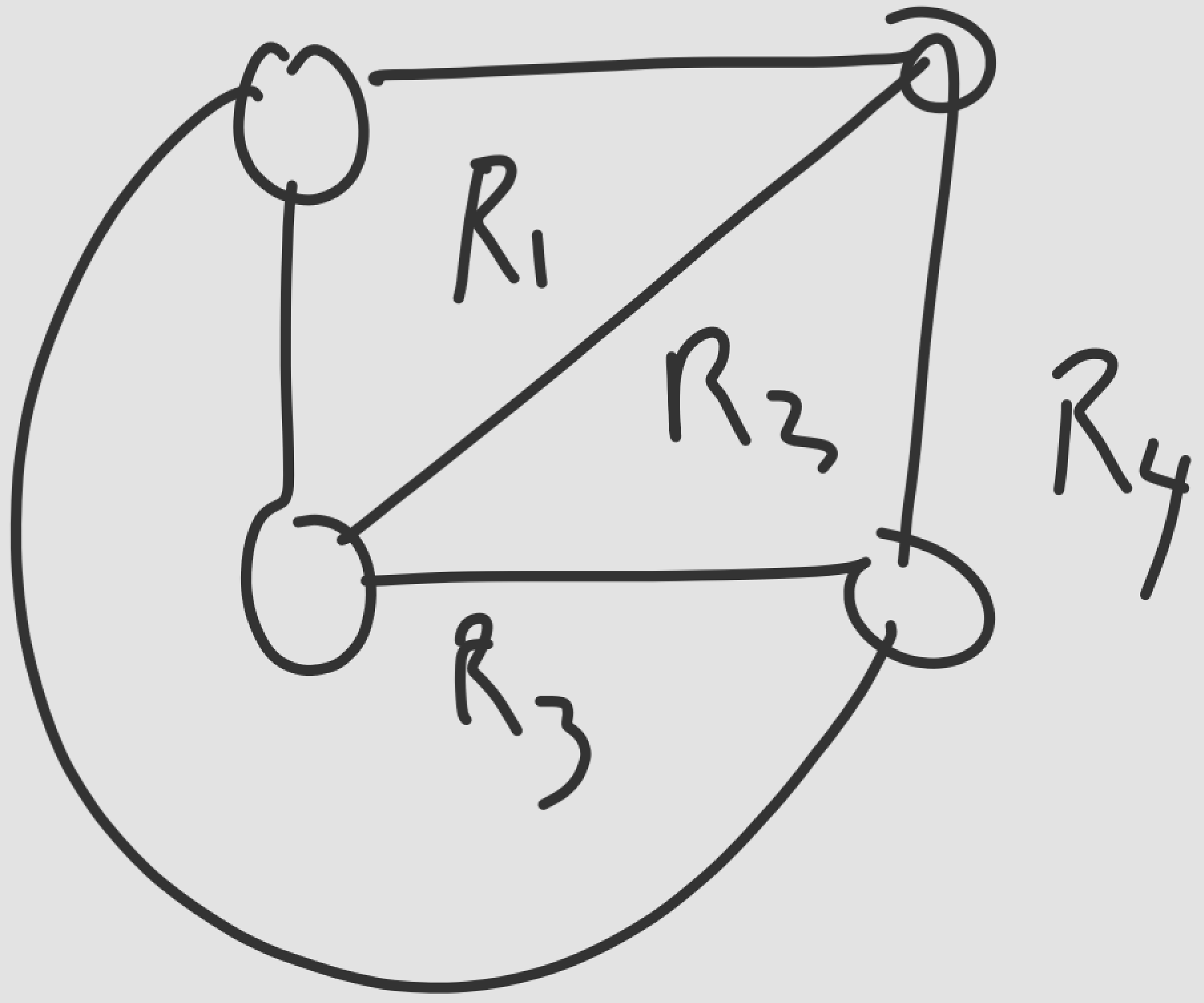
planarity Cond^n

p, q, r

V E R

$q \geqslant 3p - 6$

$E \geqslant 3V - 6$



Sum of degrees of regions =

Complete graph

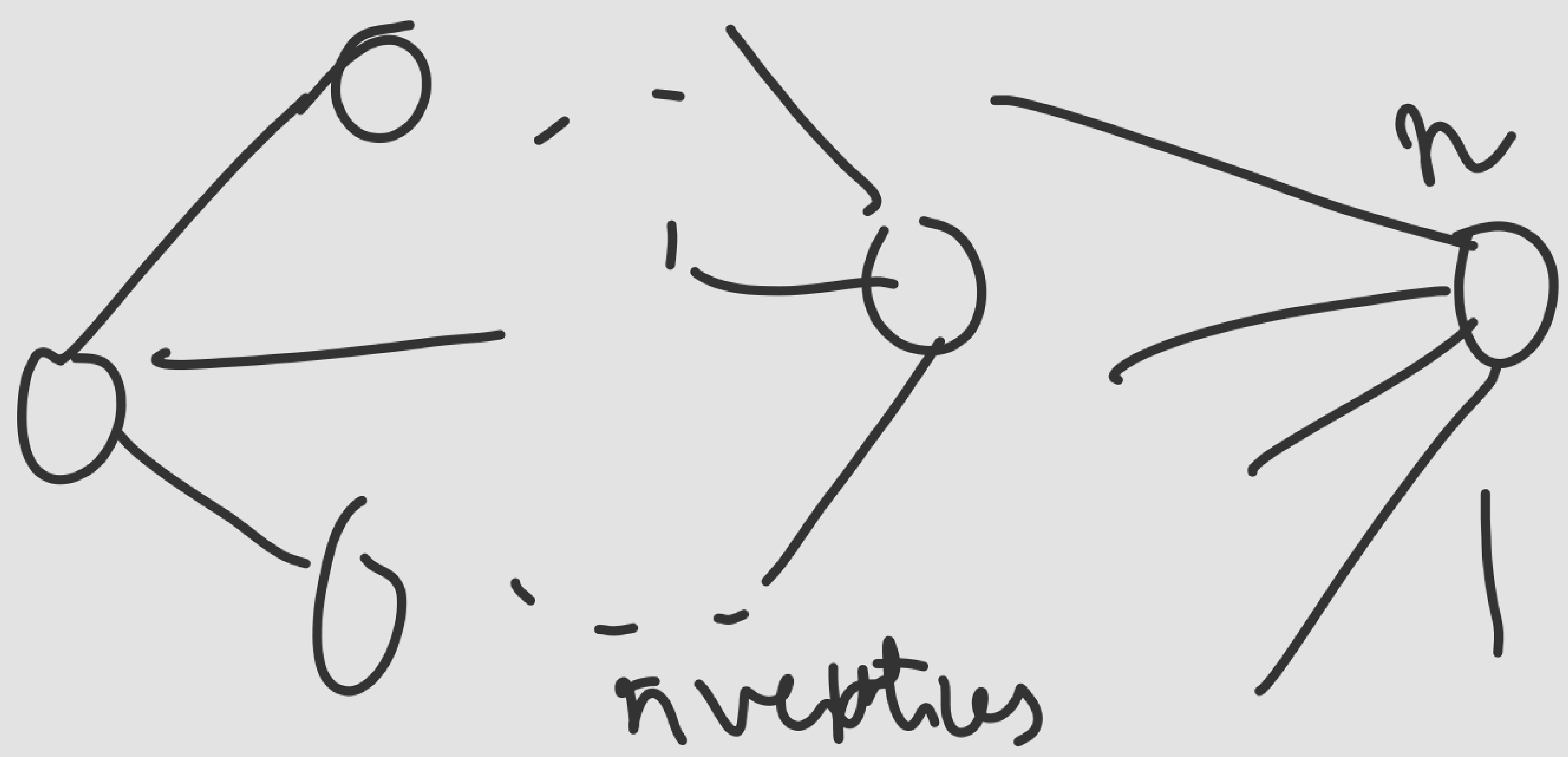
$$\frac{n(n-1)}{2}$$

$n = 0 \longrightarrow 0$ edges

$n = 1 \longrightarrow 0$ edges $= \frac{1(1-1)}{2} = 0$

$n = k \longrightarrow \frac{k(k-1)}{2}$ edges

$n = k+1 = \underbrace{\qquad\qquad}_{k \text{ vertices}} + 1$ new vertex



No of edges in graph with $k+1$

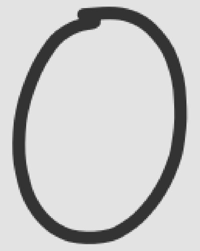
$$= \frac{k(k-1)}{2} + k$$

$$= \frac{k}{2}\left(k - 1 + 2\right) = \frac{k(k+1)}{2}$$

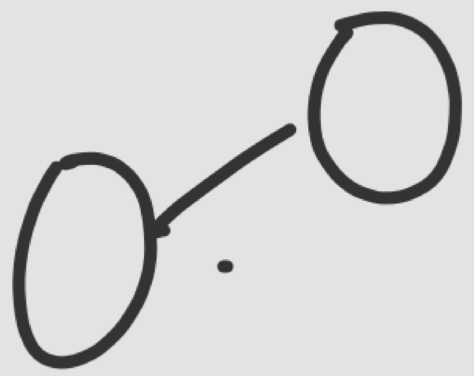
$$\frac{(k+1)(k}{2}$$

A tree has $(n-1)$ edges

Connected, acyclic

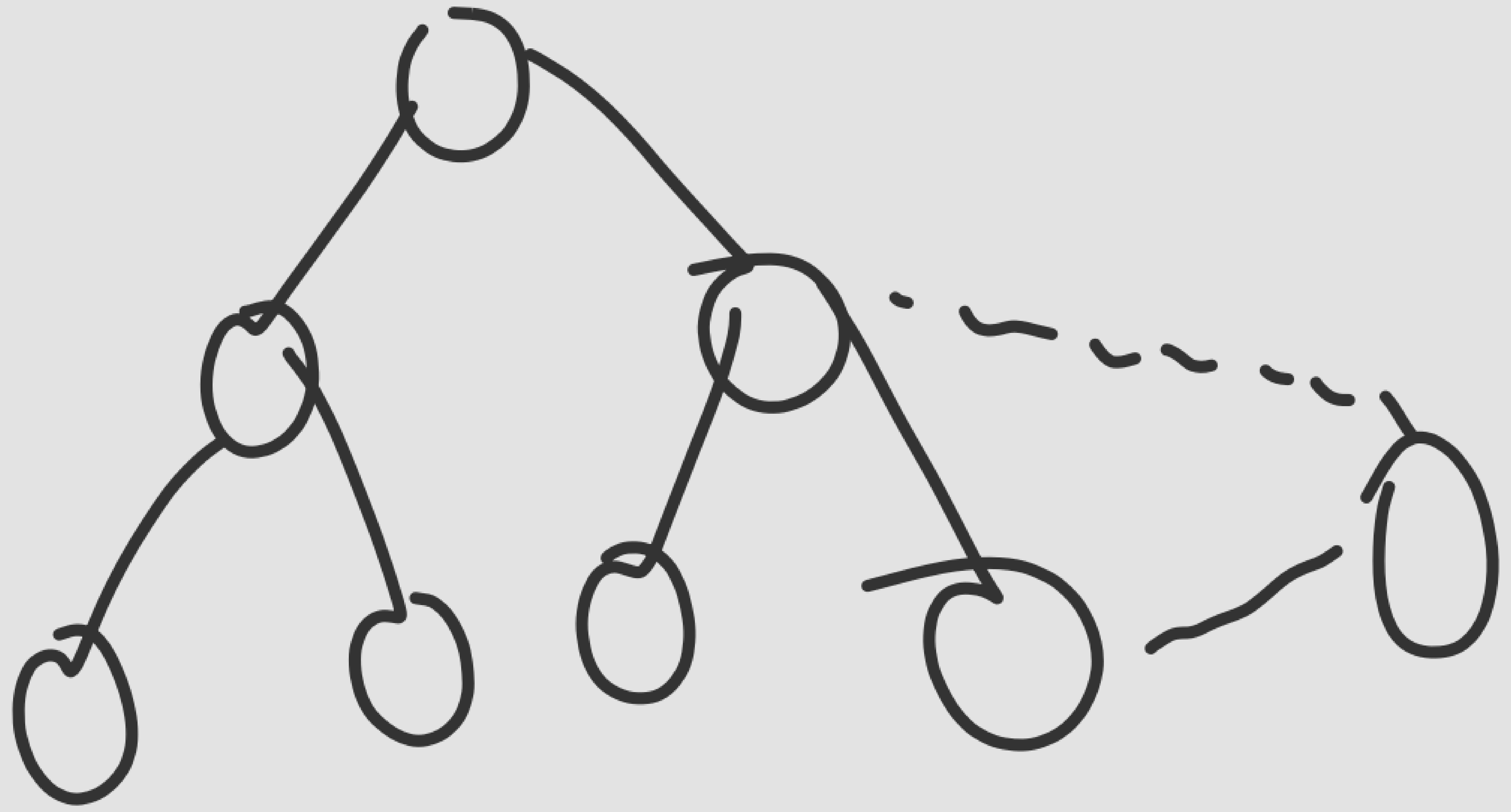


$n=1$ $\quad (0) \Leftrightarrow (1-1)$



$n=2$ $\quad$ acyclic $\Leftrightarrow 1 \Rightarrow (2-1)$

$n=k$ $\quad (k-1)$ edges

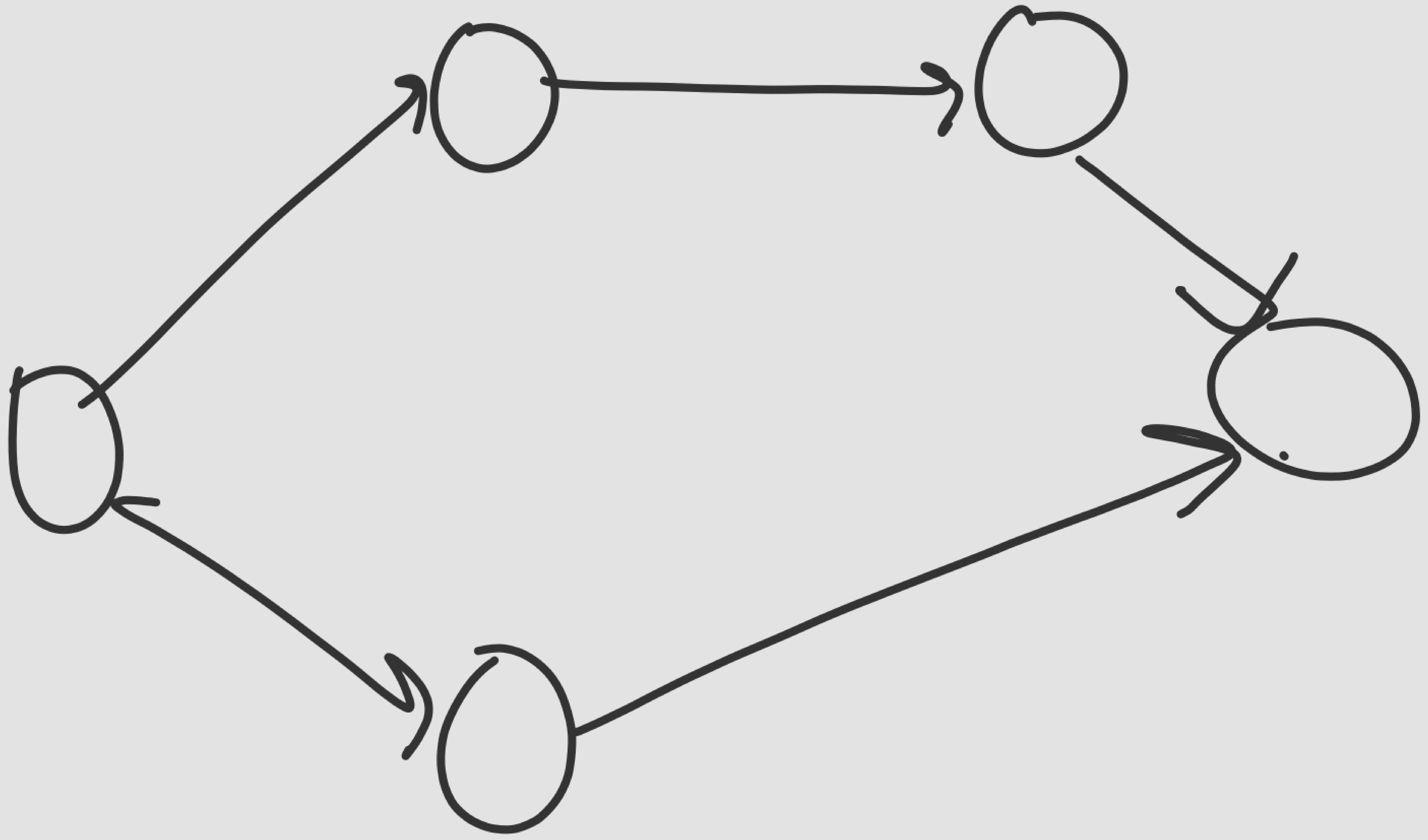


$n=k+1$

$$(k-1)+1$$

$$k$$

Directed graph



$(u, v) \neq (v, u)$

indegree → incoming degree/edges

outdegree → Outgoing degree/edges

Degree Sequence

1). (7, 6, 5, 4, 4, 3, 2, 1)

arranging degrees in ↓sing order

(1, 2, 3, 5, 4, 6, 4, 7)

Havel Hakimi Theorem

(7, 6, 5, 4, 4, 3, 2, 1)

(5, 4, 3, 3, 2, 1, 0)

(3, 2, 2, 1, 0, 0)

(1, 1, 0, 0, 0)

(0, 0, 0, 0)

Check if this is planar