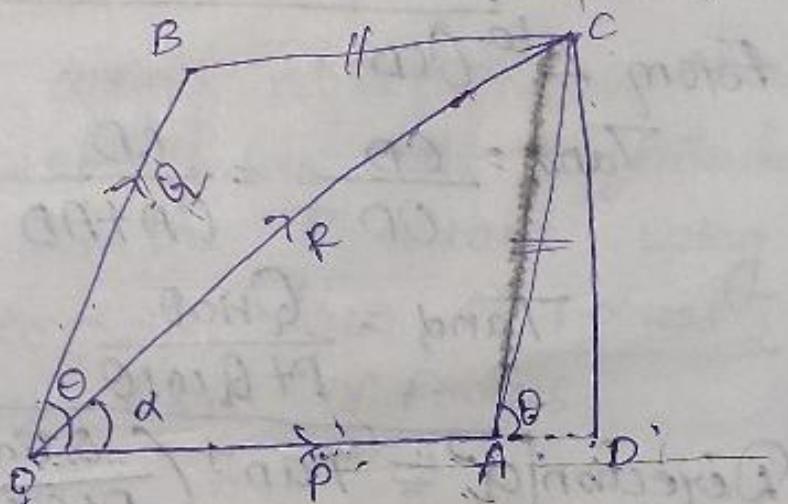


Parallelogram Law of Forces:- Two forces are acting at point O & represents Magnitude and direction by two adjacent sides of a parallelogram then the resultant represents in magnitude and direction by the diagonal sides of the parallelogram passing through that point.



Let

P, Q = Two forces acting at point 'O' (N)

R = Resultant of the forces (N)

θ = Angle between two forces

α = Angle between resultant (R) to the one of the force (P)

$$\vec{BC} = \vec{OA} = P, \vec{OB} = Q = \vec{AC}, \vec{OC} = R =$$

From $\triangle ACD$,

$$\sin \theta = \frac{CD}{AC} = \frac{CD}{AC \sin \alpha}$$

$$\cos \theta = \frac{AD}{AC} = CD = Q \sin \theta.$$

$$\Rightarrow AD = AC \cos \theta$$

$$AD = Q \cos \theta$$

from $\Delta^{le} OCD$,

$$OC^2 = OD^2 + CD^2$$

$$OC^2 = (OA + AD)^2 + CD^2$$

$$R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$

$$R^2 = P^2 + Q^2 \cos^2 \theta + 2PA \cos \theta + Q^2 \sin^2 \theta$$

$$R^2 = P^2 + Q^2 (1) + 2PQ \cos \theta$$

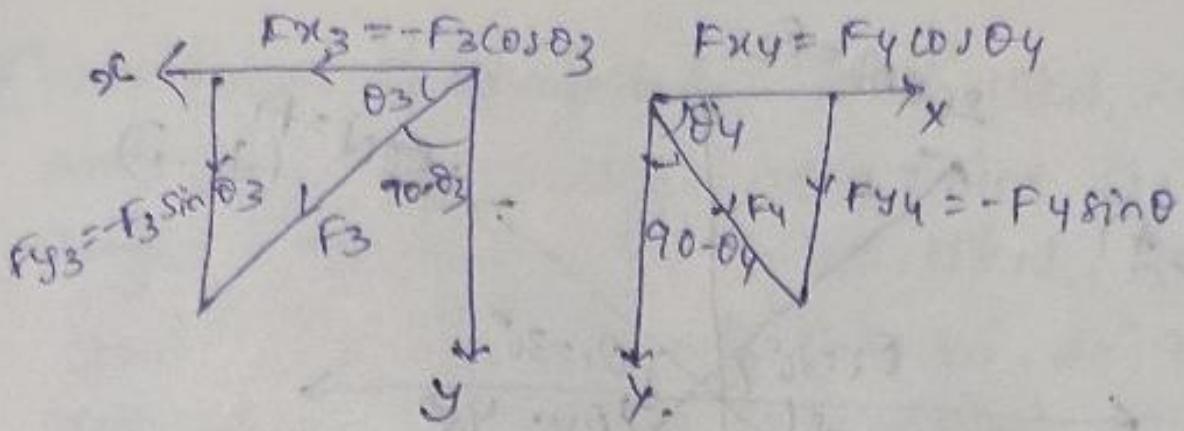
Magnitude $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

from $\Delta^{le} OCD$

$$\tan \delta = \frac{CD}{OD} = \frac{CD}{OA + AD}$$

$$\tan \delta = \frac{Q \sin \theta}{P + Q \cos \theta}$$

Direction $\alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)$



$\sum F_{x1}$ = Sum of all Horizontal forces (x-components)

$$\sum F_x = F_{x1} + F_{x2} + F_{x3} + F_{x4}$$

$$\sum F_x = F_1 \cos \theta_1 - F_2 \cos \theta_2 - F_3 \cos \theta_3 + F_4 \cos \theta_4$$

$\sum F_y$ = Sum of all Vertical forces (y-components)

$$\sum F_y = F_{y1} + F_{y2} + F_{y3} + F_{x4}$$

$$\sum F_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 - F_3 \sin \theta_3 - F_4 \sin \theta_4$$

Resultant

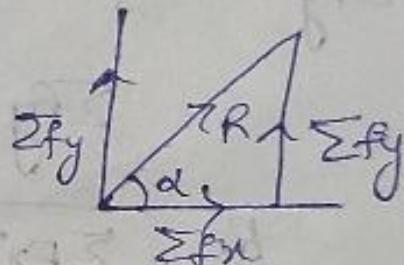
$$R^2 = \sum F_x^2 + \sum F_y^2$$

$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$\tan \alpha = \frac{\sum F_y}{\sum F_x}$$

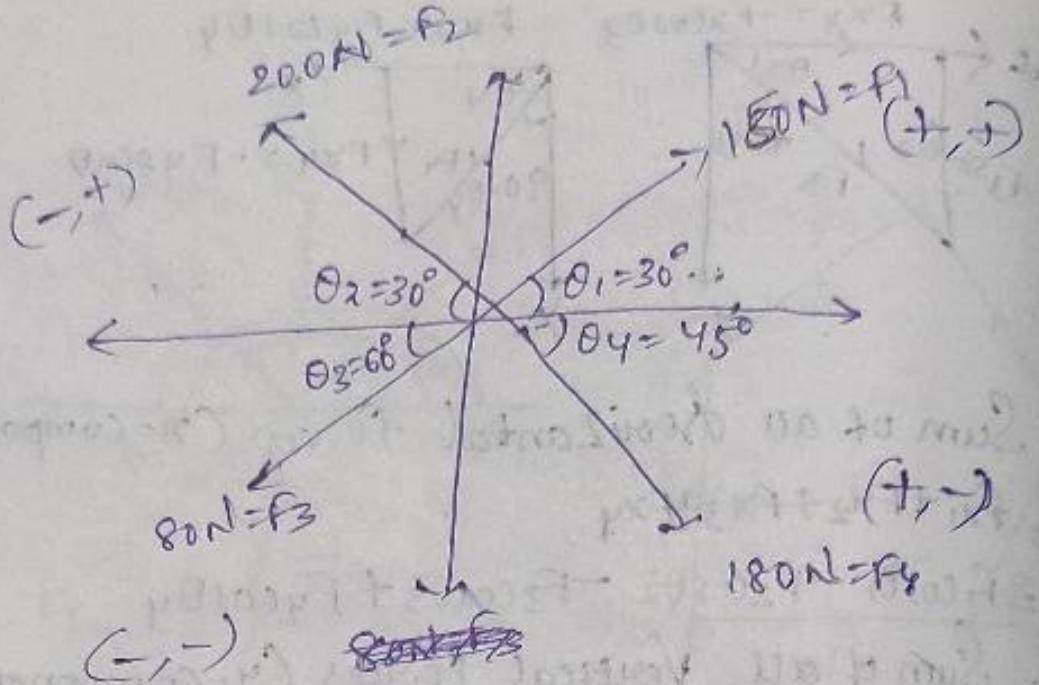
Angle b/w resultant as the x-axis

$$\alpha = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right)$$



~~Ques:-~~

Determine the Resultant of the magnitude and direction of the four forces acting on the body as shown in the figure.



$$\sum F_x = 180 \cos 30^\circ - 200 \cos 30^\circ - 80 \cos 60^\circ + 180 \cos 45^\circ$$

$$\sum F_x = 48.97\text{ N}$$

$$\sum F_y = 150 \sin 30^\circ + 200 \sin 30^\circ - 80 \sin 60^\circ - 180 \sin 45^\circ$$

$$\sum F_y = -21.561\text{ N}$$

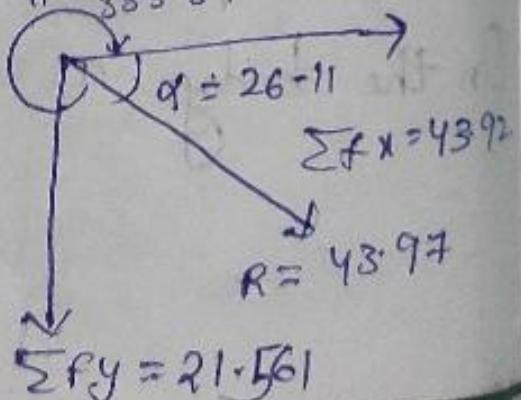
$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$\sqrt{(48.97)^2 + (-21.561)^2}$$

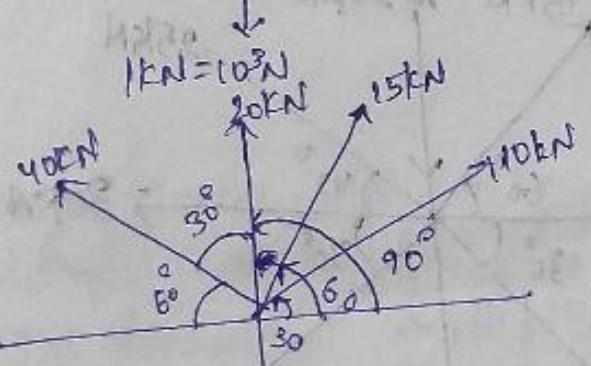
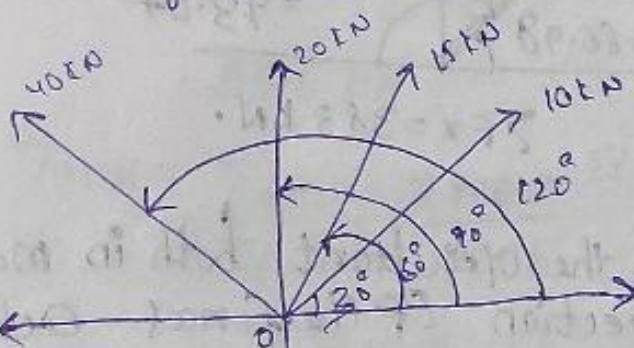
$$R = 48.97\text{ N}$$

$$\tan \alpha = \frac{\sum F_y}{\sum F_x} = \frac{-21.561}{48.97}$$

direction $\alpha = 26.11^\circ$ or 333.89°



~~Q~~ Four forces of magnitude 10kN, 15kN, 20kN and 40kN are acting at a point O. The angles made by 10kN, 15kN, 20kN and 40kN with X-axis are 30° , 60° , 90° & 120° respectively. Find the magnitude and direction of resultant force.

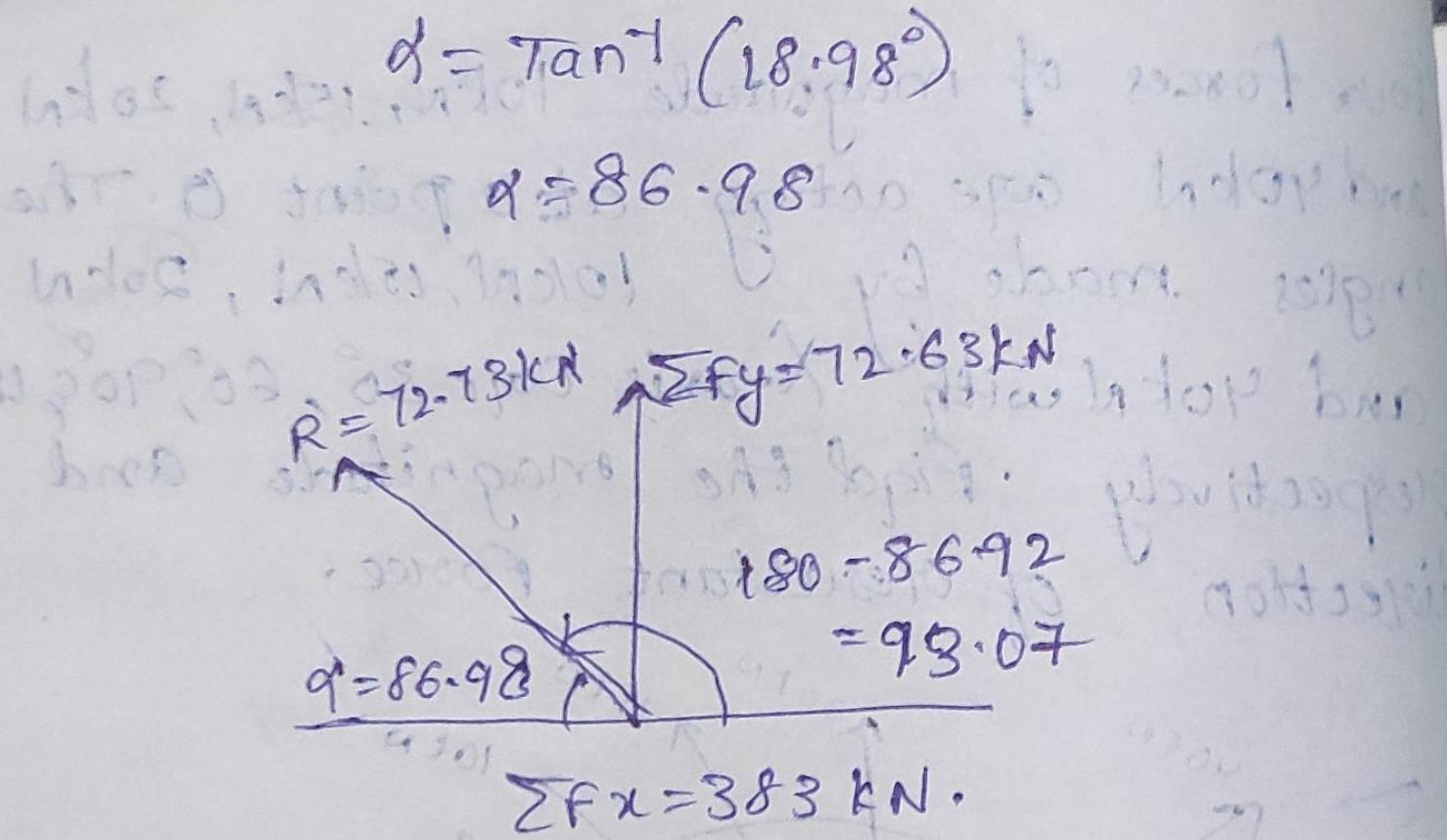


$$\sum F_x = 10\cos 30^\circ + 15\cos 60^\circ + 20\cos 90^\circ - 40\cos 60^\circ = -3.83 \text{ kN.}$$

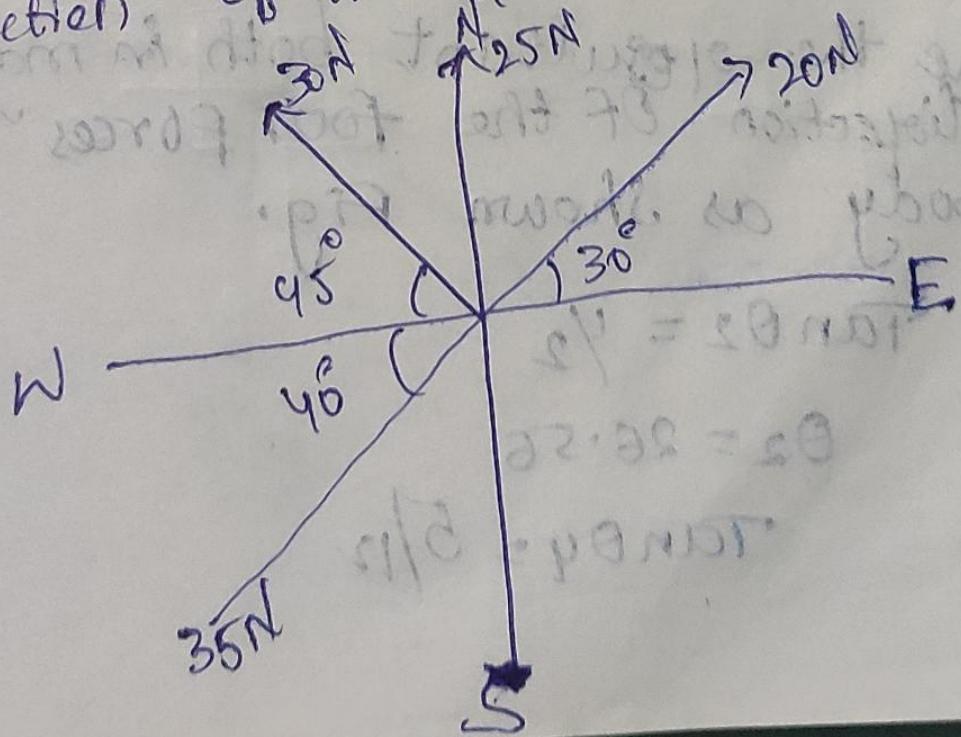
$$\sum F_y = 10\sin 30^\circ + 15\sin 60^\circ + 20\sin 90^\circ + 40\sin 60^\circ = 72.63 \text{ kN.}$$

$$R = \sqrt{\sum F_x^2 + \sum F_y^2} = \sqrt{(-3.83)^2 + (72.63)^2} \\ = 72.73 \text{ kN}$$

$$\tan \delta = \frac{\sum F_y}{\sum F_x} = \frac{72.73}{3.83} \\ \tan \delta = 18.98^\circ$$



~~The following forces~~ The following forces are acting at a point
i, 20N inclined at 30° towards the north of
east. ii, 25N towards North iii, 30N towards
North west iv, 35N inclined at 40° towards
South of west find the magnitude and
direction of the resultant force.



$$\sum F_x = 20 \cos 30^\circ - 30 \cos 45^\circ - 35 \cos 40^\circ$$

$$\sum F_x = -30.70 N$$

$$\sum F_y = 20 \sin 30^\circ + 25 + 30 \sin 45^\circ - 35 \sin 40^\circ$$

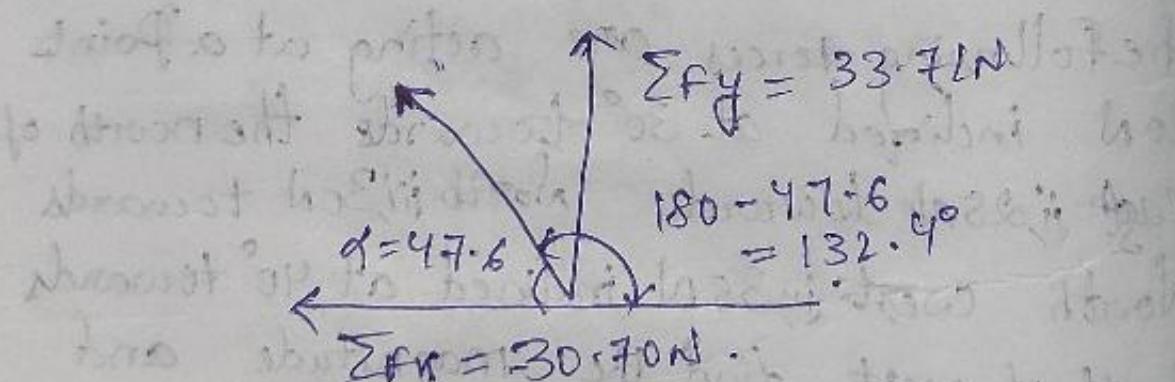
$$\sum F_y = 33.71 N$$

$$F = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$R = \sqrt{(-30.70)^2 + (33.71)^2} = 45.59 N$$

$$\tan \alpha = \frac{\sum F_y}{\sum F_x}$$

$$\alpha = 47.6^\circ$$



~~Determine the resultant both in magnitude and direction of the four forces acting on a body as shown Fig.~~

$$\tan \theta_2 = 4/2$$

$$\theta_2 = 26.56$$

$$\tan \theta_4 = 5/12$$

$$\theta_4 = 22.61$$

$$\sum F_H =$$

$$3 \cos 30 - 2.24 \cos(26.56) \\ - 2 \cos 60 + 3.9 \cos(22.61) \\ = 33.19 \text{ kN}$$

$$\sum F_Y =$$

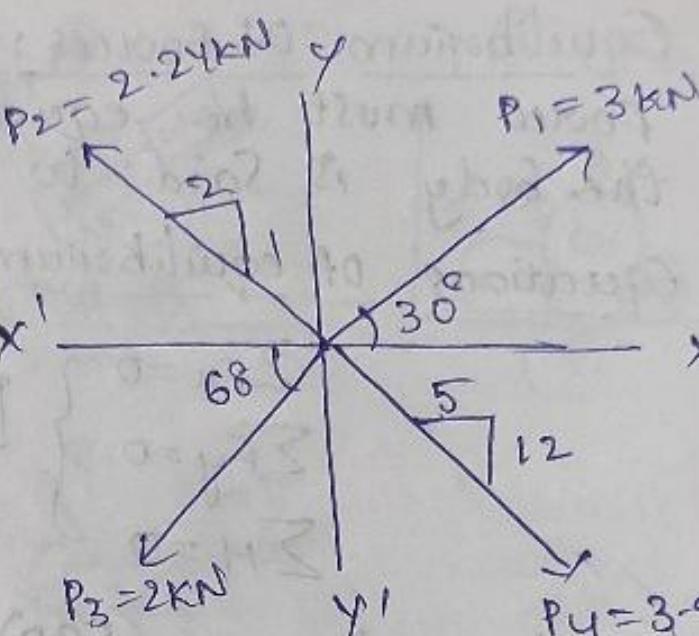
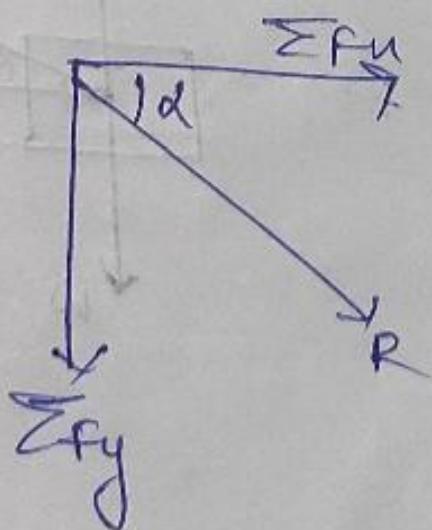
$$3 \sin 30 + 2.24 \sin 26.56 - 2 \sin 60 - 3.9 \sin 22.61 \\ = -0.429 \text{ N}$$

$$R = \sqrt{\sum F_H^2 + \sum F_Y^2} = \\ \sqrt{(33.19)^2 + (-0.429)^2}$$

$$= 3.27 \text{ N}$$

$$\alpha = \tan^{-1} \left(\frac{0.429}{3.19} \right)$$

$$\alpha = 12.87$$



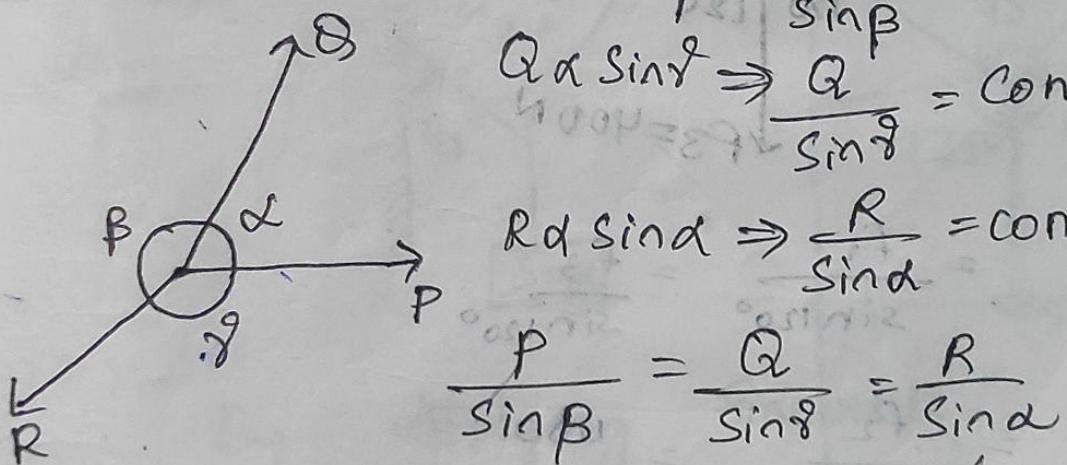
~~Noni's theorem:-~~

If three forces acting at a point at equilibrium
in each force is proportional to sine of
the angle b/w the other two forces.

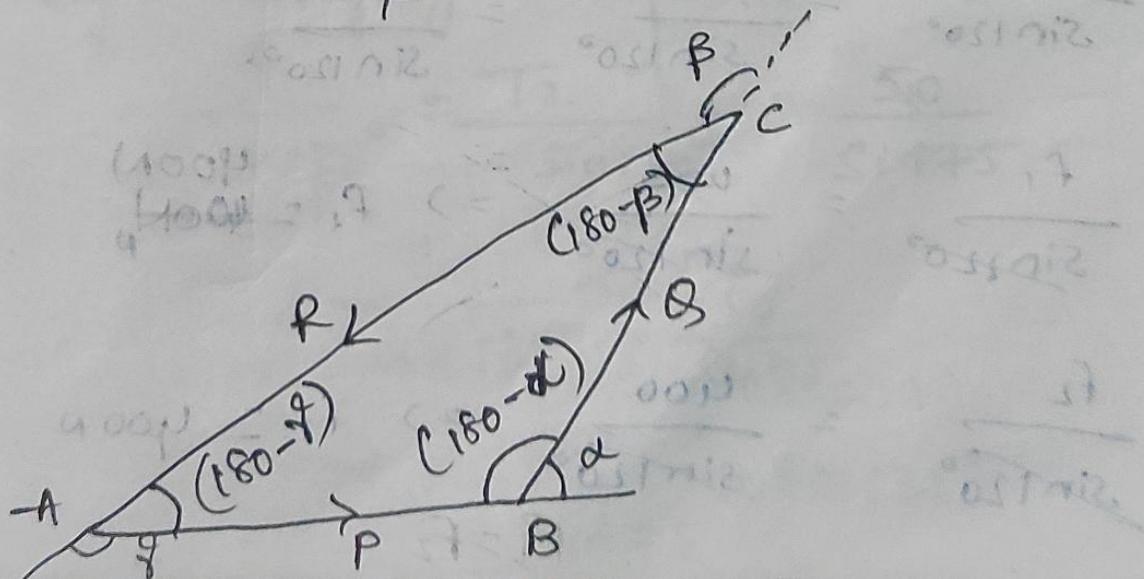
$$P \propto \sin \beta \Rightarrow \frac{P}{\sin \beta} = \text{const}$$

$$Q \propto \sin \gamma \Rightarrow \frac{Q}{\sin \gamma} = \text{const}$$

$$R \propto \sin \alpha \Rightarrow \frac{R}{\sin \alpha} = \text{const}$$



$$\frac{P}{\sin \beta} = \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha} = \text{const}$$

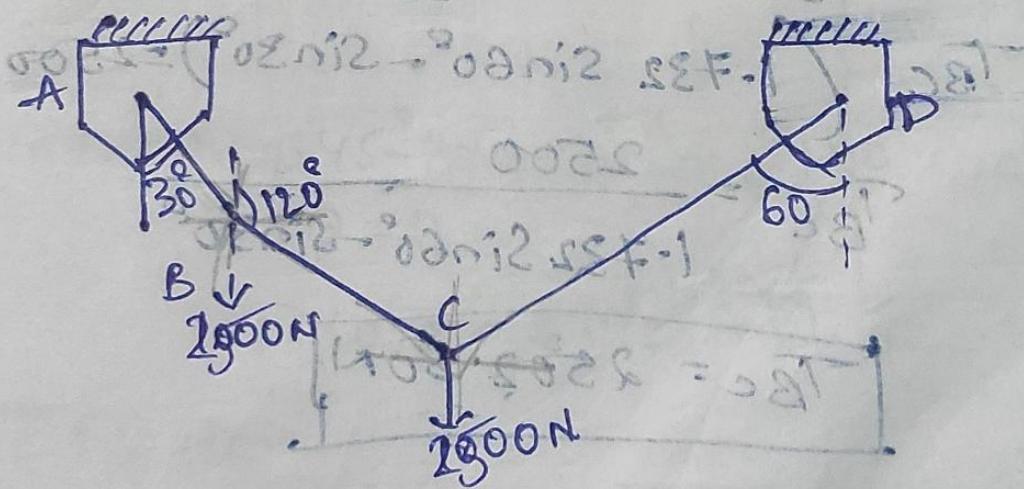


By Apply sin law

$$\frac{P}{\sin(180-\beta)} = \frac{Q}{\sin(180-\delta)} = \frac{R}{\sin(180-\alpha)}$$

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\delta} = \frac{R}{\sin\alpha}$$

~~Problem 6:-~~ Two equal loads of 2500N are supported by a flexible string. If the string ABCD makes angles of 30° , 120° , 60° and 30° at points B, C, D and A respectively. Find tension in the string AB, BC and CD.

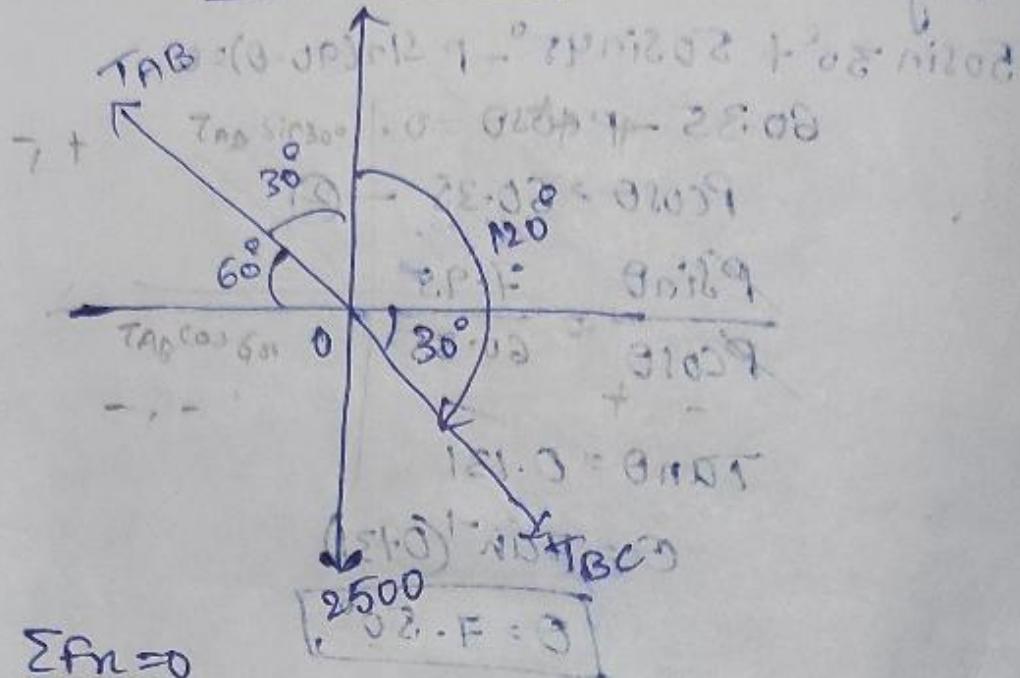


$$T_B = 1250 \times 588 = 735 \text{ N}$$

$$1250 \times 588 = 735 \text{ N}$$

ABD of Point B:

$O = p \neq S$



$$\sum F_H = 0$$

$$-T_{AB} \cos 60^\circ + T_{BC} \cos 30^\circ = 0 \quad \text{due}$$

$$T_{AB} \cos 60^\circ = T_{BC} \cos 30^\circ \quad F = 18.9$$

$$T_{AB} = T_{BC} \times \frac{\cos 30^\circ}{\cos 60^\circ}$$

$$T_{AB} = 1.732 T_{BC} \quad \boxed{1}$$

$$\sum F_V = 0 \quad \text{Lappe out} \quad \text{wall} \quad \text{against} \quad \text{point B}$$

$$T_{AB} \times \sin 60^\circ - T_{BC} \sin 30^\circ - 2500 = 0 \quad \boxed{2}$$

$$1.732 T_{BC} \times \sin 60^\circ - T_{BC} \sin 30^\circ = 2500$$

$$T_{BC} (1.732 \sin 60^\circ - \sin 30^\circ) = 2500$$

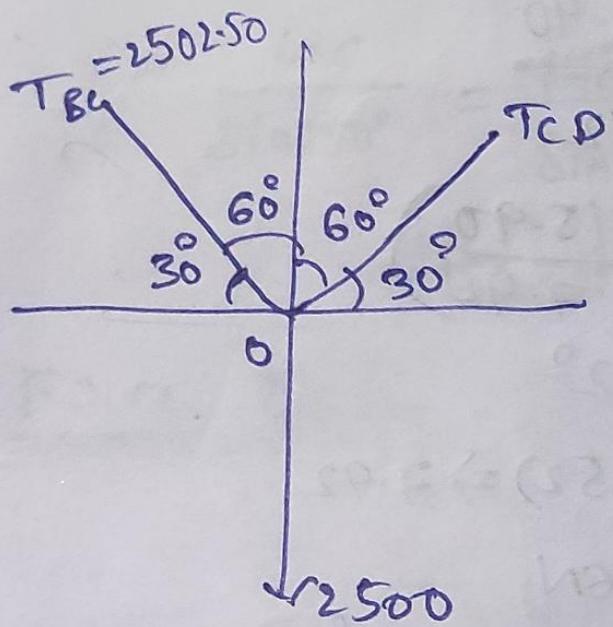
$$T_{BC} = \frac{2500}{1.732 \sin 60^\circ - \sin 30^\circ}$$

$$T_{BC} = 2502.50 \text{ N} \quad \boxed{10000}$$

$$T_{AB} = 1.732 \times 2502.50$$

$$T_{AB} = 4334.55 \text{ N} \quad \boxed{10000}$$

FBD at Point C



$$\sum F_x = 0$$

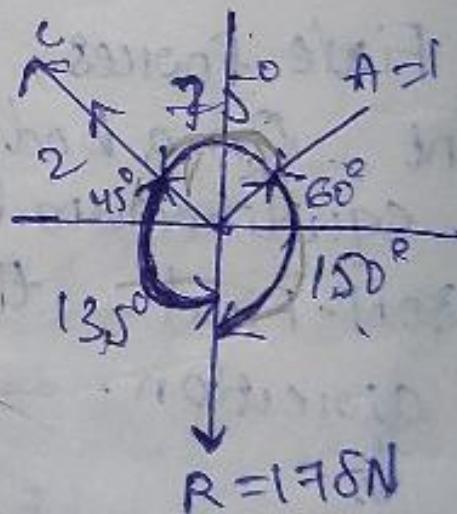
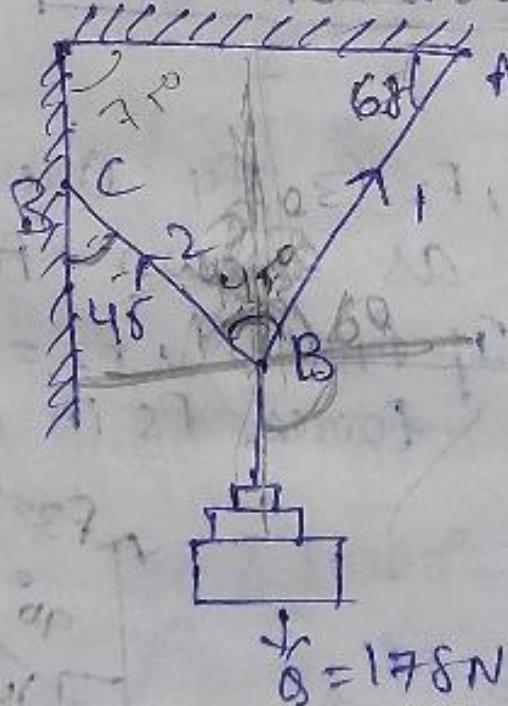
$$T_{CD} \cos 30^\circ - 2502.50 \times \cos 30^\circ$$

$$T_{CD} = \frac{2502.50 \times \cos 30^\circ}{\cos 30^\circ}$$

$$T_{CD} = 2502.50N = T_{BC}$$

~~Q~~ An equi electrical light fixture of weight $Q = 178N$ is supported as shown in the figure. Determine the tension force in the wires BA and BC if their angles of indications are as shown in figure.

$$\tan \theta = \frac{178}{R} = 0.5066 \Rightarrow \theta = 27^\circ$$



Sine law

$$\frac{AB}{\sin 135^\circ} = \frac{BC}{\sin 150^\circ} = \frac{Q}{\sin 75^\circ}$$

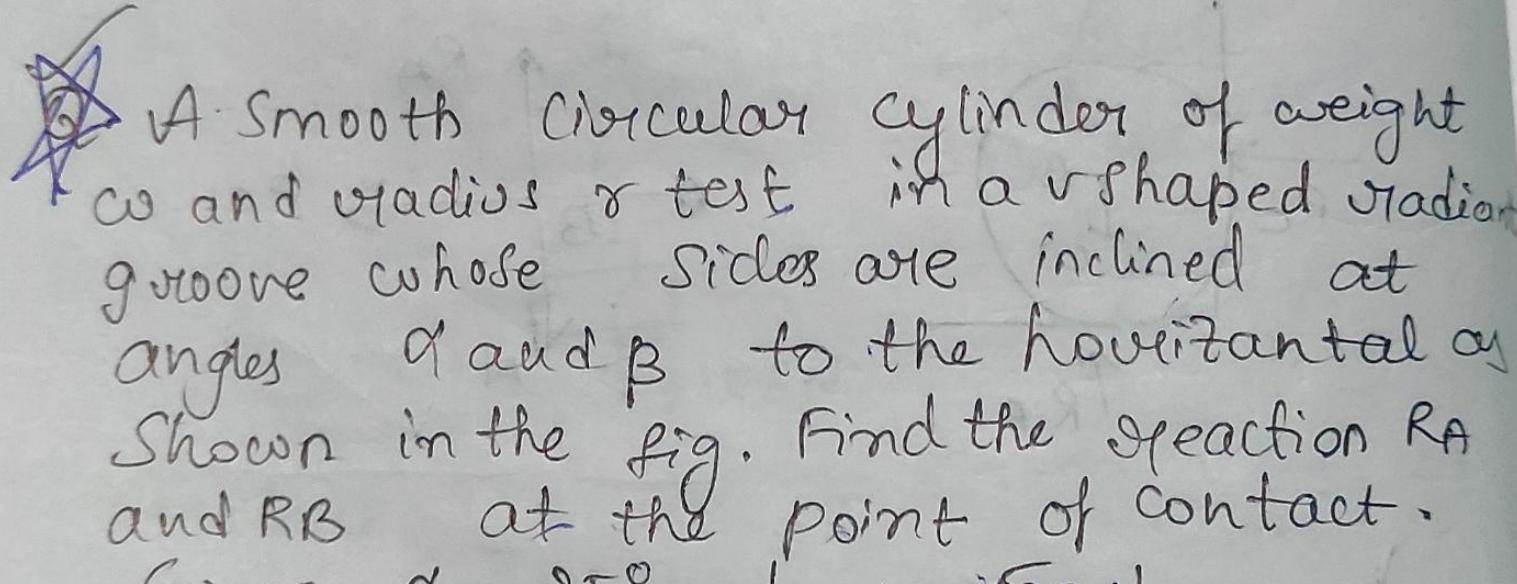
$$\frac{AB}{\sin 135^\circ} = \frac{178}{\sin 75}$$

$$AB = \frac{178}{\sin 75} \times \sin 135^\circ$$

$$AB = 130.3N$$

$$\frac{BC}{\sin 150^\circ} = \frac{178}{\sin 75^\circ}$$

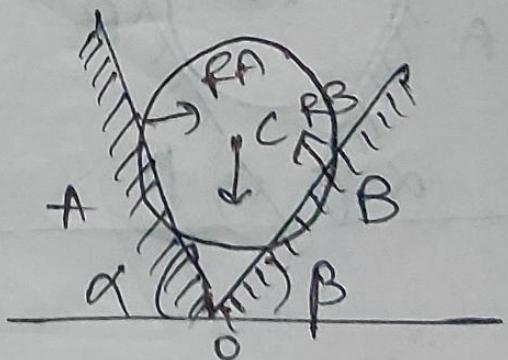
$$BC = 92.139N$$

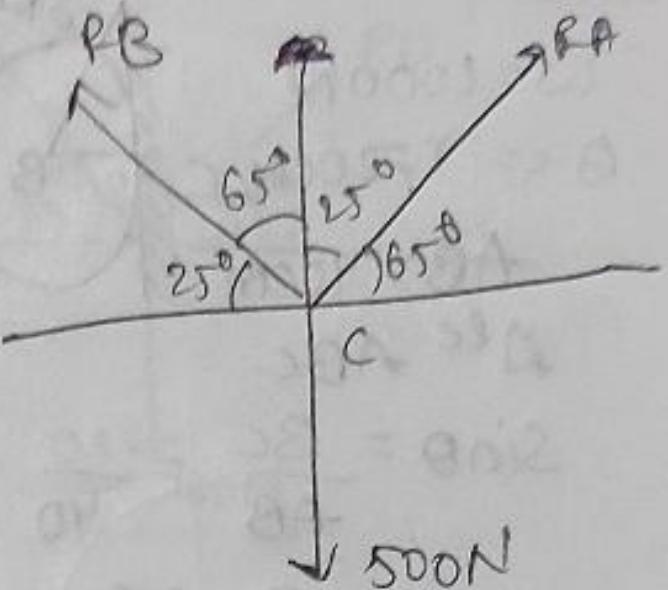
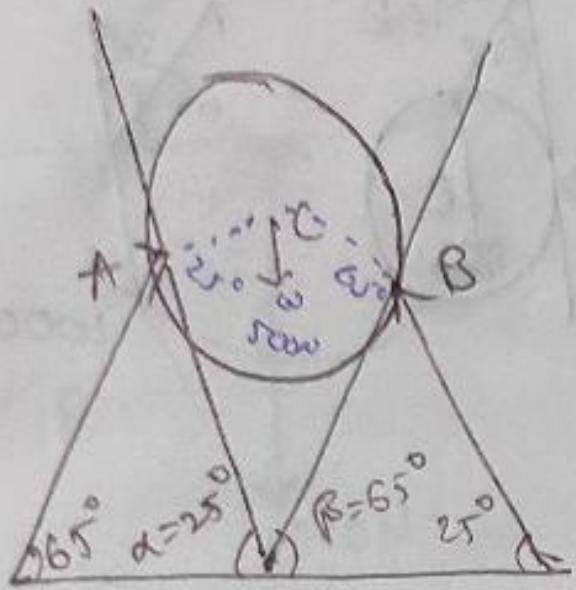

 A smooth circular cylinder of weight w and radius r rests in a V-shaped groove whose sides are inclined at angles α and β to the horizontal as shown in the fig. Find the reaction R_A and R_B at the point of contact.
 Given $\alpha = 25^\circ$ and $w = 500\text{N}$.

Sol Given that $\alpha = 25^\circ$

$\beta = 65^\circ$, $w = 500\text{N}$

$R_A, R_B = ?$





$$\sum F_x = 0$$

$$R_A \cos 65^\circ - R_B \cos 25^\circ = 0$$

$$R_A = \frac{R_B \cos 25^\circ}{\cos 65^\circ}$$

$$R_A = 2.14 R_B \quad \text{--- (1)}$$

$$\sum F_y = 0$$

~~$$R_A \sin 65^\circ + R_B \sin 25^\circ = 500$$~~

~~$$2.14 R_B \sin 65^\circ + R_B \sin 25^\circ = 500$$~~

~~$$(2.14 \sin 65^\circ + \sin 25^\circ) = 500$$~~

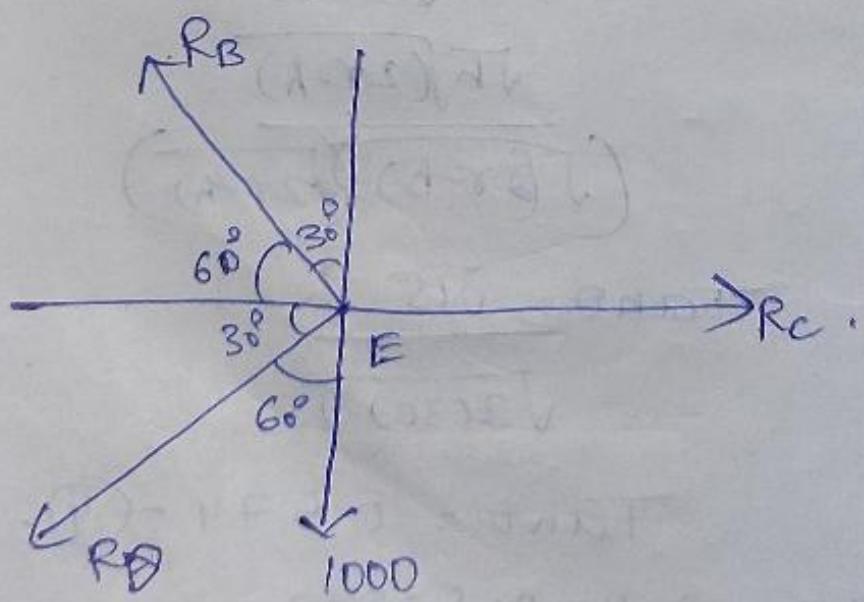
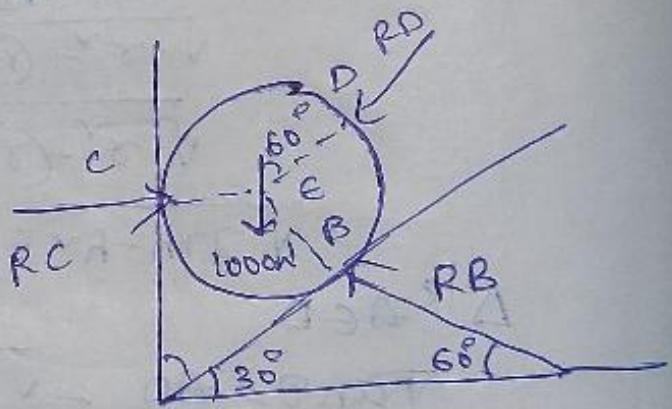
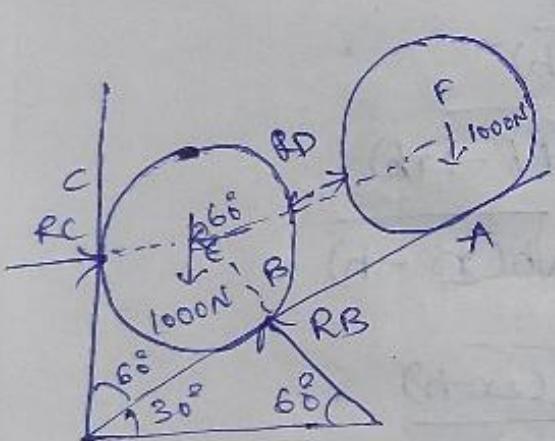
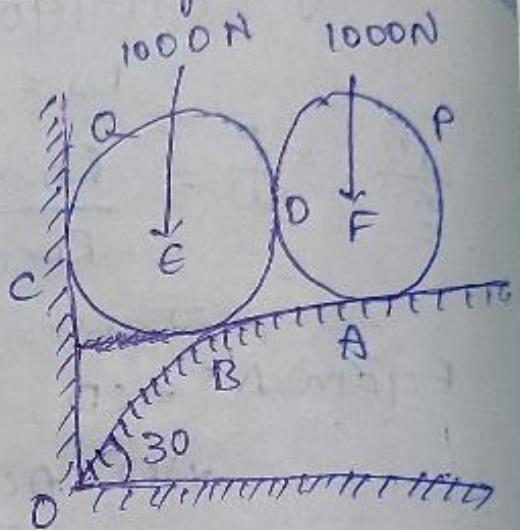
$$R_B = \frac{500}{2.14 \sin 65^\circ + \sin 25^\circ}$$

$$R_B = 211.36 N$$

$$\text{--- (1)} \Rightarrow R_A = 2.14 (211.36)$$

$$R_A = 453.36 N$$

~~Q.~~ Two identical rollers, each of weight w , are supported by an inclined plane and vertical wall as shown figure. Find the reactions at the point of supports A and C. Assume all the surfaces to be smooth.

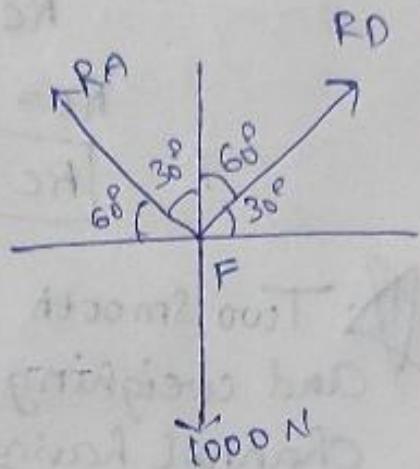
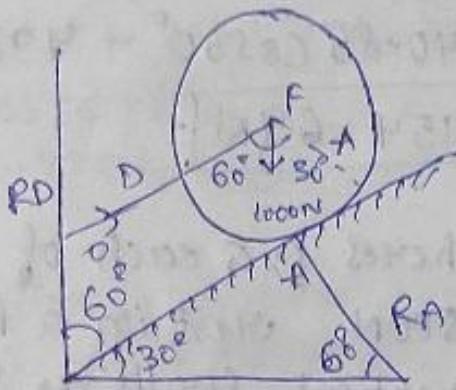


$$\sum F_n = 0$$

$$R_c - R_B \cos 60^\circ - R_D \cos 30^\circ = 0 \quad \text{--- (1)}$$

$$\sum F_y = 0 \quad R_B \sin 60^\circ - R_D \sin 30^\circ - 1000 = 0 \quad \text{--- (2)}$$

FBD of Roller-2:-



$$\sum F_n = 0$$

$$R_D \cos 30^\circ - R_A \cos 60^\circ = 0$$

$$R_D = \frac{R_A \cos 60^\circ}{\cos 30^\circ} \quad R_D = R_A [0.577]$$

$$\sum F_y = 0$$

$$R_D \sin 30^\circ + R_A \sin 60^\circ - 1000 = 0$$

$$R_A [0.57] \sin 30^\circ + R_A \sin 60^\circ - 1000 = 0$$

$$R_A [0.57 \sin 30^\circ + \sin 60^\circ] = 1000$$

$$R_A = \frac{1000}{0.57 \sin 30^\circ + \sin 60^\circ}$$

$$R_A = 866.35 \text{ N}$$

$$R_D = 866.35 (0.57)$$

$$R_D = 499.69 \text{ N}$$

From equ (2)

$$R_B \sin 60^\circ - 499.69 \sin 30^\circ + 1000$$

$$R_B \sin 60^\circ = 495.64 \sin 30^\circ + 1000$$

$$R_B = \frac{495 \cdot 64 \sin 30^\circ + 1000}{\sin 60^\circ}$$

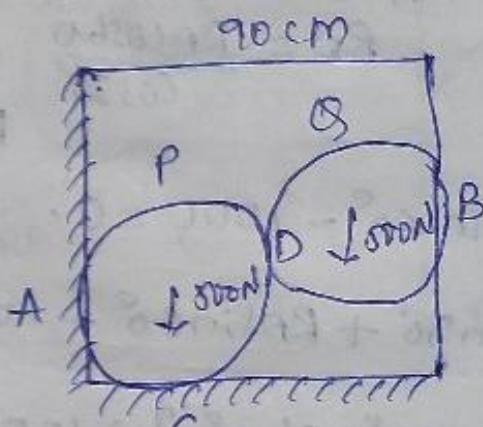
$$[R_B = 1440 \cdot 85 \text{ N}]$$

$$R_C = 1440 \cdot 85 \cos 60^\circ - 495 \cdot 64 \cos 30^\circ$$

$$R_C = 1440 \cdot 85 \cos 60^\circ + 495 \cdot 64 \cos 30^\circ$$

$$[R_C = 1154 \cdot 69 \text{ N}].$$

~~D~~. Two smooth spheres P, Q each of radius 25 cm and weighing 500N, rest in a horizontal channel having vertical walls as shown fig. If the distance between the walls is 90cm. Calculate the reactions at points of contact A, B and C.



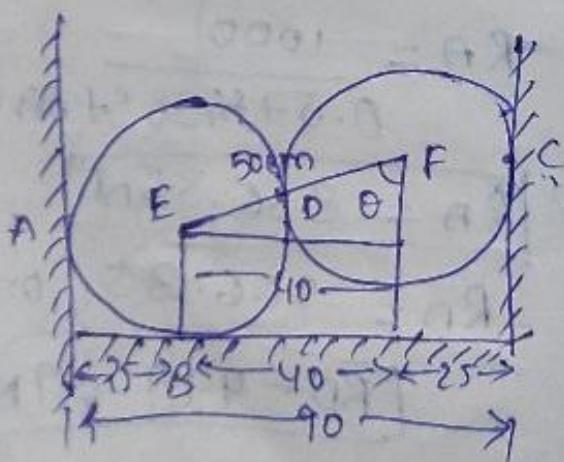
$$\sin \theta = 40/50$$

$$\text{Vertical}(\theta) = 53.13$$

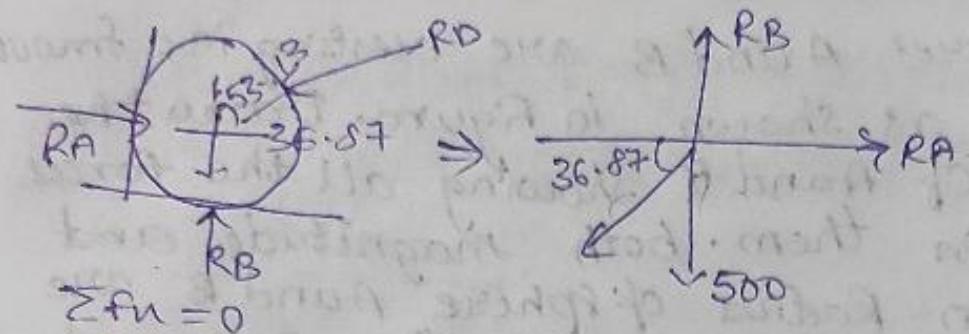
$$\text{horizontal}(\theta) =$$

$$90 - 53.13$$

$$= 36.87 \text{ cm}$$



FBD of Roller ①



$$\sum F_H = 0$$

$$R_A - R_D \cos 36.87^\circ = 0$$

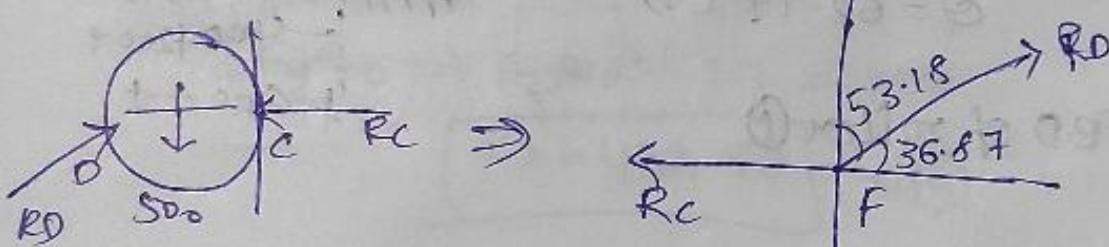
$$R_A = 0.799 R_D \quad \text{---} ①$$

$$\sum F_Y = 0$$

$$-R_B - R_D \sin 36.87 - 500 = 0$$

$$R_B = 0.600 R_D + 500 \quad \text{---} ②$$

FBD of Roller ②



$$\sum F_H = 0 \Rightarrow -R_C + R_D \cos 36.87 = 0$$

$$\sum F_Y = 0 \Rightarrow R_D \sin 36.87 - 500 = 0$$

$$R_D = \frac{500}{\sin 36.87} \Rightarrow 833.33 \text{ N} = R_D$$

$$R_C = R_D \cos 36.87$$

$$R_C = 666.66$$

$$R_A = 0.799 \times 833.33$$

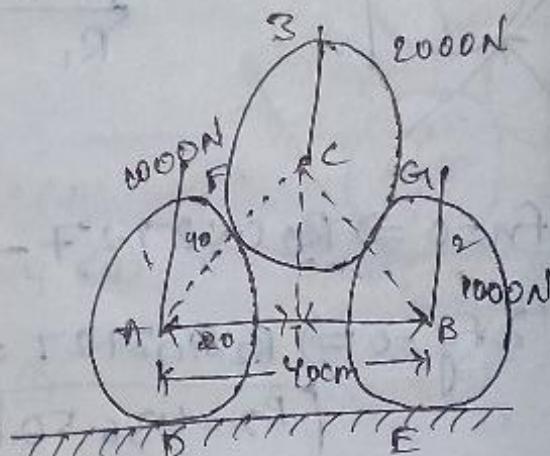
$$R_A = 665.830$$

$$R_B = 0.600 (833.33) + 500$$

$$R_B = 999.99$$

~~Diagram~~ Two Smooth Circular Cylinders, each of weight = 1000N and radius 15cm, are connected at the centres by a string AB of length = 40 cm and rest upon a horizontal plane supporting above them a third cylinder of weight 2000N and radius 15cm as shown figure. Find the force T in the string AB and the pressure produced on the floor at the point of contact D and E.

$$\begin{aligned} w_1 = w_2 &= 1000 \text{ N} \\ r_1 = r_2 &= 15 \text{ cm} \\ AB &= 40 \text{ cm} \\ w_3 &= 2000 \text{ N} \\ r_3 &= 15 \text{ cm} \end{aligned}$$

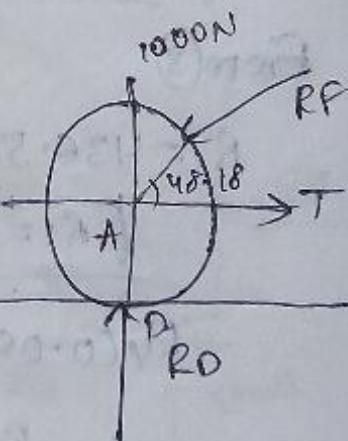


$$\cos \theta = 20/30$$

$$\theta = 48.18^\circ (\text{H})$$

$$\theta = 90 - 48.18$$

$$\theta = 41.81^\circ (\text{v})$$



$$(a) T = ?$$

$$(b) RD, RE = ?$$

FBD of cylinder :

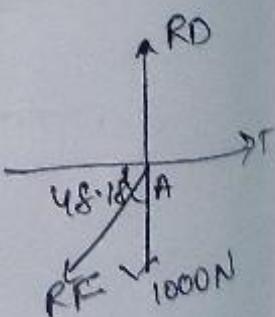
$$\sum F_x = 0$$

$$T - RF \cos 48.18 = 0$$

$$T = 0.6 RE \quad (1)$$

$$\sum F_y = 0$$

$$RD - RF \sin 48.18 - 1000 = 0$$

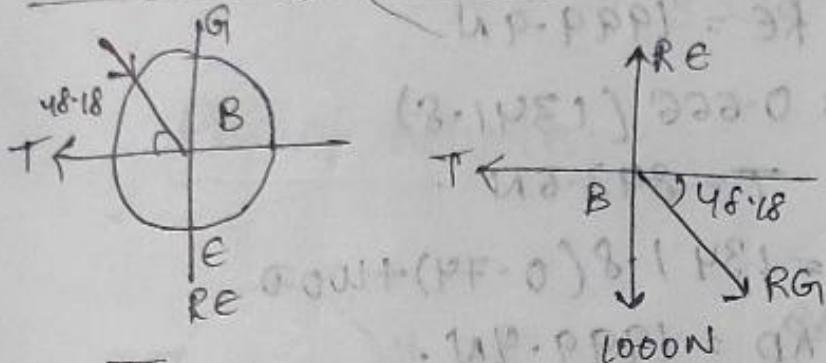


$$RD - RF \sin 48.18 = -1000$$

$$RD = RF \sin 48.18 + 1000$$

$$RD = RF(0.74) + 1000 \quad \text{--- (2)}$$

FBD of cylinder 2:



$$\sum F_x = 0$$

$$-T + RG \cos 48.18 = 0$$

$$T = RG \cos(48.18)$$

$$T = 0.666 RG \quad \text{--- (3)}$$

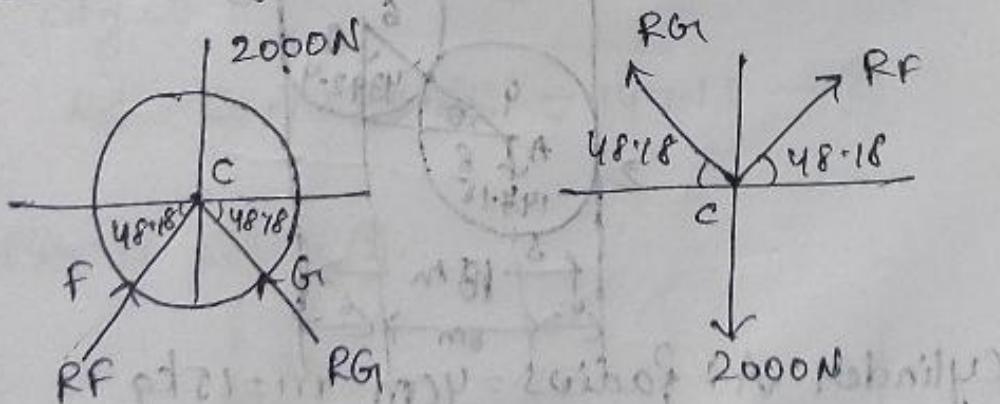
$$\sum F_y = 0$$

$$RE - 1000 - RG \sin 48.18 = 0$$

$$RG = RG \sin 48.18 + 1000$$

$$RE = RG(0.74) + 1000 \quad \text{--- (4)}$$

FBD of cylinder 3:



$$\sum F_x = 0$$

$$RF \cos 48.18 - RG \cos 48.18 = 0$$

$$RF = RG \quad \text{--- (5)}$$

$$\sum F_y = 0$$

$$RF \sin 48.18 + RG \sin 48.18 - 2000 = 0$$

$$RG_1 = 1341.8$$

$$RF = 1341.8$$

$$\textcircled{1} \rightarrow RG_1 = RF = 1341.84 \text{ N.}$$

$$\textcircled{2} \Rightarrow 1341.8 (0.74) + 1000$$

$$RE = 1999.9 \text{ N}$$

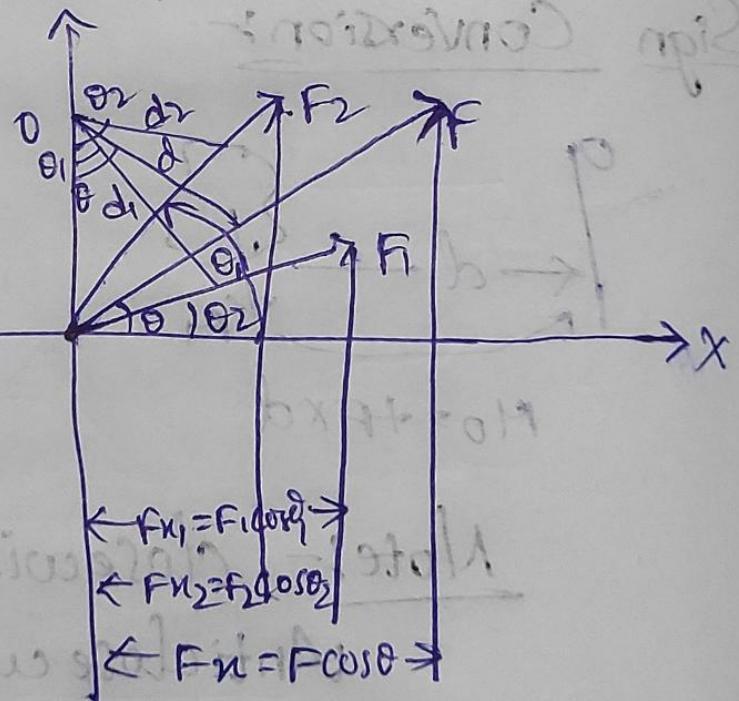
$$\textcircled{3} \Rightarrow T = 0.666 (1341.8)$$

$$T = 893.6 \text{ N}$$

$$\textcircled{4} \Rightarrow RD = 1341.8 (0.74) + 1000$$

$$RD = 1999.9 \text{ N.}$$

~~Varignon's Theorem~~: It states that the moment of a force above any point is equal to the algebraic sum of the moments of Components above same point.



The moment of force F about point $O = F \times d$
 $= F \times OA \cos \theta$

$$Fd = OAx F \cos \theta$$

$$Fd = OAx Fn - ①$$

The moment of force F_1 above point $O = F_1 d_1$

$$F_1 \times OA \cos \theta_1 \quad \left\{ \begin{array}{l} \therefore \cos \theta = d_1/OA \\ \therefore d_1 = OA \cos \theta_1 \end{array} \right.$$

$$OA \times F_1 \cos \theta_1$$

$$F_1 d_1 = OA \times F_1 \cos \theta_1 - \textcircled{2} \quad \left\{ \begin{array}{l} \cos \theta_1 = d_1/OA \\ d_1 = OA \cos \theta_1 \end{array} \right.$$

The moment of force F_2 about point $O = F_2 d_2$

$$F_2 \times OA \cos \theta_2 \quad \left\{ \begin{array}{l} \cos \theta_2 = d_2/OA \\ d_2 = OA \cos \theta_2 \end{array} \right.$$

$$F_2 \cos \theta_2 \times OA$$

$$F_2 d_2 = OA \times F_2 \cos \theta_2 - \textcircled{3}$$

$$\textcircled{2} + \textcircled{3} \Rightarrow$$

$$F_1 d_1 + F_2 d_2 = OA F_{x1} + OA F_{x2}$$

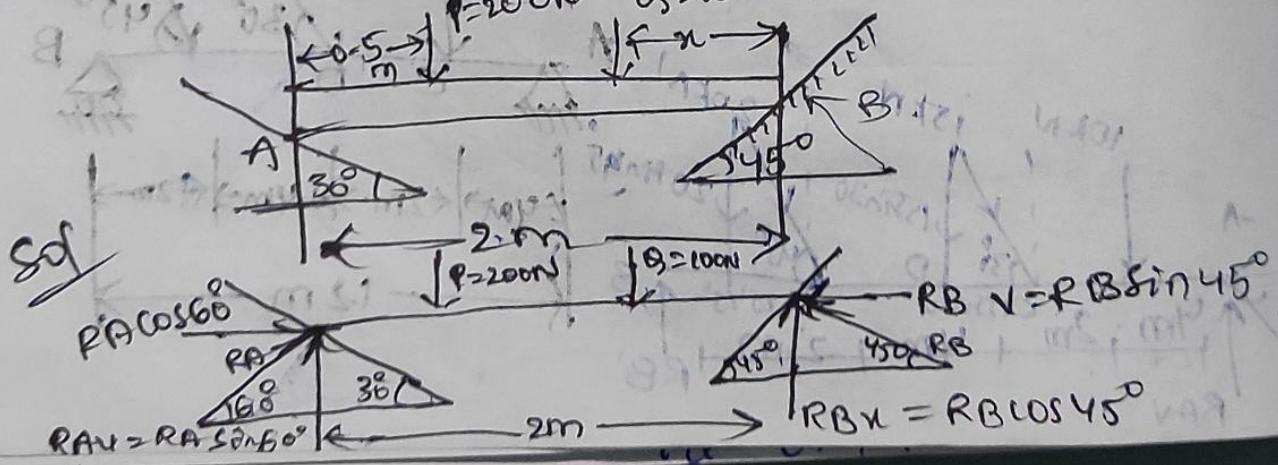
$$F_1 d_1 + F_2 d_2 = OA [F_{x1} + F_{x2}] - \textcircled{4}$$

$$F_1 d_1 + F_2 d_2 = OA \cdot F_x - \textcircled{5}$$

$$\textcircled{4} = \textcircled{5}$$

$$\boxed{Fd = F_1 d_1 + F_2 d_2}$$

~~Q~~ A bar 2m long and of negligible weight rests in horizontal position on two smooth inclined planes as shown in Fig. Determine the distance 'x' at which the load $Q=100N$ shall be placed from Point B to keep the bar horizontal.



$$\sum F_x = 0$$

$$RA \cos 60^\circ - RB \cos 45^\circ = 0$$

$$RA = \frac{RB \cos 45^\circ}{\cos 60^\circ} = 0.707 RB$$

$$RA = 1.414 RB \quad \text{(i)}$$

$$\sum F_y = 0$$

$$RA \sin 60^\circ - 200 - 100 + RB \sin 45^\circ = 0$$

$$1.414 RB \sin 60^\circ + RB \sin 45^\circ = 300$$

$$RB = \frac{300}{1.414 \sin 60^\circ + \sin 45^\circ}$$

$$RB = 155.30 N$$

$$RA = 219.60 N$$

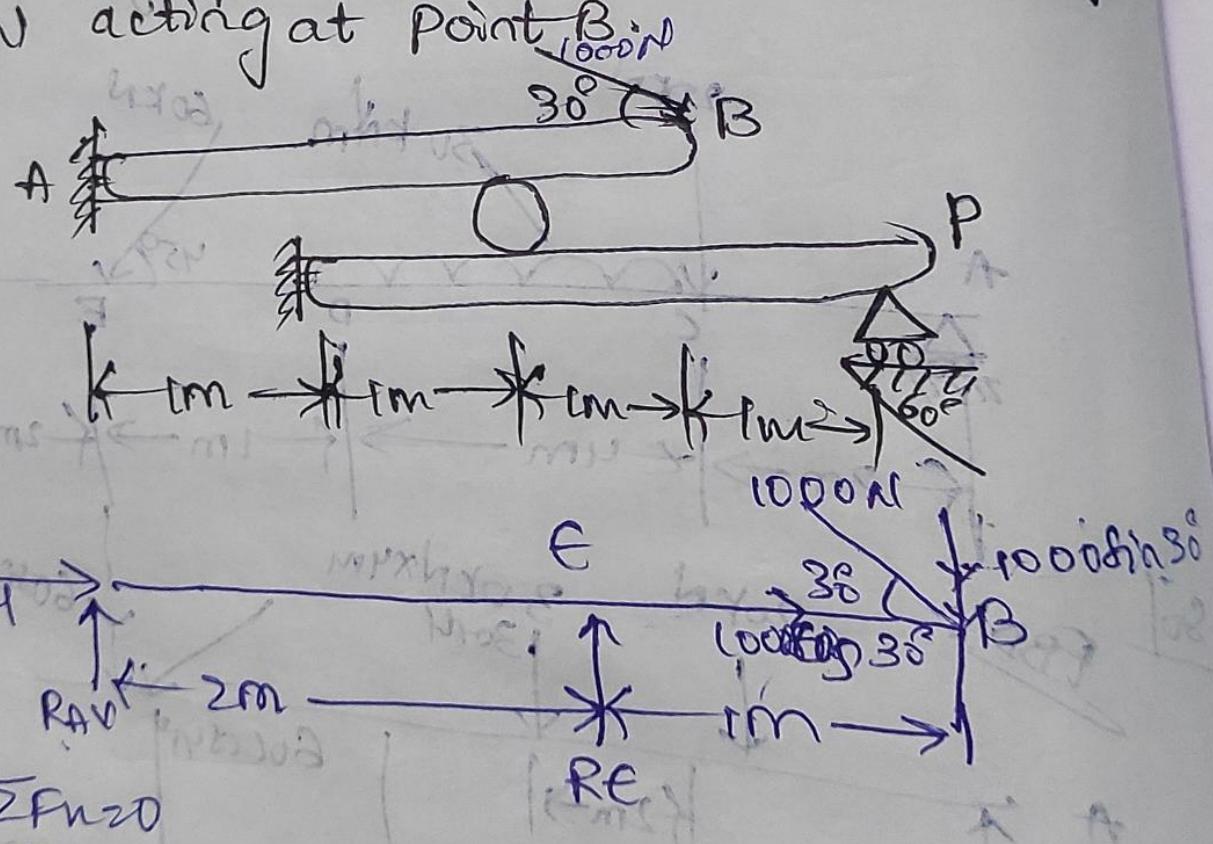
$$\sum M_B = 0$$

$$-100x2 - 200(2-0.5) + RA \sin 60^\circ \times 2 = 0$$

$$219.60 \sin 60^\circ \times 2 + 200 \times 1.5 = 100x$$

$$x = 0.80 m$$

~~Q.~~ Two beams AB and CD are arranged and supported as shown figure - find the reaction of B due to the force of 1000N acting at Point B.



$$\sum F_{H2O}$$

$$RAH + 1000 \cos 30^\circ = 0$$

$$RAH = -1000 \cos 30^\circ$$

$$RAH = -866$$

$$\sum F_y = 0$$

$$RAV + RE - 1000 \sin 30^\circ = 0$$

$$RAV + RE = 500N \quad \textcircled{1}$$

$$\sum M_A = 0$$

$$RE \times 2 + 1000 \sin 30^\circ \times 3 = 0$$

$$RE = 750N \quad \textcircled{2}$$

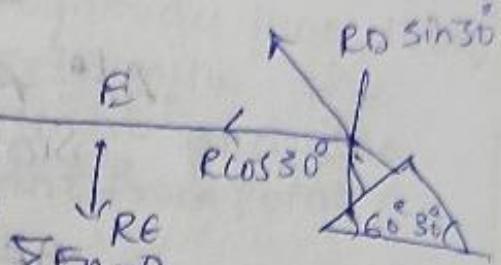
$$\textcircled{1} \Rightarrow RAV = 500 - 750$$

$$RAV = -250N$$

$$RA = \sqrt{RAV^2 + RAV^2}$$

$$\sqrt{(-866.02)^2 + (-250)^2}$$

$$= 901.38 N$$



$$RCH - RD \cos 30 = 0$$

$$RCH = RD \cos 30$$

$$Rcv - RE + RD \sin 30 = 0$$

$$Rcv + RD \sin 30$$

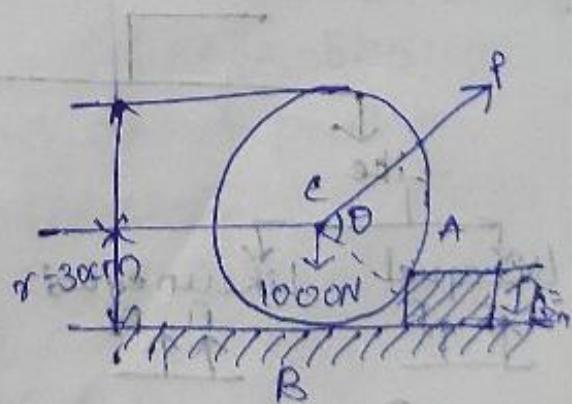
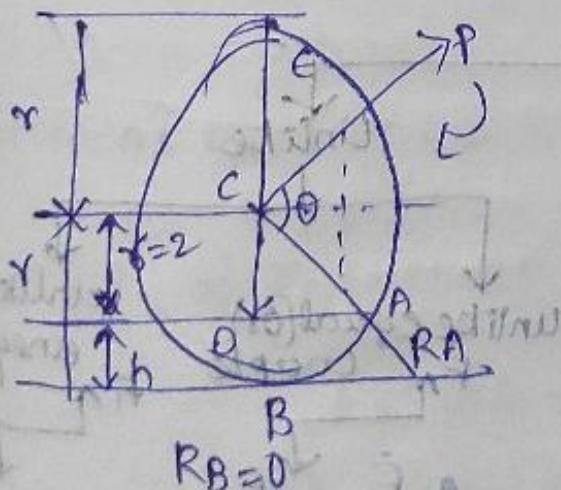
$$\sum M_A = 0$$

$$RE \times 1 - RD \sin 30 \times 3 = 0$$

$$RE = 3RD \sin 30$$

$$RE = 500 N$$

A uniform wheel of 60cm diameter weighing 15kg rests against a rectangular obstacle which is 15cm high. Find the least force required which when acting through the centre of the wheel will just turns the wheel over the corner of the block. Also find the angle θ which this least force shall make with AC as shown in fig.



$$\sum M_A = 0$$

$$-w x A D + P x A E = 0$$

$$P = \frac{\omega \times A D}{A E}$$

$\Delta^{1\circ} ACE$

$$\sin \theta = \frac{AE}{AC}$$

$$AE = AC \sin \theta$$

$$AE = r \sin \theta$$

from $\Delta^{1\circ} ADC$

$$AD^2 = AC^2 - CD^2$$

$$AD = \sqrt{AC^2 - CD^2}$$

$$AD = \sqrt{r^2 - (r-h)^2}$$

$$AD = \sqrt{r^2 - (r^2 + h^2 - 2rh)}$$

$$AD = \sqrt{2rh - h^2}$$

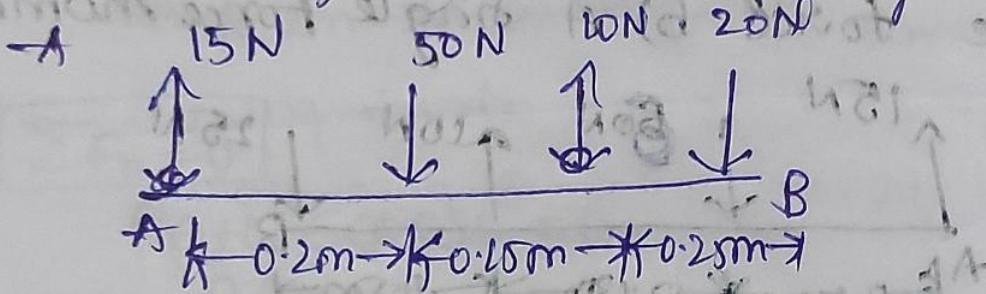
$$\sin \theta = \frac{1}{2} \alpha \\ \theta = 90^\circ \cdot P = \frac{\omega \times \sqrt{2rh - h^2}}{r \sin \theta}$$

$$P = 1000 \times \sqrt{2 \times 3.0 \times 15 - 15^2}$$

$$30 \times 1$$

$$P_{mm} = 866.02 N$$

10. A Rigid bar is subjected to a system of parallel force as shown in figure. Reduce the system of
 a) A single force
 b) A single force-moment system at A
 c) A single force-moment system at B.



a) Single force (Resultant)

$$R = -15 + 50 - 10 + 20$$

$$R = -45 \text{ (down)} - R = 45 \text{ N (up)}$$

$$Fd = F_1d_1 + F_2d_2 + F_3d_3 + F_4d_4$$

The moment of 15N about point A

$$15 \times 0.2 = 3 \text{ Nm}$$

The moment of 50N about point B

$$B = 50 \times 2.0 = 10 \text{ Nm}$$

The moment of 10N about point C

$$= -10 \times 0.35 = -3.5 \text{ Nm}$$

The moment of 20N about point D

$$= 20 \times 0.6 = 12 \text{ Nm}$$

b) The moment of Resultant = 45×2

A Single moment System of at A

~~in bsp. ↓ and 0.28, 0.41 m apart~~

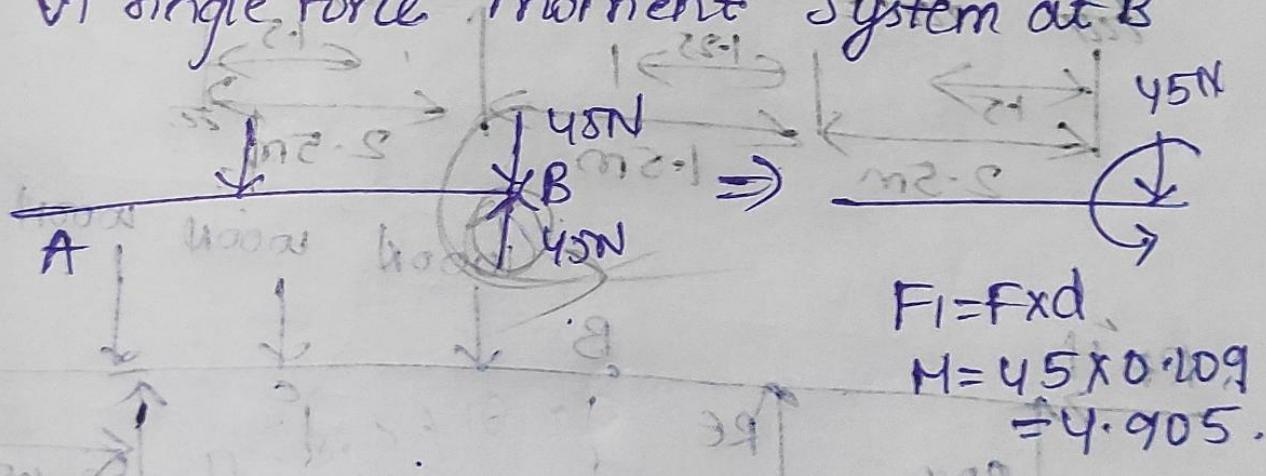
$$45 \times x = 18.5$$

$$x = 18.5 \quad \text{m}$$

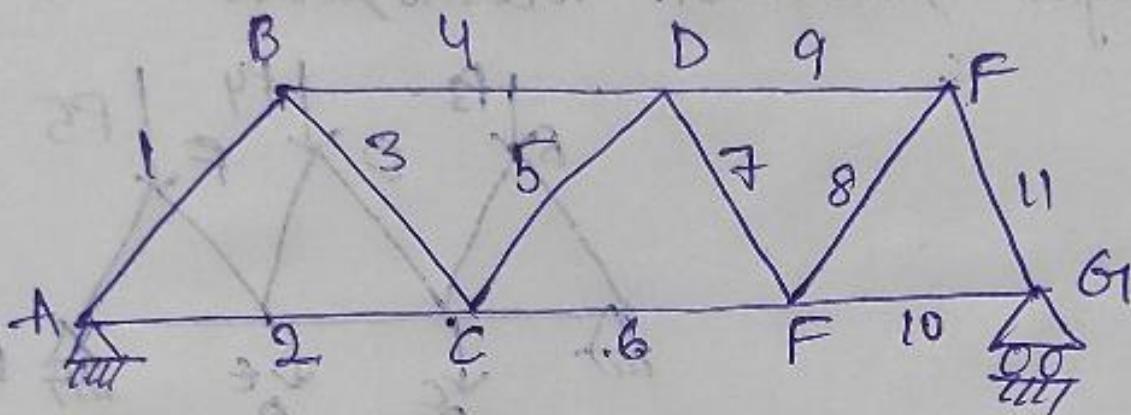
~~At } x=0.412 \rightarrow 0.169 \rightarrow \text{ node } \leftarrow 0.6 \rightarrow \text{ node}~~

$$\Rightarrow \begin{array}{c} 45N \\ | \\ 45N \\ \text{horizontal} \\ 45N \\ | \\ 0.411 \end{array} \quad \begin{array}{c} B \\ \downarrow \\ 45N \\ M = F \times d \\ = 45 \times 0.411 = \\ 18.495 \text{ Nm} \end{array}$$

c) A single force moment system at B



Assumption of Perfect Truss



$$J = 7$$

$$m = 11$$

$$m = 2J - 3$$

(Perfect truss)

$$11 = 11$$

- * All the members are in One plane.
- * The external loads are applied at the joints.
- * The weight of members are negligible.
- * The cross section of members are uniform.
- * The truss is statically determined.

$$\sum F_x = 0, \sum F_y = 0, \sum M = 0.$$

- * All the joints of a truss, ideally hinged (On) roller.
- * The forces in the member are only axial that is either tensile or compressive.

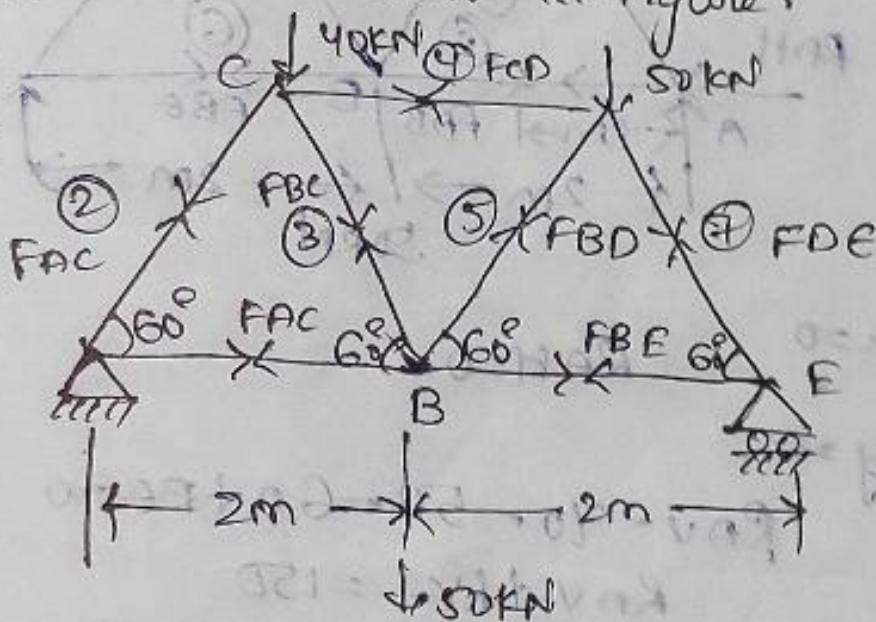
Methods Of Analysis of Trusses

I. Method of Joints

ii) Method of Sections (not in Syllabus)

Problem:-

- Q. Determine the axial forces included in the members at a truss Using method of joints as shown in figure.



$$\text{Members} = 7$$

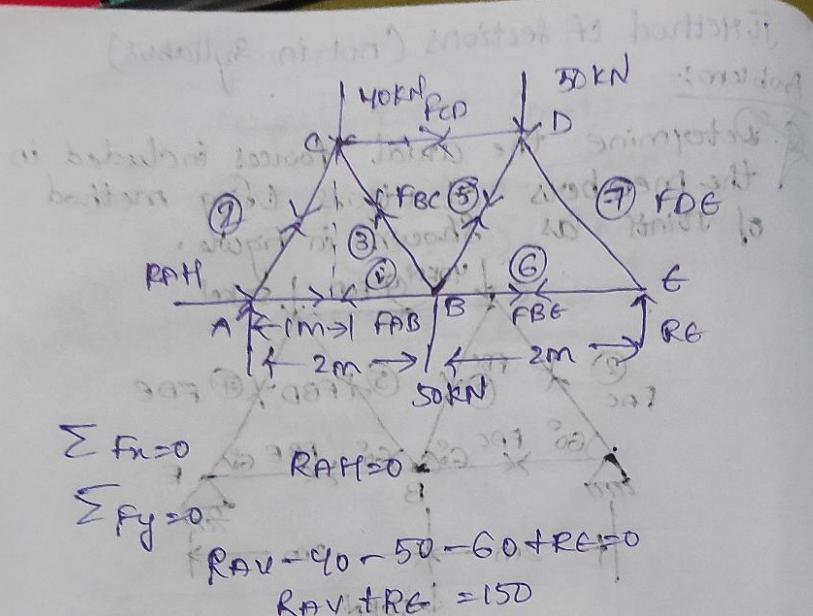
$$\text{Joints} = 5$$

$$m = 2 \times 3 - 2J - 3$$

$$= 2 \times 5 - 3$$

Perfect truss

SNo	member	forces
1	AB	F _{AB}
2	AC	F _{AC}
3	BC	F _{BC}
4	CD	F _{CD}
5	BD	F _{BD}
6	BE	F _{BE}
7	DE	F _{DE}



$$\sum M_A = 0$$

$$40 \times 1 + 60 \times 2 + 50 \times 3 - RG \times 4 = 0$$

$$RG = 77.5 \text{ kN}$$

$$RAV = 72.5 \text{ kN}$$

Joint A reaction
RAV = 72.5 kN

$$\sum F_x = 0$$

$$F_{AB} + F_{BC} \cos 60^\circ = 0 \quad \text{---(1)}$$

$$\sum F_y = 0$$

$$72.5 + F_{AC} \sin 60^\circ = 0$$

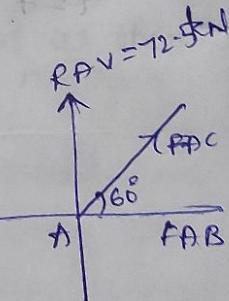
$$F_{AC} = \frac{-72.5}{\sin 60}$$

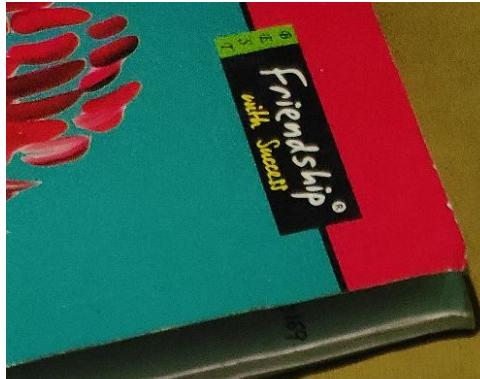
$$F_{AC} = -83.7 \text{ kN}$$

FAC = 83.7 (compression)

$$F_{AB} = -F_{AC} \cos 60^\circ$$

$$F_{AB} = 41.85 (+) \text{ kN}$$





Joint E:-

$$\sum F_n = 0$$

$$-F_{BE} - F_{DE} \cos 60^\circ = 0$$

$$\sum F_y = 0$$

$$72.5 + F_{DE} \sin 60^\circ = 0$$

$$F_{DE} = \frac{72.5}{\sin 60^\circ}$$

$$F_{DE}$$

$$= -89.4 \text{ kN (C)}$$

$$F_{BE} = 44.74 \text{ kN (T)}$$

Joint D:- $F_{BE} = 44.74 \text{ kN (T)}$ and $F_{DE} = -89.4 \text{ kN (C)}$

$$\sum F_n = 0$$

$$F_{CD} - (-83.7 \cos 60^\circ) + F_{BC} \cos 60^\circ$$

$$F_{CD} + 41.85 + F_{BC} \cos 60^\circ = 0$$

$$\sum F_y = 0$$

$$-40 - (-83.75 \sin 60^\circ) - F_{BC} \sin 60^\circ = 0$$

$$-40 + 83.75 \sin 60^\circ = F_{BC} \sin 60^\circ$$

$$F_{BC} = 37.51 \text{ kN (T)}$$

$$F_{CD} = -41.85 - 37.51 \cos 60^\circ$$

$$F_{CD} = -60.605 \text{ kN}$$

$$F_{CD} = 60.605 \text{ kN (C)}$$

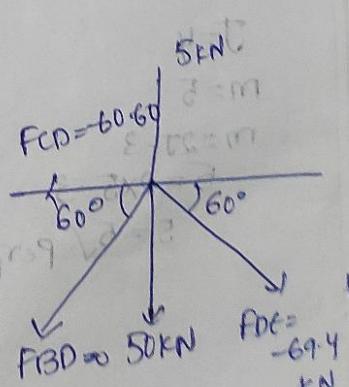
Joint D:-

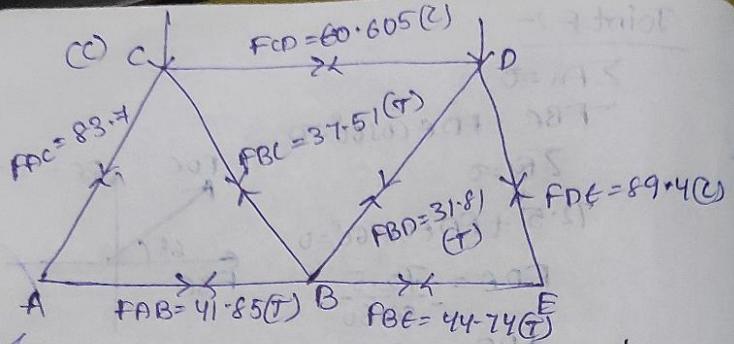
$$\sum F_n = 0$$

$$-(60.605) - F_{BD} \cos 60^\circ +$$

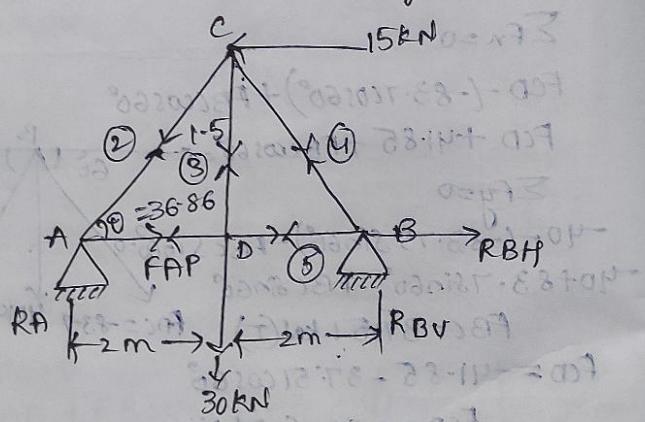
$$(-89.4 \cos 60^\circ) = 0.$$

$$F_{BD} = 31.81 \text{ kN (T)}$$





~~* Determine the magnitude and nature of forces in the various members of the triangular truss loaded and supported as shown figure.~~



$$J=4$$

$$M=5$$

$$M=2J+3$$

$$5=2 \times 4 + 3$$

$5=5$ ✓ perfect

5 member force

1 AD FAD

2 AC FAC

3 CD FCD

4 BC FBC

5 BD FBD

$$\tan \theta = \frac{1.5}{2}$$

$$\theta = 36.86^\circ$$

$$\sum F_n = 0 \quad 81 + 82 - 83 - 87 = -9.87$$

$$RBH - 15 = 0 \quad RBH = 15 \text{ kN}$$

$$RBH = 15 \text{ kN} \quad RV = 15 \times 1.5 = 22.5 \text{ kN}$$

$$\sum F_y = 0$$

$$RA = 30 + RV = 0$$

$$\sum M_A = 0$$

$$30 \times 25 - 15 \times 1.5 - RV \times 4 = 0$$

$$RV = 9.375 \text{ kN}$$

$$RA = 20.625 \text{ kN}$$

Joint A :-

$$\sum F_n = 0$$

$$FAD + FAC \cos 36.86^\circ = 0$$

$$\sum F_y = 0$$

$$20.625 + FAC \sin 36.86^\circ = 0$$

$$\frac{FAC}{\sin 36.86^\circ} = 20.625$$

$$FAC = 34.38$$

$$FAC = 34.38 \text{ (G)}$$

$$FAD = 27.50 \text{ kN (T)}$$

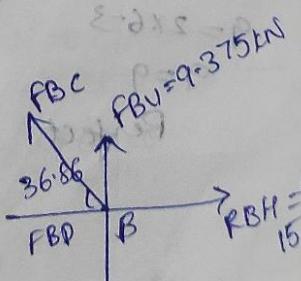
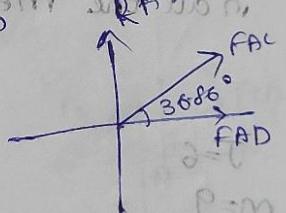
Joint B :-

$$\sum F_n = 0$$

$$15 - FBD - FBC \cos 36.86^\circ = 0$$

$$\sum F_y = 0$$

$$9.375 + FBC \sin 36.86^\circ = 0$$



$$F_{BC} = -15.62$$

$$F_{BC} = 15.62 \text{ (C)}$$

$$F_{BD} = -F_{BC} \cos 36.86 + 15 = 47.3$$

$$F_{BD} = 15.62 \cos 36.86 + 15 = 47.3$$

$$F_{BD} = 27.49 \text{ kN (T)}$$

Joint D:-

$$\sum F_x = 0$$

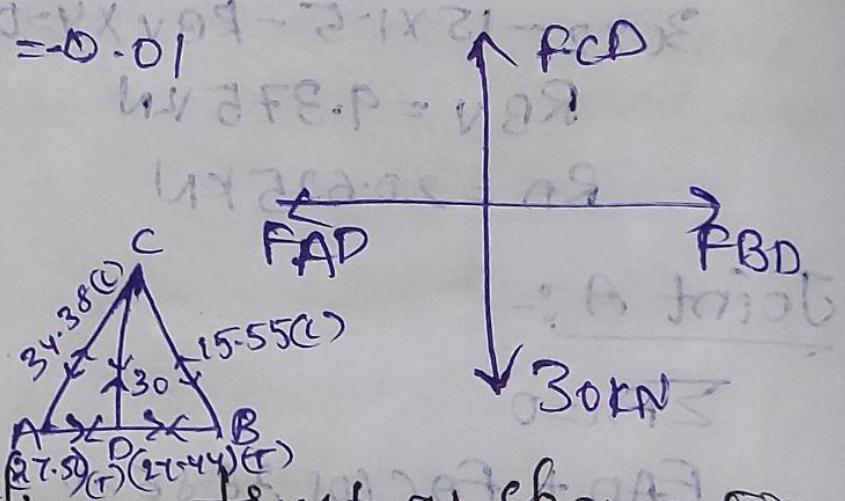
$$F_{CD} - F_{AD} = 0$$

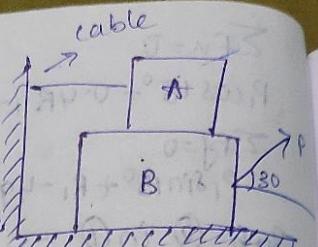
$$27.49 - 27.50 = -0.01$$

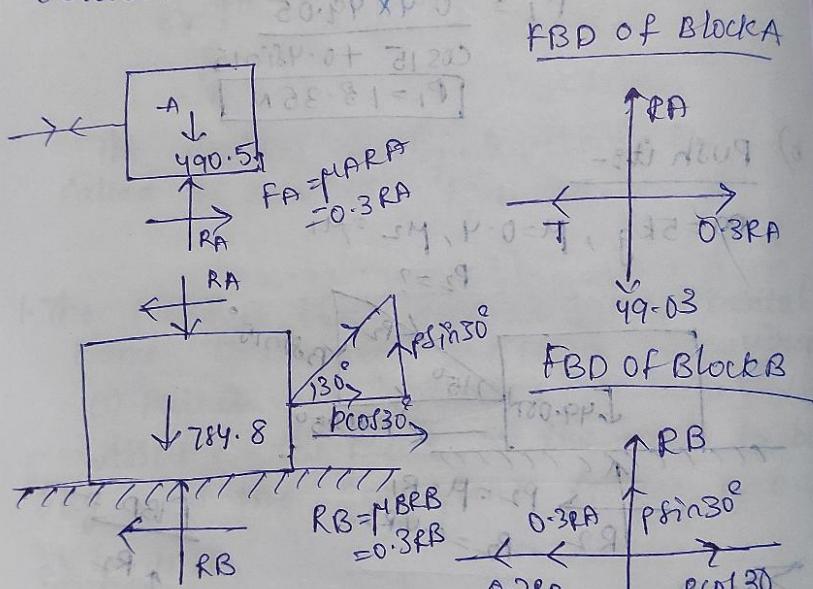
$$\sum F_y = 0$$

$$F_{CD} - 30 = 0$$

$$\boxed{F_{CD} = 30 \text{ T}}$$



 Two blocks A and B of masses 50kg and 80kg respectively are in equilibrium as shown in figure. Determine the force P required to move the lower block B and the tension in the cable A. Assume the coefficient of friction between two surfaces can be taken as 0.3



$$\sum F_x = 0$$

$$0.3RA - T = 0$$

$$T = 0.3RA$$

$$\sum F_y = 0$$

$$RA - 490.5 = 0$$

$$RA = 490.5N$$

$$T = 0.3RA \Rightarrow 0.3 \times 490.5$$

$$T = 147.15N$$

$$\sum F_x = 0$$

$$P \cos 30 - 0.3(R_A + R_B) = 0$$

$$P = \frac{0.3(490.5 + R_B)}{\cos 30}$$

$$P = \frac{147.15 + 0.3R_B}{\cos 30}$$

$$R_B = \frac{P \cos 30 - 147.15}{0.3}$$

$$\sum F_y = 0, 0.3R_B = P \cos 30 - 0.3(490.5)$$

$$-0.3R_B - 0.3 \times 490.5 + P \cos 30 = 0 \quad (1)$$

$$\sum F_y = 0, 0.3R_B = P \sin 30 - 184.8 + P \sin 30 = 0$$

$$R_B - 490.5 - 184.8 + P \sin 30 = 0$$

$$R_B = 1275.3 - P \sin 30 \quad (2)$$

$$-0.3(1275.3 - P \sin 30) - 0.3 \times 490.5 + P \cos 30 = 0$$

$$P[0.3 \sin 30 + \cos 30] = 0.3 \times 1257.3 + 0.3 \times 490.5$$

$$P(1.0160) = 524.34$$

$$\boxed{P = 527.419 N}$$

~~A.~~ Two blocks of weight $w_1 = 50\text{N}$ and $w_2 = 50\text{N}$ rest on a rough incline plane and connected by string as shown in Fig. The coefficient of

Friction b/w the inclined plane and the weight w_1 and w_2 are 0.3 and 0.2 respectively. Determine the inclination of the plane for which slipping will be imminent.

$$\sum F_n = 0$$

$$-T + w_1 \sin \alpha - \mu_1 R_1 = 0 \quad \text{--- (1)}$$

$$\sum F_y = 0$$

$$R_1 - w_1 \cos \alpha = 0$$

$$R_1 = w_1 \cos \alpha \quad \text{--- (2)}$$

$$\sum F_n = 0$$

$$-T + w_2 \sin \alpha - \mu_2 R_2 = 0$$

$$\sum F_y = 0 \quad \text{--- (3)}$$

$$R_2 - w_2 \cos \alpha = 0$$

$$R_2 = w_2 \cos \alpha$$

$$\text{--- (4)}$$

$$\Rightarrow T + w_1 \sin \alpha - \mu_1 R_1 - T + w_2 \sin \alpha - \mu_2 R_2 = 0$$

$$(w_1 + w_2) \sin \alpha - \mu_1 w_1 \cos \alpha - \mu_2 w_2 \cos \alpha = 0$$

$$\text{lookin } \alpha = (0.3 \times 50 + 0.2 \times 50) \cos \alpha$$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{15 + 10}{100} = \frac{25}{100} = \frac{1}{4}$$

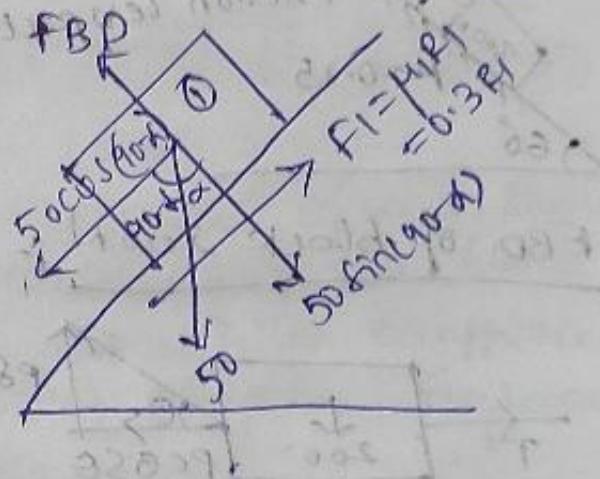
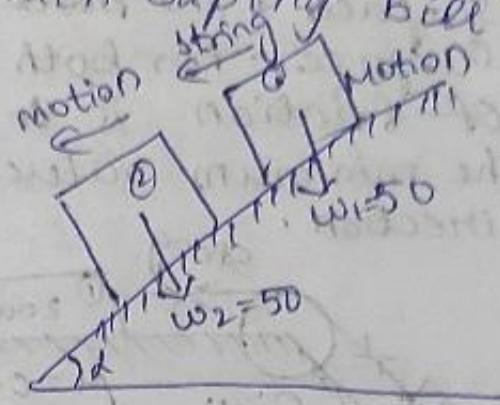
$$\alpha = \tan^{-1}(\frac{1}{4})$$

$$\alpha = 14.03^\circ$$

$$\text{--- (1)} \Rightarrow T = \mu_1 R_1 - w_1 \sin \alpha$$

$$= 0.3 \times 50 \cos(14.03^\circ) - 50 \sin(14.03^\circ)$$

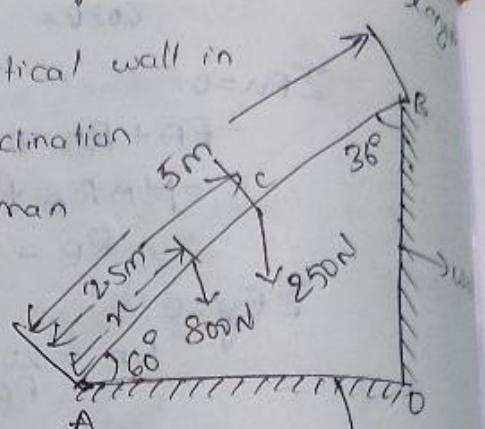
$$\boxed{T = 2.43 \text{ N}} \quad \text{--- (1)}$$



~~Q~~ Uniform ladder of weight 250N and

5m is placed on a vertical wall in the position where its inclination to the vertical is 30° . A man weight 800N climbs the ladder. At what position will be induced slipping?

Assume the coefficient of friction at both ends of the ladder can be taken as 0.2



$$\mu = 0.2 - \mu_A = \mu_B \quad \mu_B$$

$$[\mu = ?]$$

$$\sum F_n = 0$$

$$\mu_A = \mu_B = 0.2$$

$$-F_A + R_B = 0$$

$$-0.2\mu_A R_A + R_B = 0$$

$$R_B = 0.2 R_A \quad \text{---(1)}$$

$$\sum F_y = 0$$

$$R_A + R_B - 250 - 800 = 0$$

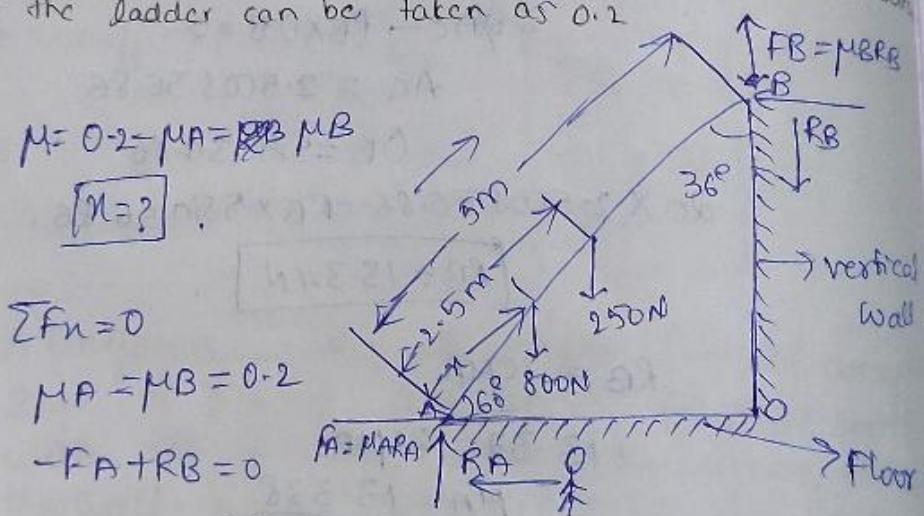
$$R_A + 0.2 R_B - 1050 = 0$$

$$R_A + 0.2 \times 0.2 R_A = 1050$$

$$R_A (1 + 0.4) = 1050$$

$$R_A = 1009.61 \text{ N}$$

$$R_B = 201.92 \text{ N}$$



$$\sum MA = 0$$

$$250 \times 0.75$$

$$\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{2}$$

$$\Rightarrow 250 \times AE + 800 \times AF - RB \times OB - FB \times OA = 0$$

$$\Rightarrow 250 \times 2.5 \cos 60^\circ + 800 \times \cos 60^\circ - 201.92 \times 5 \sin 60^\circ - 0.2 \times 201.92 \times 5 \cos 60^\circ = 0$$

$$\Rightarrow 312.5 + 800 \times \cos 60^\circ - 874.3 - 100.96 = 0.$$

$$\Rightarrow 800 \times \cos 60^\circ = 975.26 - 312.5$$

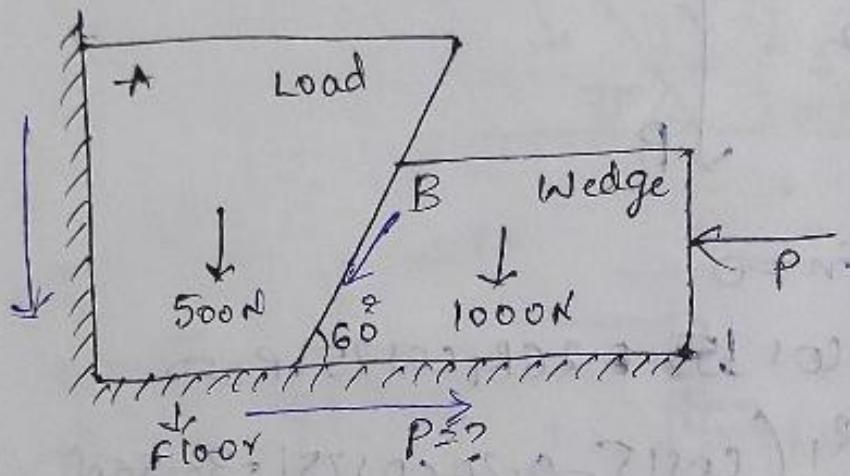
$$\Rightarrow 800 \times \cos 60^\circ = 662.76$$

$$\cos 60^\circ = 0.8284.$$

$$x = \frac{0.828}{\cos 60^\circ}$$

$$x = 1.656 \text{ m}.$$

~~(Q)~~ Two blocks A & B resting against a wall and floor as shown in Fig. Determine the value of horizontal force P applied to the lower block that will hold the system in equilibrium. The coefficient of friction is 0.25 at the floor, 0.3 at the wall and 0.2 between the blocks.

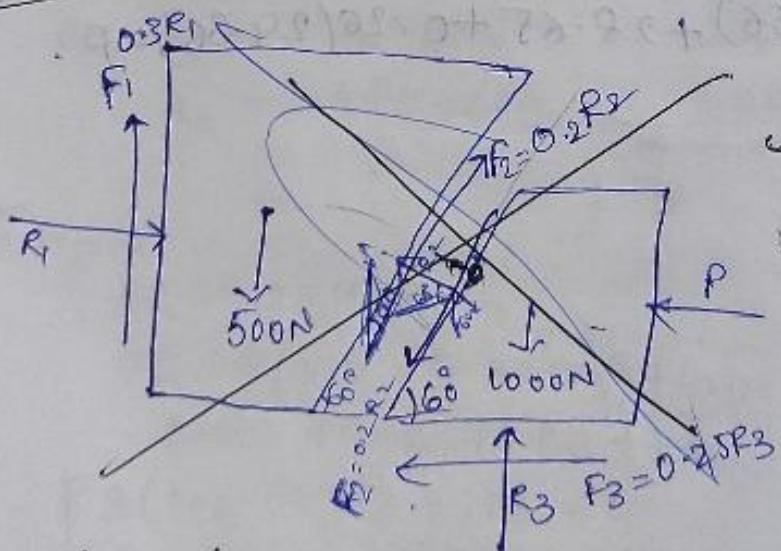


$$\mu_3 = (\mu)_{\text{floor}} = 0.25$$

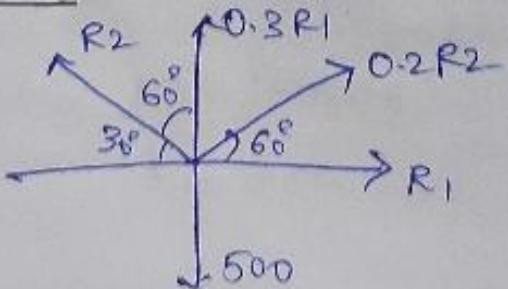
$$\mu_1 = (\mu)_{\text{wall}} = 0.3$$

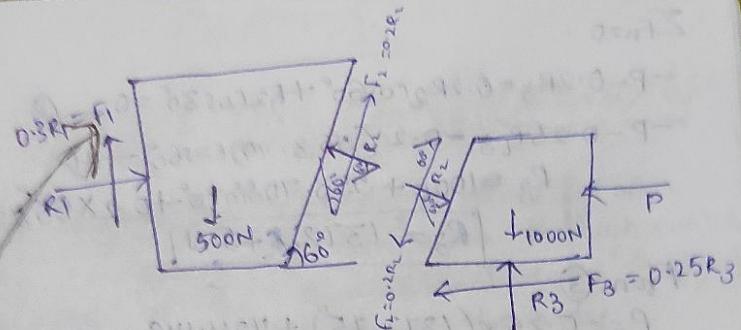
$$\mu_2 = (\mu)_{\text{block}} = 0.2$$

FBD:

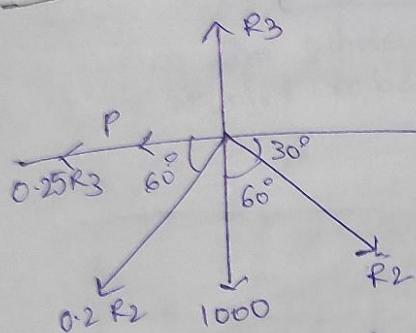


FBD of Load:





FBD of wedges:-



$$\begin{aligned}\sum F_x &= 0 \\ R_1 + 0.2R_2 \cos 60^\circ - R_2 \cos 30^\circ &= 0 \\ R_1 + (0.2 \cos 60^\circ - \cos 30^\circ) R_2 &= 0 \\ R_2 = \frac{R_1}{0.766} &\Rightarrow R_1 = 0.766 R_2 \quad \text{(1)}\end{aligned}$$

$$\begin{aligned}\sum F_y &= 0 \\ 0.2R_2 \sin 60^\circ + 0.3R_1 + R_2 \sin 30^\circ - 500 &= 0 \\ 0.2R_2 \sin 60^\circ + 0.3[0.766 R_2] + R_2 \sin 30^\circ &= 500 \\ R_2 [0.2 \sin 60^\circ + 0.2298 + \sin 30^\circ] &= 500\end{aligned}$$

$$R_2 = \frac{500}{0.9030}$$

$$R_2 = 553.70 \text{ N}$$

$$\text{(1)} \Rightarrow R_1 = 424.14 \text{ N}$$

$$\sum F_n = 0$$

$$-P - 0.2R_3 - 0.2R_2 \cos 60^\circ + R_2 \cos 30^\circ = 0 \quad \text{--- (3)}$$

$$-P - 0.25R_3 = 0.2(553.70) \cos 60^\circ \quad \text{--- (4)}$$

$$R_3 = 1000 + 553.70 \sin 30^\circ + 0.2 \times 553.70 \sin 60^\circ$$

$$R_3 = 1372.75 \text{ N}$$

From (3),

$$P = -0.25 \times (1372.75) + 424.148$$

$$P = 80.96 \text{ N}$$

