UNIT-V DYNAMICS

In statics we considerd The bodies at rest. Now we shall begin with The study of dynamics. Dynamics is The part of mechanics that deals with The analyses of body in motion. while The study of statics is very old science, dynamics is a comparatively new one.

Dynamics → 1) Kinematics
→ 2) Kinetics

Kinematics: Kinematics is The study of The relationships b/w The displacement, velocity, accelration and time of a given motion, without considering the forces That cause The motion.

Kinetics: Kinetics is The study of The relationship b/w The forces acting on a body, The mass of The body and The motion of The body.

Types of motion: when a particles Moves in space it describes a curve, called path. This path can be straight or curved.

1) **Rectilinear Motion**: when the particle Moves along a Path which is a straight line, it is called rectilinear motion.

2) **curvilinear motion**: when The particle moves along a curved path it is called curvilinear motion.

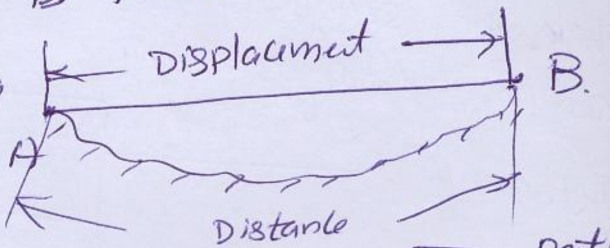
Kinematics : (Rectilinear motion)

Let us consider a motion of a particle along a st. line.

Displacement : (S)

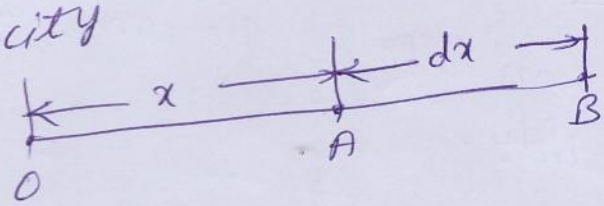
The change of position of a particle or a body w.r.t certain fixed reference point is termed as displacement

Consider a body that moves along a curved path and takes time t to move from position A to B.



The total distance covered by the body along the path followed is called distance and the shortest distance b/w the two positions is called displacement

Velocity (V) The rate of change of position of a body w.r.t the time is called velocity



$$\boxed{V = \frac{dx}{dt}}$$

units : m/sec

Accelration : (A) The rate of change of velocity of a body w.r.t time is called accelration

$$\boxed{a = \frac{dV}{dx}}$$

units : m^2/sec

Equations of Rectilinear Motion:

when The body moves in a st. line with uniform accelration, the equations of motion are

$$\boxed{\begin{aligned} &(a)\ V = u + at \\ &(b)\ V^2 - u^2 = 2as \\ &(c)\ S = ut + \tfrac{1}{2}at^2 \end{aligned}}$$

where

u = Initial velocity

V = Final Velocity

a = accelation of b

S = Distance trave in time t

and Distance travelled in nth second is

$$(d)\ S_n = u + \frac{a}{2}(2n-1)$$

(a) $\boxed{V = u + at}$

Accelration $a = \frac{dv}{dt}$.

$$dV = a\,dt$$

$$\int_u^v dv = \int_0^t a\,dt$$

$$V - u = at$$

$$\boxed{V = u + at}$$

(b) Distance travelled = Avg velocity * time

$$= \frac{v+u}{2} * t$$

$$= \frac{[(u+at) + u]t}{2} \qquad (\because v = u + at)$$

$$= ut + \frac{at^2}{2}$$

$$\boxed{S = ut + \tfrac{1}{2}at^2}$$

$$\therefore\ V = \frac{dx}{dt}$$

$$dx = v\,dt \Rightarrow dx = (u + at)\,dt$$

$$\int_0^s dx = \int_0^t (u+at)\,dt$$

$$s-0 = ut + \frac{at^2}{2}$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

(3) $$a = \frac{dv}{dt} = \frac{dv}{dx} * \frac{dx}{dt} = \frac{dv}{dx} * v.$$

$$a\,dx = v\,dv$$

$$a\int_0^s dx = \int_u^v v\,dv$$

$$as = \frac{v^2-u^2}{2}$$

$$\boxed{v^2-u^2 = 2as}$$

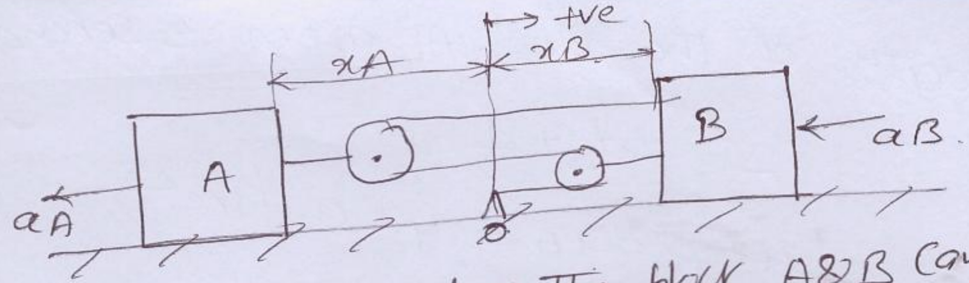
Problems:

1) Two blocks A and B resting on a horizontal plane and connected by a cord as shown Fig. A block A starts from rest and moves to the left with a constant acceleration. It was observed that the block B attains a velocity of 12 cm/s after moving a distance 24 cm.

Determine (a) Acceleration of block A & B

(b) Velocity and Position of block A after 5 sec.

Sol:



The length of the cored connecting the block A&B can be expressed in terms of distances x_A and x_B of blocks A&B from a fixed point O

$$3x_B + 2x_A = \text{const}$$

Giving an incremet Δx_A to the block A

$$3\Delta x_B + 2\Delta x_A = 0$$

$$3\frac{dx_B}{dt} + \frac{2dx_A}{dt} = 0 \quad \text{--- (1)}$$

$$3V_B + 2V_A = 0$$

$$\frac{3d^2x_B}{dt^2} + \frac{2d^2x_A}{dt^2} = 0 \quad \text{--- (2)}$$

$$3a_B + 2a_A = 0$$

For block B using the eq.,

$$v^2 - u^2 = 2as.$$

$$u = 0, \; V = 12\text{ cm/sec}, \; S = 24\text{ cm}$$

$$12^2 - 0^2 = 2 * a_B * 24$$

$$a_B = \frac{12^2}{2 \times 24} = 3\text{ cm/sec}^2$$

For block A using the eq., (2)

$$2 * 3 + 2 * a_A = 0$$

$$a_A = -\frac{3}{2} * 3 = -4.5\text{ cm/sec}^2$$

$$\overleftarrow{a_A} = 4.5\text{ cm/sec}^2$$

velocity of The block A after 5 seconds

$$S = ut + \frac{1}{2}at^2$$

$$u = 0, \quad a_A = 4.5\ cm/sec^2$$

$$S_A = 0 + \frac{1}{2} \times 4.5 \times 5^2$$

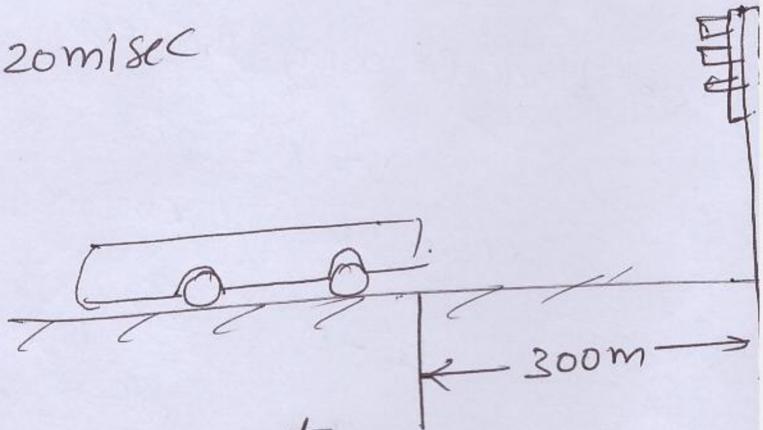
$$S_A = 56.25\ \cancel{m}\ cm$$

2) Driver of a car travelling at 72 km/hour observes the traffic light 300m. ahead him turning red. The traffic light is timed to remain red for 20 sec before it turns green. If The motorist wishes to pass the lights without stopping to wait for it to turn green Determine

(a) The required accelation of the car.

(b) The speed with which the motorist crosses the traffic light

$$72\ km/hour = \frac{70 \times 1000}{60 \times 60} = 20\ m/sec$$



$u = 20\ m/sec$

$S = 300m$

$t = 20 sec$

$$S = ut + \frac{1}{2}at^2$$

$$300 = 20 \times 20 + \frac{1}{2} \times a \times 20^2$$

$$\boxed{a = -0.5\ m/sec^2}$$

$$V = u + at$$

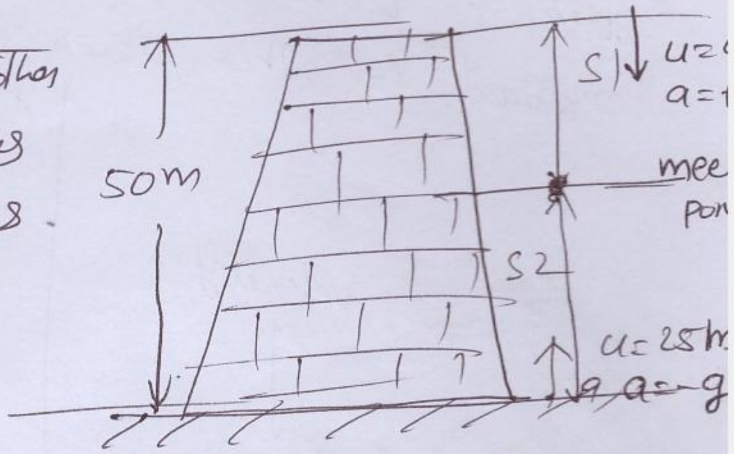
$$V = 20 - 0.5 * 20$$

$$V = 10\ m/sec$$

$$V = \frac{10 \times 3600}{1000}$$

$$V = 36\ km/hour$$

3) A stone is dropped from the top of the tower 50m high. At the same time another stone is thrown up from the foot of the tower with a velocity of 25m/sec. At what distance from the top and after how much time the two stones cross each other.



The two stones to cross each other is that the sum of the distances S_1 & S_2 travelled by two stones at the time of crossing

$$S_1 + S_2 = 50m$$

$$S = ut + \frac{1}{2}at \quad (u=0)$$

$$S_1 = \frac{1}{2}gt^2 \rightarrow \text{First stone}$$

$$S_2 = ut + \frac{1}{2}at^2 \quad \begin{pmatrix} u = 25\,m/sec \\ a = -g \end{pmatrix}$$

$$S_2 = 25t - \frac{1}{2}gt^2$$

$$S_1 + S_2 = 50$$

$$\cancel{\frac{1}{2}gt^2} + 25t - \cancel{\frac{1}{2}gt^2} = 50$$

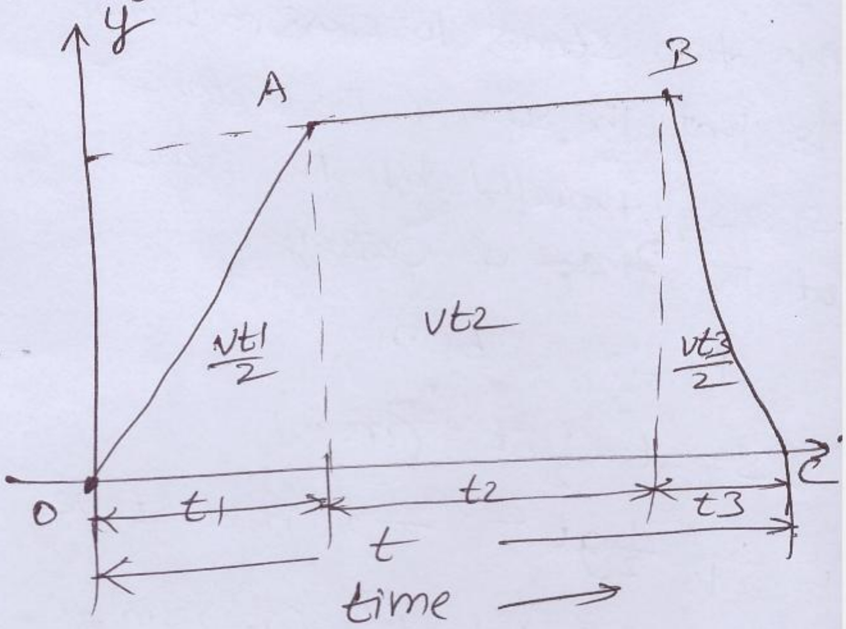
$$25t = 50$$

$$t = 2\,sec$$

$$S_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.81 \times 4$$

$$S_1 = 19.6\,m \text{ from the top}$$

4) A train start from rest and incresed its speed from zero to v m/s with a constant accelration of a_1 m/s², runs at this speed for some time and finally comes to rest with a constant deceleration a_2 m/s². If the total distance travelled is x mt. Find the total time t required for this Journey.



OA → 0 to v m/sec.

AB → const velocity v

BC → Decelration from velocity v to 0

Portion OA:

$V = u + at$ Initial velocity $u = 0$

$V = 0 + a_1 t_1$ Accelration $a = a_1$

$V = a_1 t_1$ time $t = t_1$

$t_1 = \frac{V}{a_1}$

Portion BC: $V = u + at$ Final velocity is zero, $V =$

$a = -a_2$

$V = V - a_2 t_3$ $\cancel{t_2}\ t_3$

$t_3 = \frac{V}{a_2}$.

Total distance travelled by the train (x)

$$x = \frac{1}{2}vt_1 + vt_2 + \frac{1}{2}vt_3$$

$$\frac{x}{v} = \frac{t_1}{2} + t_2 + \frac{t_3}{2}$$

$$t_2 = \frac{x}{v} - \frac{t_1}{2} - \frac{t_3}{2}$$

Total time travel $t = t_1 + t_2 + t_3$

$$t = \frac{v}{a_1} + \left(\frac{x}{v} - \frac{t_1}{2} - \frac{t_3}{2}\right) + \frac{v}{a_2}$$

$$t = \frac{v}{a_1} + \frac{x}{v} - \frac{v}{2a_1} - \frac{v}{2a_2} + \frac{v}{a_2}$$

$$\boxed{t = \frac{x}{v} + \frac{v}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)}$$

5) Motion of particle along a st. line is given by the equation $a = t^2 - 2t + 2$

where a = Accelration in m/s^2

t = time in sec

After 1 sec the distance travelled by the particle and the velocity of the article were found to be 14.75 m and 6.33 m/sec Find

(a) Distance travelled (b) velocity

(c) Accelration of the particle after 2sec.

Sol: At $t = 1$sec, $x = 14.75$mt; $v = 6.33$ m/sec

Given $a = t^2 - 2t + 2$.

$$a = \frac{dv}{dt} = t^2 - 2t + 2 \quad \text{--- (1)}$$

$$\int dv = \int a\, dt$$

$$\int dv = \int (t^2 - 2t + 2)\, dt$$

$$v = \frac{t^3}{3} - \frac{2t^2}{2} + 2t + C_1$$

$$6.33 = \frac{1}{3} - 1 + 2 + C_1$$

$$6.33 = \frac{1}{3} + 1 + C_1$$

$$C_1 = 5$$

$$v = \frac{t^3}{3} - \frac{2t^2}{2} + 2t + 5 \quad \text{—— (2)}$$

$$v = \frac{dx}{dt} = \frac{t^3}{3} - \frac{2t^2}{2} + 2t + 5$$

$$\int dx = \int v\, dt = \int \left(\frac{t^3}{3} - \frac{2t^2}{2} + 2t + 5\right) dt$$

$$x = \frac{t^4}{12} - \frac{\cancel{2}t^3}{\cancel{2}\times 3} + \frac{2t^2}{2} + 5t + C_2$$

$t = 1$, $x = 14.75$ m

$$14.75 = \frac{1}{12} - \frac{1}{3} + 1 + 5 + C_2$$

$$C_2 = 9$$

$$\therefore x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 + 5t + 9 \quad \text{—— (3)}$$

Distance

$$x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 + 5t + 9$$

$t = 2$

$$x = \frac{2^4}{12} - \frac{2^3}{3} + 2^2 + 5(2) + 9$$

$$x = 21.67 \text{ mt}$$

velocity: $v = \dfrac{t^3}{3} - \dfrac{2t^2}{2} + 2t + 5$

$t = 2$ $\quad v = \dfrac{2^3}{3} - 2^2 + 2(2) + 5$

$v =$ [illegible] m/sec

Accelration

$$a = t^2 - 2t + 2$$

$t = 2$

$$a = 2^2 - 2(2) + 2$$

$$a = 2 \text{ m/sec}^2$$

6) A particle starts with v_0. Its accelration and velocity related by the equation $a = -KV$ 6

where K = constant
V = velocity of the particle
a = Accelration of the particle

Find the Displacement, time relation

Sol:

$$a = -KV$$

$$\because a = \frac{dv}{dt}$$

$$a = \frac{dv}{dt} = -KV.$$

$$\int_{v_0}^{v} \frac{dv}{v} = -\int_0^t K\,dt$$

$$\ln\left(\frac{v}{v_0}\right) = -Kt$$

$$\frac{v}{v_0} = e^{-Kt}$$

$$v = v_0 e^{-Kt}$$

$$\because v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = v_0 e^{-Kt}$$

$$\int_0^x dx = \int_0^t v_0 e^{-Kt}\,dx$$

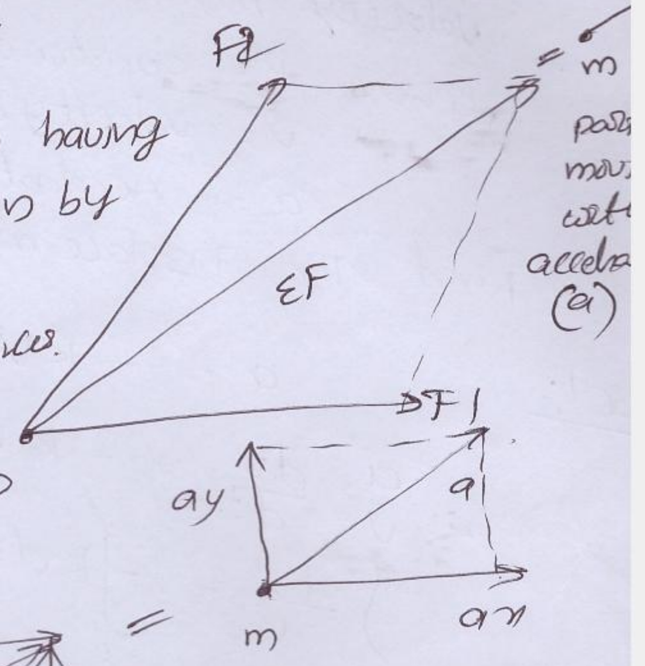
$$x = \left(v_0 \frac{e^{-Kt}}{-K}\right)_0^t$$

$$x = -\frac{v_0}{K}\left(e^{-Kt} - 1\right)$$

$$\boxed{x = \frac{v_0}{K}\left(1 - e^{-Kt}\right)}$$

KINETICS (RECTILINEAR MOTION)

Equations of Rectilinear Motion:

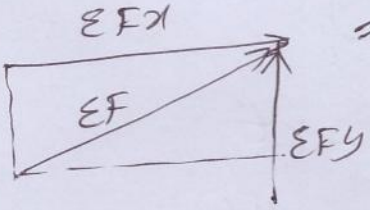


consider a particle P of mass m having an accelation a when acted upon by several forces F_1 and F_2.
Let ΣF be the resultant of these forces.

Apply Newton second law

$$\Sigma F = ma$$

$$\boxed{\begin{aligned} \Sigma F_x &= m a_x \\ \Sigma F_y &= m a_y \end{aligned}}$$



The eq. written in the above form are called equations of motion of the particle

$$\Sigma F_x = m a_x$$

$$\boxed{\Sigma F_x = m\ddot{x}}$$

$$\ddot{x} = a = \frac{d^2x}{dt^2}$$

Equation of Dynamic equilibrium:

The equations of motion of the particle P

$$\Sigma F = ma$$

$$\boxed{\Sigma F - ma = 0}$$

where ΣF = Resultant of the external Force
m = Mass
a = accelation
$(-ma)$ = Inertia force

The magnitude of Inertia force is equal to the product of the mass and accelration the particle and it acts in a direction opposite to the direction of accelration of the particle.

$$\Sigma F - ma = 0$$

$$\Sigma F + (-ma) = 0$$

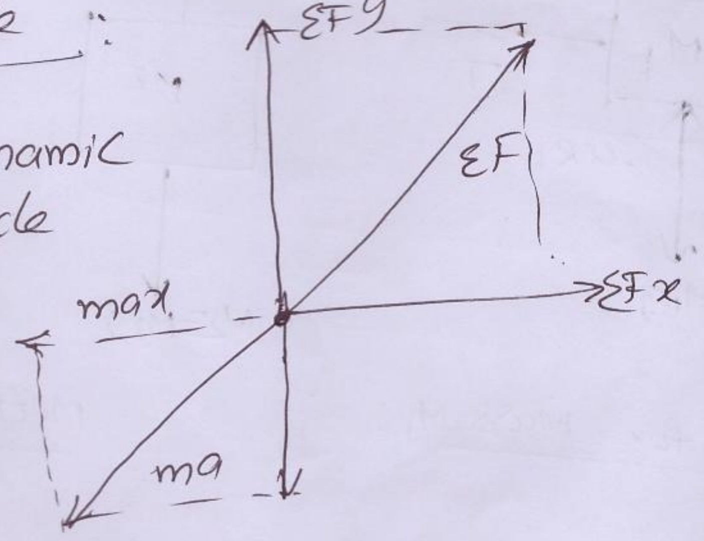
(Inertia Force)

The component form

$$\boxed{\begin{aligned} \Sigma F_x + (-ma_x) &= 0 \\ \Sigma F_y + (-ma_y) &= 0 \end{aligned}}$$

The above equations are called Dynamic equibrium of the particle

D'Alembert's principle :



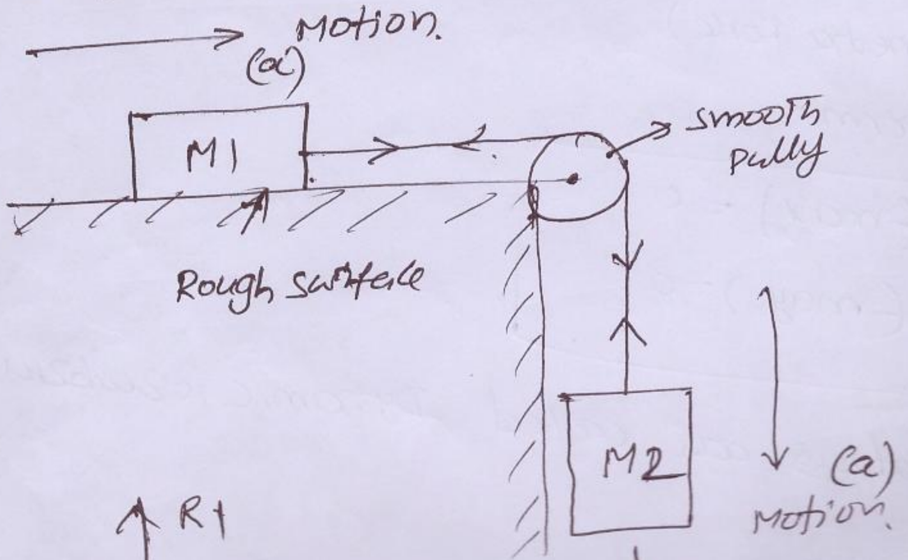
To write eq.. of dynamic equibrium of a particle add Inertia force to external acting on a particle and equate the sum to zero.

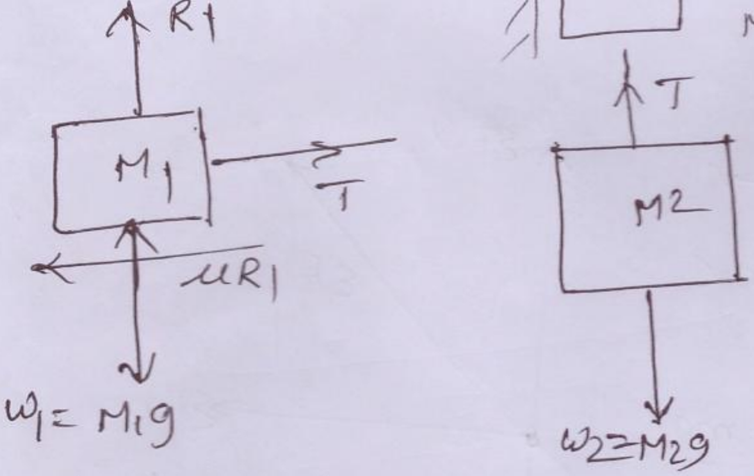
This concept is known as D'Alembert's principle.

$$\boxed{\Sigma F - ma = 0}$$

problems:

1) Two blocks of masses M_1 & M_2 are connected by a flex but inextensible string as shown fig. Assuming the coeff. of friction b/w block M_1 and the horizontal surface to be μ. Find the accelration of the masses and tension in the string. Assume $M_1 = 10\,kg$, $M_2 = 5\,kg$, $\mu = 0.25$





Motion for Mass M_1:

$\Sigma F_x = M a_x$

$\Sigma F_y = M a_y$

$T - \mu R_{1} = M_1 a$ — (1)

$M_1 g - R = 0$ — (2)

From (1) $T - \mu M_1 g = M_1 a$

$T = \mu M_1 (\cancel{a} + g)$ — (A)
$+ M_1 a$

Motion for Mass M2:

$\Sigma F_x = M a_x$ $\quad a_x = \cancel{0}$

$\Sigma F_y = M a_y$ $\quad a_y = 0$

$M_2 g - T = M_2 a$.

$T = M_2 (g - a)$ — (B)

(A) = (B)

$$M_1 a + \mu M_1(\cancel{a} + g) = M_2(g-a)$$

$$M_1 a + M_2 a = M_2 g - \mu M_1 g$$

$$a\,(M_1 + M_2) = M_2 g - \mu M_1 g$$

$$\boxed{a = \frac{g\,(M_2 - \mu M_1)}{M_1 + M_2}}$$

$$T = M_2(g - a.)$$

$$T = M_2\left[g - \frac{g\,(M_2 - \mu M_1)}{M_1 + M_2}\right]$$

$$T = \frac{M_2 g}{M_1 + M_2}\left(M_1 + \cancel{M_2} - \cancel{M_2} + \mu M_1\right)$$

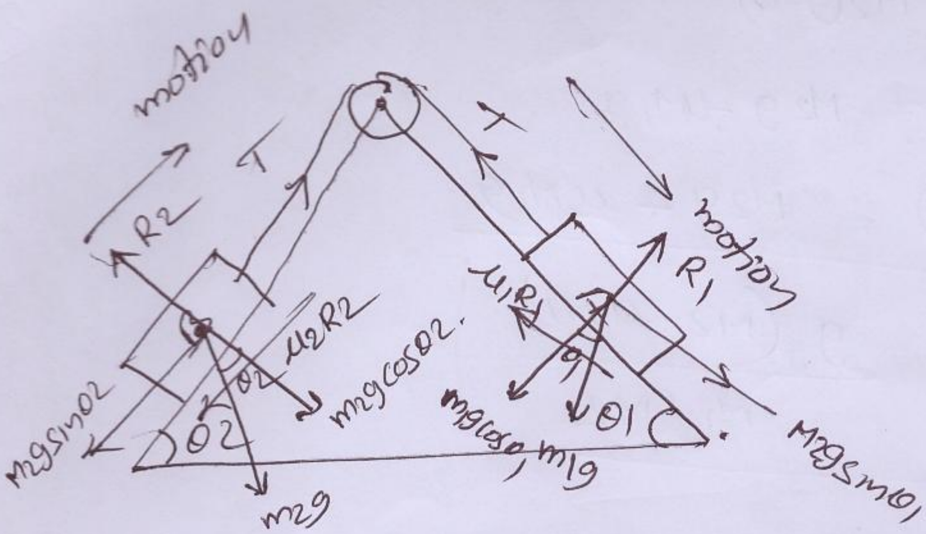
$$\boxed{T = \frac{M_1 M_2 g\,(1+\mu)}{M_1 + M_2}}$$

$$a = \frac{9.81\,(5 - 0.25 * 10)}{10+5} = 1.635 \text{ m/sec}^2$$

$$T = \frac{10 \times 5 \times 9.81\,(1 + 0.25)}{10+5} = 40.875 \text{ N}$$

2) Two blocks of masses M_1 and M_2 are placed on two inclined planes of elevation θ_1 and θ_2 are connected by a string as shown Figure. Find the accelration of the masses. The coeff. of friction b/w the blocks and the planes is μ. Assume $M_1 = 5\text{kg}$; $\theta_1 = 30^\circ$, $\mu = 0.33$

$M_2 = 10\text{kg}$; $\theta_2 = 60^\circ$



Motion of ~~Motion~~ Mass M2 :

$\Sigma F_x = Ma_x$

$\Sigma F_y = Ma_y$

$T - M_2 g\sin\theta_2 - \mu_2 R_2 = M_2 a$ — ①

$M_2 g\cos\theta_2 - R_2 = 0$

$R_2 = M_2 g\cos\theta_2$ — ②

$\therefore T = M_2 g\sin\theta_2 + \mu_2 M_2 g\cos\theta_2 + M_2 a$ — (A)

Motion of Mass M1 ,

$\Sigma F_x = Ma_x$

$\Sigma F_y = Ma_y$

$M_2 g\sin\theta_1 - T - \mu_1 R_1 = m$

$m_1 g\cos\theta_1 - R_1 = 0$

$T = M_1 g\sin\theta_1 - \mu m$
$- m_1 a$ — (B)

$M_2 g\sin\theta_2 + \mu_2 M_2 g\cos\theta_2 + M_2 a = M_1 g\sin\theta_1 - \mu_1 m_1 g\cos\theta - m_1 a$

$a(M_1 + M_2) = M_1 g\sin\theta_1 - M_2 g\sin\theta_2 - \mu_1 m_1 g\cos\theta_1 - \mu_2 M_2 g\cos\theta_2$

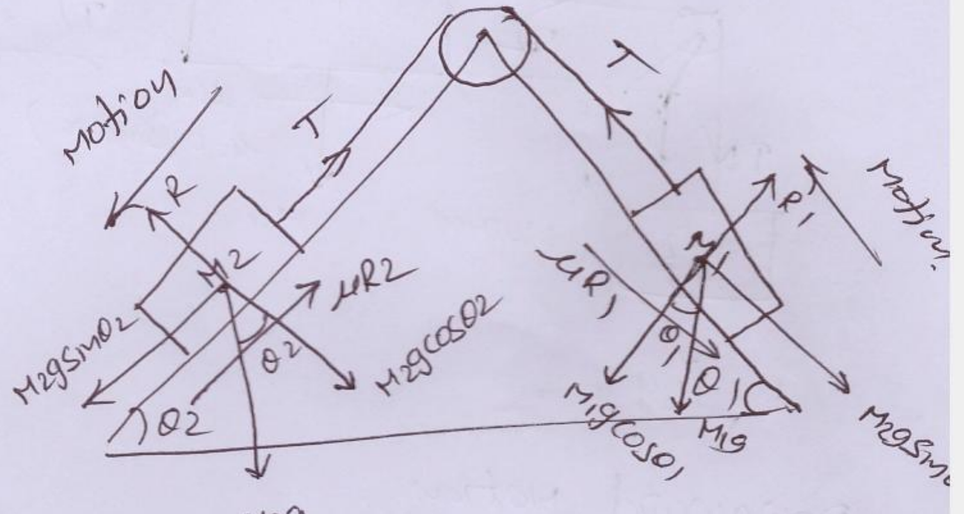
$a(M_1 + M_2) = gM_1(\sin\theta_1 - \mu\cos\theta_1) - gM_2(\sin\theta_2 + \mu\cos\theta_2)$

$a = \frac{g}{M_1 + M_2}\left[M_1(\sin\theta_1 - \mu\cos\theta_1) - M_2(\sin\theta_2 + \mu\cos\theta_2)\right]$

$a = 6.0423 \; m/sec^2 \downarrow$.

2) Two blocks of masses M_1 and M_2 are placed on two inc planes elevation θ_1 and θ_2 are connected by a string as shown Figure. Find the accelration of the masses. The coeff. of friction b/w the blocks and the planes is μ

Assume $M_1 = 5\,kg$, $\theta_1 = 30^\circ$, $M_2 = 10\,kg$, $\theta_2 = 60^\circ$, $\mu_1 = \mu_2 = 0.3$



Motion of Mass M2:

$\Sigma F_x = m a_x \quad (a_x = a)$

$T - M_2 g \sin\theta_2 + \mu R_2 = M_2 a \quad \text{--- (1)}$

$\Sigma F_y = m a_y \quad (a_y = 0)$

$M_2 g \cos\theta_2 - R_2 = 0$

$R_2 = M_2 g \cos\theta_2 \quad \text{--- (2)}$

From (1) & (2)

$T = M_2 a + M_2 g \sin\theta_2 - \mu M_2 g \cos\theta_2 \quad \text{--- (A)}$

Motion of Mass M1:

$\Sigma F_x = M a_x \quad (a_x = a)$

$M_2 g \sin\theta_2 - T + \mu R_1 = M_1 a \quad \text{--- (3)}$

$\Sigma F_y = M a_y \quad (a_y = 0)$

$M_1 g \cos\theta_1 - R_1 = 0$

$R_1 = M_1 g \cos\theta_1 \quad \text{--- (4)}$

From (3) & (4)

$T = M_1 g \sin\theta_2 + \mu M_1 g \cos\theta_1 - M_1 a \quad \text{--- (B)}$

(A) = (B)

$M_2 a + M_2 g \sin\theta_2 - \mu M_2 g \cos\theta_2 = M_1 g \sin\theta_2$

$a(M_1 + M_2) = M_2 g \sin\theta_2 + \mu M_1 g \cos\theta_1 - M_1 g \sin\theta_1 + \mu M_2 g$

$a(M_1 + M_2) = M_2 g(\sin\theta_2 + \mu c$
$+ M_1 g\,(\mu\cos\theta_1 - s$

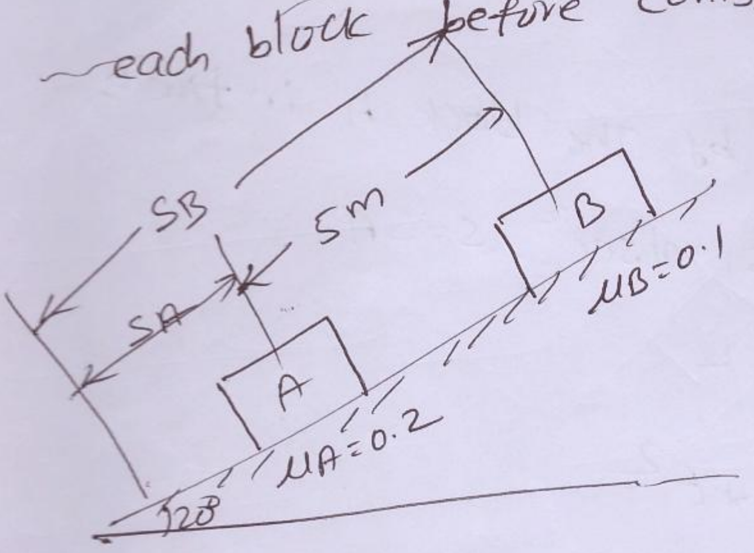
$a = \frac{g}{M_1 + M_2}\left\{M_2(\sin\theta_2 + \mu\cos\theta\right.$
$+ M_1(\mu\cos\theta_1 -$

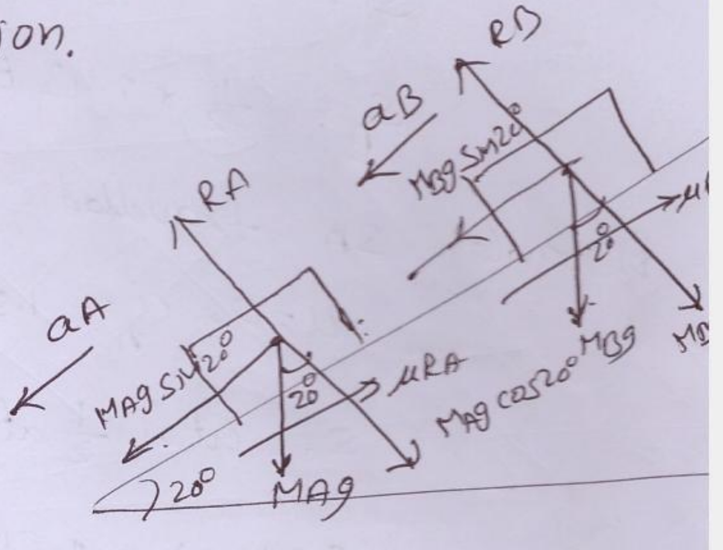
$a = \frac{9.81}{5+10}\left[10 * (\sin 60^\circ + 0.33\right.$
$+ 5(0.33\cos 30^\circ - s$

$a = \frac{9.81}{15}(10.3102 - 1.071$

$a = 6.0423\ m/sec^2$

(4) Two block A & B are held on a inclined plane 5m apart as shown Fig. The coeff. of friction b/w the block and B and the inclined plane are 0.2 and 0.1 respectively. If the blocks begain to slide down the plane simultaneo[usly] calculate the time and distance travelled by the each block before collision.





Eq. of motion of the block-A

$\Sigma F_x = ma_x \Rightarrow M_A g \sin 20° - \mu_A R_A = M_A a_A$ $(a_x = a_A)$

$\Sigma F_y = ma_y \Rightarrow R_A - M_A g \cos 20° = 0$ $(a_y = 0)$

$\cancel{M_A} g \sin 20° - 0.2(\cancel{M_A} g \cos 20°) = \cancel{M_A} a_A$

$a_A = 9.81 \sin 20° - 0.2 \times 9.81 \cos 20°$

$a_A = 1.510 \text{ m/sec}^2$

Equations of motion of the block-B

$\Sigma F_x = ma_x \Rightarrow M_B g \sin 20° - \mu_B R_B = M_B a_B$

$\Sigma F_y = ma_y \Rightarrow R_B - M_B g \cos 20° = 0$

$\cancel{M_B} g \sin 20° - 0.1 \cancel{M_B} g \cos 20° = \cancel{M_B} a_B$

$a_B = 9.81 \sin 20° - 0.1 \times 9.81 \times \cos 20°$

$a_B = 2.43 \text{ m/sec}^2$

Let The blocks collide after a time t of release

Distance SB travelled by The block B in time t

$$a_B = 2.43 \text{ m/sec}^2, \quad u = 0, \quad S = S_B$$

$$S = ut + \frac{1}{2}at^2$$

$$S_B = \frac{1}{2} * 2.43\, t^2$$

Distance SA travelled by The block A in time t

$$u = 0, \quad a_A = 1.51 \text{ m/sec}^2, \quad S = S_A$$

$$S = ut + \frac{1}{2}at^2$$

$$S_A = \frac{1}{2} * 1.51 * t^2$$

For The blocks to collide

$$S_B - S_A = 5$$

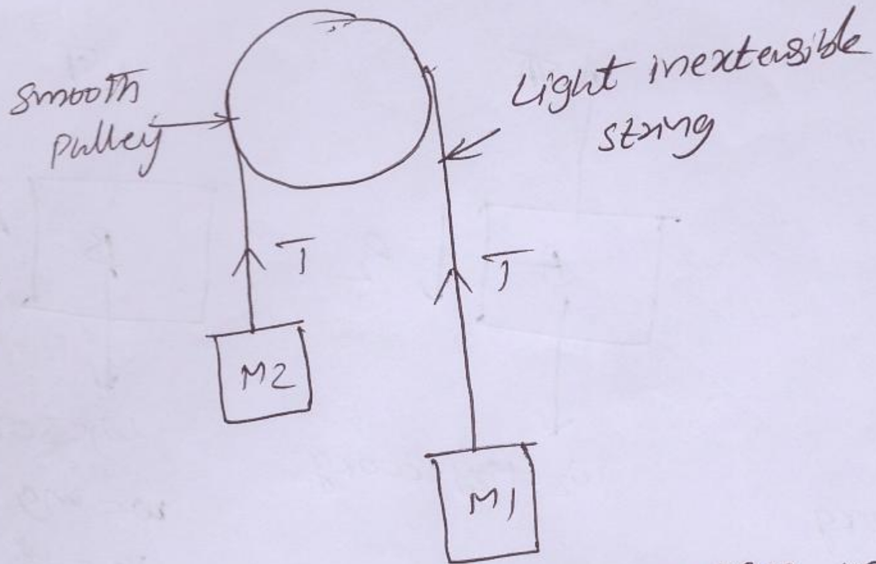
$$\frac{1}{2} * 2.43 t^2 - \frac{1}{2} * 1.51 t^2 = 5$$

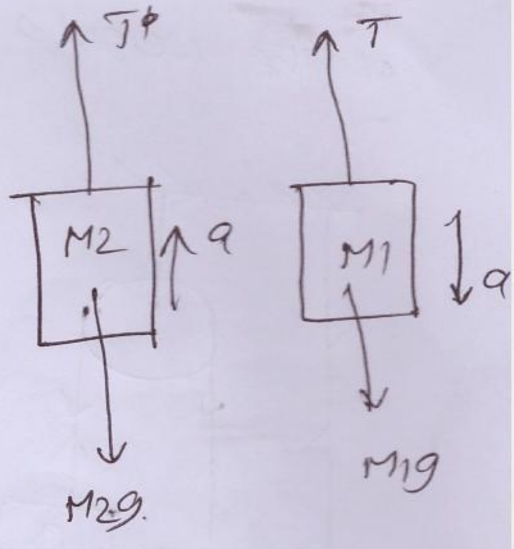
$$t = 3.35 \text{ sec.}$$

$$\therefore \quad S_A = \frac{1}{2} * 1.51 * 3.35^2 = 8.20 \text{m}$$

$$S_B = \frac{1}{2} * 2.43 * 3.35^2 = 13.20 \text{mt}$$

Motion of two bodies connected by a string passing over a smooth pulley:





consider two bodies of masses m_1 and m_2 connected at the end of a light inextensible string that passes over a smooth frictionless pulley. If $m_1 > m_2$ then the mass m_1 moves downwards and m_2 moves upward

$\Sigma F_x = ma_x$

$\Sigma F_y = ma_y$

$M_1g - T = M_1a$ —— (1) Downward motion of mass m_1

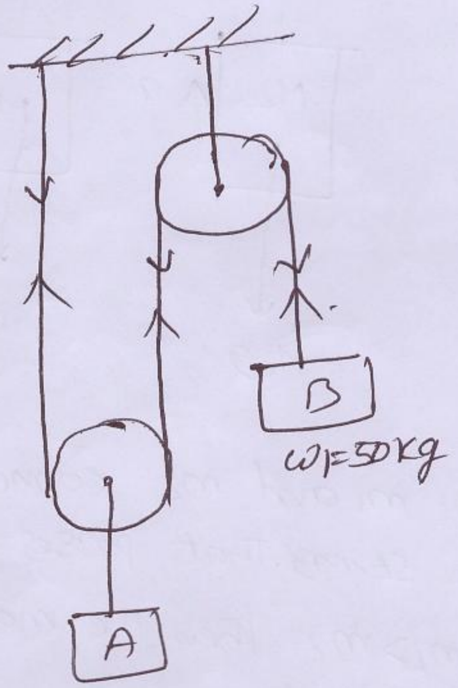
$T - M_2g = M_2a$ —— (2) upward motion of mass m_2

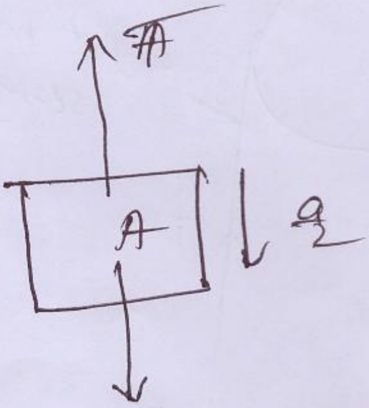
(1) + (2) $M_1g - \cancel{T} + \cancel{T} - M_2g = M_1a + M_2a$

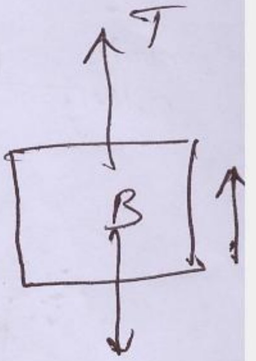
$$(M_1 - M_2)\, g = (M_1 + M_2)\, a$$

$$a = \frac{M_1 - M_2}{M_1 + M_2} * g.$$

5) A system of frictionless pulley carries two weig[hts] hung by inextensible cords as shown Fig. Make calculations for the tension in the cords and accelera[tion] of the weights







$w = mg$

$m = \frac{w}{g}$

Given arrangement that when weight W_1 travels a unit distance, the weight traves $\frac{1}{2}$ distance.

Accelration is proportional to distance travelled.

$\therefore$ a is accelration of weight W_1, the weight W_2 wi[ll] have accelration equal to $\frac{a}{2}$. The weight W_1 moves up & the weight W_2 moves downward.

$$T - 50 = \frac{50}{9.81} * a \quad \text{(upward)}$$

$$T = 50 + \frac{50}{9.81} a \quad \text{— (1)}$$

$$200 - T = \frac{200}{9.81} * \frac{a}{2} \quad \text{(downward)}$$

$$T = 100 - \frac{50}{9.81} a \quad \text{— (2)}$$

from (1) = (2)

$$50 + \frac{50}{9.81} a = 100 - \frac{50}{9.81} a$$

$$\frac{100}{9.81} a = 50$$

$$a = \frac{50 \times 9.81}{100}$$

$$a = 4.905 \ m/sec^2$$

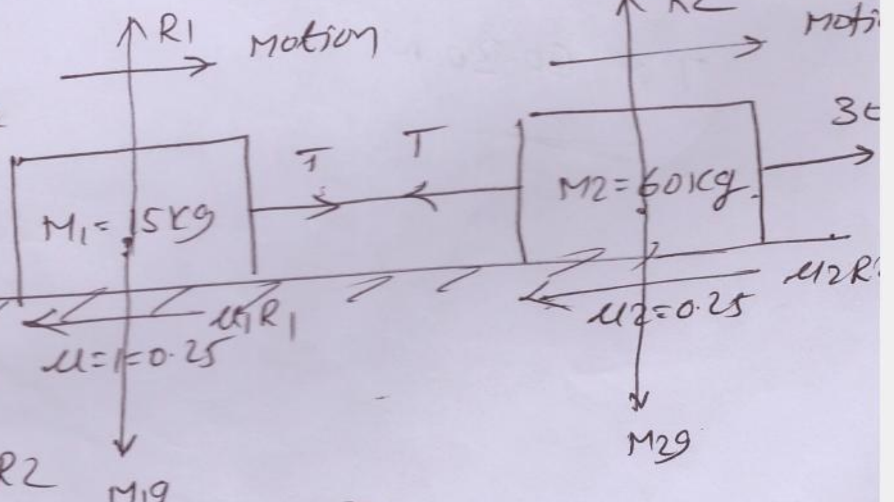
$$T = 50 + \frac{50}{9.81} \times 4.905 = 75N$$

6) Two blocks of mass 60 kg and 15 kg are connected a string and move along a rough horizontal surface wh a force of 300N is applied to the block of 60 kg Mass as shown Fig. Apply D'Alembert principle To determine the accelration of the blocks and tension in the string. The coeff. of friction b/w the sliding surface of the blocks and the plane is 0.25

using D'Alembert principle

$\Sigma F_x + (-ma_x) = 0 \ (a_x = a)$

$\Sigma F_y + (-ma_y) = 0 \ (a_y = 0)$



$$300 - \cancel{T} + \cancel{T} - \mu_1 R_1 - \mu_2 R_2 + -(m_1 + m_2) a = 0 \quad \text{— (1)}$$

$M_1 g - R_1 = 0 \Rightarrow R_1 = M_1 g.$

$M_2 g - R_2 = 0 \Rightarrow R_2 = M_2 g$

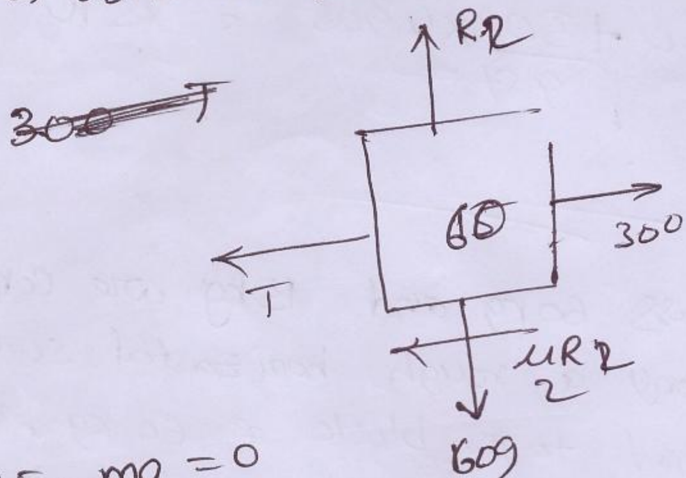
From (1)

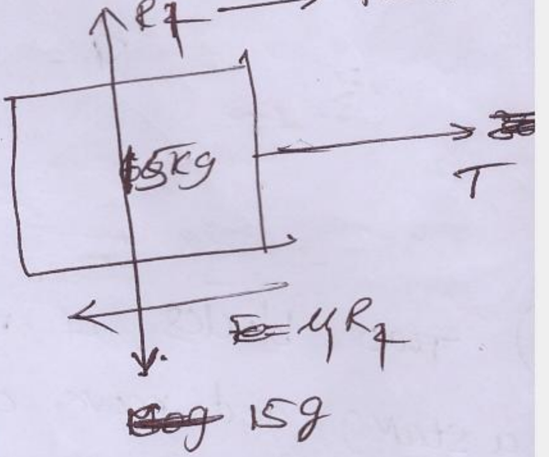
$$300 - 0.25 * 15 \times 9.81 - 0.25 \times 60 * 9.81 - 75a = 0$$

$$300 - 183.94 - 75a = 0$$

$$\boxed{a = 1.547 \text{ m/sec}^2}$$

Consider dynamic equilibrium of the block of mass 60kg





$\Sigma F - ma = 0$

$(300 - T - \mu_2 R_2) - ma = 0$

$300 - T - 0.25 * 60 * 9.81 - 60 * 1.547 = 0$

$T = 60.30$ N

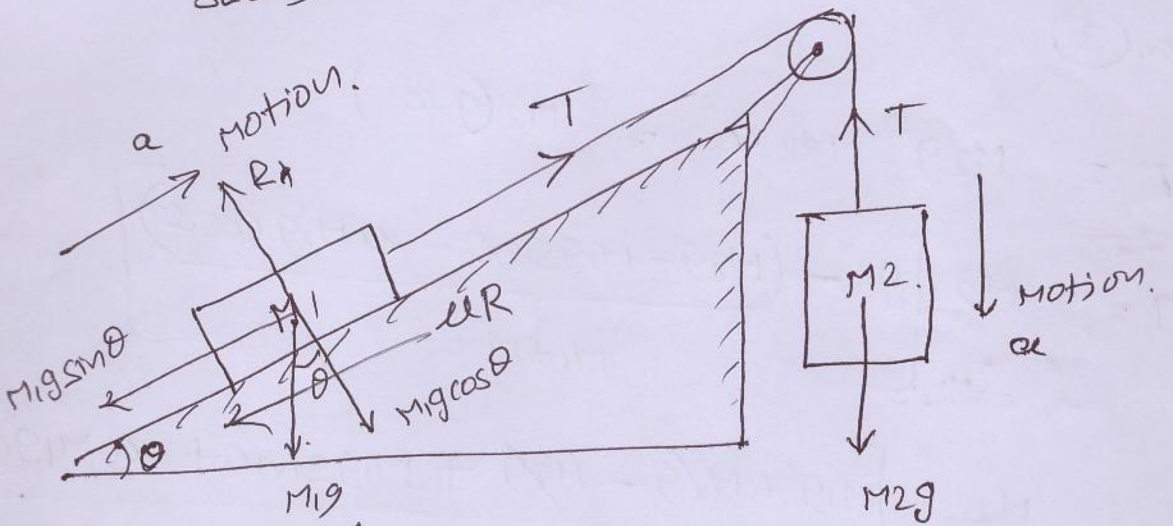
$\Sigma F = ma$

$T - \mu_1 R_1 - ma = 0$

$T - 0.25 * 15 \times 9.8 - 15 \times 1.547$

$T = 59.98$ N

7) A block of mass M_1 lying on an inclined plane of angle θ is pulled up by an angle an another block M_2 connected by a string as shown figure. The coefficient of friction b/w the inclined plane and the block M_1 is μ. Find the
(a) Accelaration of the mass M_2 and the tension in the string



$$\boxed{\begin{array}{l} \Sigma F_x - Ma_x = 0 \\ \Sigma F_y - Ma_y = 0 \end{array}}$$

The Motion of the block-M_1 :

$(a_x = a)$ $T - M_1 g\sin\theta - \mu R = M_1 a$ —①

$(a_y = 0)$ $M_1 g\cos\theta - R = 0$

$R = M_1 g\cos\theta$ —②

$\therefore T - M_1 g\sin\theta - \mu M_1 g\cos\theta = M_1 a$

$T = M_1 g\sin\theta + \mu M_1 g\cos\theta + M_1 a$ —Ⓐ

The motion of the block-

$a_y = a$

$a_x = 0$

$M_2 g - T = M_2 a$ —

$T = M_2 g - M_2$

—Ⓑ

Ⓐ = Ⓑ

$M_1 g\sin\theta + \mu M_1 g\cos\theta + M_1 a = M_2 g - M_2 a$

$M_1 g\sin\theta + \mu M_1 g\cos\theta + M_1 a = M_2 g - M_2 a$

$M_1 a + M_2 a = M_2 g - M_1 g\sin\theta - \mu M_1 g\cos\theta$

$$(M_1+M_2)\,a = M_2 g - M_1 g\sin\theta - \mu M_1 g\cos\theta$$

$$\boxed{a = \frac{M_2 g - M_1 g\sin\theta - \mu M_1 g\cos\theta}{M_1+M_2}} \rightarrow \text{acce}$$

From eqn (3)

$$T = M_2 g - M_2 a = M_2(g-a)$$

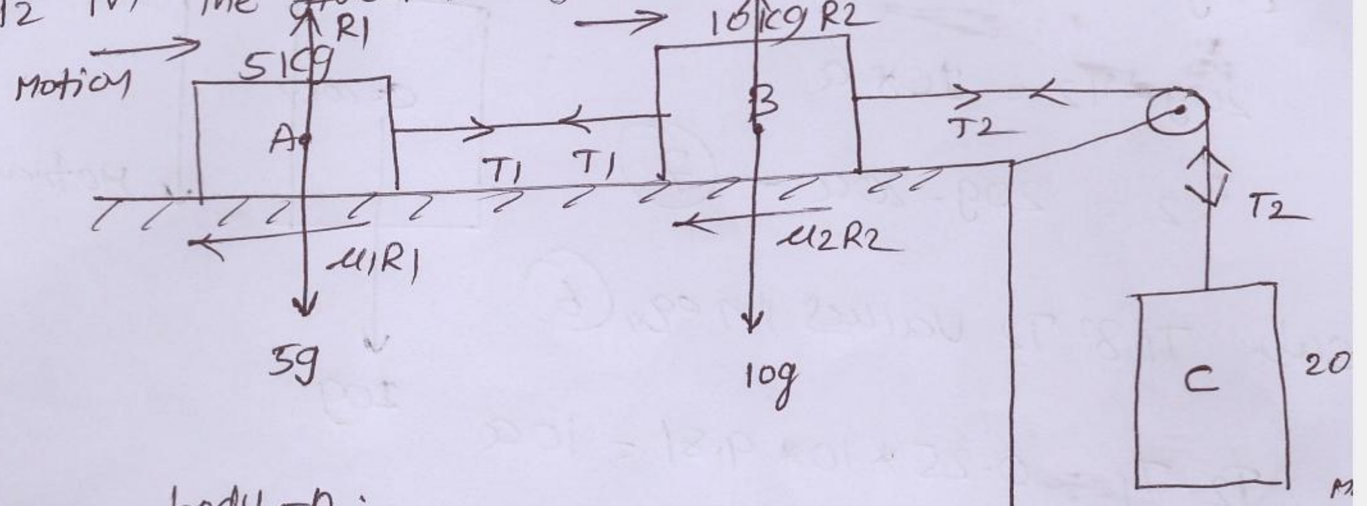
$$T = M_2\left[g - \frac{(M_2 g - M_1 g\sin\theta - \mu M_1 g\cos\theta)}{M_1+M_2}\right]$$

$$T = M_2\left[\frac{M_1 g + \cancel{M_2 g} - \cancel{M_2 g} + M_1 g\sin\theta + \mu M_1 g\cos\theta}{M_1+M_2}\right]$$

$$T = M_2\,\frac{\left[M_1 g(1+\sin\theta+\mu\cos\theta)\right]}{M_1+M_2}$$

$$\boxed{T = \frac{M_1 M_2 g\,(1+\sin\theta+\mu\cos\theta)}{M_1+M_2}} \rightarrow \text{Tension}$$

8) Three blocks A, B, C are connected as shown Fig. Find acceleration of the masses and the tension T_1 and T_2 in the ~~given~~ string. Given $\mu_1 = 0.2$ and $\mu_2 = 0.25$



Motion of a body -A :

$\Sigma F_x = M a_x$

$T_1 - \mu_1 R_1 = M_1 a$

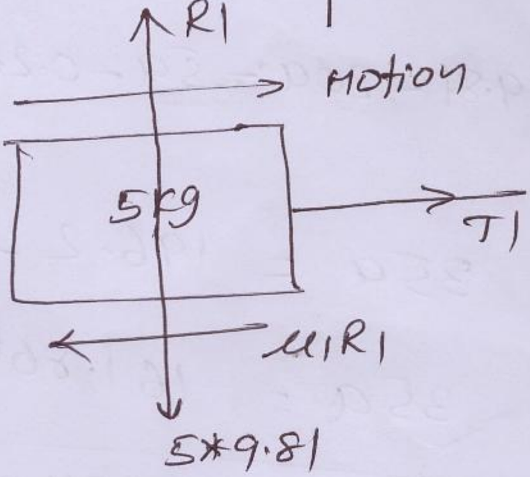
$T_1 - \mu_1 R_1 = 5a$ — (1)

$\Sigma F_y = M a_y$

$R_1 - 5g = 0$

$R_1 = 5g$ — (2)

$\therefore \; T_1 = 5a + 0.2 * 5 \times 9.81$ — (3)



Motion of a body B :

$\Sigma F_x = M a_x$

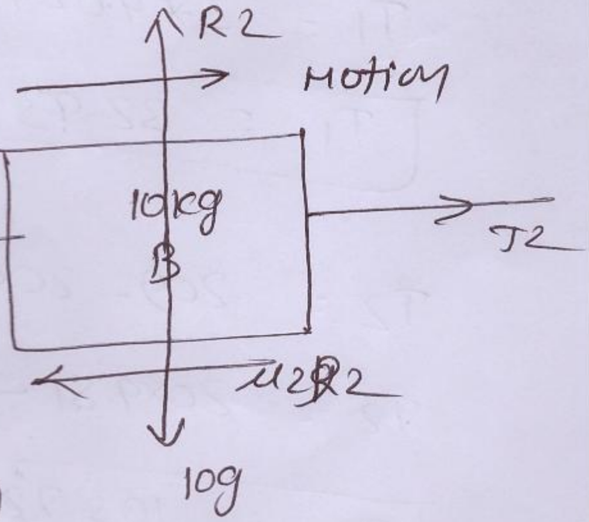
$T_2 - T_1 - \mu_2 R_2 = M_2 * a$

$T_2 - T_1 - 0.25 R_2 = 10a$ — (4)

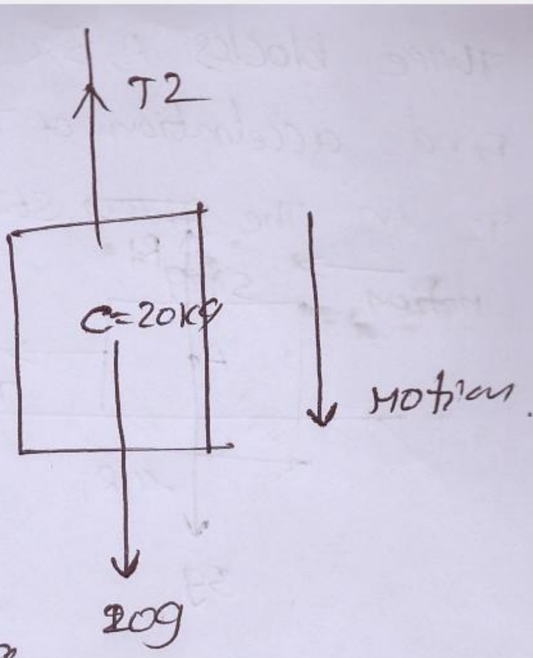
$\Sigma F_y = M a_y$

$R_2 - 10g = 0 \Rightarrow R_2 = 10g$ — (5)

$\therefore \; T_2 - T_1 - 0.25 * 10 * 9.81 = 10a$ — (6)



Motion of a body C.



$\Sigma F_y = M a_y$

$20g - T_2 = 20 * a$

$T_2 = 20g - 20a \quad (7)$

Sub. T_1 & T_2 values in eq. (6)

$T_2 - T_1 - 0.25 * 10 * 9.81 = 10a$

$20*9.81 - 20a - 5a - 0.2*5*9.81 - 0.25*10*9.81 = 10a$

$35a = 196.2 - 9.81 - 24.525$

$35a = 161.865$

$\boxed{a = 4.624 \text{ m/sec}^2}$

$T_1 = 5a + 0.2*5*9.81$

$T_1 = 5*4.624 + 0.2*9.81*5$

$\boxed{T_1 = 32.93 \text{ N}}$

$T_2 = 20g - 20a$

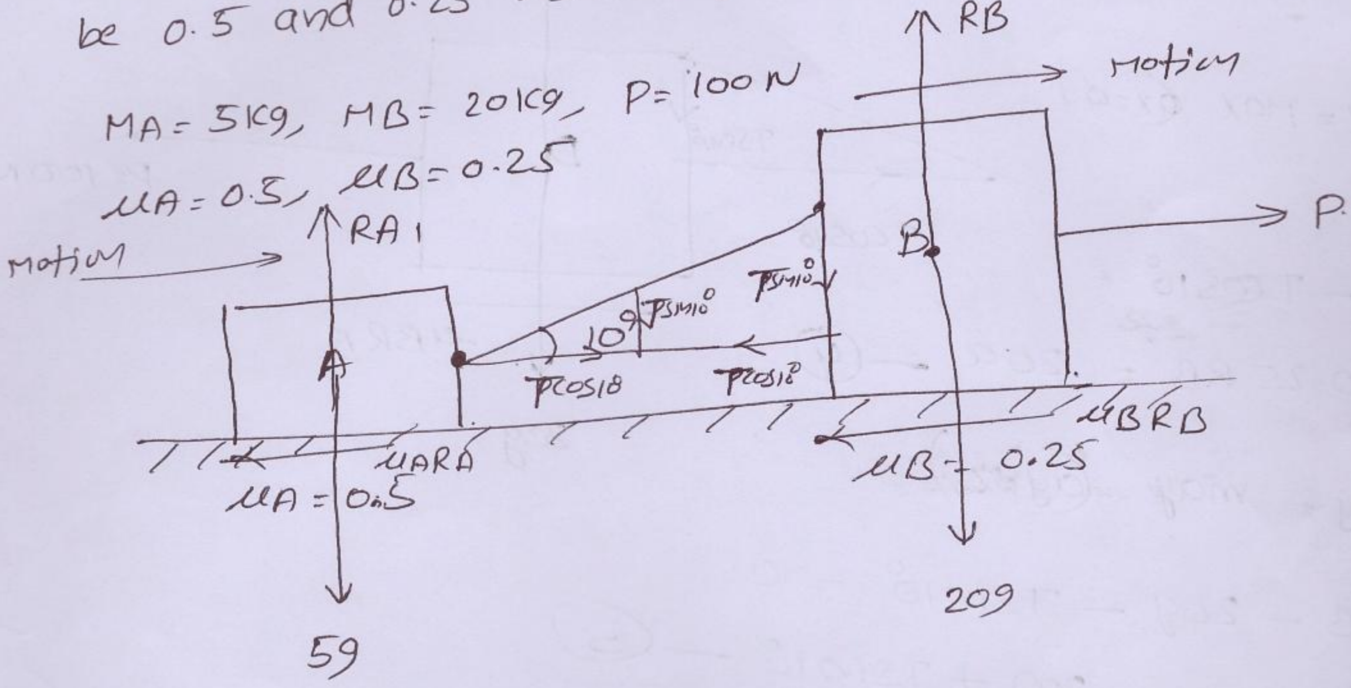
$T_2 = 20\times9.81 - 20*4.624$

$\boxed{T_2 = 103.72 \text{ N}}$

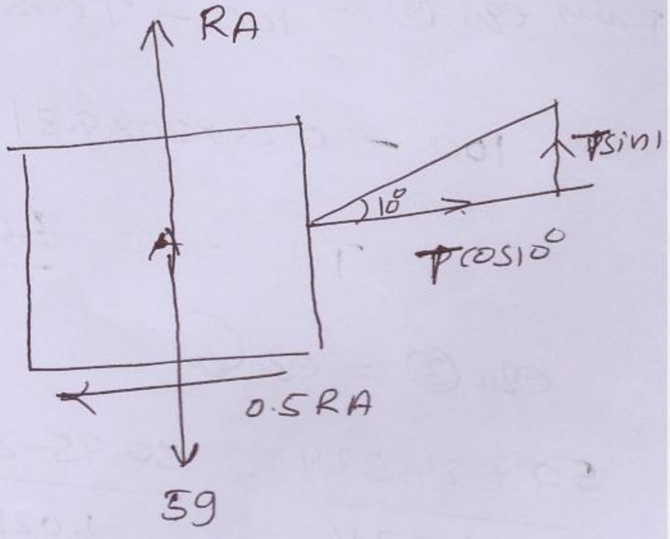
9) Two blocks A and B of masses 5kg and 20kg are connected by an inclined string. A horizontal force P of 100N is applied to the block B as shown Figure. Determine the tension in the string and the accelration of the system. Assume the coeff. of friction b/w the plane and the blocks A and B to be 0.5 and 0.25 respectively.

$M_A = 5kg$, $M_B = 20kg$, $P = 100N$

$\mu_A = 0.5$, $\mu_B = 0.25$



Motion of the ~~mass~~ body A :



$\Sigma F_x = ma_x \quad (a_x = a)$

$T\cos 10° - 0.5 R_A = M_A a$

$T\cos 10° - 0.5 R_A = 5a$ —— (1)

$\Sigma F_y = ma_y \quad (a_y = 0)$

$R_A - 5g + T\sin 10° = 0$

$R_A = 5g - T\sin 10°$ —— (2)

∴ From eqn (1)

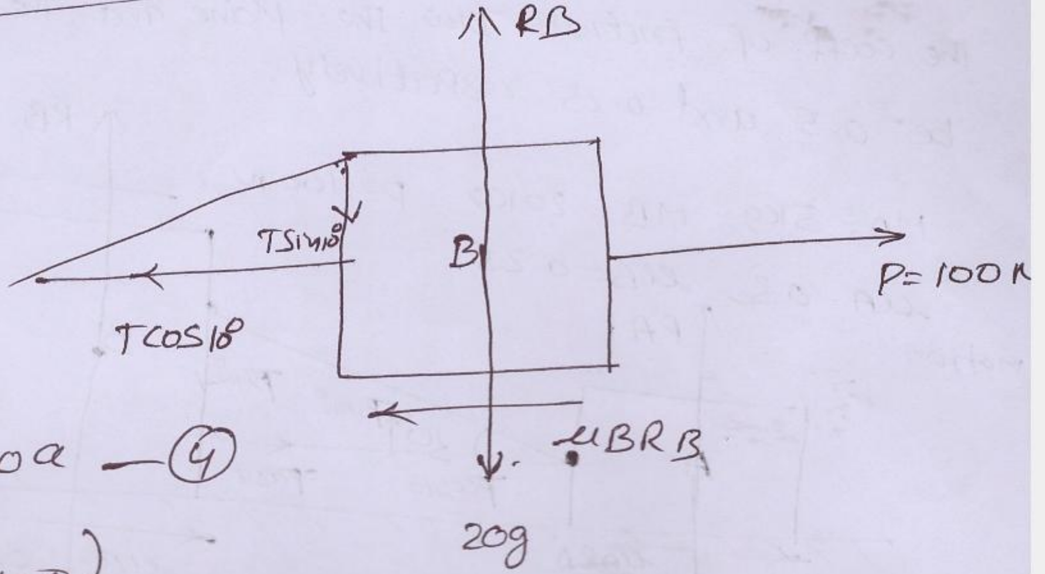
$T\cos 10° - 0.5(5g - T\sin 10°) = 5a$

$T\cos 10° - 0.5 * 5 * 9.81 + 0.5 * T * \sin 10° = 5a$

$T(\cos 10° + 0.5 \sin 10°) = 5a + 0.5 * 5 * 9.81$

$$T = \frac{5a + 24.525}{1.0716} \quad \text{--- (3)}$$

Motion of the body - B :



$\Sigma Fx = Max \quad (ax=a)$

$100 - T\cos 10°$

$- 0.25\, RB = 20a \quad \text{--- (4)}$

$\Sigma Fy = may \quad (ay=0)$

$RB - 20g - T\sin 10° = 0$

$RB = 20g + T\sin 10° \quad \text{--- (5)}$

From eq. (4) $\quad 100 - T\cos 10° - 0.25(20g + T\sin 10°) = 20a$

$100 - 0.25 \times 20 \times 9.81 - 20a = T(\cos 10° + 0.25\sin 10°)$

$$T = \frac{50.95 - 20a}{1.0282} \quad \text{--- (6)}$$

eq. (3) = eq. (6)

$$\frac{5a + 24.524}{1.0716} = \frac{50.95 - 20a}{1.0282} \Rightarrow \boxed{a = 1.105 \text{ m/sec}^2}$$

$$T = \frac{5 * 1.105 + 24.525}{1.0716}$$

$$\boxed{T = 28.042\ N}$$

curvilinear Motion: when a particle moves along a curved path is called curvilinear motion.

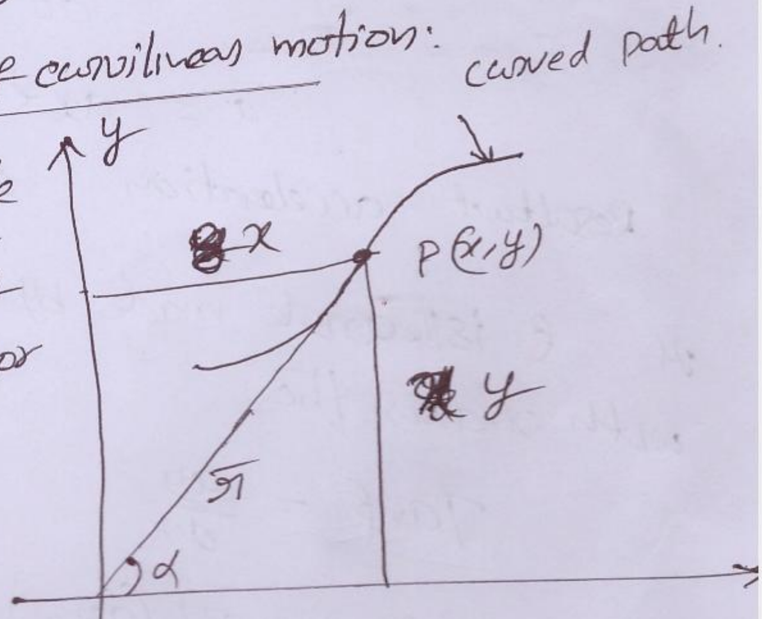
Examples of the curvilinear motion are

1) An Automobile vehicle negotiating turn on the road.
2) projectile motion of a bullet fired from a gun
3) motion of satellite around the earth.

The curvilinear Motion may be two Dimensional (plane curvilinear) or three dimensional (space curvilinear). The study of the plane curvilinear motion is made with reference to the following coordinate systems.

Rectangular components of curvilinear motion:

The Position of The particle on the curved path at any instant by the Position vector $\bar{r}$



$\bar{r} = xi + yj$

where i, j are the unit vector

magnitude $r = |\bar{r}| = \sqrt{x^2+y^2}$

velocity vector $\bar{V} = \frac{d\bar{r}}{dt} = \frac{d}{dt}(xi+yj)$

$$\bar{V} = i\frac{dx}{dt} + j\frac{dy}{dt}$$

$$\bar{V} = v_x i + v_y j$$

Resultant velocity $\quad v = \sqrt{v_x^2+v_y^2}$

The direction of velocity is tangential to the path of motion of the particle. If α is the angle made by the resultant with x-axis

$$\tan\alpha = \frac{v_y}{v_x}$$

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Further acceleration $\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(v_x i + v_y j)$

$$\bar{a} = \frac{d}{dt}\left(\frac{dx}{dt} i + \frac{dy}{dt} j\right)$$

$$\bar{a} = \frac{d^2x}{dt^2} i + \frac{d^2y}{dt^2} j$$

$$\bar{a} = a_x i + a_y j$$

Resultant acceleration $a = \sqrt{a_x^2 + a_y^2}$

If β is the angle made by the resultant acceleration with x-axis, then

$$\tan\beta = \frac{a_y}{a_x}$$

$$\beta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

For a curvilinear motion in space (Three Dimensional)

$$\boxed{\begin{aligned} \vec{r} &= xi + yj + zk \\ \vec{v} &= v_x i + v_y j + v_z k \\ \vec{a} &= a_x i + a_y j + a_z k \end{aligned}}$$

problem :

1) The motion of a particle is defined by the relation $x = t^2 + 3t$ and $y = t^3 - 8t^2 + 3$ where x & y are in meters and t is in sec (a) write the eq. defining the motion of the particle in vectorial form

(b) calculate the velocity and acceleration of the particle at time $t = 2$ seconds.

sol:

$x = t^2 + 3t$; $y = t^3 - 8t^2 + 3$

$v_x = \dfrac{dx}{dt} = 2t + 3$; $v_y = \dfrac{dy}{dt} = 3t^2 - 16t$

$a_x = \dfrac{dv_x}{dt} = 2$; $a_y = \dfrac{dv_y}{dt} = 6t - 16$

(a)

position vector : $\vec{r} = xi + yJ = (t^2+3t)\,i + (t^3 - 8t^2 + 3)$

velocity vector $\vec{V} = v_x i + v_y J = (2t+3)\,i + (3t^2 - 16t)\,J$

Accelration vector $\vec{a} = a_x i + a_y J = 2i + (6t - 16)\,J$

(b) At $t = 2$ seconds

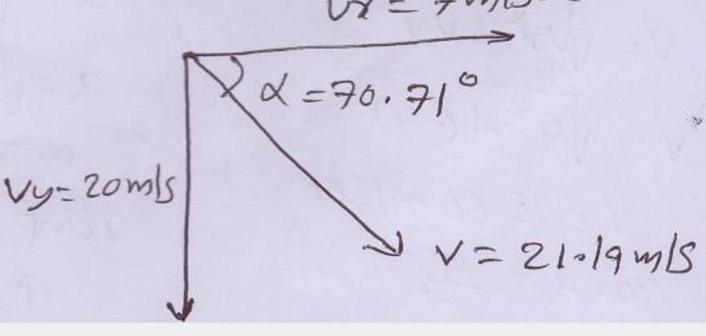
$\vec{V} = (2\times2 + 3)\,i + (3\times2^2 - 16\times2)\,J = 7i - 20J$

$\vec{V} = v_x i + v_y J$ $\quad v_x = 7$, $v_y = -20$

Resulted velocity $V = \sqrt{v_x^2 + v_y^2} = \sqrt{7^2 + (20)^2} = 21.19$ m/s

$\operatorname{Tan}\alpha = \dfrac{v_y}{v_x} = \dfrac{20}{7} = 2.857$ $\quad$ v_x is +ve, v_y is −ve the V lies 4th Quadrant

$\alpha = 70.71^\circ$



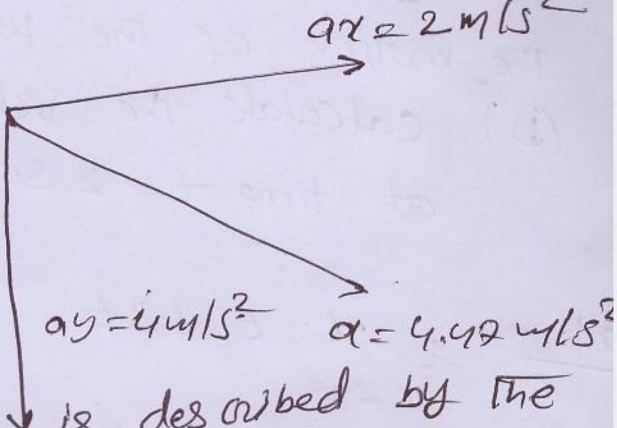
$a = \sqrt{a_x^2 + a_y^2}$

$\bar{a} = 2i + (6 \times 2 - 16)J = 2i - 4J$

$a = \sqrt{(2)^2 + (-4)^2} = 4.47\ m/s^2$

$\tan\beta = \frac{4}{2} = 2$

$\beta = 63.45^0$



2) The motion of the particle is described by the following eq., $x = 2(t+1)^2$; $y = 2(t+1)^{-2}$ so that the path travelled by the particle is a rectangular hyperbola. Find also, the velocity and acceleration of the particle at $t=0$.

sol:

$x = 2(t+1)^2 \;;\; y = 2(t+1)^{-1}$

$xy = 2 * 2$

$xy = 4 \rightarrow$ which represents a rectangular hyperbola.

$x = 2(t+1)^2$

$y = 2/(t+1)^2 = \frac{(t+1)^2 d(2) - 2 \times}{((t+1)^2)^2}$

$v_x = \frac{dx}{dt} = 4(t+1)$

$v_y = \frac{dy}{dt} = -4(t+1)^{-3}$

$a_x = \frac{dv_x}{dt} = 4$

$a_y = \frac{dv_y}{dx} = 12(t+1)^{-4}$

At $t=0$

$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{4^2 + (-4)^2}$

$v = 5.66\ m/sec$

$\tan\alpha = \frac{4}{4} =$

At $t=0$

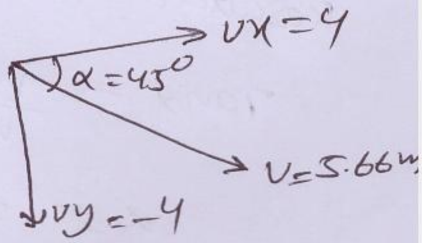
$a = \sqrt{a_x^2 + a_y^2}$

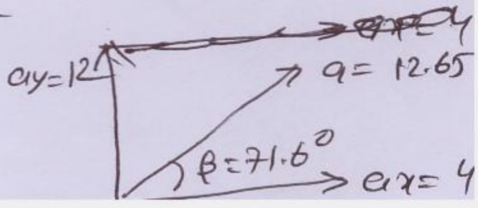
$a = \sqrt{4^2 + 12^2}$

$a = 12.65\ m/sec$

$\tan\beta = \frac{12}{4}$

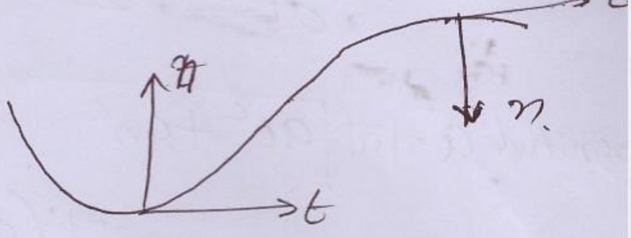
$\beta = 71.6^0$



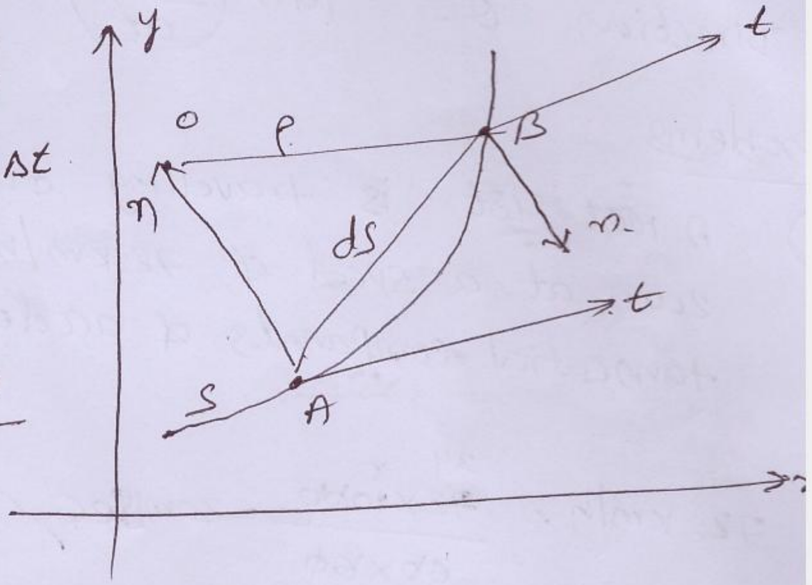


Components of acceleration : (Normal and Tangential)

In This system the velocity and acceleration of a moving partical are expressed in tangential(t) and Normal components (n)



Consider a particle That moves along a curved path from Point A to B and transverse an infinitely small distance ds in small interval of time Δt



$$V_t = \frac{\text{Distance moved along The tangential direction}}{\text{time interval}}$$

$$\boxed{V_t = \frac{ds}{dt}}$$

$$V_n = \frac{\text{Distance moved along The Normal direction.}}{\text{time interval}}$$

$$\boxed{V_n = 0}$$

$$V = V_t + V_n$$

$$a = a_t + a_n.$$

tangential component: It is defined as rate of change of the speed of the particle

$$a_t = \frac{dV}{dt} =$$

Normal component: it is ~~the~~ defined as The particle at a point equal to square of The speed divided by The radious of curvature

$$a_n = \frac{V^2}{\rho}$$

$\bar{a} = a_t + a_n$

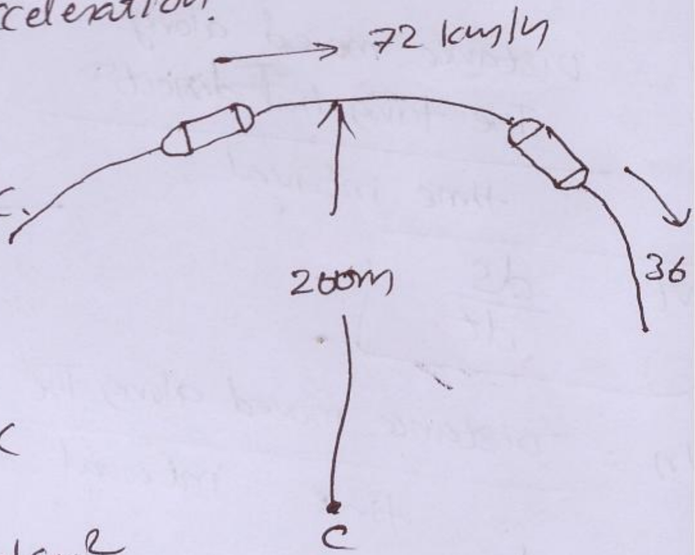
$$a = \frac{dv}{dt} + \frac{v^2}{2}$$

Magnitude $a = |\bar{a}| = \sqrt{a_t^2 + a_n^2}$

Direction $\theta = \tan^{-1}\left(\frac{a_n}{a_t}\right)$

Problems :

1) A Motorist is travelling on a curved road of radio 200m at a speed of 72 km/hour. Find the normal and tangential componets of acceleration.



$$72\ km/h = \frac{\overset{34}{\cancel{72}} \times \overset{5}{\cancel{1000}}}{\underset{3}{\cancel{60}} \times \underset{3}{\cancel{60}}} = 20\ m/sec.$$

$36\ km/h = 10\ m/sec$

constant speed $v = 20\ m/sec$

$$a_n = \frac{v^2}{r} = \frac{20^2}{200} = 2\ m/sec^2$$

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(20) = 0$$