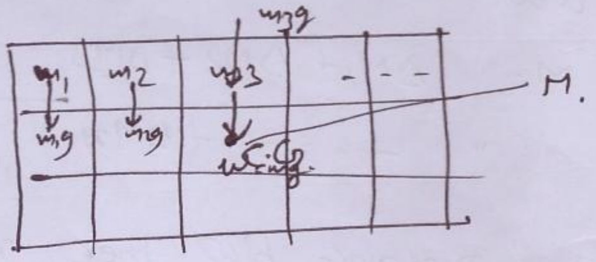
CENTRIODS AND CENTERE OF GRAVITY

very offen it is required to define a point such that the length of wire_ the area of the plate_ the mass or gravitional force acting on a body may be assumed to be concentrated at that point

Centre of gravity :

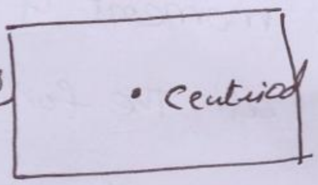
centre of gravity of a body is a point through which the distributed resultant gravitional force passing through it.



Centre of mass : It is a point through which entire mass concentrated at that point.

The C.G and C.M ~~mass be~~ of a body are differnt only when the gravitational field is not uniform are parallel. For most practicle purpose they are assigned to be same.

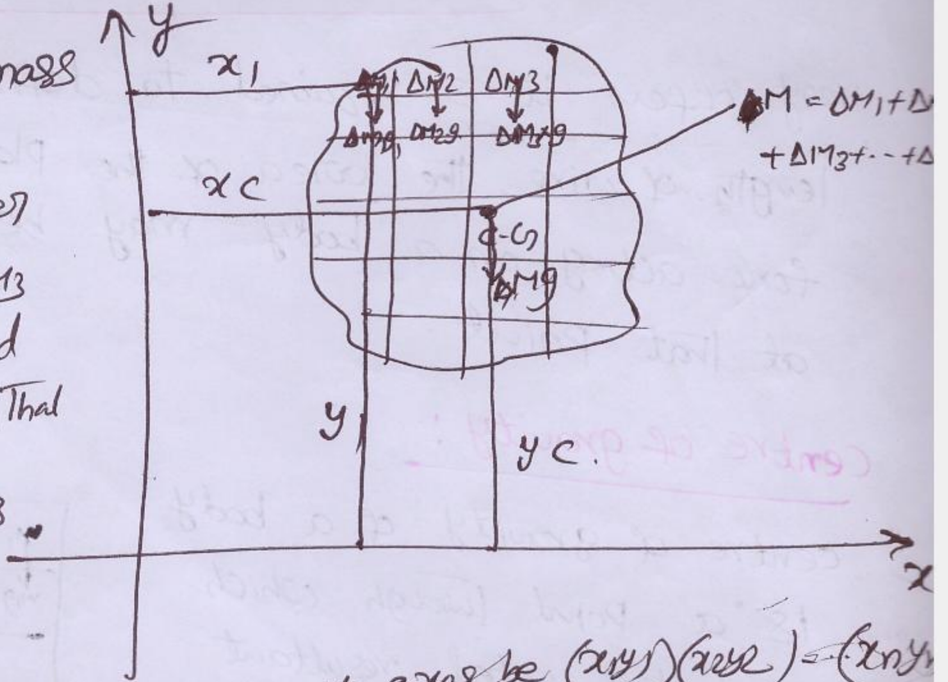
Centriod : For a 2 dimensional body only having area there is no thickness so that to locate the centre point of a two dimensional body is called centriod.



1) Centre of gravity of a body : (3-D) Method of Moments

Consider a body of mass M. Let This body be composed of 'n' number of masses $\Delta M_1, \Delta M_2, \Delta M_3$ ---- ΔM_n distributed with in The body such That

$M = \Delta M_1 + \Delta M_2 + \Delta M_3 + \cdots + \Delta M_n$



The Distance b/w These masses w.r.t axes be $(x_1 y_1)(x_2 y_2) \cdots (x_n y_n)$

Let The C.G of the whole mass M lie at a distance $(x_c y_c)$ w.r.t The x, y axes.

Let us assume That The gravitational field is uniform and parallel.

Gravitational force acting on The mass $\Delta M_1 = \Delta M_1 * g$

$\Delta M_2 = \Delta M_2 * g$

$\vdots$

$\Delta M_n = \Delta M_n * g$

Apply The principle of moments

The moment of the resultant of all The forces about y-axis = The sum of The moments of all forces about y axis

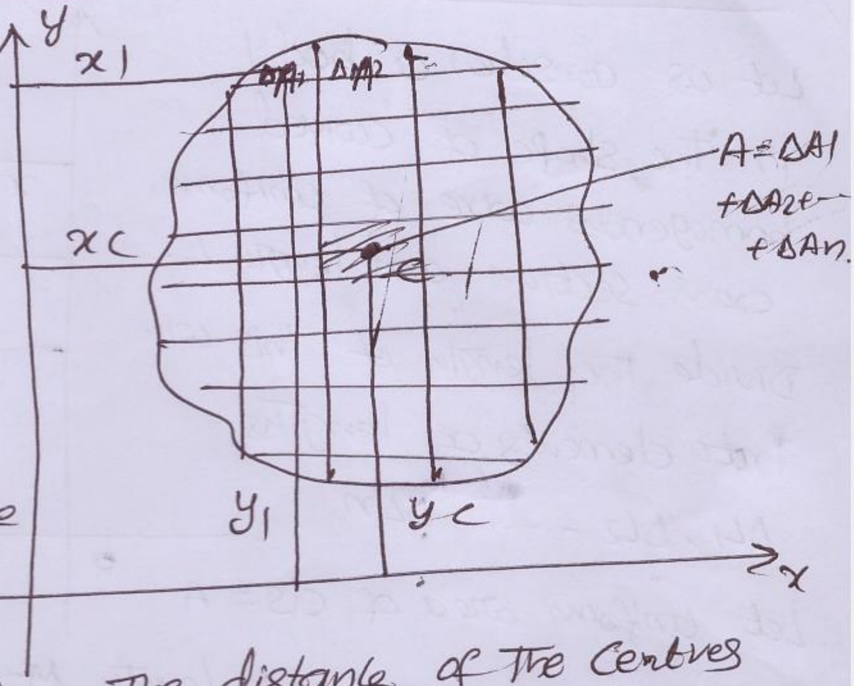
$$M g * x_C = \Delta M_1 g * x_1 + \Delta M_2 g * x_2 + \cdots + \Delta M_n g * x_n$$

$$x_C = \frac{\Delta M_1 x_1 + \Delta M_2 x_2 + \Delta M_3 x_3 + \cdots + \Delta M_n x_n}{M}$$

$$x_C = \frac{\Delta M_1 x_1 + \Delta M_2 x_2 + \cdots + \Delta M_n x_n}{\Delta M_1 + \Delta M_2 + \cdots + \Delta M_n}$$

$$\boxed{x_C = \frac{\Sigma \Delta M_i x_i}{\ }} ; \boxed{y_C = \frac{\Sigma \Delta M_i y_i}{\ }}$$

2) Centriod of two dimensional body:



Let us consider the case of homogenous plate or lamina of uniform thickness (t), density (ρ), and total area A.

Divide the area of the plate into elements of areas $\Delta A_1, \Delta A_2, \Delta A_3$ ---- ΔA_n. The distance of the centres of these areas w.r.t the axes be $(x_1 y_1)(x_2 y_2)$ --- $(x_n y_n)$

The centriod of the whole area A lie at a distance $(x_c y_c)$ w.r.t the x,y axes.

Mass of the plate $M = (At) * \rho$ (volume * density) $= M_1 + M_2 + \dots + M_n$

Mass ΔM_1 of the element $\Delta M_1 = (\Delta A_1 * t)\rho$

$$\Delta M_2 = (\Delta A_2 * t)\rho$$

$$\vdots$$

$$\Delta M_n = (\Delta A_n * t)\rho$$

$$\Sigma \Delta M = \Delta M_1 + \Delta M_2 + \dots + \Delta M_n$$

Apply principle of moment

The moment of the resultant of all the forces about y-axis = The sum of the moments of all forces about y-axis

$$M . x_c = \Delta M_1 x_1 + \Delta M_2 x_2 + \dots + \Delta M_n x_n$$

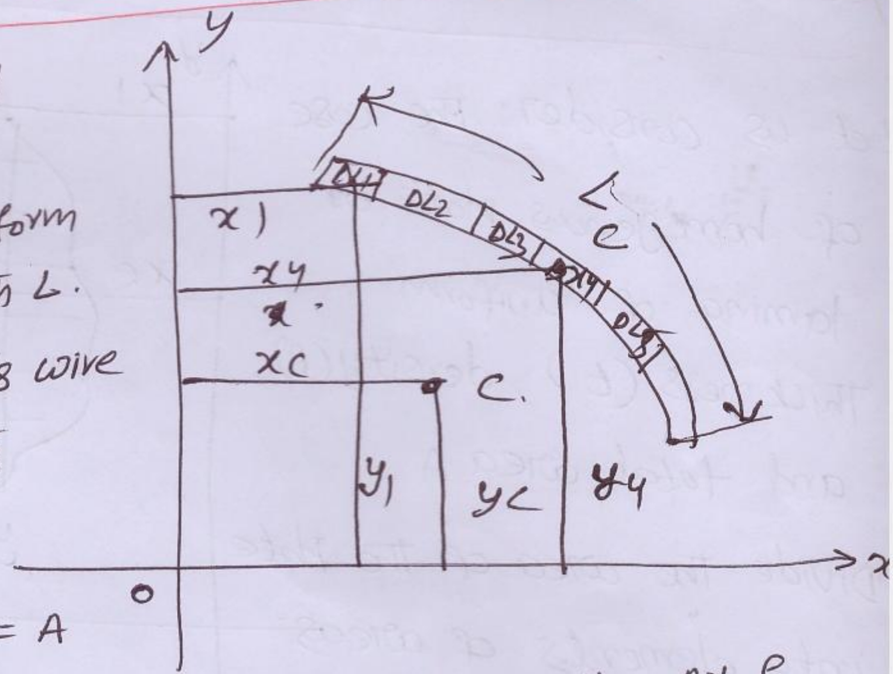
$$(M_1 + M_2 + \dots + M_n) x_c = \frac{\Delta A_1 \cancel{t} \cancel{\rho} x_1 + \Delta A_2 \cancel{t} \cancel{\rho} x_2 + \dots + \Delta A_n \cancel{t} \cancel{\rho} x_n}{\Delta A_1 \cancel{t} \cancel{\rho} + \Delta A_2 \cancel{t} \cancel{\rho} + \dots + \Delta A_n \cancel{t} \cancel{\rho}}$$

$$\boxed{x_c = \frac{\Sigma \Delta A_i x_i}{\Sigma \Delta A_i}} \quad \text{III}^{\text{ly}} \quad \boxed{y_c = \frac{\Sigma \Delta A_i y_i}{\Sigma \Delta A_i}}$$

3) Centroid of one dimensional body (Line segment):

Let us consider a body in the shape of curved homogenous wire of uniform cross section and length L.

Divide the length of this wire into elements of lengths $\Delta L_1, \Delta L_2 --- \Delta L_n$



Let uniform area of $C/S = A$

The Mass M of the wire length $M = (AL) * \rho$.

$M = AL\rho$

$= m^2 * m \times \frac{kg}{m^3}$

Density of the wire $= \rho$

The Mass of the length $\Delta L_1 = \Delta M_1$

$\Delta M_1 =$ vol. $*$ density $= (A\Delta L_1) * \rho$

$\Delta M_2 = (A\Delta L_2) * \rho$

$\vdots$

$\Delta M_n = (A\Delta L_n) * \rho$

Apply the principle of moment

The moment of the resultant of all the forces about the y-axis $=$ The sum of the moments of all the forces about y-axis

$M x_c = \Delta M_1 x_1 + \Delta M_2 x_2 + --- + \Delta M_n x_n$

$(AL\rho)(x_c) = (A\Delta L_1\rho) x_1 + (A\Delta L_2\rho) x_2 + --- + (A\Delta L_n\rho)$

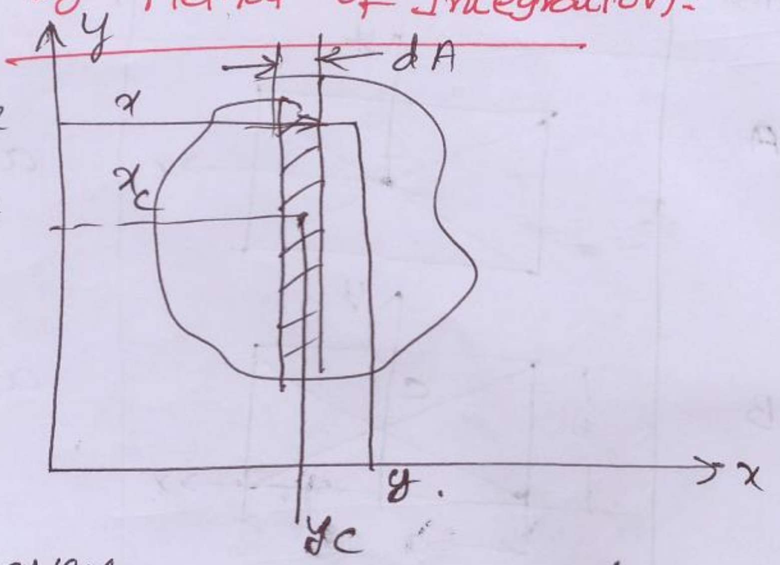
$$x_c = \frac{\Delta L_1 x_1 + \Delta L_2 x_2 + --- + \Delta L_n x_n}{\Delta L_1 + \Delta L_2 + --- + \Delta L_n}$$

$$\boxed{x_c = \frac{\Sigma \Delta L_i x_i}{\Sigma \Delta L_i}}$$

|||ly

$$\boxed{y_c = \frac{\Sigma \Delta L_i y_i}{\Sigma \Delta L_i}}$$

Determination of Centriod by Method of Integration:



Let us Consider a infinite small element of area dA is a distance of x, y from x, y axes,

Let x_c, y_c = coordinates of centriod of a given section of area A.

Apply principle of moment.

$$x_c \int dA = \int x \, dA$$

2-D.

$$\boxed{x_c = \frac{\int x \, dA}{\int dA} \qquad y_c = \frac{\int y \, dA}{\int dA}}$$

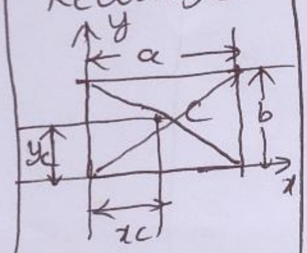
$\int x \, dA \rightarrow$ First Moment of area w.r.t y-axis

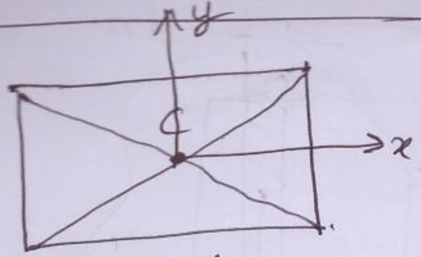
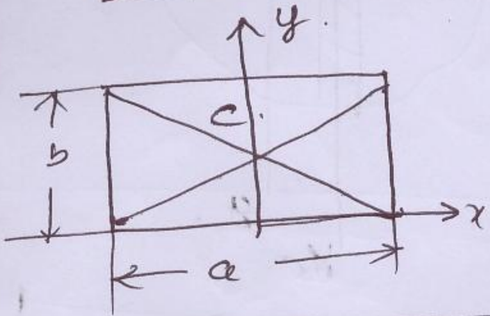
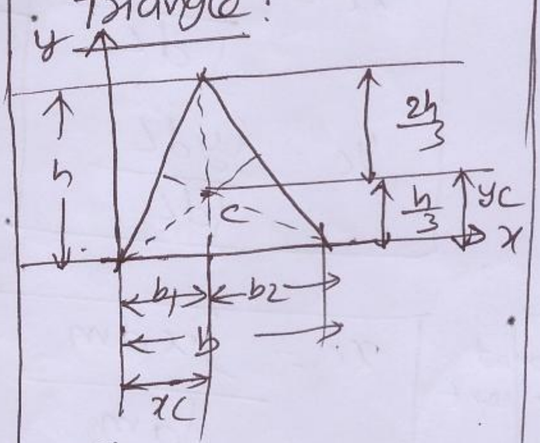
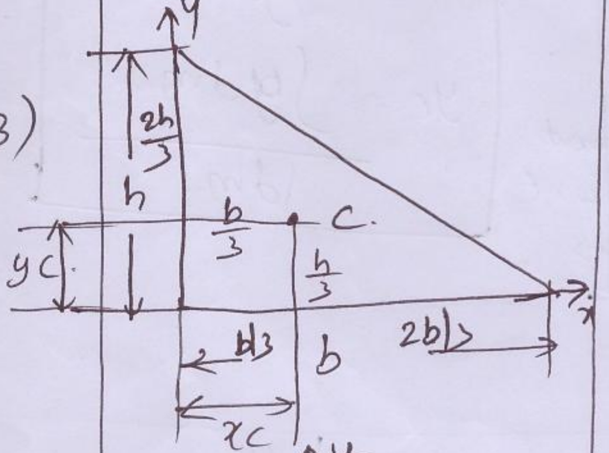
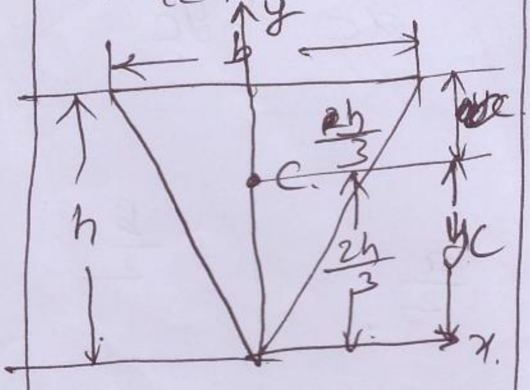
$\int y \, dA \rightarrow$ First Moment of area w.r.t x-axis

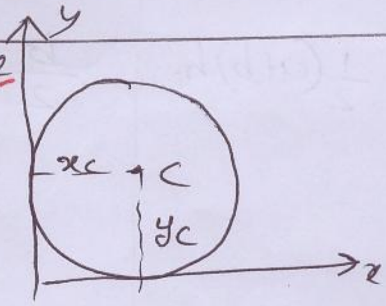
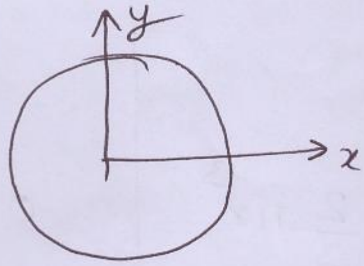
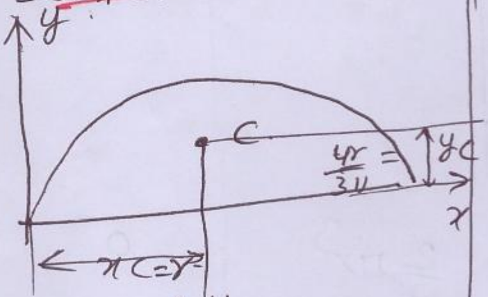
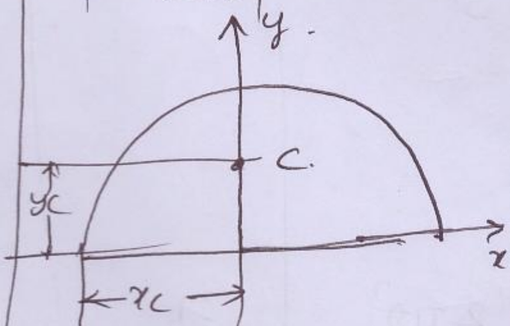
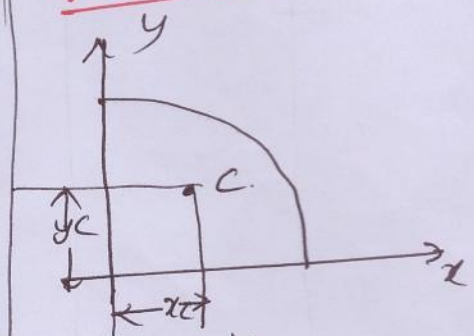
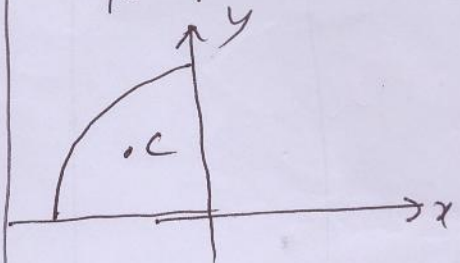
$$\boxed{x_c = \frac{\int x \, dL}{\int dL} \qquad y_c = \frac{\int y \, dL}{\int dL}} \rightarrow \text{1D.}$$

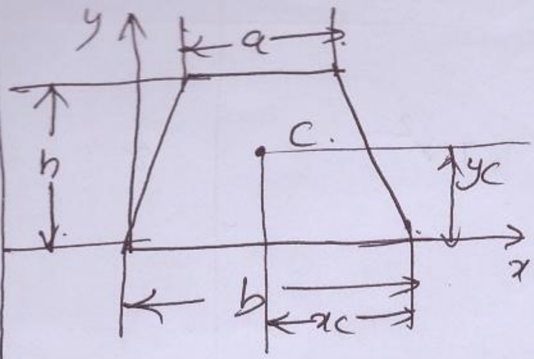
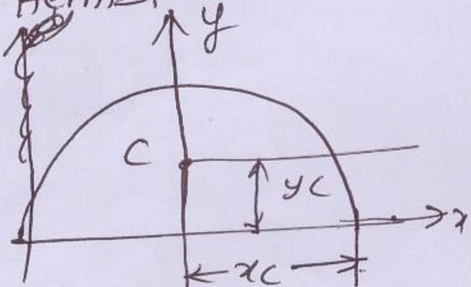
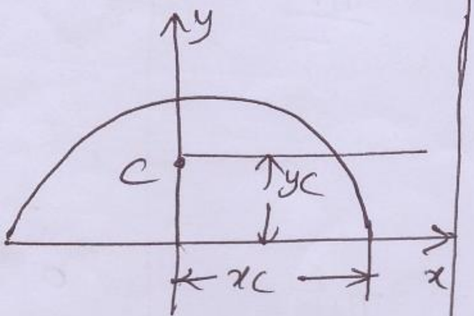
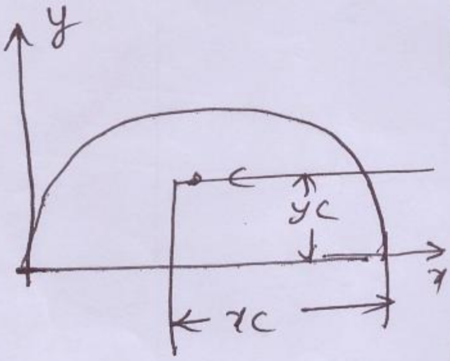
$$\boxed{x_c = \frac{\int x \, dm}{\int dm} \qquad y_c = \frac{\int y \, dm}{\int dm}} \rightarrow \text{3D}$$

Centriods of various shapes:

S.NO	Shape	Area	x_c	y_c
1)	Rectangle 	ab	$\frac{a}{2}$	$\frac{b}{2}$

S.No	shape	Area	x_c	y_c
1A		ab	0	0.
1B		ab	0	$\frac{b}{2}$
2) Triangle:				
(2A)		$\frac{1}{2} * bh$	$\cancel{\ }$ b_1	$\frac{h}{3}$
(2B)		$\frac{1}{2} * bh.$	$\frac{b}{3}$	$\frac{h}{3}$
(2C)		$\frac{1}{2} bh$	0	$\frac{2h}{3}$

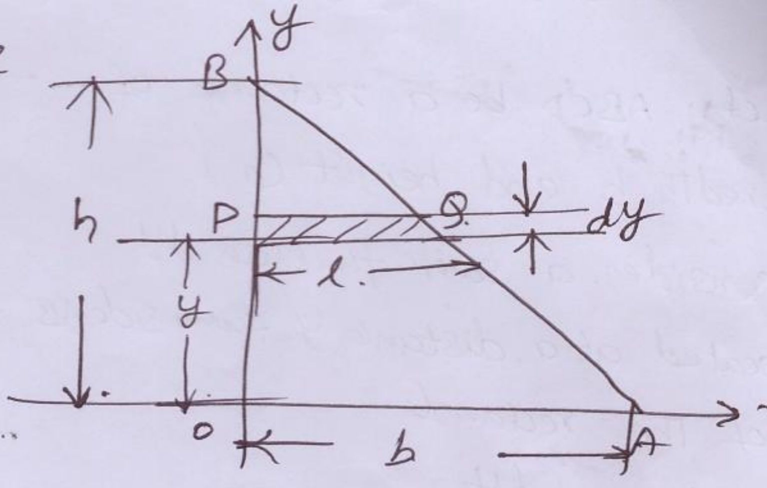
S.NO	Shape	Area	x_c	y_c
(3) 3A	circle 	πr^2	r	r.
3B		πr^2	0	0
(4A)	Semi circle			
(4A)		$\frac{\pi r^2}{2}$	r.	$\frac{4r}{3\pi}$
(4B)		$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$
(5)	Quadrant: 	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
		$\frac{\pi r^2}{4}$	$\frac{-4r}{3\pi}$	$\frac{4r}{2\pi}$

6)	Trapezodial lamina 	$\frac{1}{2}(a+b)h$	$\frac{b}{2}$	$\frac{h}{3}\left(\frac{b+2a}{b+a}\right)$
7)	Hemisphere 	$\frac{2}{3}\pi r^3$	0	$\frac{3}{8}r$
		$\frac{2}{3}\pi r^3$	0	$\frac{3}{8}r$
		$\frac{2}{3}\pi r^3$	r	$\frac{3}{8}r$

2) Determine Centriod of The area of a Δle w.r.t its base.

Let OAB be Δle of base b and height h.

Consider an elementary strip of width l, thickness dy and located at a distance y from base OA of The Δle.



Area $dA = l\,dy$

Δle OAB & PQB are similar triangles.

$$\frac{l}{b} = \frac{h-y}{h}$$

$$l = \left(\frac{h-y}{h}\right) b$$

$$l = \frac{hb-yb}{h}$$

$$dA = l\,dy = \left(\frac{hb-yb}{h}\right) dy$$

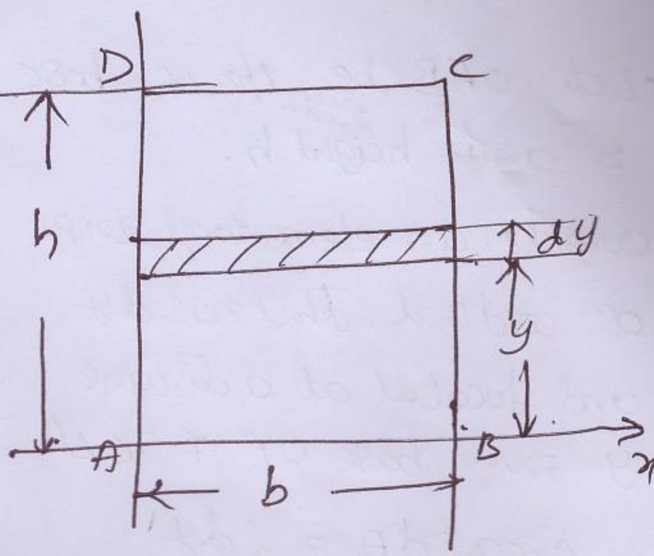
$$\therefore\ y_c = \frac{\int y\,dA}{\int dA} = \frac{\int_0^h y\left(\frac{bh-by}{h}\right)dy}{\int_0^h \left(\frac{bh-by}{h}\right)dy}$$

$$y_c = \frac{\int_0^h \left(\frac{ybh-by^2}{h}\right)dy}{\int_0^h \left(\frac{bh-by}{h}\right)dy} = \frac{\left(\frac{y^2}{2}\cancel{b} - \frac{\cancel{b}}{h}\frac{y^3}{3}\right)_0^h}{\left(\cancel{b}y - \frac{\cancel{b}}{h}\frac{y^2}{2}\right)_0^h}$$

$$y_c = \frac{\frac{h^2}{2} - \frac{h^{\cancel{3}}}{3}}{h - \frac{h^{\cancel{2}}}{2\cancel{h}}} = \frac{\frac{3h^2-2h^2}{6_3}}{\frac{2h-h}{\cancel{2}}} = \frac{h^{\cancel{2}}}{3\cancel{h}}$$

$$\boxed{y_c = \frac{h}{3}}$$

Problems: i) Determine centroid of area of reactangle of bredth b and height h.



Let ABCD be a rectangle of bredth b and height (h)
consider a strip thickness dy located at a distance y from side AB of the rectangle.

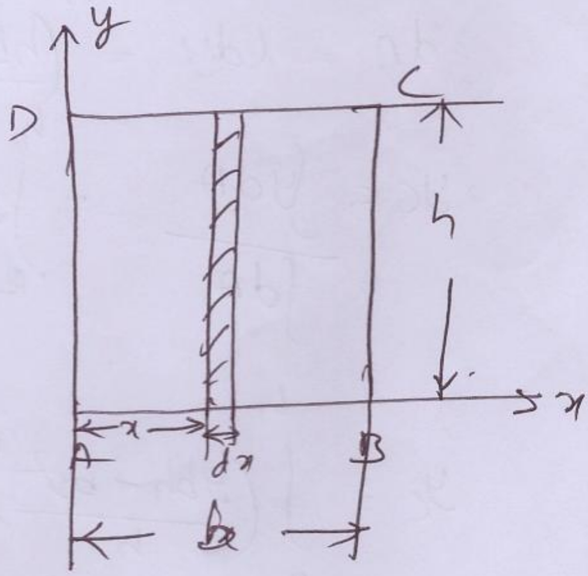
$$dA = b\,dy$$

Apply moment principle

$$y_c = \frac{\int y\,dA}{\int dA} = \frac{\int y b\,dy}{\int dA}$$

$$y_c = \frac{b\int_0^h y\,dy}{b\int_0^h dy} = \frac{\cancel{b}\frac{h^{\cancel{2}}}{2}}{\cancel{b}\cancel{h}}$$

$$\boxed{y_c = \frac{h}{2}}$$



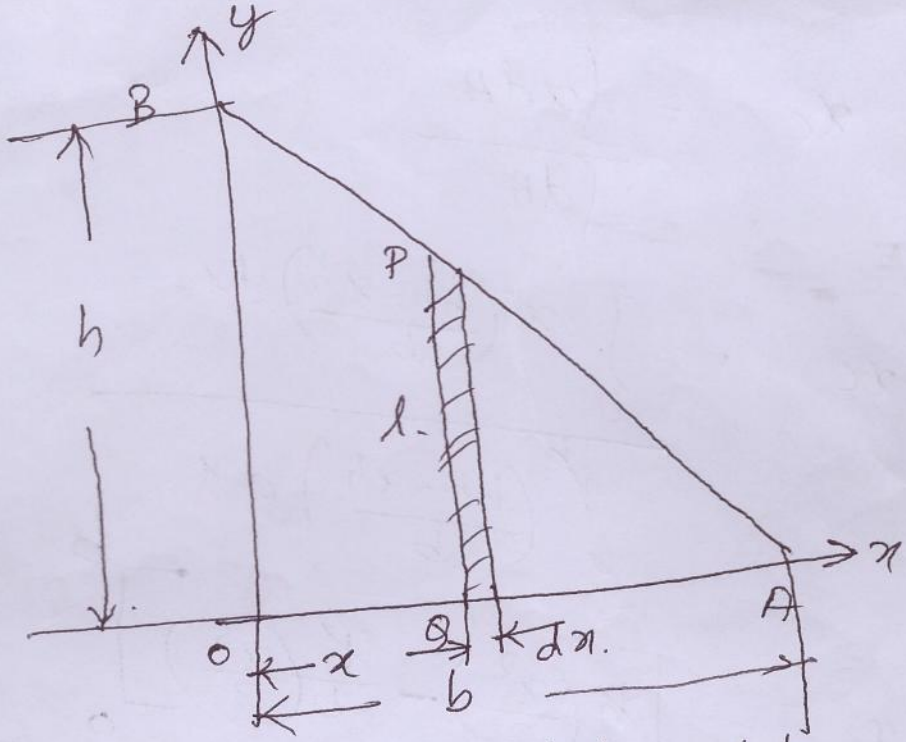
$$x_c = \frac{\int x\,dA}{\int dA}$$

$$dA = h\,dx.$$

$$x_c = \frac{\int_0^b x h\,dx.}{\int_0^b h\,dx} = \frac{h\left(\frac{x^2}{2}\right)_0^b}{(hx)_0^b}$$

$$x_c = \frac{\cancel{h}\frac{b^{\cancel{2}}}{2}}{\cancel{h}\cancel{b}}$$

$$\boxed{x_c = \frac{b}{2}}$$



Let OAB be Δle of base b and height h.

consider an elementary strip of width l, thickness dx and located at a distance x from OB From Δle

Δle OAB & AQP ||ler Δle

$$\frac{b-x}{b} = \frac{l}{h}$$

$$l = \left(\frac{b-x}{b}\right) h$$

Area $dA = l\,dx$.

$$dA = \left(\frac{b-x}{b}\right) h\,dx$$

$$dA = \left(\frac{hb-xh}{b}\right) dx.$$

$$x_c = \frac{\int x\,dA}{\int dA}$$

$$x_c = \frac{\int_0^b x\left(\frac{hb - xh}{b}\right)dx}{\int_0^b \left(\frac{hb - xh}{b}\right)dx}$$

$$x_c = \frac{\left[\cancel{h}\left(\frac{x^2}{2}\right) - \frac{\cancel{h}}{b}\left(\frac{x^3}{3}\right)\right]_0^b}{\left[\cancel{h}(x) - \frac{\cancel{h}}{b}\left(\frac{x^2}{2}\right)\right]_0^b}$$

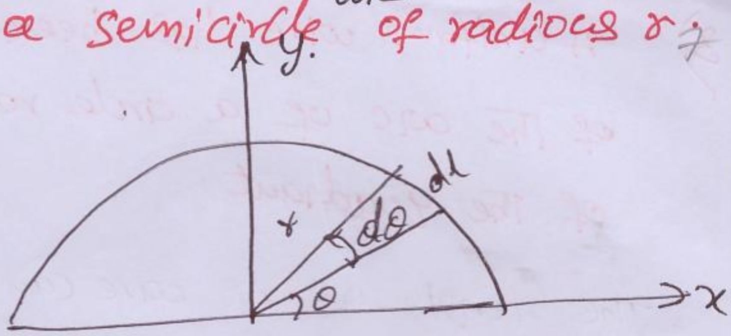
$$x_c = \frac{\frac{b^2}{2} - \frac{b^2}{3}}{b - \frac{b}{2}}$$

$$x_c = \frac{\frac{3b^2 - 2b^2}{6}}{\frac{2b - b}{2}}$$

$$x_c = \frac{\frac{b^{\cancel{2}}}{\cancel{6}\,3}}{\frac{\cancel{b}}{\cancel{2}}}$$

$$\boxed{x_c = \frac{b}{3}}$$

4) Determine the Centriod of a semicircle arc of radious r ;



$$x_c = \frac{\int x\,dl}{\int dl}$$

polar coordinates

$x = r\cos\theta$, $y = r\sin\theta$, $dl = r\,d\theta$

$x = r\cos\theta \rightarrow \theta = 0 \text{ to } 180^\circ$

$y = r\sin\theta \rightarrow \theta = 0 \text{ to } 180^\circ$

$$x_c = \frac{\int r\cos\theta * r\,d\theta}{\int r\,d\theta} = \frac{\int_0^{\pi} r^2 \cos\theta\,d\theta}{\int_0^{\pi} r\,d\theta}$$

$$x_c = \frac{r^2 \left(+\sin\theta\right)_0^{\pi}}{r\left(\pi - 0\right)} = \frac{r^2\left[+\left(\sin\pi - \sin 0\right)\right]}{r\pi}$$

$$\boxed{x_c = 0}$$

$$y_c = \frac{\int y\,dl}{\int dl} = \frac{\int r\sin\theta * r\,d\theta}{\int r\,d\theta} = \frac{r^2 \int_0^{\pi/2} \sin\theta\,d\theta}{r\int_0^{\pi/2} d\theta}$$

$$y_c = \frac{r^2 - \left(\cos\frac{\pi}{\cancel{7}} - \cos 0\right)}{r\left(\frac{\pi}{\cancel{7}} - 0\right)} = \frac{r^{\cancel{2}} - \cdot(-1-1)}{\frac{\cancel{\pi} r}{2}}$$

$$\boxed{y_c = \frac{2r}{\pi}}$$

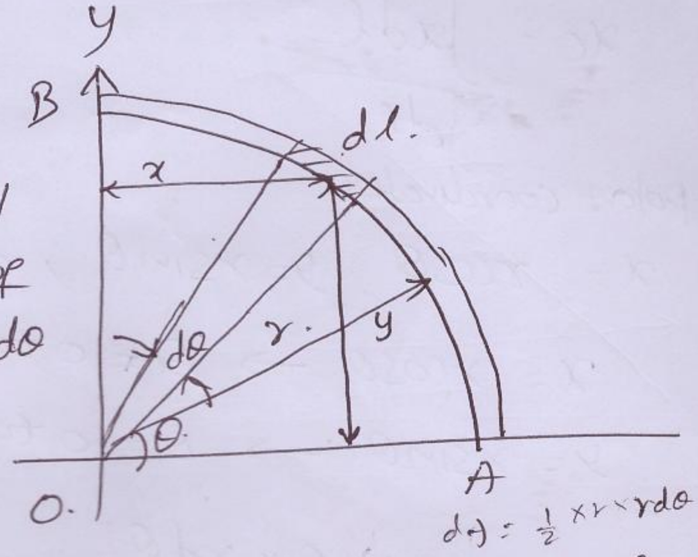
5) ~~Determine the Centriod of a lamina in the shape of a circular sector radious r and central angle 2α.~~

cone

~~height~~ h.

3) A uniform wire has been bent in the form of a quadrant of the arc of a circle radius r. determine the centroid of the quadrant.

The length AB of wire can be divided into no. of small elements, one such element of length dl subtends an angle dθ at O.



$dA = \frac{1}{2} \times r \times r d\theta$

$x = \frac{2}{3} r \cos\theta$

$$\bar{x}_C = \frac{\int x\,dl}{\int dl}$$

$x = r\cos\theta$, $y = r\sin\theta$, $dl = r\,d\theta$.

$$\bar{x}_C = \frac{\int x\,dl}{\int dl} = \frac{\int_0^{\pi/2} r\cos\theta * r\,d\theta}{\int_0^{\pi/2} r\,d\theta} = \frac{\int_0^{\pi/2} r^2\,d\theta \cos\theta}{\int_0^{\pi/2} r\,d\theta}$$

$$\bar{x}_C = \frac{r^2 (\sin\frac{\pi}{2} - \sin 0)}{\frac{r\pi}{2}} = \frac{r^{\cancel{2}} (1-0)}{\frac{\cancel{r}\pi}{2}}$$

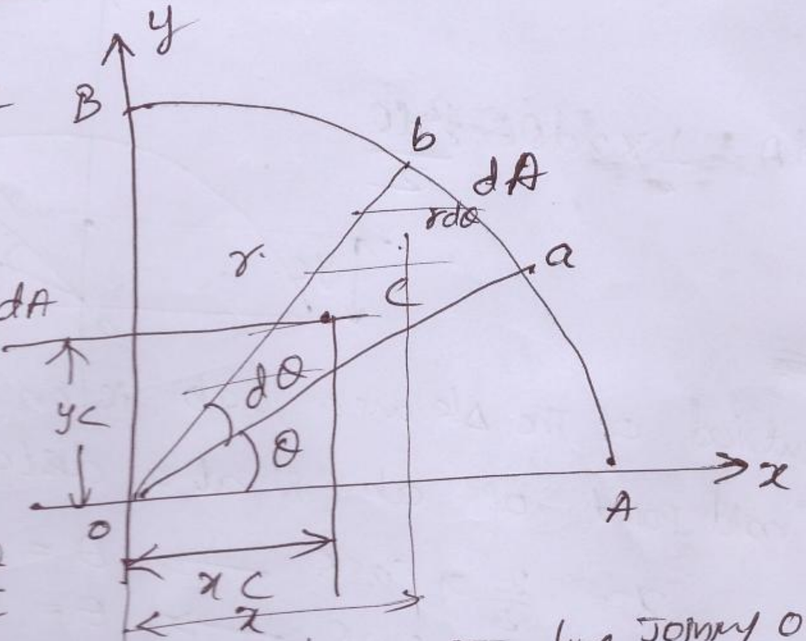
$$\boxed{\bar{x}_C = \frac{2r}{\pi}}$$

IIly $$\bar{y}_C = \frac{\int y\,dl}{\int dl} = \frac{\int_0^{\pi/2} r\sin\theta * r\,d\theta}{\int_0^{\pi/2} r\,d\theta} = \frac{r^2 \int_0^{\pi/2} \sin\theta\,d\theta}{r \int_0^{\pi/2} d\theta}$$

$$\bar{y}_C = \frac{r^2 (\cos\frac{\pi}{2} - \cos 0)}{r(\frac{\pi}{2} - 0)} = \frac{r^{\cancel{2}} (1)}{\frac{\pi \cancel{r}}{2}}$$

$$\boxed{\bar{y}_C = \frac{2r}{\pi}}$$

4A) Quadrant of the circle :



consider the ~~length~~ Area AB of wire can divided into no. of small element one such element of area dA subtends an angle dθ at o.

Area of the Δle element

$$dA = \frac{1}{2} \times r \times r\,d\theta = \frac{r^2 d\theta}{2}$$

centriod of the triangle area oab lie on the line Joining o to the mid point of ab at a distance $\frac{2}{3}r$ from Point o

$x = \frac{2}{3} r \cos\theta \rightarrow \theta$ various o to $\pi/2$

$y = \frac{2}{3} r \sin\theta \longrightarrow \theta$ various o to $\pi/2$

$$x_c = \frac{\int x\,dA}{\int dA}$$

$$x_c = \frac{\int_0^{\pi/2} \frac{\cancel{2}}{3} r\cos\theta * \frac{r^2 d\theta}{\cancel{2}}}{\int_0^{\pi/2} \frac{r^2 d\theta}{2}}$$

$$x_c = \frac{\frac{1}{3} r^{\cancel{3}} \left(\sin\frac{\pi}{2} - \sin 0\right)}{\frac{\cancel{r^2}}{2}\left(\frac{\pi}{2} - 0\right)}$$

$$x_c = \frac{\frac{r}{3}(1-0)}{\frac{1}{2}\left(\frac{\pi}{\cancel{2}}\right)}$$

$$\boxed{x_c = \frac{4r}{3\pi}}$$

$$y_c = \frac{\int y\,dA}{\int dA}$$

$$y_c = \frac{\int_0^{\pi/2} \frac{\cancel{2}}{3} r \sin\theta * \frac{r^2 d\theta}{\cancel{2}}}{\int_0^{\pi/2} \frac{r^2 d\theta}{2}}$$

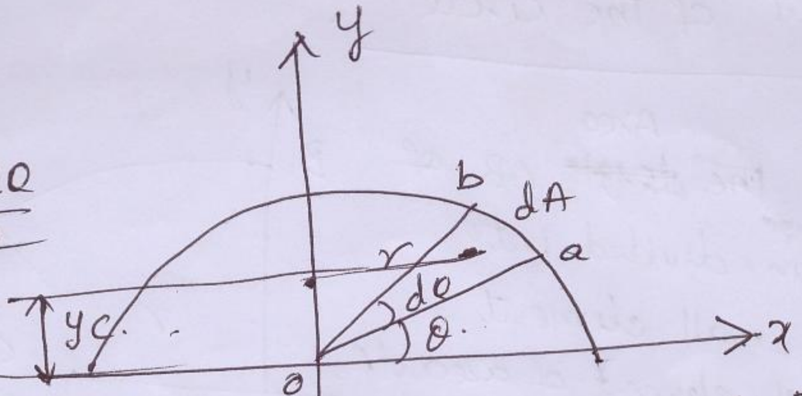
$$y_c = \frac{\frac{1}{3} r^{\cancel{3}} \left[-\left(\cos\frac{\pi}{2} - \cos 0\right)\right]}{\frac{\cancel{r^2}}{2}\left(\frac{\pi}{2}\right)}$$

$$y_c = \frac{\frac{r}{3}\left[-(0-1)\right]}{\frac{\pi}{4}}$$

$$\boxed{y_c = \frac{4r}{3\pi}}$$

4B) semi circle :

$$dA = \frac{1}{2} \times r \times r\,d\theta = \frac{r^2 d\theta}{2}$$



Centriod of The Δle area oab lie on the Joining o to the mid point of ab & at a distance $\frac{2}{3}r$

$x = \frac{2}{3} r\cos\theta \longrightarrow \theta = 0 \text{ to } \pi$

$y = \frac{2}{3} r\sin\theta \longrightarrow \theta = 0 \text{ to } \pi$

$$x_c = \frac{\int x\,dA}{\int dA}$$

$$x_c = \frac{\int_0^{\pi} \frac{\cancel{2}}{3} r\cos\theta \times \frac{r^2 d\theta}{\cancel{2}}}{\int_0^{\pi} \frac{r^2 d\theta}{2}}$$

$$x_c = \frac{\frac{1}{3} r^3 (\sin\pi - \sin 0)}{\frac{r^2}{2}(\pi - 0)}$$

$$\boxed{x_c = 0}$$

$$y_c = \frac{\int y\,dA}{\int dA}$$

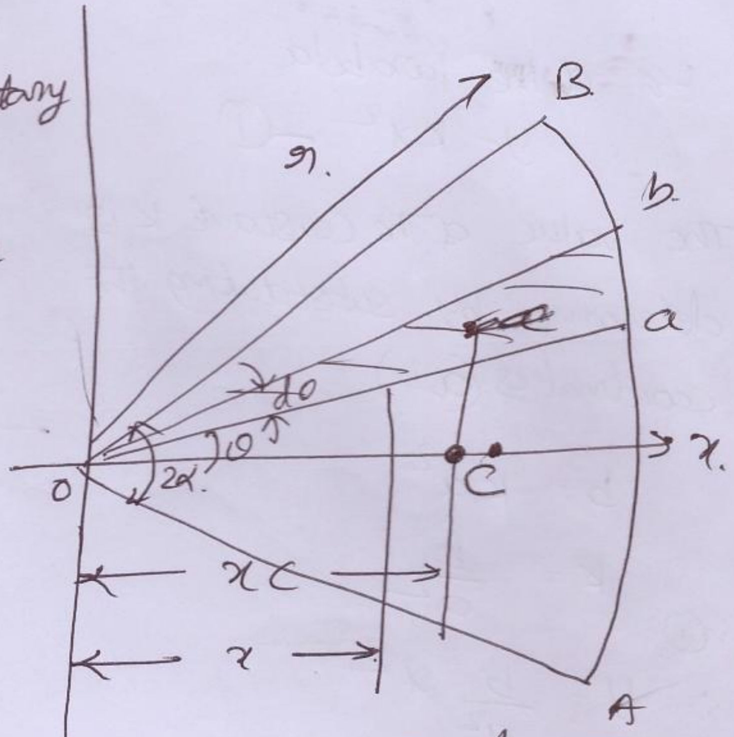
$$y_c = \frac{\int_0^{\pi} \frac{\cancel{2}}{3} r\sin\theta \times \frac{r^2 d\theta}{\cancel{2}}}{\int_0^{\pi} \frac{r^2 d\theta}{2}}$$

$$y_c = \frac{\frac{1}{3} \cancel{r^3} \left[-(\cos\pi - \cos 0)\right)}{\frac{\cancel{r^2}}{2}(\pi)}$$

$$y_c = \frac{\frac{r}{3}\left[-(1-1)\right)}{\frac{\pi}{2}}$$

$$\boxed{y_c = \frac{4r}{3\pi}}$$

Prob-5 Determine the coordinate of centroid of a lamina in the shape of a circular sector of radius r and central angle 2α.



Let us consider an elementary strip oab that subtends an angle $d\theta$ at the centre and has base $ab = r\,d\theta$

Area of the triangular element

$$oab = \frac{1}{2}(r\,d\theta)\,r.$$

$$oab = \frac{1}{2}r^2 d\theta = dA$$

Centroid of the triangular area oab lie on the line joining o to the mid point of ab at a distance $\frac{2}{3}r$ from point

Let $x = \frac{2}{3}r\cos\theta$

$$x_C = \frac{\int x\,dA}{\int dA}$$

$$x_C = \frac{\cancel{2}\int_0^\alpha \left(\frac{2}{3}r\cos\theta\right)\frac{r^2 d\theta}{\cancel{2}}}{\cancel{2}\int_0^\alpha \frac{r^2 d\theta}{2}}$$

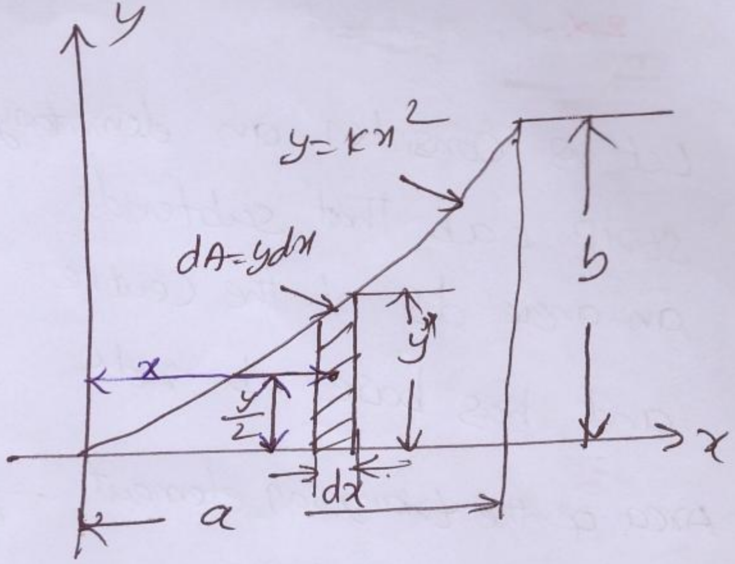
$$x_C = \frac{\frac{r^3}{3}\int_0^\alpha \cos\theta\, d\theta}{\frac{r^2}{2}\int_0^\alpha d\theta} = \frac{2r}{3}\frac{(\sin\alpha - \sin 0)}{\alpha - 0}$$

$$\boxed{x_C = \frac{2r\sin\alpha}{3\alpha} \qquad y_C = 0}$$

$$y_C = \frac{\int y\,dA}{\int dA} = \frac{\int_{-\alpha}^{\alpha}\frac{2}{3}r\sin\theta\,\frac{r^2 d\theta}{2}}{\int_{-\alpha}^{\alpha}\frac{r^2 d\theta}{2}}$$

$$y_C = \frac{\frac{r^3}{3}\int_0^\alpha \sin\theta}{r^2(\alpha - 0)} = 0$$

6) Determine the Centroid of the Parabolic Spandrel as shown Fig. The eq. of the Parabola is given by $y = kx^2$



Eq. of the parabola

$$y = kx^2 \quad \text{---} (1)$$

The value of the constant k is determine by substituting the coordinates (a, b)

$$b = ka^2$$

$$k = \frac{b}{a^2}$$

From (1)

$$\therefore y = \frac{b}{a^2}x^2$$

$$x^2 = \frac{ya^2}{b}$$

$$x = \frac{a\sqrt{y}}{\sqrt{b}}$$

Vertical strip : $dA = y\,dx$.

$$x_c = \frac{\int x\,dA}{\int dA}$$

$$x_c = \frac{\int_0^a xy\,dx}{\int y\,dx}$$

$$x_c = \frac{\int_0^a x\left(\frac{b}{a^2}x^2\right)dx}{\int_0^a \frac{b}{a^2}x^2\,dx}$$

$$x_c = \frac{\frac{b}{a^2}\left(\frac{x^4}{4}\right)_0^a}{\frac{b}{a^2}\left(\frac{x^3}{3}\right)_0^a}$$

$$x_c = \frac{\cancel{\frac{b}{a^2}}\ \frac{a^4}{4}}{\cancel{\frac{b}{a^2}}\ \frac{a^3}{3}}$$

$$\boxed{x_c = \frac{3}{4}a}$$

$x = x,\ y = \frac{y}{2}$

Distance of the centroid of strip from x-axis $= \frac{y}{2}$

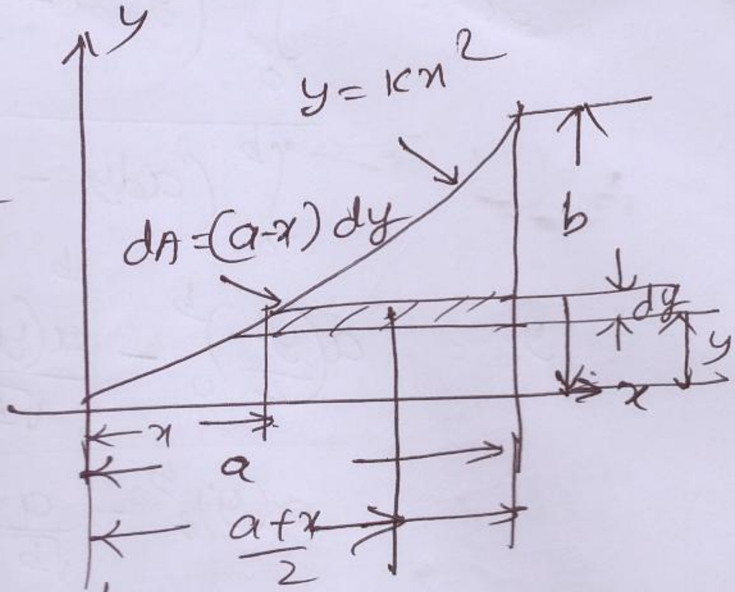
$$y_c = \frac{\int y\,dA}{\int dA} = \frac{\int_0^a \frac{y}{2} \times y\,dx}{\int y\,dx}$$

$$y_c = \frac{\int_0^a \frac{1}{2}\left(\frac{b}{a^2}x^2\right)^2 dx}{\int_0^a \frac{b}{a^2}x^2\,dx}$$

$$y_c = \frac{\frac{b^2}{2a^4}\left(\frac{x^5}{5}\right)_0^a}{\frac{b}{a^2}\left(\frac{x^3}{3}\right)_0^a} = \frac{\frac{b^{\cancel{2}}}{2\cancel{a^4}\,a^2}\ \frac{\cancel{a^5}\,a^3}{5}}{\frac{\cancel{b}}{\cancel{a^2}}\ \frac{\cancel{a^3}}{3}}$$

$$\boxed{y_c = \frac{3b}{10}}$$

Horizontal strip :



$dA = (a-x)\,dy$

Distance of the Centriod of the element from y-axis $= \frac{a+x}{2}$

$$x_c = \frac{\int x\,dA}{\int dA}$$

$$x_c = \frac{\int_0^b \left(\frac{a+x}{2}\right)(a-x)\,dy}{\int_0^b (a-x)\,dy} = \frac{\int_0^b \left(\frac{a^2-x^2}{2}\right)dy}{\int_0^b (a-x)\,dx}$$

$$y = kx^2$$
$$b = ka^2$$
$$k = \frac{b}{a^2}$$
$$y = \frac{b}{a^2}x^2$$
$$x =$$

$$\cancel{x_c = \frac{1}{2}\,\frac{a^2 - b\,x^2}{a^2}}$$

$$x_c = \frac{\frac{1}{2}\int_0^b \left(a^2 - \frac{a^2}{b}y\right)dy}{\int_0^b \left(a - \frac{a}{\sqrt{b}}\sqrt{y}\right)dy}$$

$$x_c = \frac{\frac{1}{2}\left[a^2 b - \frac{a^2}{b}\left(\frac{b^2}{2}\right)\right]}{ab - \frac{a}{\sqrt{b}}\,\frac{b^{3/2}}{\frac{3}{2}}} = \frac{\frac{1}{2}\left(\frac{a^2 b}{2}\right)}{ab - \frac{2ab}{3}}$$

$$x_c = \frac{\frac{a^2 b}{4}}{\frac{ab}{3}} \Rightarrow \boxed{x_c = \frac{3}{4}a}$$

$$y_c = \frac{\int y\,dA}{\int dA} = \frac{\int_0^b y\,(a-x)\,dy}{\int_0^b (a-x)\,dy} = \frac{\int_0^b y\left(a - \frac{a}{\sqrt{b}}\sqrt{y}\right)dy}{\int_0^b \left(a - \frac{a}{\sqrt{b}}\sqrt{y}\right)dy}$$

$$y_c = \frac{\int_0^b \left(y\,a - \frac{y a \sqrt{y}}{\sqrt{b}}\right) dy}{\int_0^b \left(a\,dy - \frac{a}{\sqrt{b}}\sqrt{y}\right) dy}$$

$$y_c = \frac{a\left(\frac{y^2}{2}\right)_0^b - \int_0^b \frac{a\left(y^{3/2}\right)_0^b}{\sqrt{b}}}{a(y)_0^b - \frac{a}{\sqrt{b}}\,\frac{y^{3/2}}{\frac{3}{2}}}$$

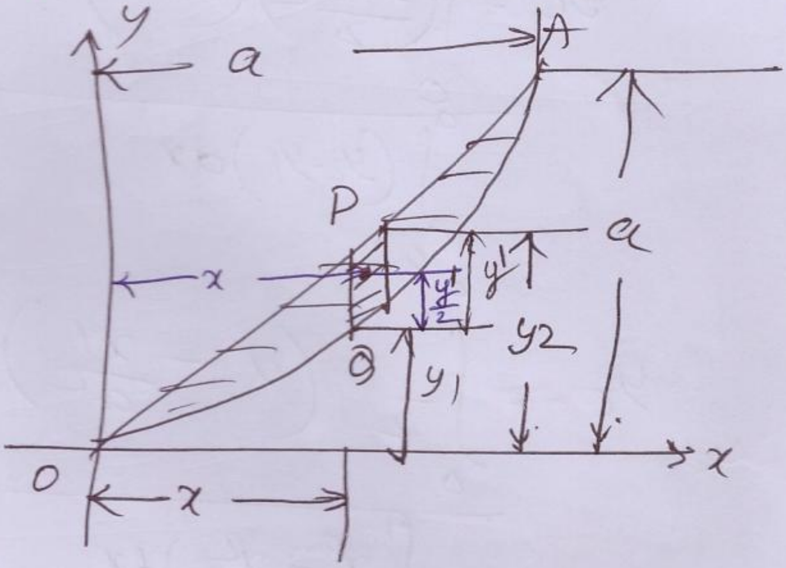
$$y_c = \frac{\frac{ab^2}{2} - \frac{a}{\sqrt{b}}\,\frac{(b)^{5/2}}{\frac{5}{2}}}{ab - \frac{a}{\sqrt{b}}\,\frac{(b)^{3/2}}{\frac{3}{2}}}$$

$$y_c = \frac{\frac{ab^2}{2} - \frac{2ab^2}{5}}{ab - \frac{2ab}{3}} = \frac{\frac{5ab^2 - 2ab^2}{10}}{\frac{3ab - 2ab}{3}}$$

$$y_c = \frac{3\cancel{a}b^{\cancel{2}}}{10\cancel{ab}}$$

$$\boxed{y_c = \frac{3b}{10}}$$

7) Determine the coordinates of the Centriod of the Shaded area formed by the intersection of a st. line and parabola as shown Fig. The eq., of the Parabola is given by $y = \frac{x^2}{a}$ & st line by $y = x$.



$y_1 = \frac{x^2}{a}$ — (1)

$y_2 = x$ — (2)

$dA = y' dx$

$dA = (y_2 - y_1)\, dx$

$$x_C = \frac{\int x\, dA}{\int dA}$$

$$x_C = \frac{\int_0^a x\left(x - \frac{x^2}{a}\right) dx}{\int_0^a \left(x - \frac{x^2}{a}\right) dx} = \frac{\int_0^a \left(x^2 - \frac{x^3}{a}\right) dx}{\int_0^a \left(x - \frac{x^2}{a}\right) dx}$$

$$x_C = \frac{\left(\frac{x^3}{3}\right)_0^a - \left(\frac{x^4}{4a}\right)_0^a}{\left(\frac{x^2}{2}\right)_0^a - \left(\frac{x^3}{3a}\right)_0^a} = \frac{\frac{a^3}{3} - \frac{a^3}{4}}{\frac{a^2}{2} - \frac{a^2}{3}}$$

$$x_C = \frac{\frac{4a^3 - 3a^3}{12}}{\frac{3a^2 - 2a^2}{6}} = \frac{\frac{a^3}{12}}{\frac{a^2}{6}}$$

$$\boxed{x_C = \frac{a}{2}}$$

$$y_C = \frac{\int y\,dA}{\int dA}$$

$$y = \cancel{\frac{y_1+y_2}{2}}\ \frac{y_2-y_1}{2} + y_1$$

$$y = \frac{y_2-y_1+2y_1}{2} = \frac{y_1+y_2}{2}$$

$$y_C = \frac{\int_0^a \left(\frac{y_1+y_2}{2}\right)(y_2-y_1)\,dx}{\int_0^a (y_2-y_1)\,dx} = \frac{\int_0^a \left(\frac{y_2^2-y_1^2}{2}\right)dx}{\int_0^a (y_2-y_1)\,dx}$$

$$y_C = \frac{\int_0^a \frac{1}{2}\left(x^2-\frac{x^4}{a^2}\right)dx}{\int_0^a \left(x-\frac{x^2}{a}\right)dx} = \frac{\frac{1}{2}\left(\frac{x^3}{3}-\frac{x^5}{5a^2}\right)_0^a \cancel{dx}}{\left(\frac{x^2}{2}-\frac{x^3}{3a}\right)_0^a}$$

$$y_C = \frac{\frac{1}{2}\left(\frac{a^3}{3}-\frac{a^{\cancel{5}3}}{5\cancel{a^2}}\right)}{\left(\frac{a^2}{2}-\frac{a^{\cancel{3}2}}{3\cancel{a}}\right)} = \frac{\frac{1}{2}\left(\frac{5a^3-3a^3}{15}\right)}{\left(\frac{3a^2-2a^2}{6}\right)}$$

$$y_C = \frac{\frac{1}{\cancel{2}} * \frac{\cancel{2}a^{\cancel{3}}}{\cancel{15}\,5}}{\frac{\cancel{a^2}}{\cancel{6}\,2}}$$

$$\boxed{y_C = \frac{2a}{5}}$$

Centriods of Composite Figures:

1) Determine the Centriod of ds area of unequal I-section?

Centriod of the total Figure lies on y-axis

$\therefore \quad x_c = 0$

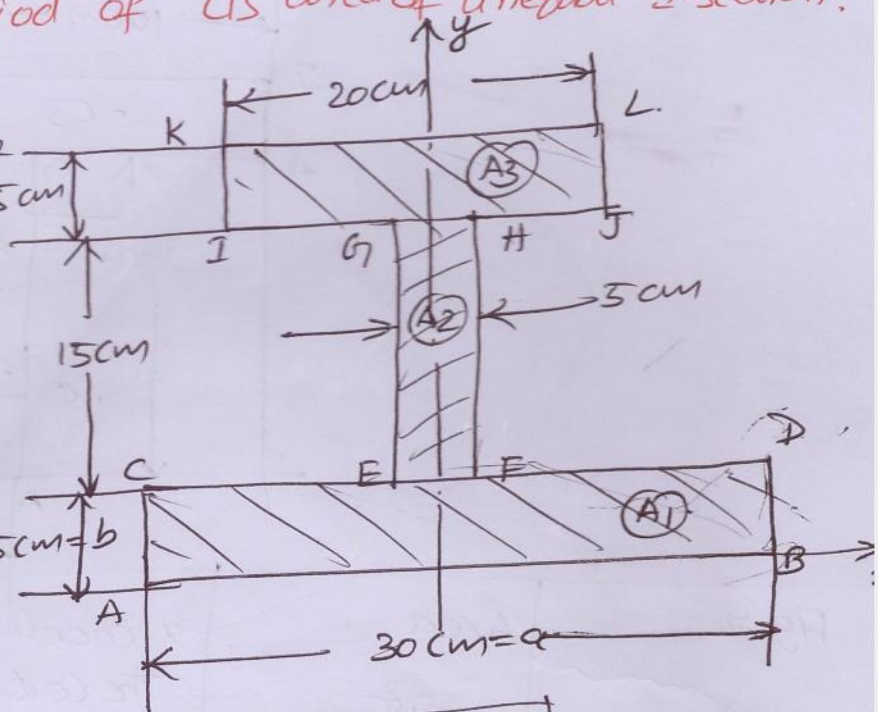


Figure	Area cm^2	x (cm)	y (cm) $\left(\frac{b}{2}\right)$
□le ABCD	$A_1 = 5 \times 30 = 150$	0	$y = \frac{5}{2} = 2.5$
□le EFGH	$A_2 = 5 \times 15 = 75$	0	$5 + \frac{15}{2} = 12.5cm$
□le IJKL	$A_3 = 5 \times 20 = 100$	0	$5 + 15 + \frac{5}{2} = 22.5cm$

$$y_c = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{150 \times 2.5 + 75 \times 12.5 + 100 \times 22.5}{150 + 75 + 100}$$

$$y_c = \frac{3562.5}{325}$$

$$\boxed{y_c = 10.96\,cm}$$

2) Find the Centriod of the c/s area of a Z-section as shown Figure

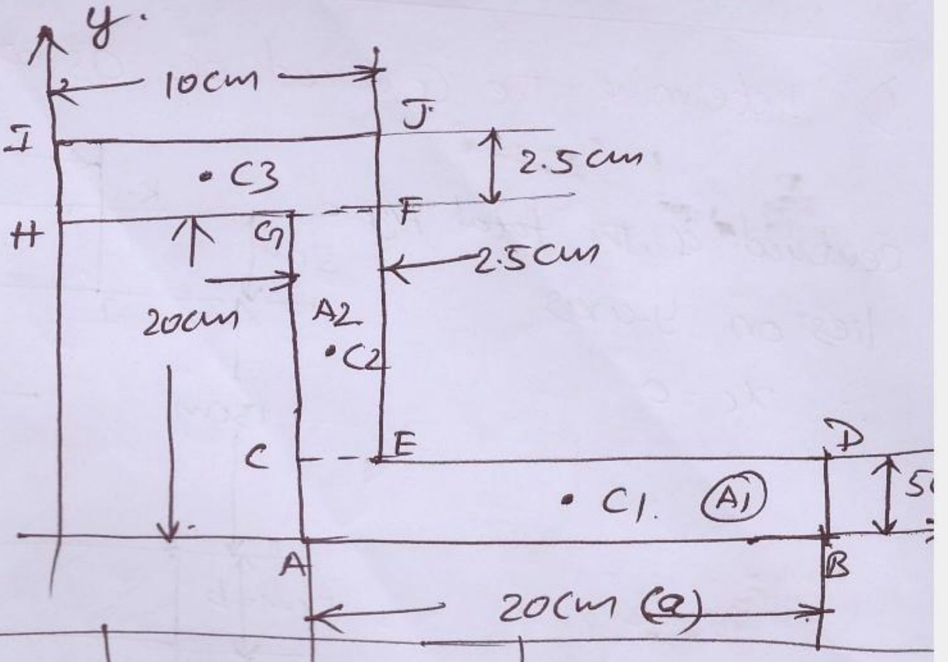


Figure	Area cm^2	x-coordinate of The Centriod (x_c) (cm)	y-coordinate of The Centriod (y_c) (cm)
□le ABCD	$A_1 = 5 \times 20$ $A_1 = 100$	$x_1 = 10 - 2.5 + \frac{20}{2}$ $= 17.5$	$y_1 = \frac{5}{2} = 2.5$
□le CEFG	$A_2 = 2.5 \times 15$ $A_2 = 37.5$	$x_2 = 10 - \frac{2.5}{2}$ $= 8.75$	$y_2 = 5 + \frac{15}{2} = 12.5$
□le ~~H~~FIJ	$A_3 = 2.5 \times 10$ $A_3 = 25$	$x_3 = \frac{10}{2} = 5$	$y_3 = 5 + 15 + \frac{2.5}{2}$ $= 21.25$

$$x_c = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{100 \times 17.5 + 37.5 \times 8.75 + 25 \times 5}{100 + 37.5 + 25}$$

$$x_c = 13.557\,cm$$

$$y_c = \frac{100 \times 2.5 + 37.5 \times 12.5 + 25 \times 21.25}{100 + 37.5 + 12.5}$$

$$\boxed{y_c = 7.69\,cm}$$

3) Determine The Centriod of The c/s area of section as shown Figure. 13

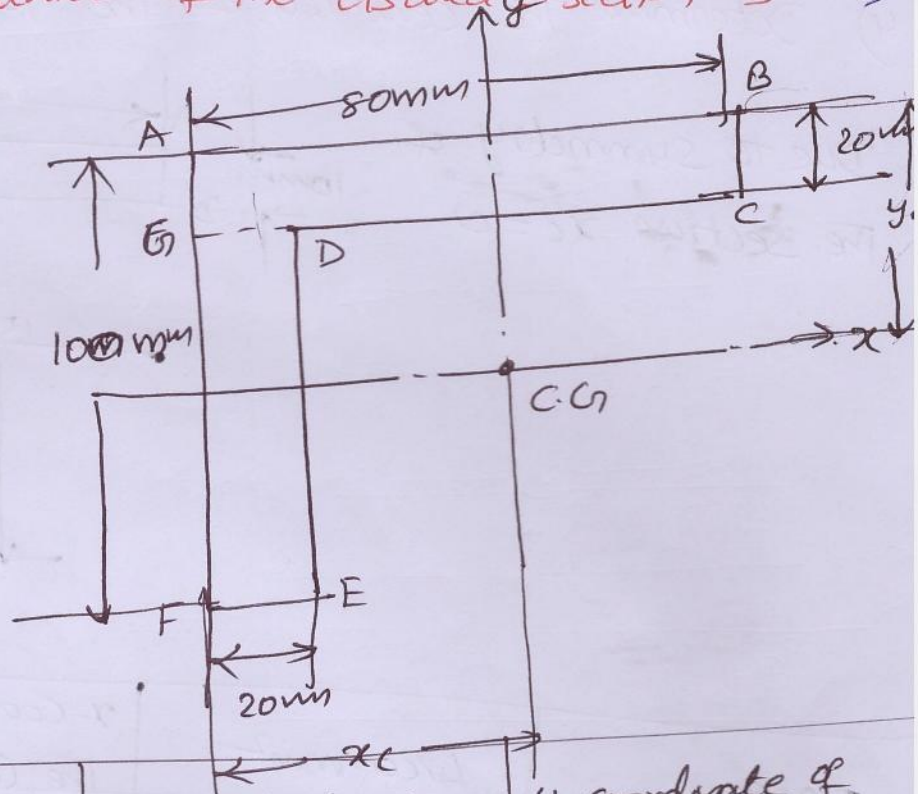


Figure	Area mm^2	x-Coordinate of The Centriod mm	y-Coordinate of The Centriod. mm
□le ABCG	$A_1 = 80 \times 20 = 1600 mm^2$	$x_1 = \frac{80}{2} = 40$	$y_1 = \frac{20}{2} = 10$
□le DEFG.	$A_2 = 80 \times 20 = 1600 mm^2$	$x_2 = \frac{20}{2} = 10$	$y_2 = 80 + \frac{80}{2} = 90mm$

$$x_C = \frac{A_1x_1 + A_2x_2 + \cancel{A_3x_3}}{A_1 + A_2 + \cancel{A_3}} = \frac{1600 \times 40 + 1600 \times 10}{1600 + 1600} = 25 mm$$

$$y_C = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{1600 \times 10 + 1600 \times 90}{1600 + 1600} = 50mm$$

4) Determine the centriod of the T-section as shown Figure

Due to symmetry of the section $x_c = 0$

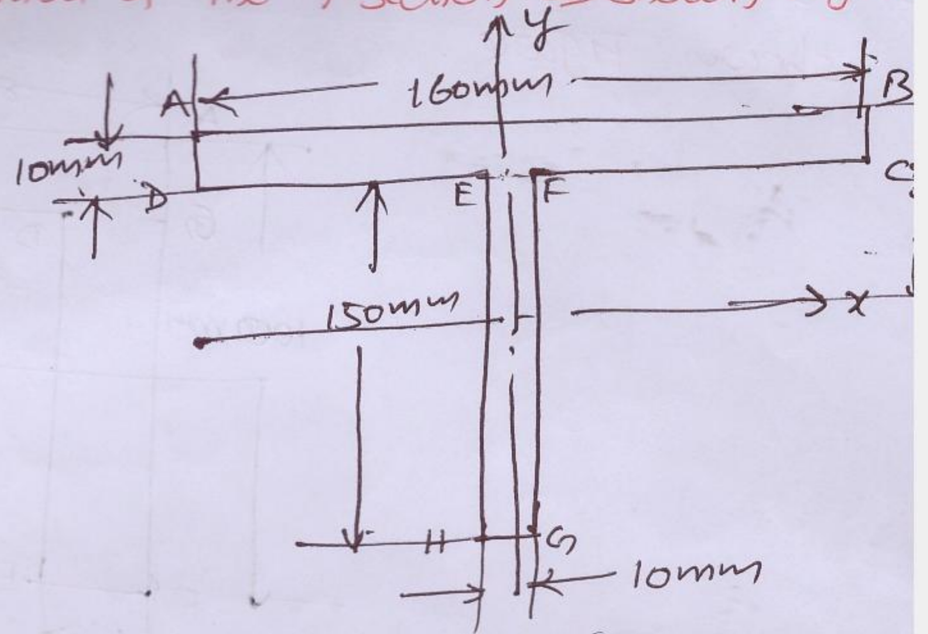


Figure	Area mm^2	x-Coordinate of the Centriod (x_c)	y-coordinate of the Centriod
▭le ABCD	$A_1 = 160 \times 10$ $= 1600$	0	$y_1 = \frac{10}{2} = 5\,mm$
▭le EFGH	$A_2 = 150 \times 10$ $= 1500$	0	$y_2 = 10 + \frac{150}{2}$ $y_2 = 85\,mm$

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{1600 \times 5 + 1500 \times 85}{1600 + 1500}$$

$$y_c = 43.71\ mm$$

$$x_c = 0$$

5) Locate the Centriod of The shaded area obtained by removing a Semicircle of diameter "a" from a quadrant of a circle of radious a.

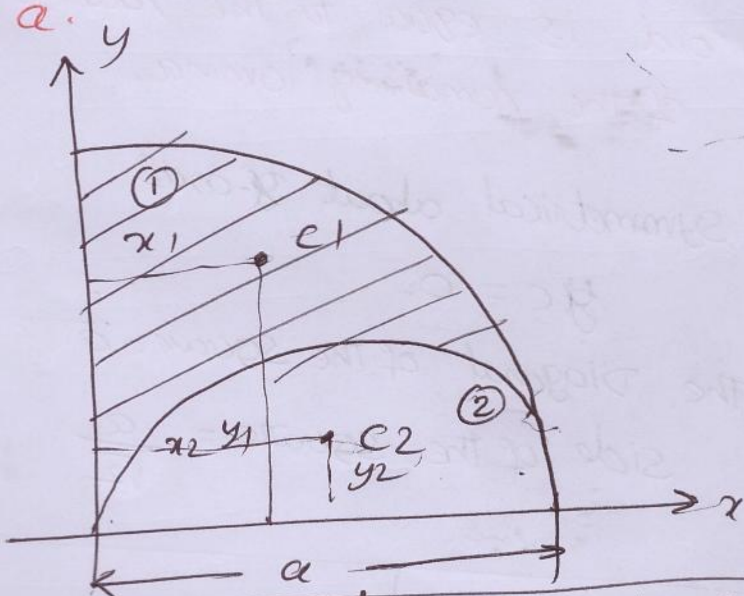


Figure	Area mm^2	x-coordinate of the Centriod mm	y-coordinate of The Centriod mm
Quadrant $A=\frac{\pi}{4}r^2$ Radious $=a$	$A_1=\frac{\pi}{4}a^2$	$x_1=\frac{4r}{3\pi}=\frac{4a}{3\pi}$	$y_1=\frac{4r}{3\pi}=\frac{4a}{3\pi}$
Semi circle $A=\frac{\pi}{2}r^2$ Radious $=\frac{a}{2}$	$A_2=\frac{\pi}{2}\left(\frac{a}{2}\right)^2=\frac{\pi a^2}{8}$	$x_2=r=\frac{a}{2}$	$y_2=\frac{4r}{3\pi}=\frac{4(a/2)}{3\pi}=\frac{2a}{3\pi}$

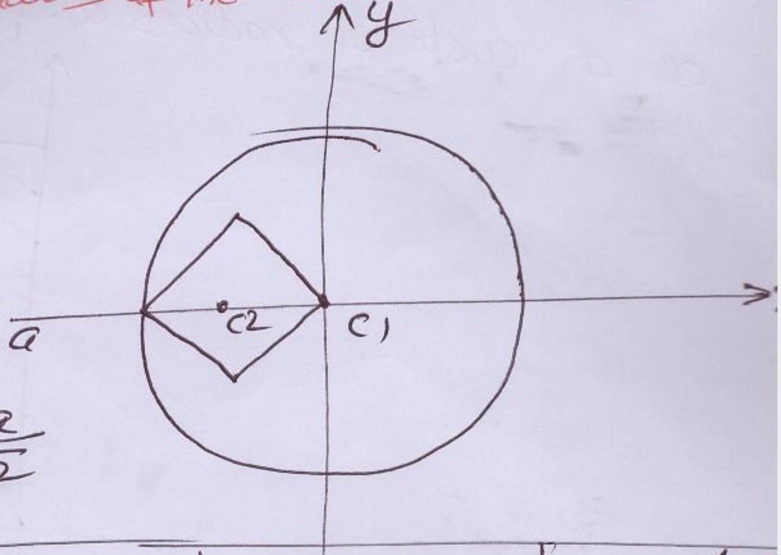
$$x_c=\frac{A_1x_1-A_2x_2}{A_1-A_2}=\frac{\frac{\pi}{4}a^2\left(\frac{4a}{3\pi}\right)-\frac{\pi a^2}{8}\left(\frac{a}{2}\right)}{\frac{\pi a^2}{4}-\frac{\pi a^2}{8}}$$

$$x_c=\frac{\frac{a}{3\pi}-\frac{a}{16}}{\frac{1}{8}}=8a\left(\frac{1}{3\pi}-\frac{1}{16}\right)$$

$$y_c=\frac{A_1y_1-A_2y_2}{A_1-A_2}=\frac{\frac{\pi}{4}a^2\left(\frac{4a}{3\pi}\right)-\frac{\pi a^2}{8}\left(\frac{2a}{3\pi}\right)}{\frac{\pi a^2}{4}-\frac{\pi a^2}{8}}=\frac{\frac{a}{3\pi}-\frac{a}{12\pi}}{\frac{1}{8}}$$

$$\boxed{y_c=\frac{2a}{\pi}}$$

6) A square hole is punched out of a circular lamina as shown Fig. The diagonal of the square which is punched out is equal to the radious of the circle. Find the Centroid of the remaining lamina.



symmetrical about x-axis

$y_C = 0$

The Diagonal of the square $= a$

side of the square $= \frac{a}{\sqrt{2}}$

Figure	Area	x-coordinate of the Centriod	y-coordinate of the Cent
circle of radious a	$A_1 = \pi a^2$	$x_1 = 0$	$y_1 = 0$
square of side $= \frac{a}{\sqrt{2}}$	$A_2 = \frac{a}{\sqrt{2}} * \frac{a}{\sqrt{2}}$ $= \frac{a^2}{2}$	$x_2 = -\frac{a}{2}$	$y_2 = 0$

$$x_C = \frac{A_1x_1 - A_2x_2}{A_1 - A_2} = \frac{\pi a^2 * 0 - \frac{a^2}{2} * \left(-\frac{a}{2}\right)}{\pi a^2 - \frac{a^2}{2}}$$

$$x_C = \frac{\frac{a^3}{4}}{a^2\left(\pi - \frac{1}{2}\right)} = \frac{a}{4(\pi - 0.5)}$$

$$\boxed{x_C = 0.095a}$$

$$\boxed{y_C = 0}$$

7) Determine the coordinates of the Centroid of a L-section as shown in figure

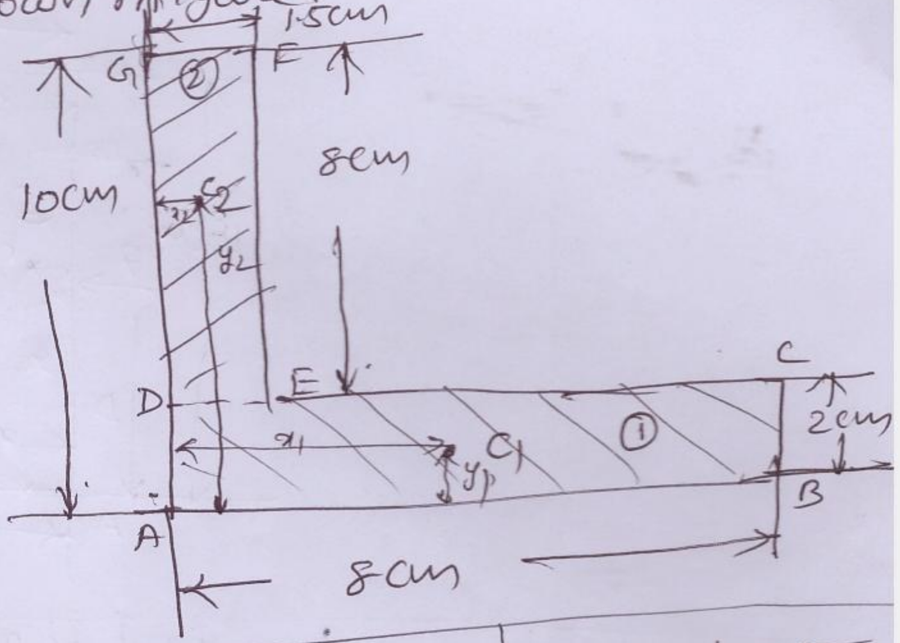


Figure	Area cm^2	Centroid of the x-coordinate cm	Centroid of the y-coordinate cm
▭le ABCD	$A_1 = 8 \times 2 = 16$	$x_1 = \frac{8}{2} = 4$	$y_1 = \frac{2}{2} = 1$
▭le DEFG	$A_2 = 8 \times 1.5 = 12$	$x_2 = \frac{1.5}{2} = 0.75$	$y_2 = 2 + \frac{8}{2} = 6$

$$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{16 \times 4 + 12 \times 0.75}{16+12} = \cancel{20}\ 2.607$$

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{16 \times 1 + 12 \times 6}{16+12} = 3.142 \text{ cm}$$

8) Find the centroid of a channel as shown Fig

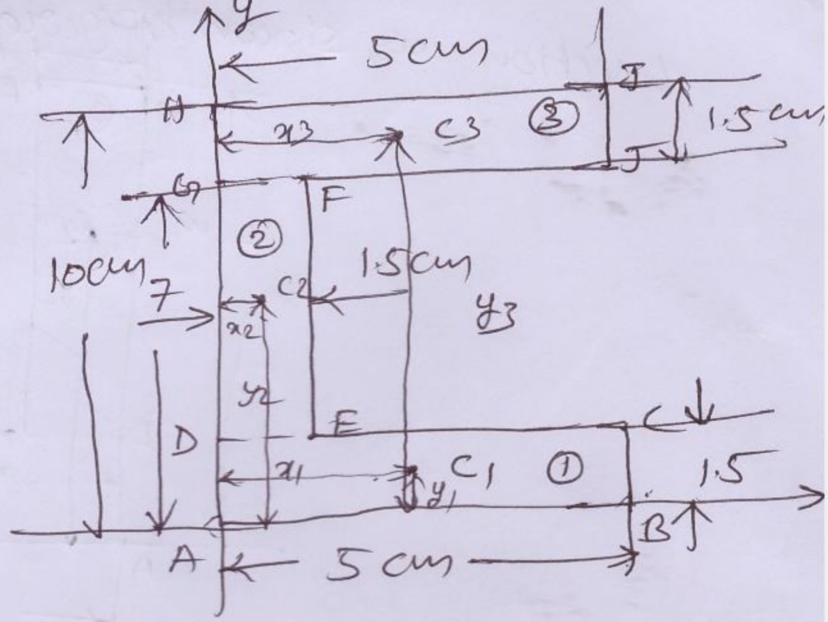


Figure	Area (cm^2)	Centroid of the x-coordinate (cm)	Centroid of the y-coordinate (cm)
$\square^{le}$ ABCD	$A_1 = 5 \times 1.5 = 7.5$	$\frac{5}{2} = 2.5$	$\frac{1.5}{2} = 0.75$
$\square^{le}$ DEFG	$A_2 = 7 \times 1.5 = 10.5$	$\frac{1.5}{2} = 0.75$	$1.5 + \frac{7}{2} = 5$
$\square^{le}$ GHIJ	$A_3 = 5 \times 1.5 = 7.5$	$\frac{5}{2} = 2.5$	$1.5 + 7 + \frac{1.5}{2} = 9.25$

$$x_c = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{7.5 \times 2.5 + 10.5 \times 0.75 + 7.5 \times 2.5}{7.5 + 10.5 + 7.5}$$

$$x_c = 1.779 \text{ cm}$$

$$y_c = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{7.5 \times 0.75 + 10.5 \times 5 + 7.5 \times 9.25}{7.5 + 10.5 + 7.5}$$

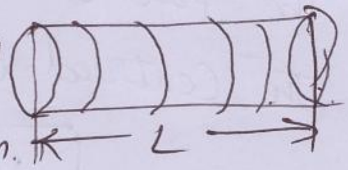
$$y_c = \underline{5 \text{ cm}}$$

Theorem of pappus and Guldinus:

There are two Theorems for determining The surface area and volume generated by revolving respectively a plane curve or a plane area about a non intersecting axis lying in its plane.

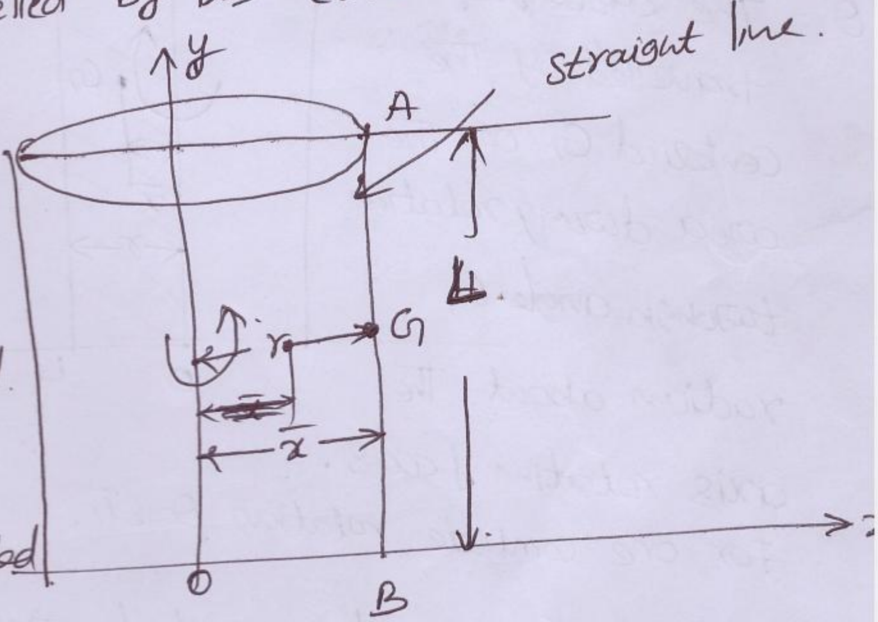
$A = (2\pi r) * L =$ Surface area

$V = \pi r^2 * L =$ volume of The cylinder.



Theorem-I :

The area of the surface generated by a revolving a plane curve about a non intersecting axis in a plane of the curve is equal to product of the length(L) of the curve and the distance travelled by its centriod.



$$\boxed{A = L(\bar{x} * \theta).}$$

where

A = Surface area generated.

L = Length of the curve.

$\bar{x}\theta$ = The distance travelled by the centriod G of the curve during rotation through angle θ radian about the axis of rotation. (y-axis) for complete rotation $\theta = 2\pi$

Consider a st. line AB of length L at a distance r from y axis be rotated about the same axis.

$L = L, \ \theta = 2\pi, \ \bar{x} = r.$

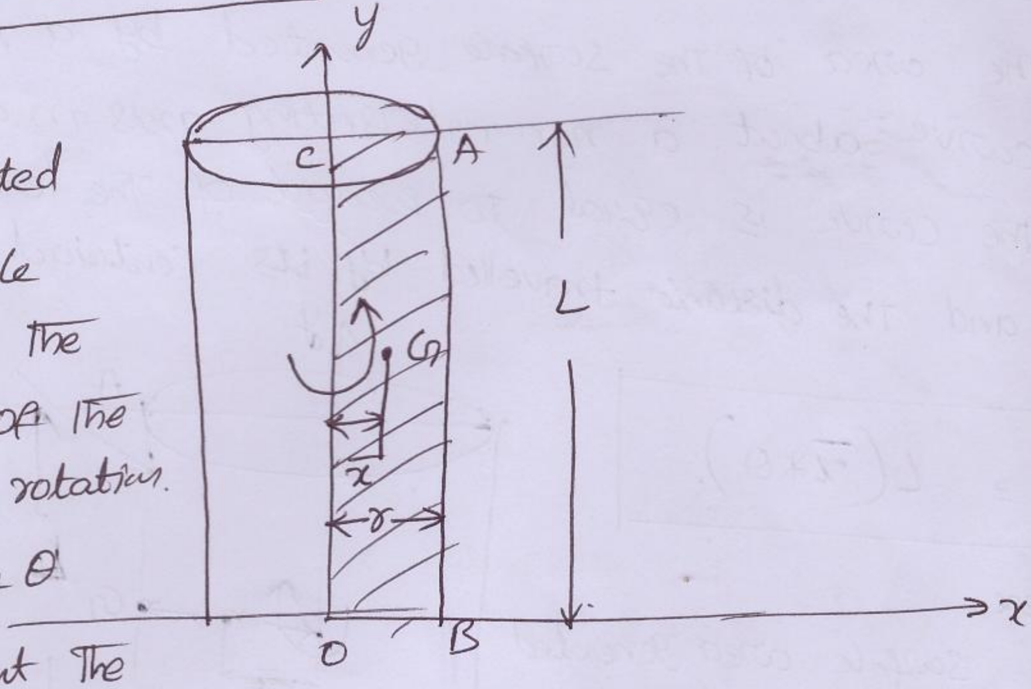
Area of the surface cylinder $A = L * (r * 2\pi)$

$$\boxed{A = 2\pi r L.}$$

Theorem - II: (Guldincs)

The volume of the solid generated by revolving a plane area about a non intersecting axis in its plane is equal to the product of the area and the length travelled by the Centriod G of the area during the revolution.

$$\boxed{V = A * \bar{x}\theta}$$



where

V = volume generated

$\bar{x}\theta$ = The distance travelled by the Centriod G of the area during rotation through angle θ radian about the axis rotation y-axis.

For one complete rotation $\theta = 2\pi$

Consider a rectangular shaded area ABOC rotated about y-axis

Area of ABOC $A = r * L$.

The Centriod G is at a distance $\frac{r}{2}$ from y-axis

$$\bar{x} = \frac{r}{2}, \quad \theta = 2\pi$$

volume of the cylinder generated $V = A * \bar{x}\theta$

$$V = (r * L) * \frac{r}{\cancel{2}} * \cancel{2}\pi$$

$$\boxed{V = \pi r^2 L}$$