MONASH University

ENG1013: Engineering Smart Systems

Week 6

Topics:

Electrical fundamentals

Presenters: Jonathan Li

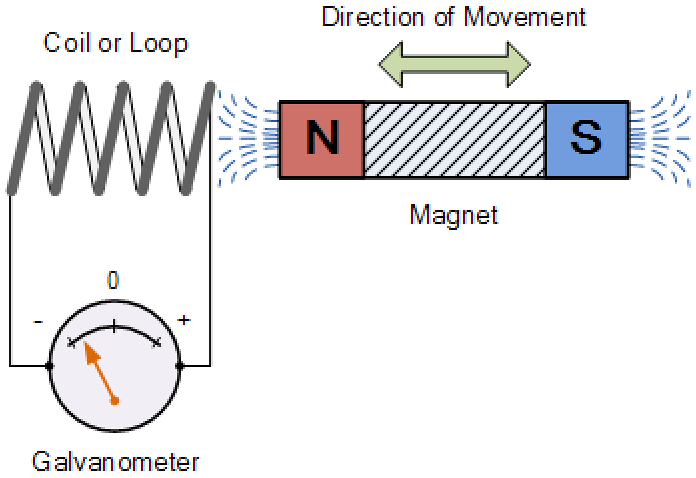
Electrical fundamentals

- **Objective:** To be able to perform basic **circuit analysis**

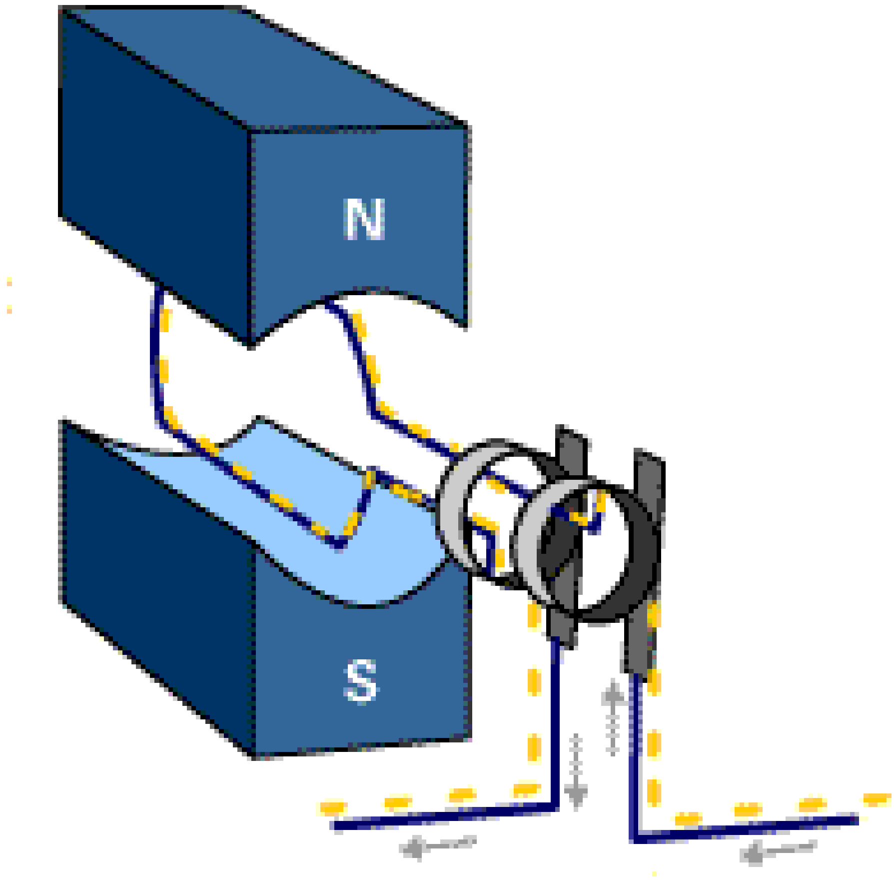
- **Concepts required:**
  - Voltage, current, power
  - Circuits and Circuit elements
  - Kirchhoff’s Laws
  - Basic circuit analysis techniques
  - Multimeter

Electrical energy generation

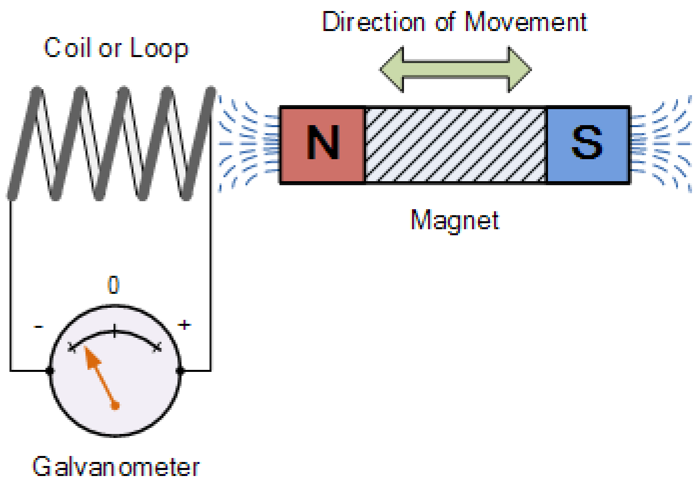
Electrical energy generation



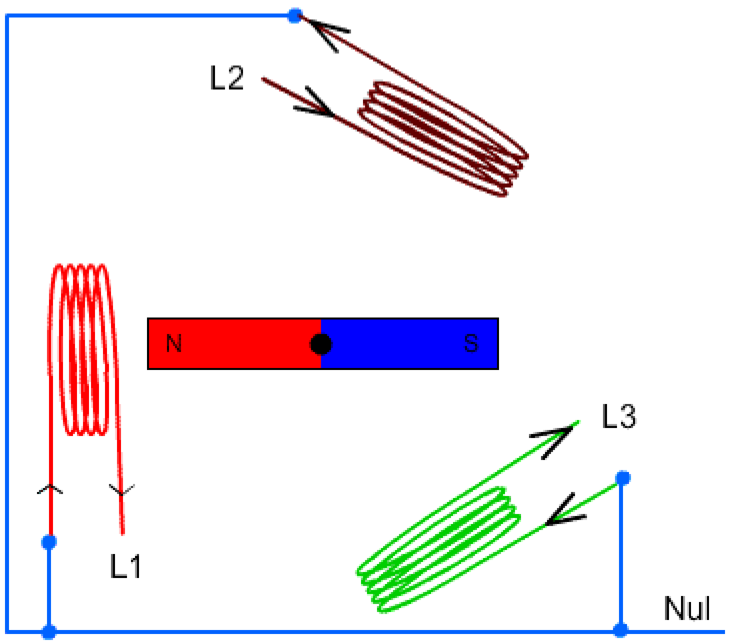
Electrical energy generation



Electrical energy generation

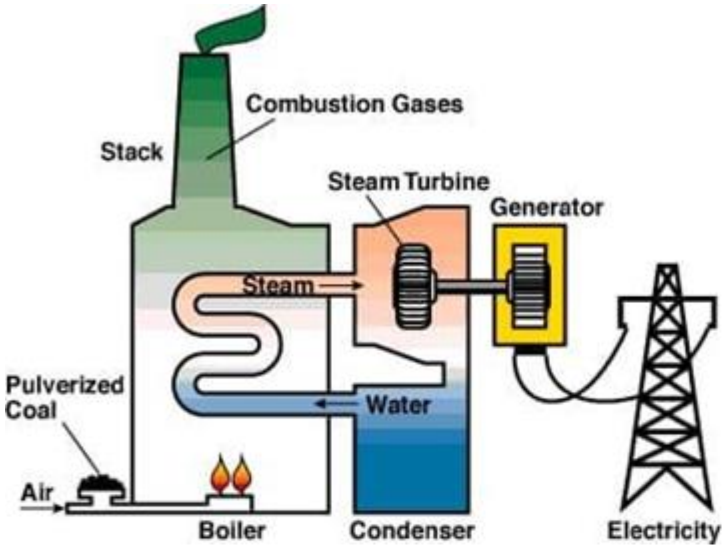


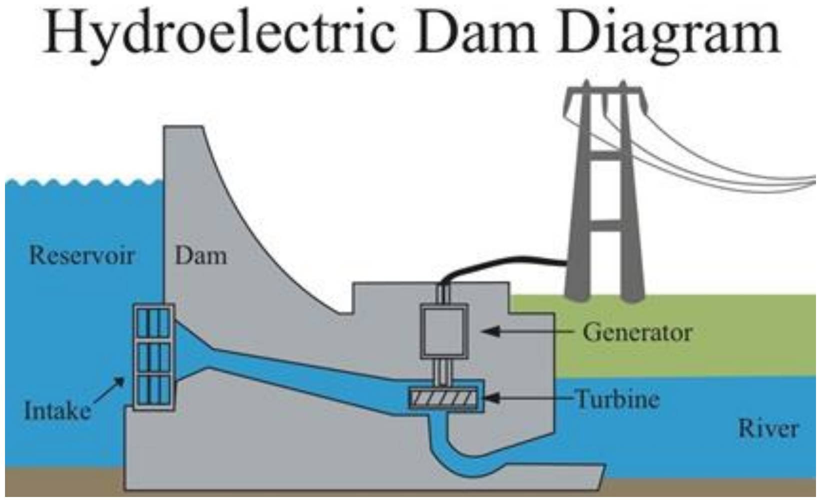
Electrical energy generation

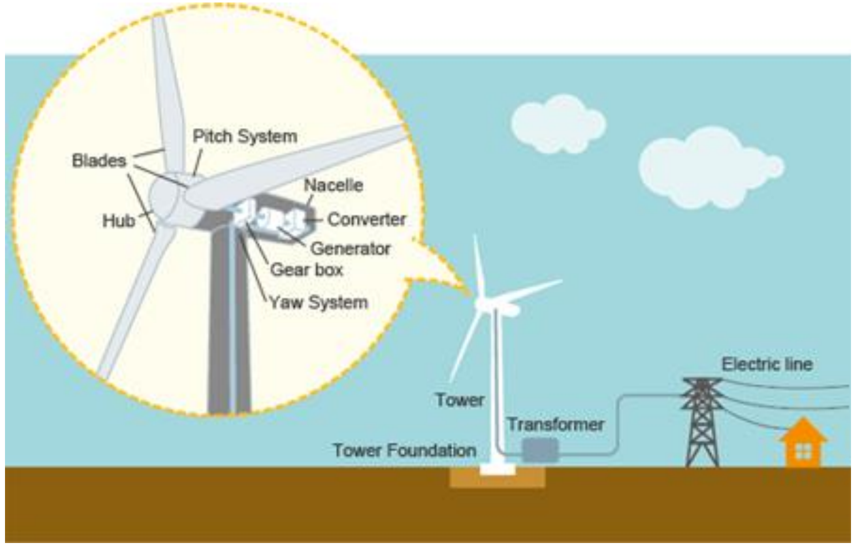


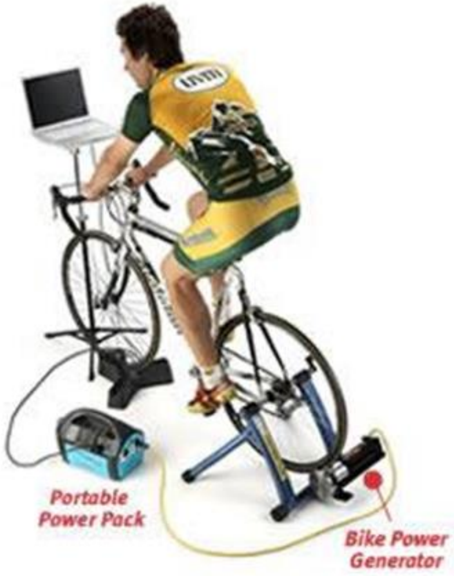
Turbines

Turbines

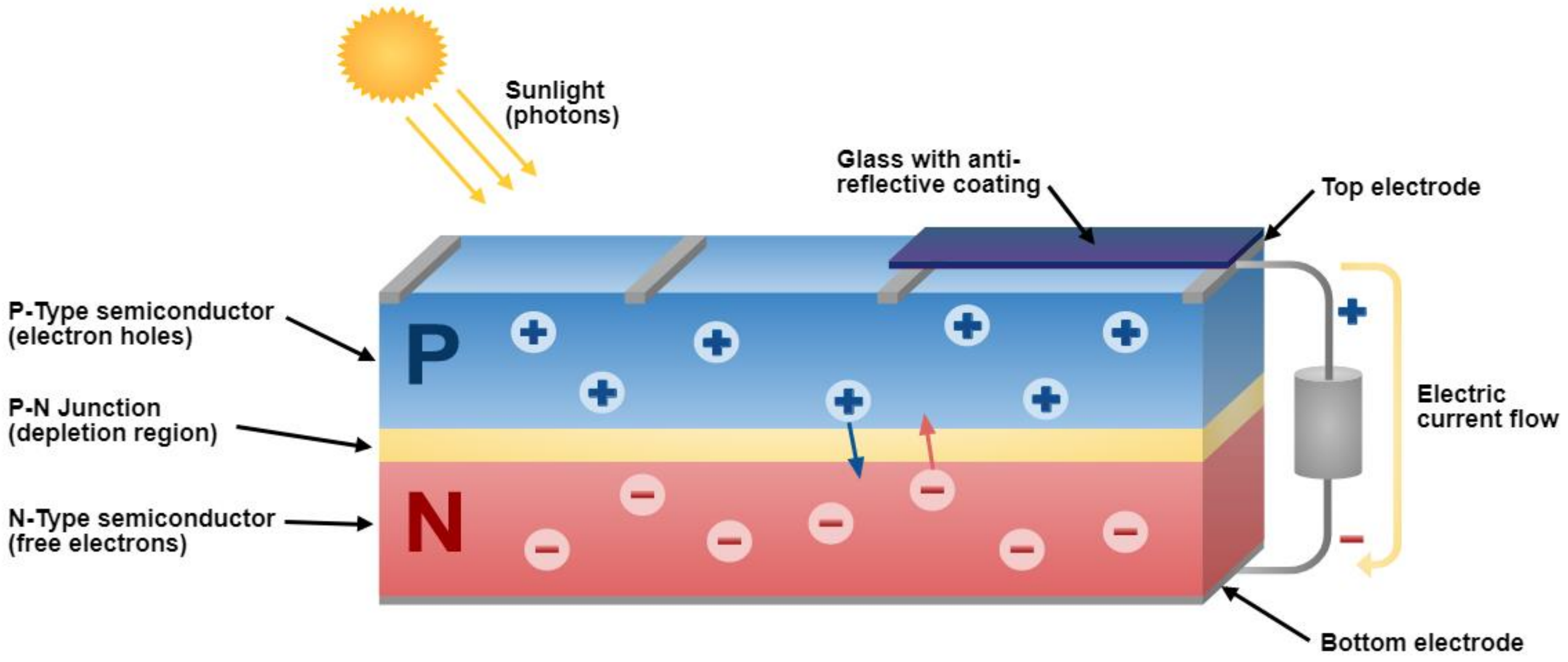

Combustion Gases
Stack
Steam Turbine
Generator
Steam
Water
Pulverized Coal
Air
Boiler
Condenser
Electricity


Hydroelectric Dam Diagram
Reservoir
Dam
Generator
Intake
Turbine
River

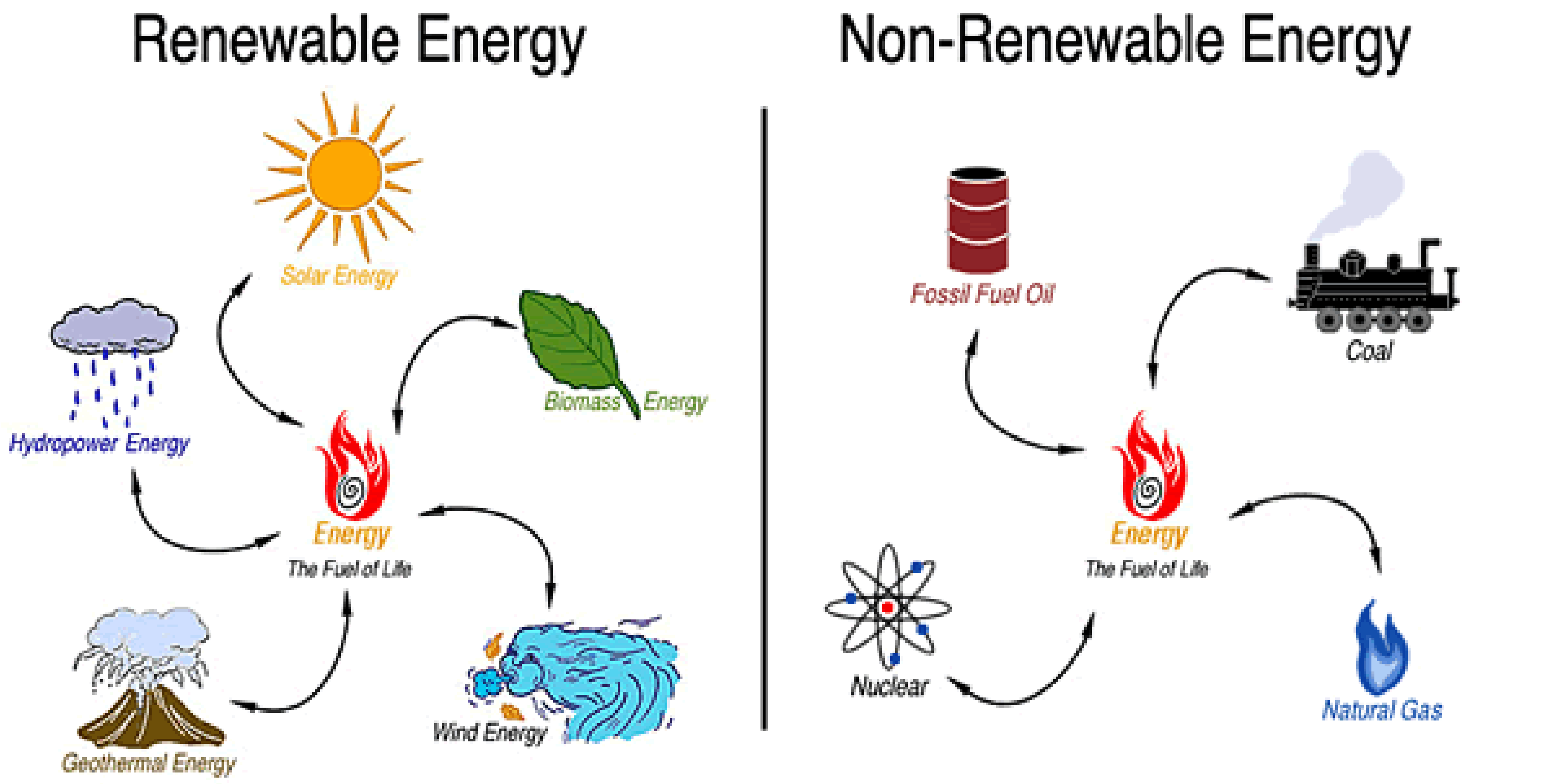

Pitch System
Blades
Nacelle
Hub
Converter
Generator
Gear box
Yaw System
Electric line
Tower
Transformer
Tower Foundation


Portable Power Pack
Bike Power Generator

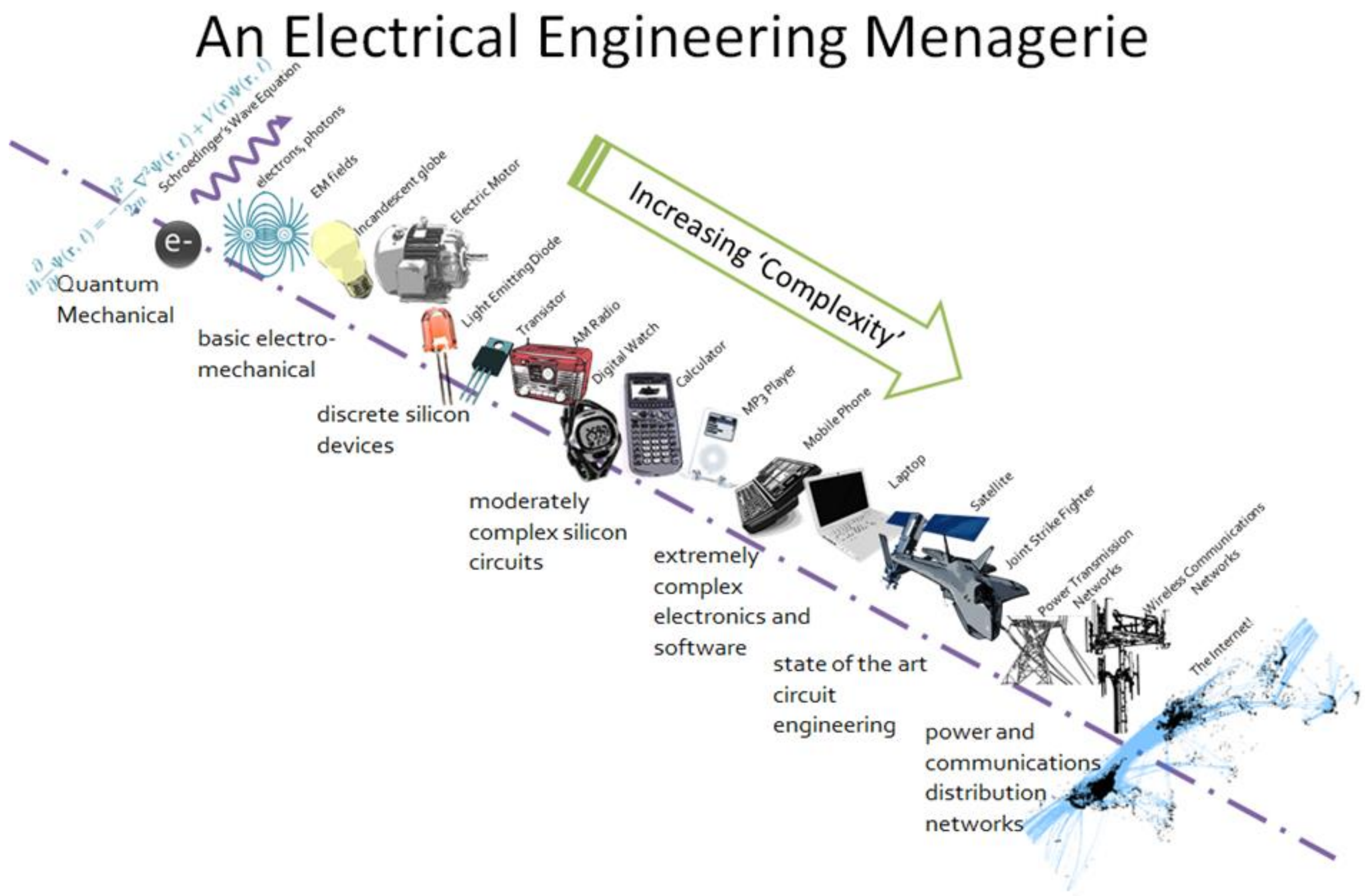
Photovoltaic cells



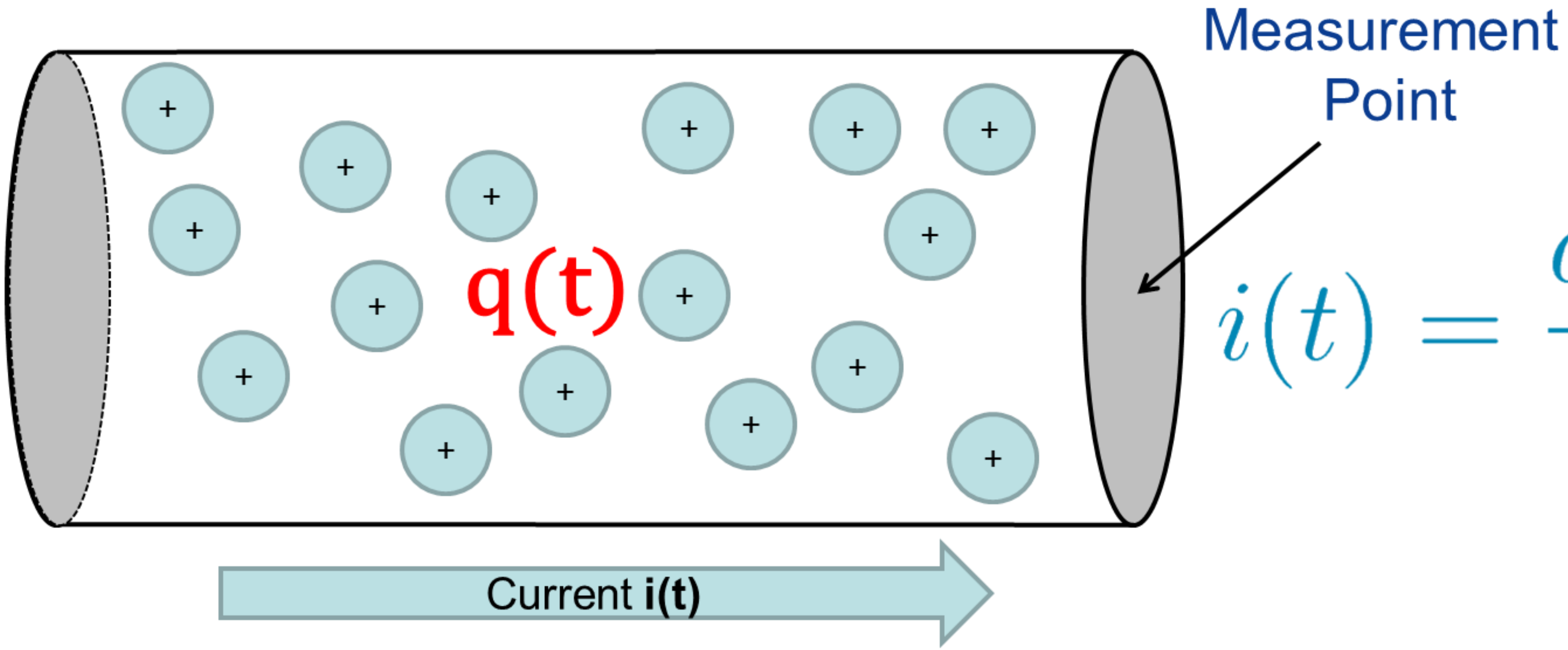
Renewable energy



What is electrical engineering?

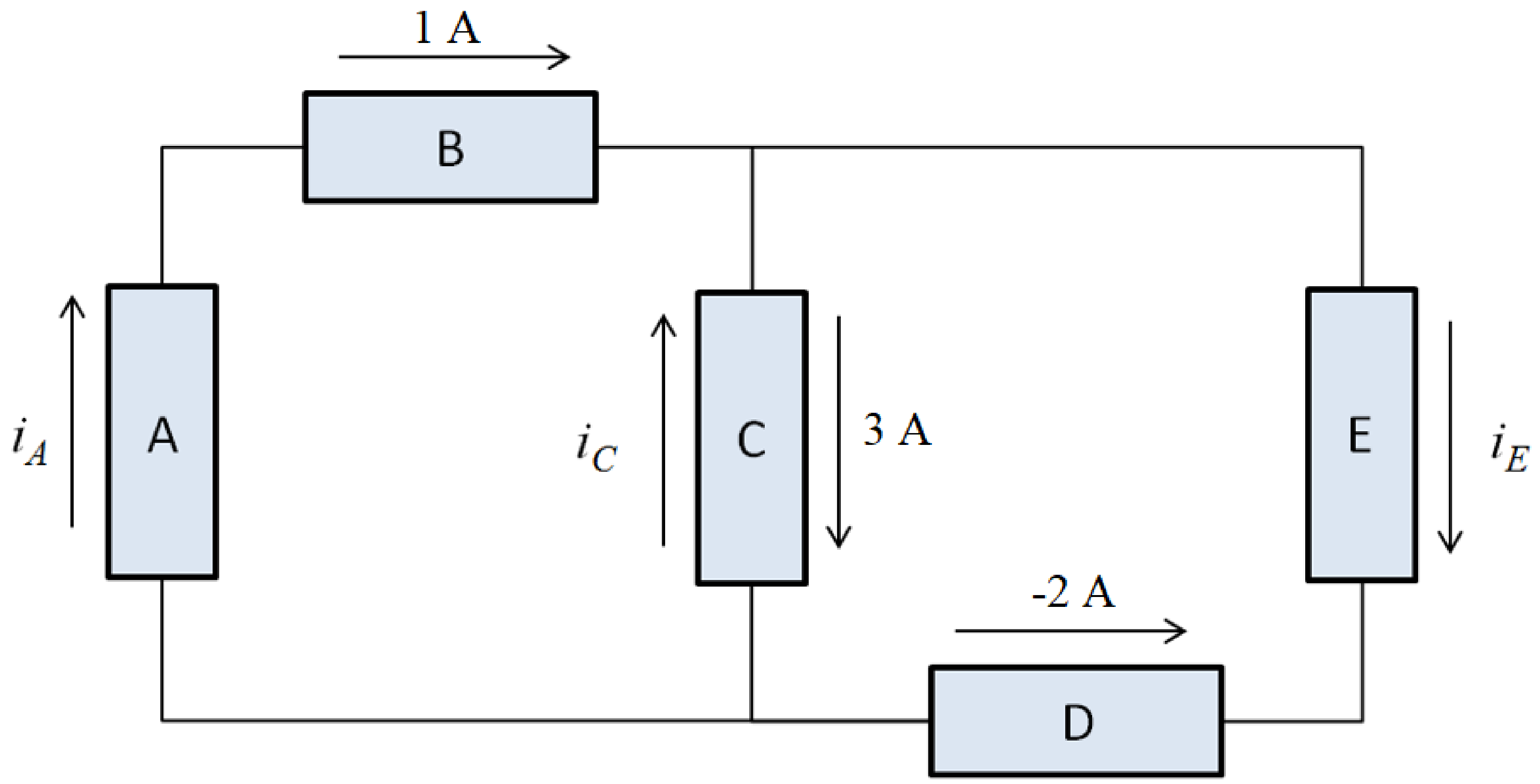


Electrical current



$$i(t) = \frac{dq(t)}{dt}$$

Electrical current



Charge and current relationship

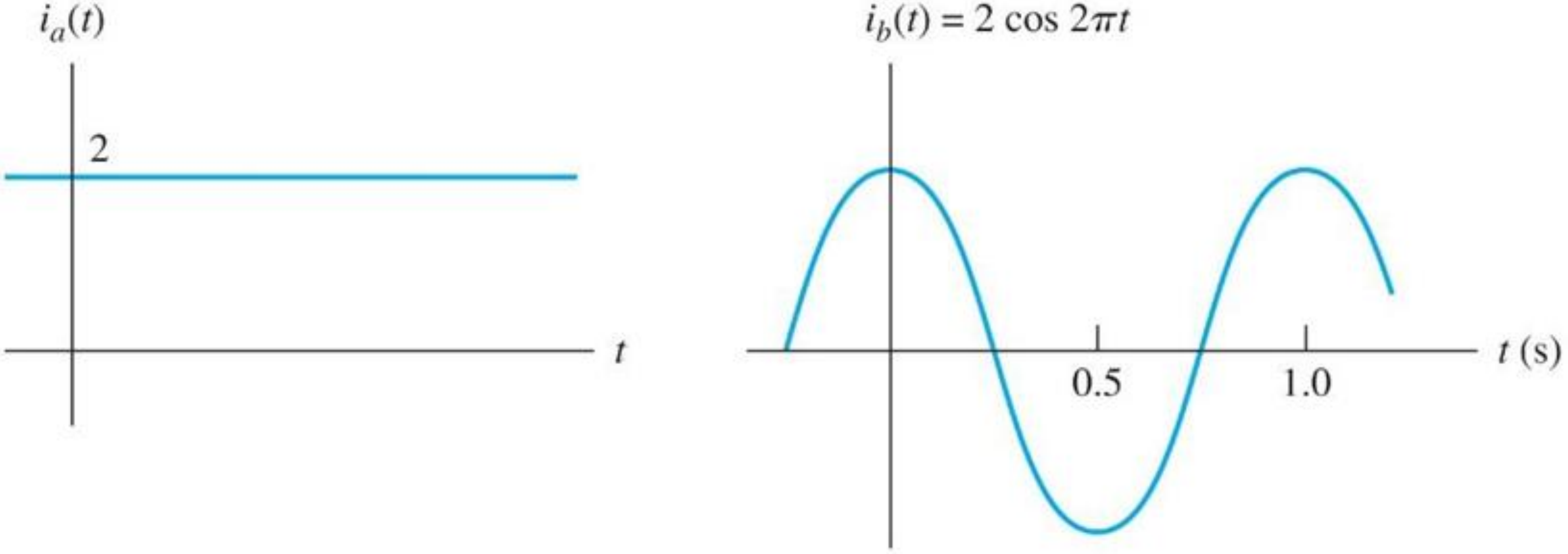
- Given a record of charge q(t), current i(t) can be found as:

$$i(t) = \frac{dq(t)}{dt}$$

- Given a record of current i(t) through an element, the amount of charge moved can be found as:

$$q(t) = \int_{t_0}^{t} i(\tau)d\tau + q(t_0)$$

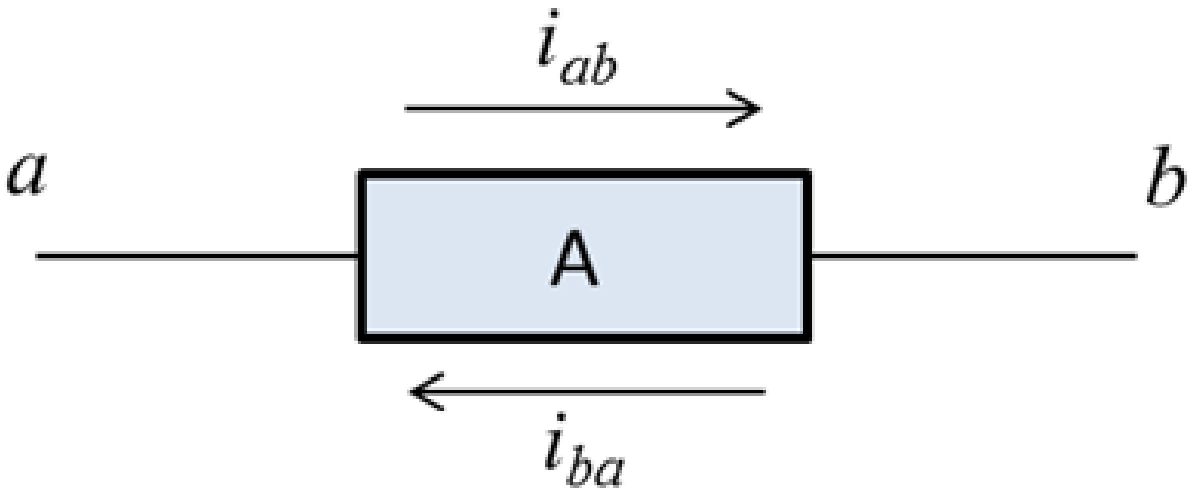
Direct Current (DC) and Alternating Current (AC)



(a) Dc current

(b) Ac current

Double subscript notation

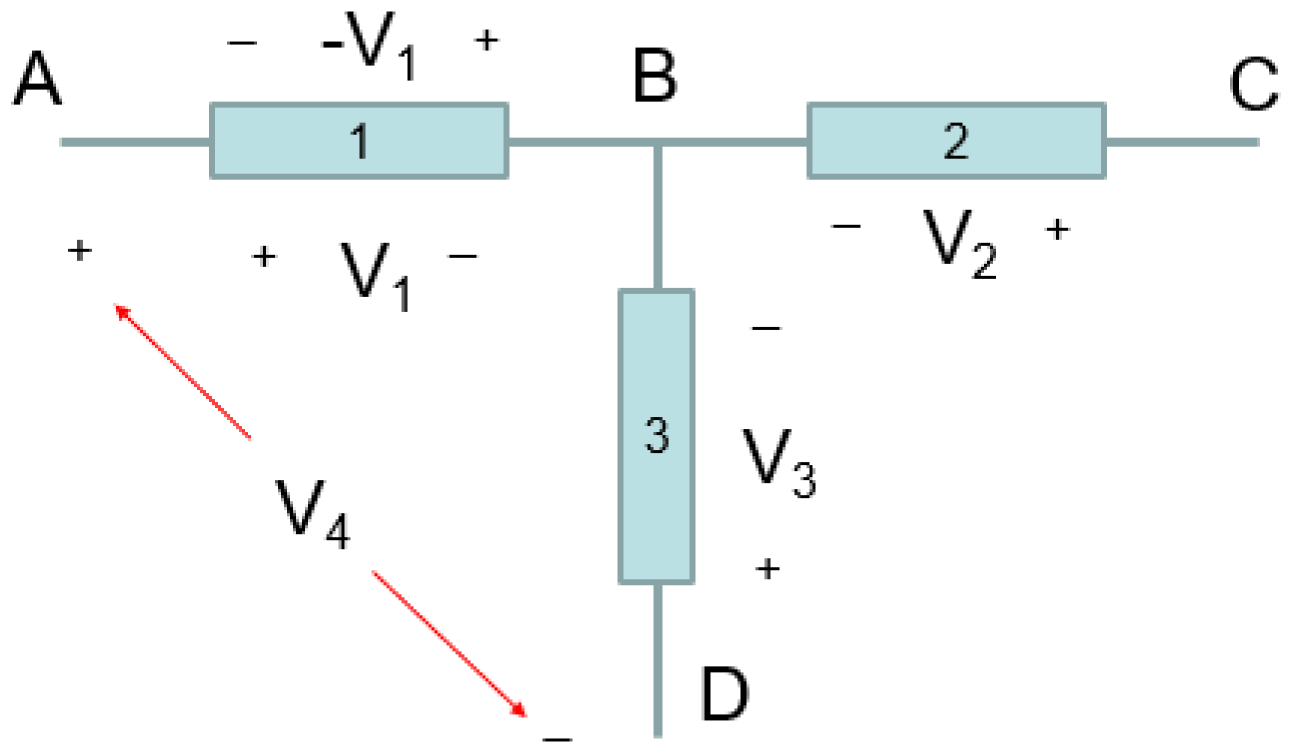


Electrical voltage

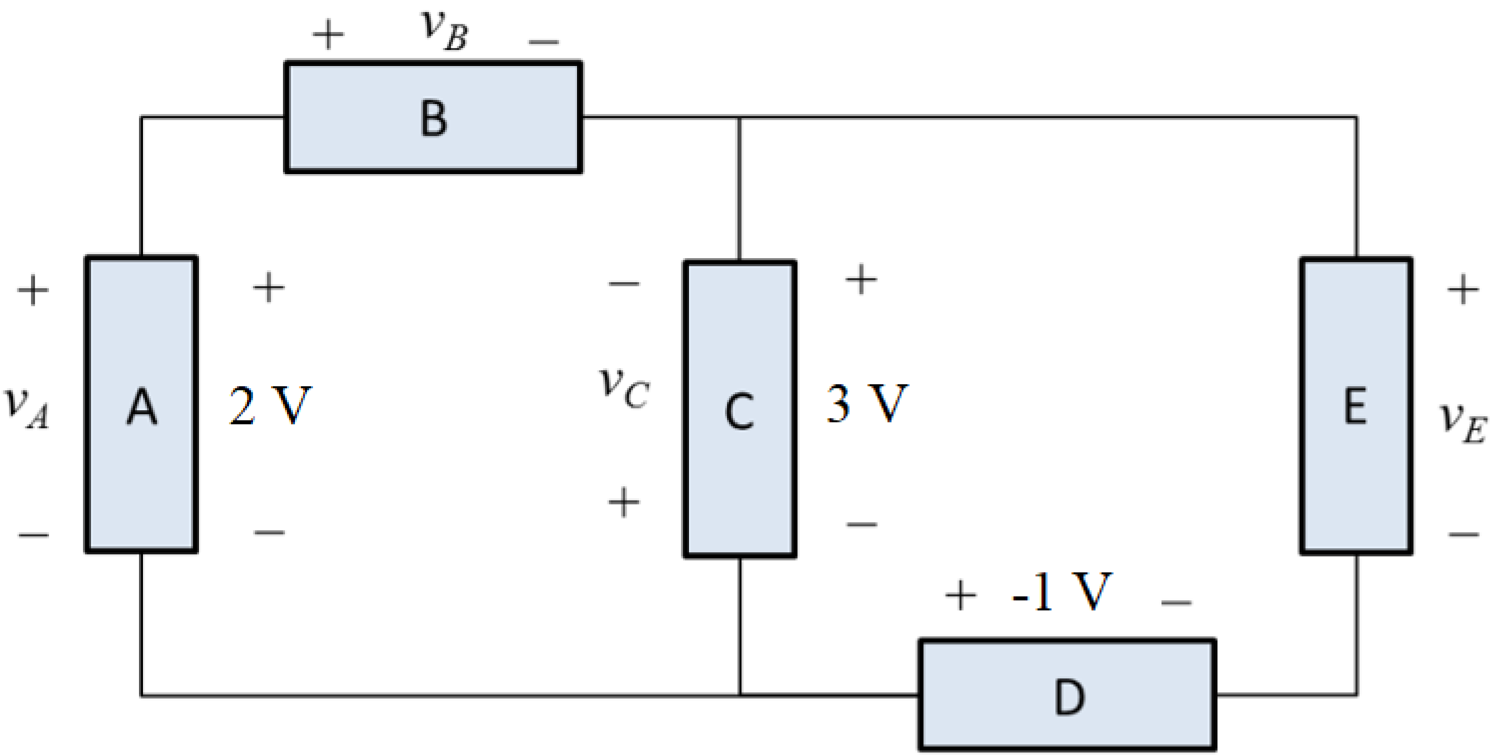
- **Voltage** is a measure of the difference in electrical potential energy **between two locations**

- Common notations to describe voltage
  - Voltage difference
  - Node voltage
  - Double-subscript

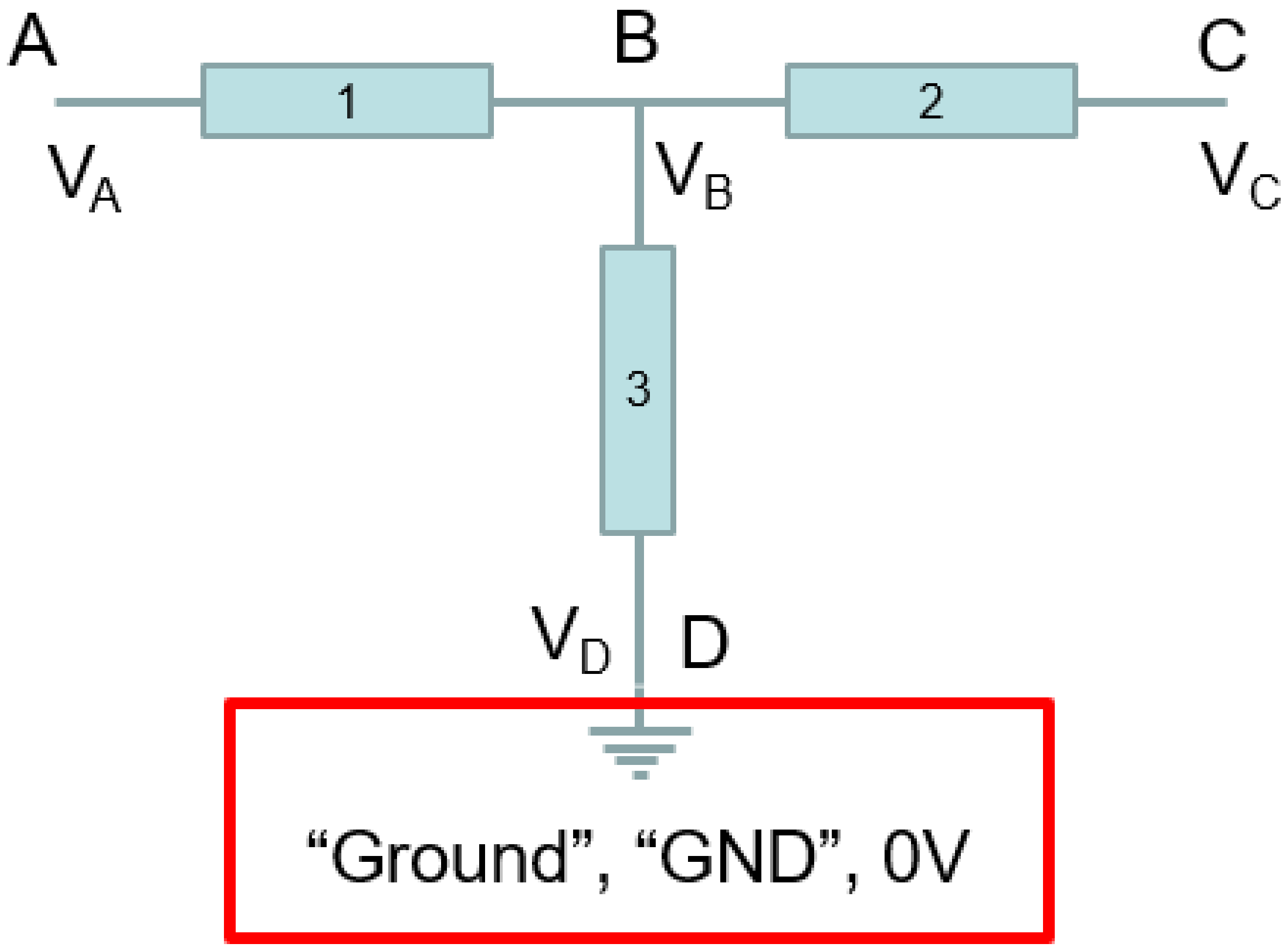
Voltage difference notation (+/- specified)



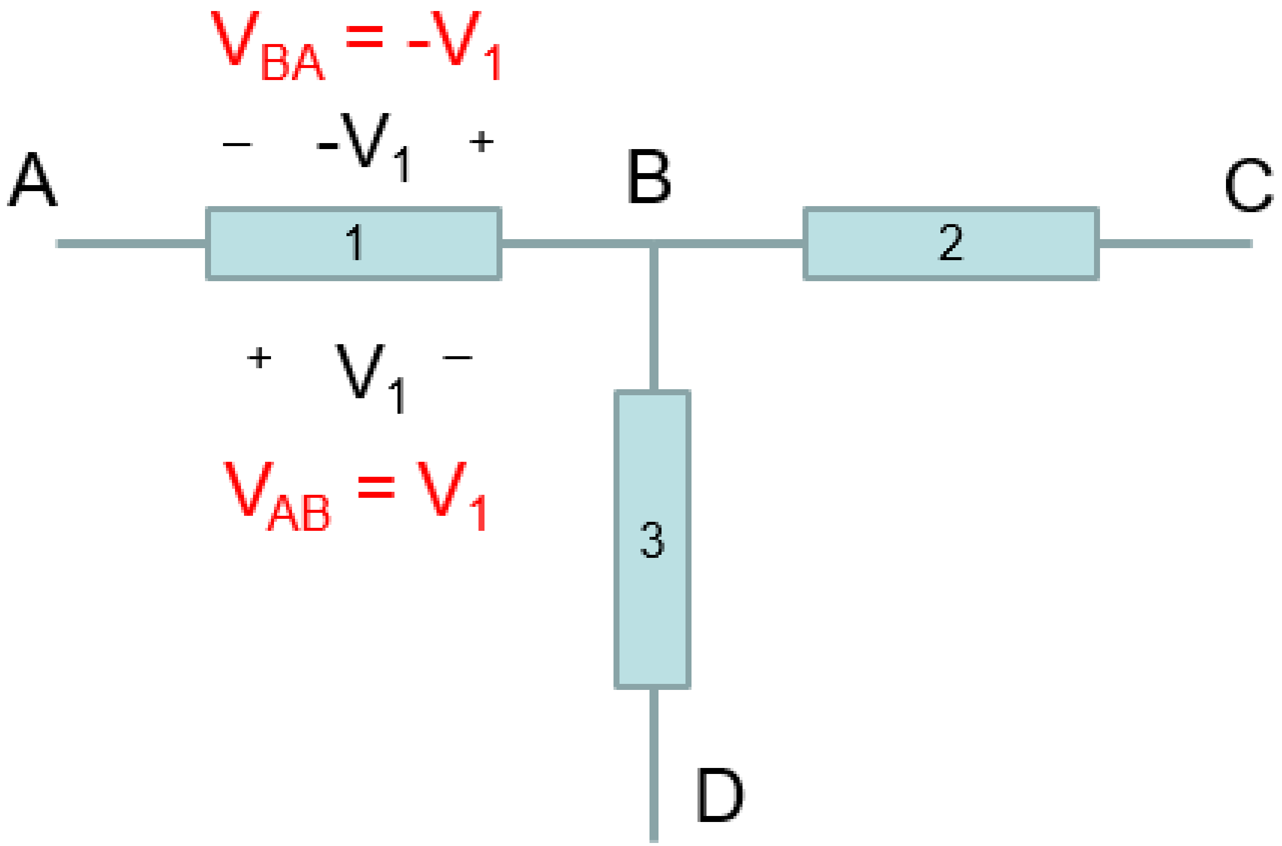
Voltage difference notation (+/- specified)



Node voltage



Double-Subscript Notation



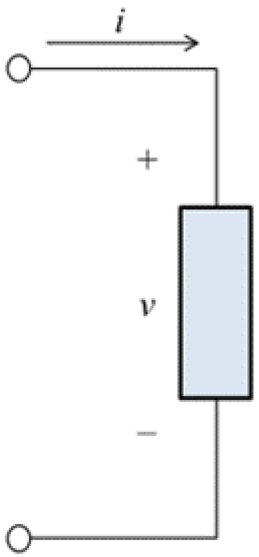
Electrical Power & Passive Reference Convention

- **Power** is a measure of the amount of energy (in Joules) consumed per unit time (in seconds), typically expressed in a unit called **Watts** (W).




Electrical power

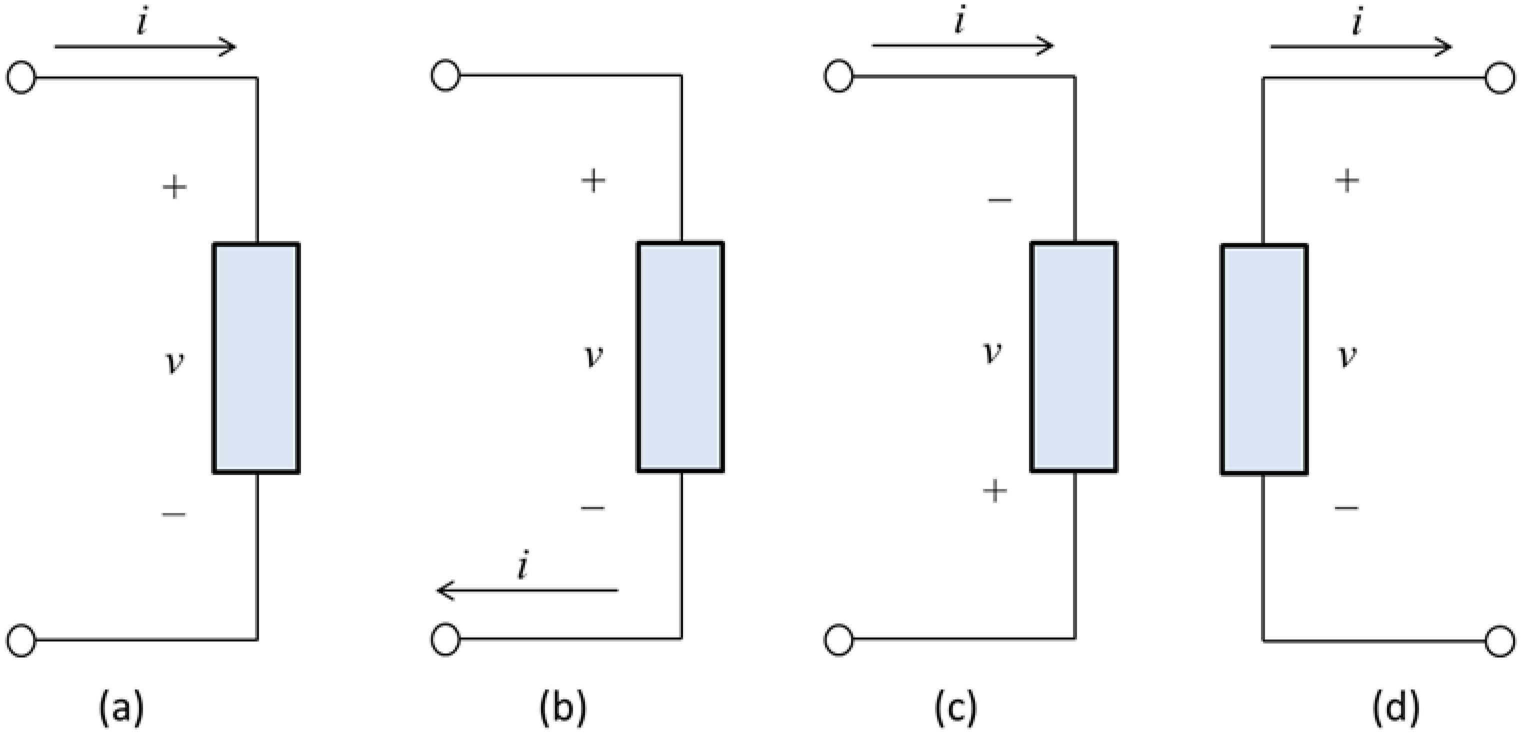
$$p = i \times v$$



$$\frac{Coulombs}{second} \times \frac{Joules}{Coulomb} = \frac{Joules}{second} = Watts$$

Passive reference convention

- Calculate the power from the associated current and voltage using a **standard combination of reference direction for the current and reference polarity for the voltage**.
- **Check if the value for calculated power is positive or negative**. If the value is positive, the element is absorbing power. If the value is negative, the element is supplying power.



Tellegen’s Theorem

- Total energy (and hence power) supplied by all circuit elements must be absorbed by other circuit elements.
- This is because the elements that supply power will have negative values for power and those that absorb power will have positive values. For any valid circuit, this law must be satisfied!

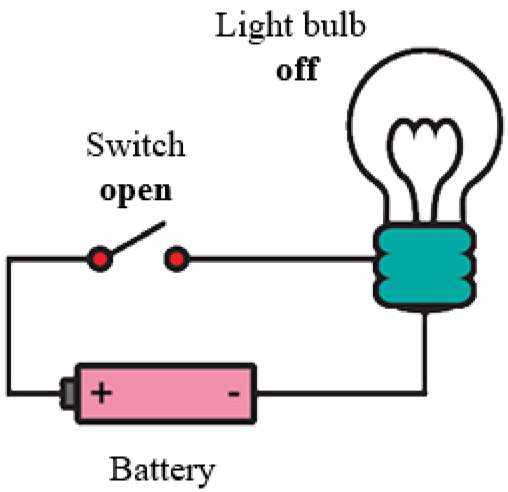
$$\sum_{i=1}^{n} p_i = 0$$

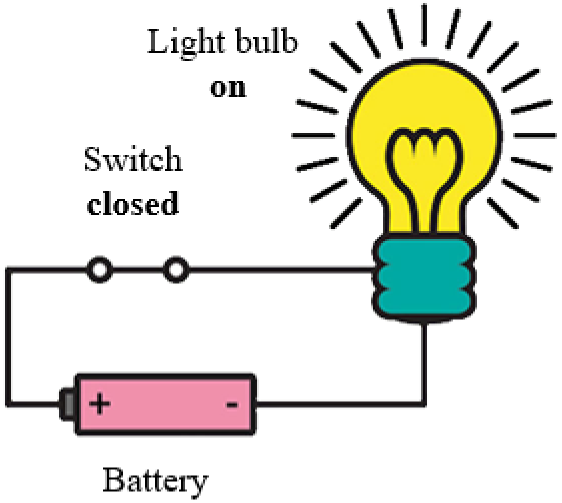
Electrical circuits

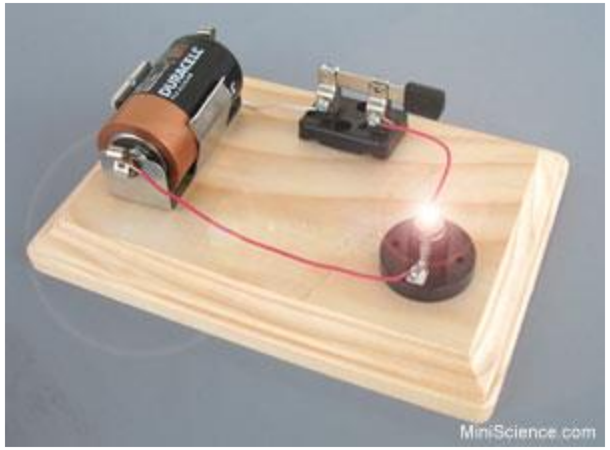
- At the core of all electrical engineering is the concept of an electrical circuit.

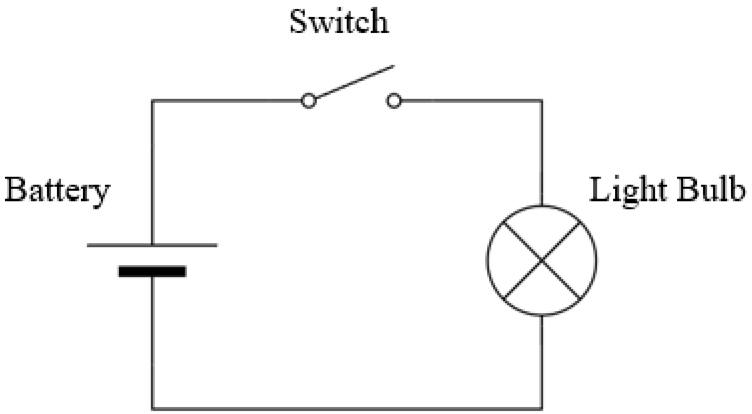
- For an electrical circuit to work properly, there must be a **closed path** that charge can move through.

Electrical circuits

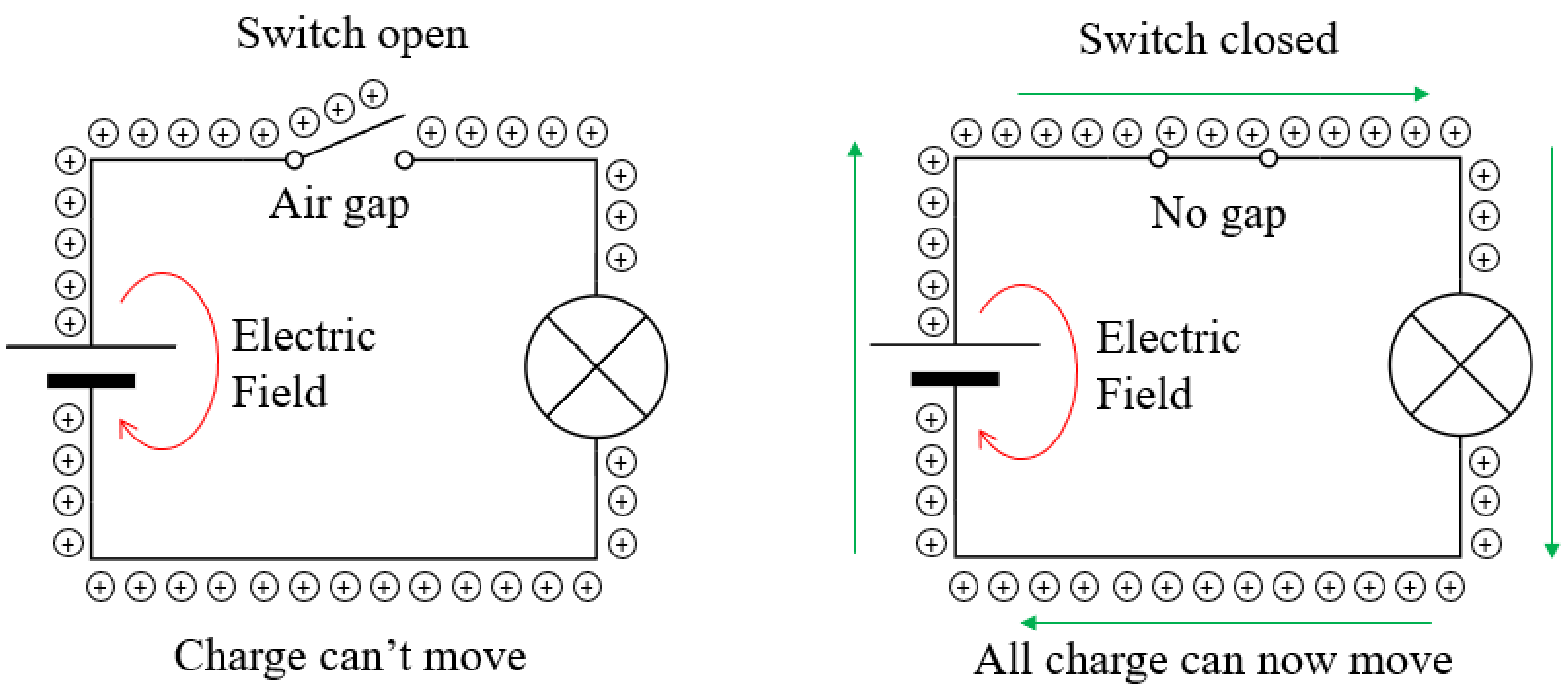




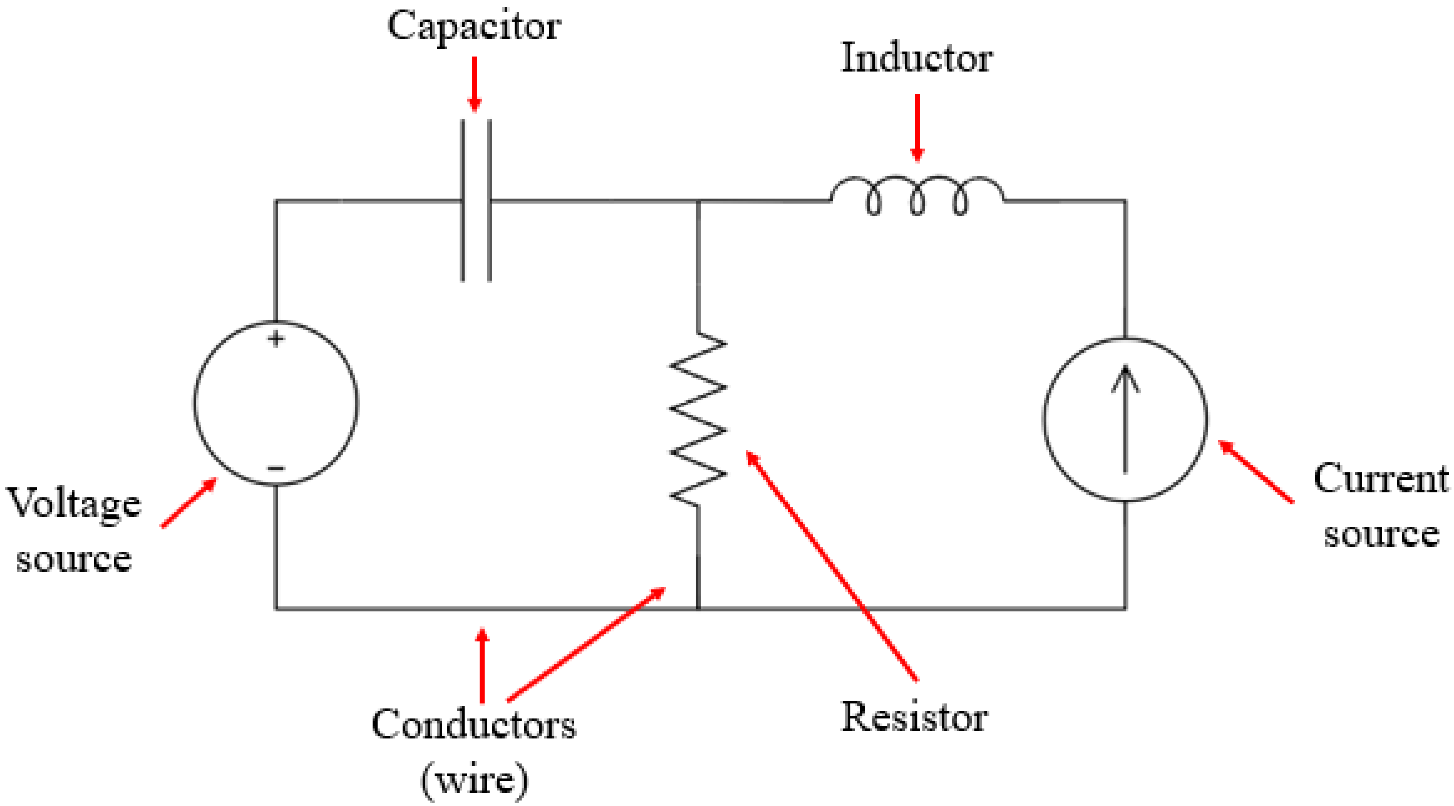




Why switches work



Circuit elements



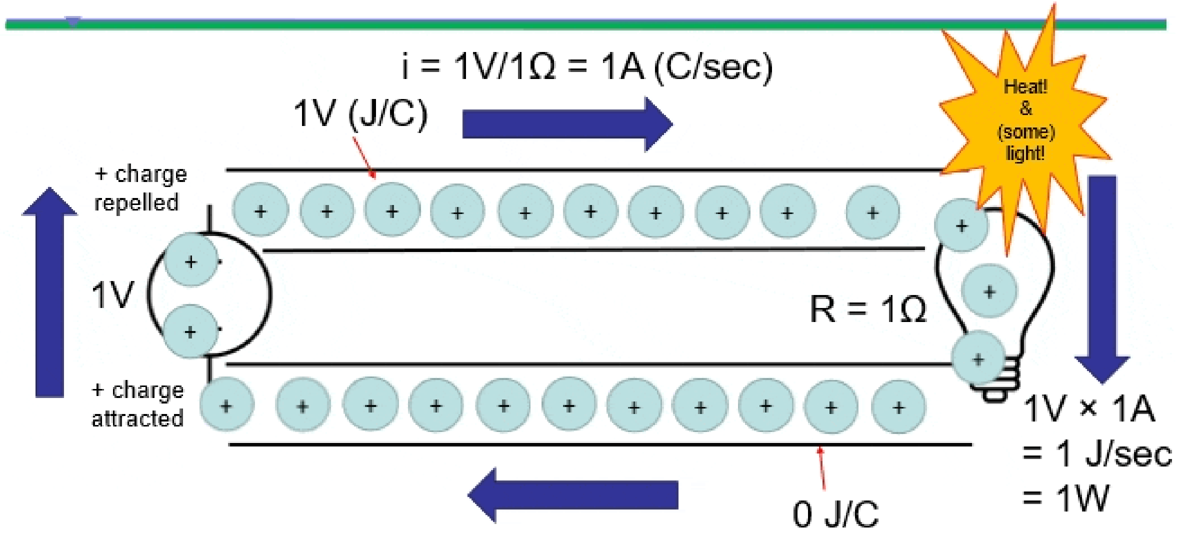
Common misconception

- Charge is "given energy" at an energy source
  → carried through the circuit by the electrical charge
  → given to energy consumer.
- **This is incorrect!**

- Charge in copper moves with a drift velocity of approximately **23 μm/sec = 8.3 cm/hour…** energy would never reach your house in time from a power station!

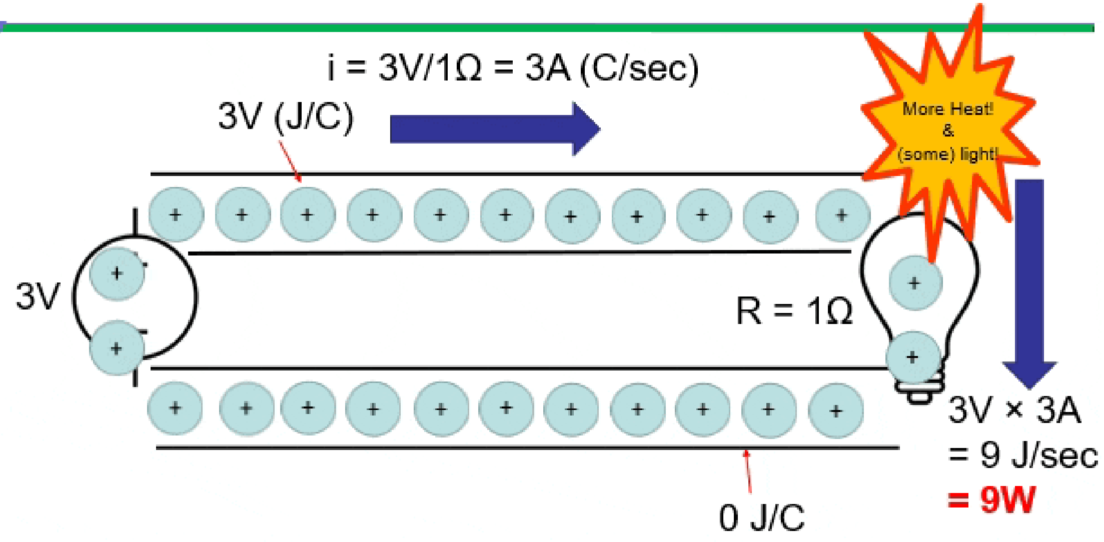
- Mobile charge in the conductors move together as they push on one another (like water in pipes), creates an electromagnetic field that propagates at close to the speed of light around (and outside of) the circuit to transfer energy

Visualisation



NOTE: animation sped up by ~ 100000 times

Visualisation



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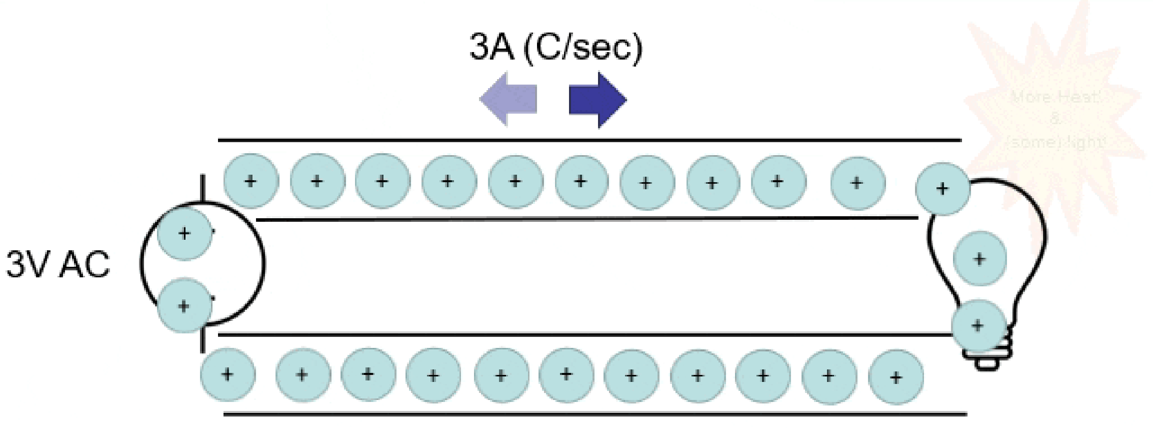
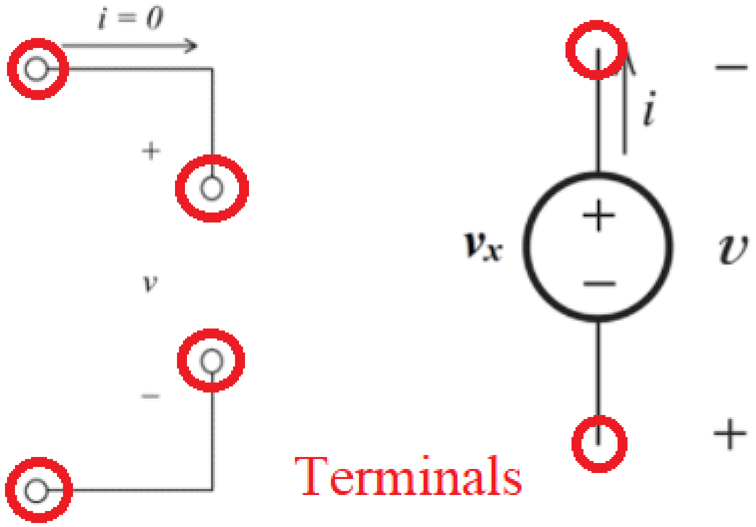
Visualisation

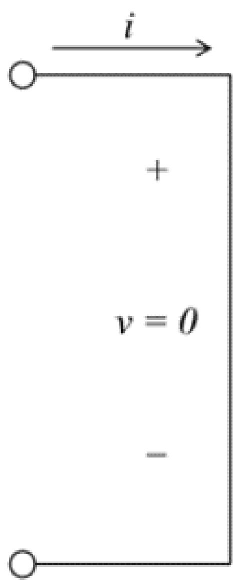


NOTE: animation sped up by ~ 100000 times

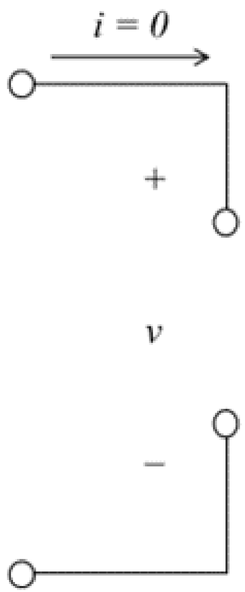
Terminals



Conductors ("wire", "short circuit") and Open Circuits



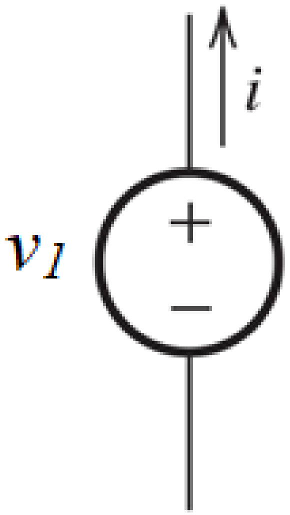
Conductors ("wire", "short circuit")



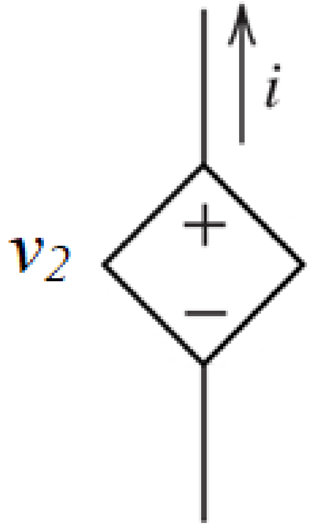
“Open Circuit”

Voltage sources

- The two symbols below are called (ideal) voltage sources. They have **(+/-)** symbols within them.
- The function of a voltage source is to **maintain a set voltage between "+" and "-" sides**



independent voltage source (circle symbol)



dependent voltage source (diamond symbol)

$$v_2 = a.i_x$$

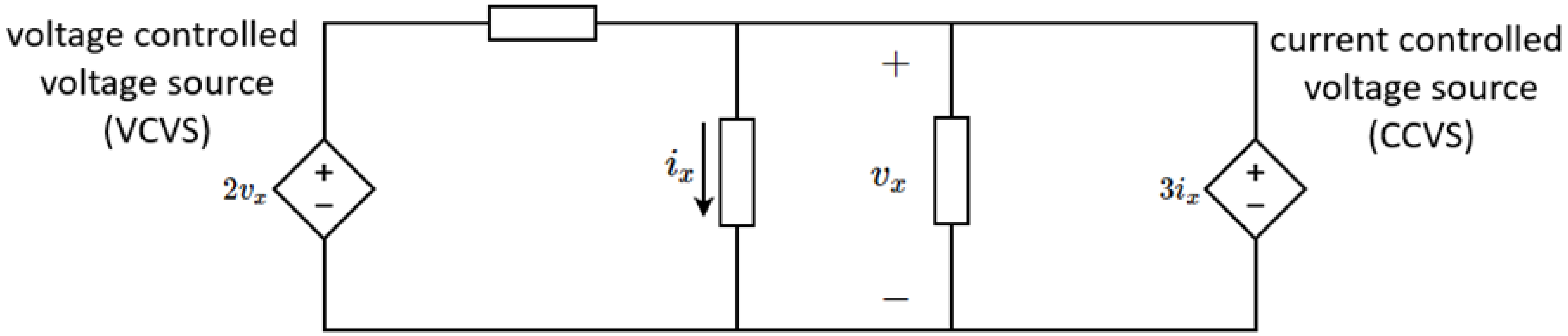
a must have the units of $\frac{[V]}{[A]}$

or

$$v_2 = b.v_x$$

b must be dimensionless

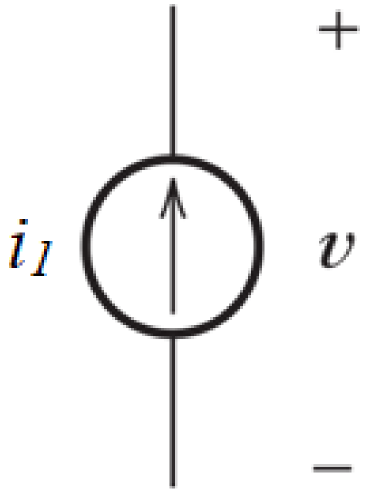
Example



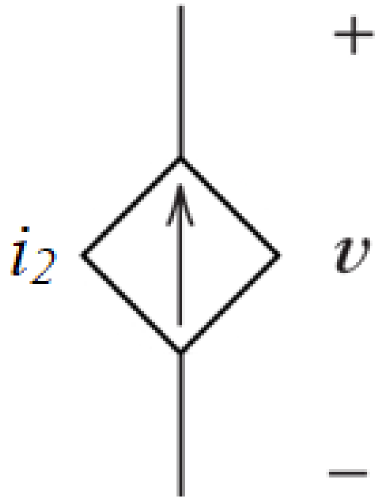
voltage controlled
voltage source
(VCVS)
$2v_x$
i_x
v_x
+
−
$3i_x$
current controlled
voltage source
(CCVS)

Current sources

- The two symbols below are called (ideal) current sources. They have an **arrow** symbol within them.
- The function of a current source is to maintain a set current through the source



independent current source (circle symbol)



dependent current source (diamond symbol)

$$i_2 = a.i_x$$

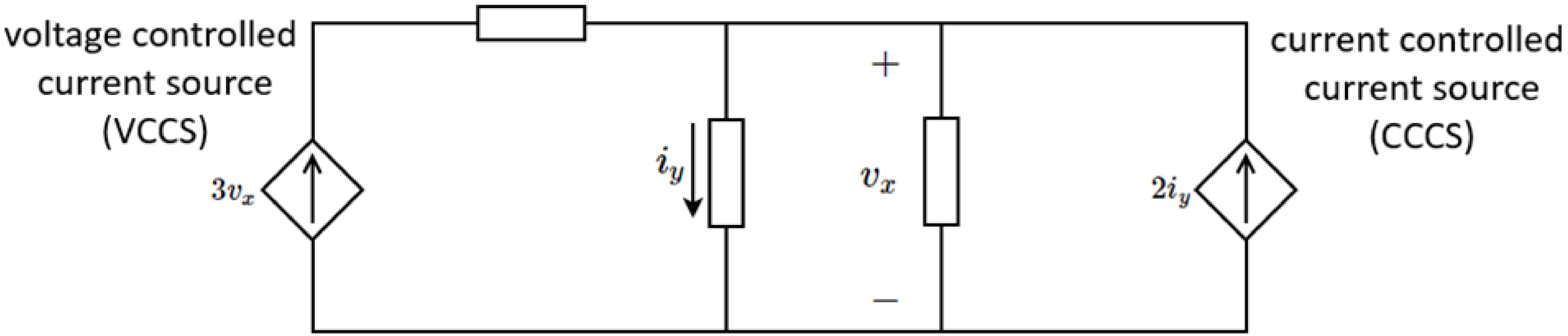
a must be must be dimensionless

or

$$i_2 = b.v_x$$

b must have the units of $\frac{[A]}{[V]}$

Example

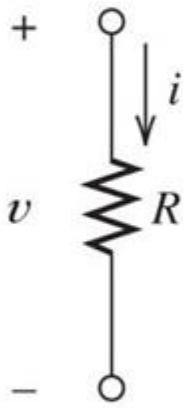

voltage controlled
current source
(VCCS)
$3v_x$
i_y
+
v_x
−
$2i_y$
current controlled
current source
(CCCS)

Resistors and Ohm’s Law

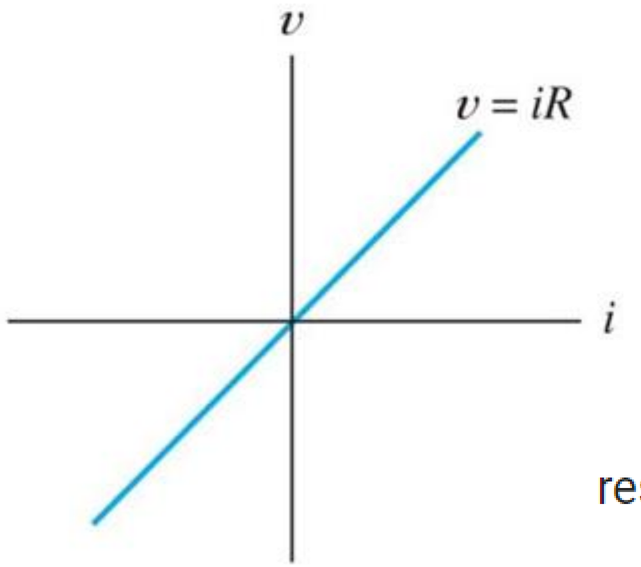
- Resistors are real devices that exhibit a property called "Resistance" and are a useful circuit element because they exhibit a simple relationship between the voltage and current ("V-I relationship") associated with them, called **Ohm's Law**.

- Many devices can be modelled as a resistance, especially if overall they exhibit V-I relationships that obeys Ohm's Law.

Resistors and Ohm's Law



(a) Resistance symbol



(b) Ohm's law

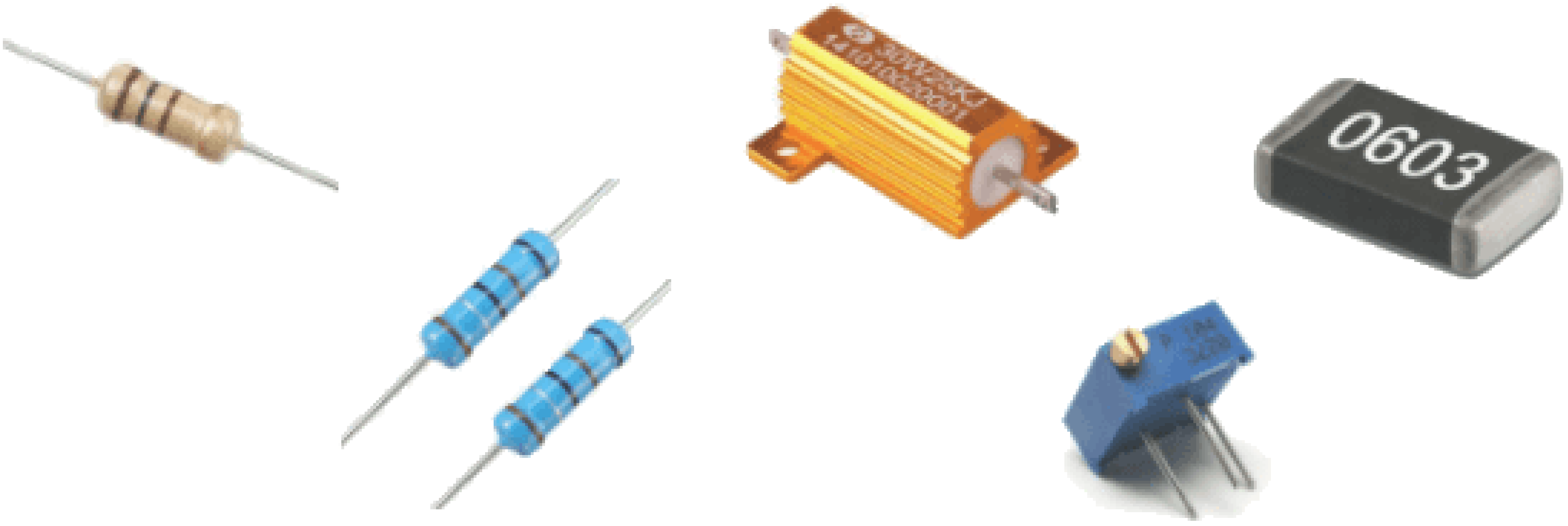
Ohm's Law

$v = iR$ or $i = v/R$

resistance R is measured in the unit of **Ohms (Ω) =** $\frac{Volts}{Ampere}$

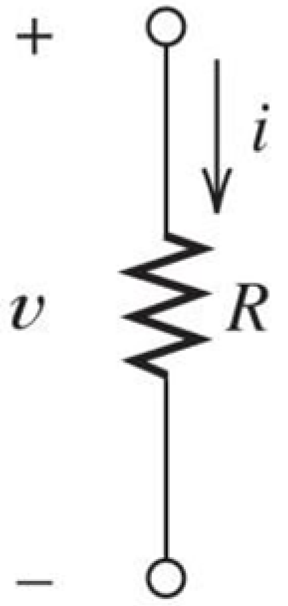
⚠ **Note:** This relationship is only valid for a **voltage measured across a single resistor**. It does not hold if the voltage is measured across multiple resistors. In these cases, to use Ohm's Law across multiple resistors requires that those resistors be replaced with a **single equivalent resistance**.

Real resistors



Power dissipation in resistors

- Resistors **always absorb** electrical power
- Many resistors have a **power rating**, exceeding this can damage the resistor
- Resistors have a **tolerance**



$$p = iv$$

Shortcuts if only current or power known

$$p = i^2 R \qquad p = \frac{v^2}{R}$$

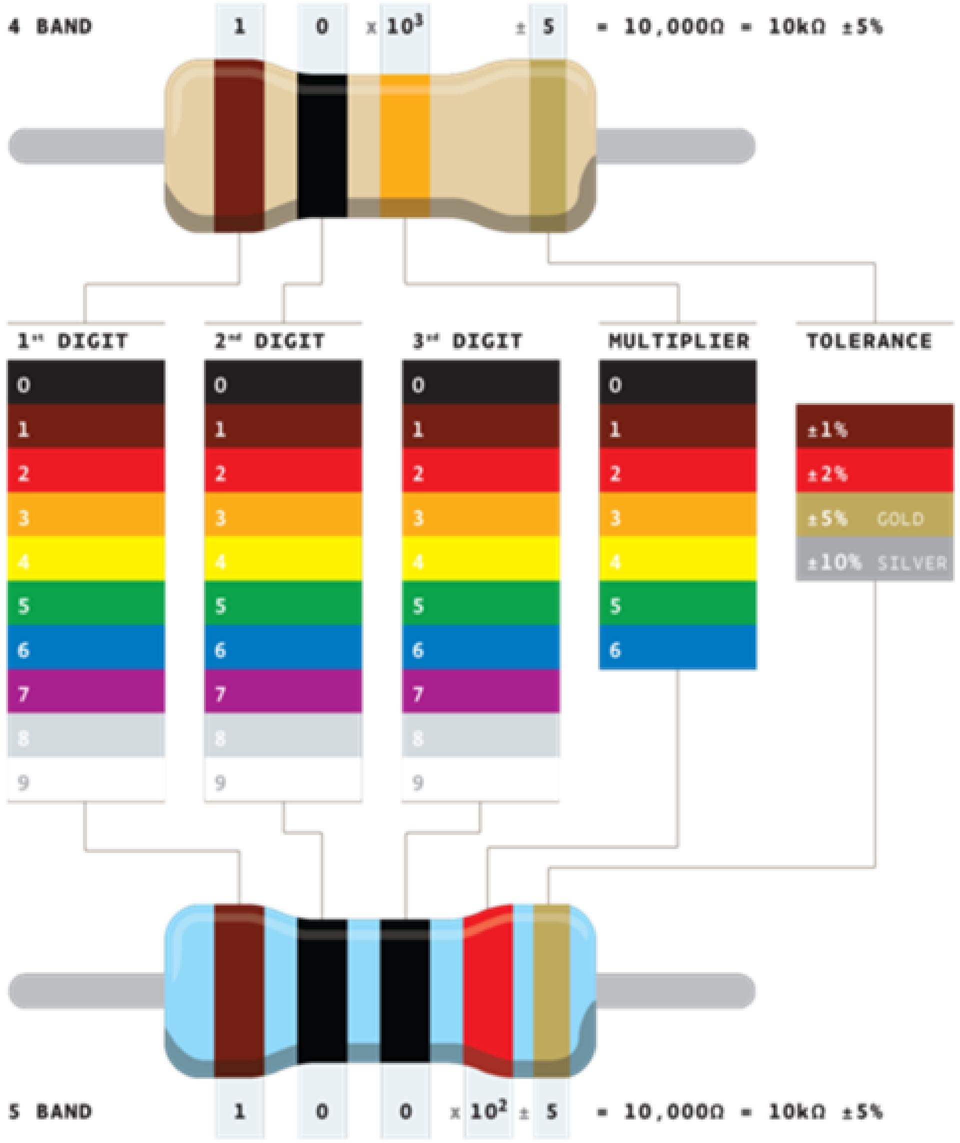
Standard resistor values

- E-series values:
  - E6 20% tolerance
  - E12 10%
  - E24 5% (also available with 1%)
  - E48 2%
  - E96 1%
  - E192 0.5% (also used for resistors with 0.25% and 0.1%)

- E-12 values:
  - 10, 12, 15, 18, 22, 27, 33, 39, 47, 56, 68, 82 ($\times 10^n$ where n is an integer)

- Standard notation: R = "Ω", k = 1,000, M = 1,000,000
  - e.g. "100R" = 100 Ω, "2k2" = "2.2k" = 2,200Ω, "1M2" = 1.2 MΩ

Reading resistor values



Nodes and Loops

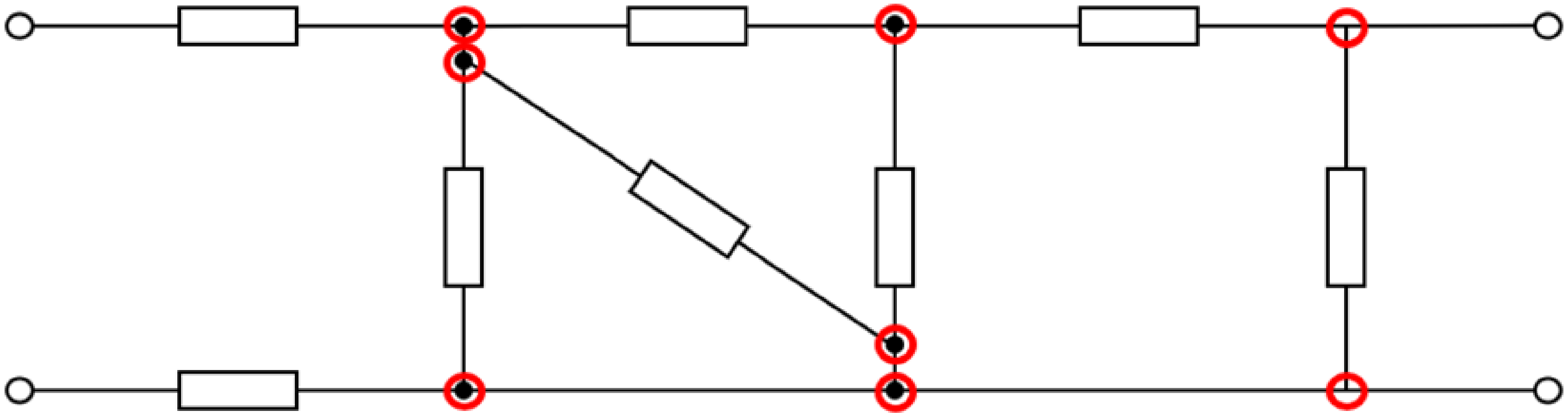
- Circuit terminology

- **It is extremely important that you comfortably identify nodes and loops, so that you can perform circuit analysis.**

- **Make sure you can do this proficiently with practise, otherwise the rest of the electrical content will be very confusing for you.**

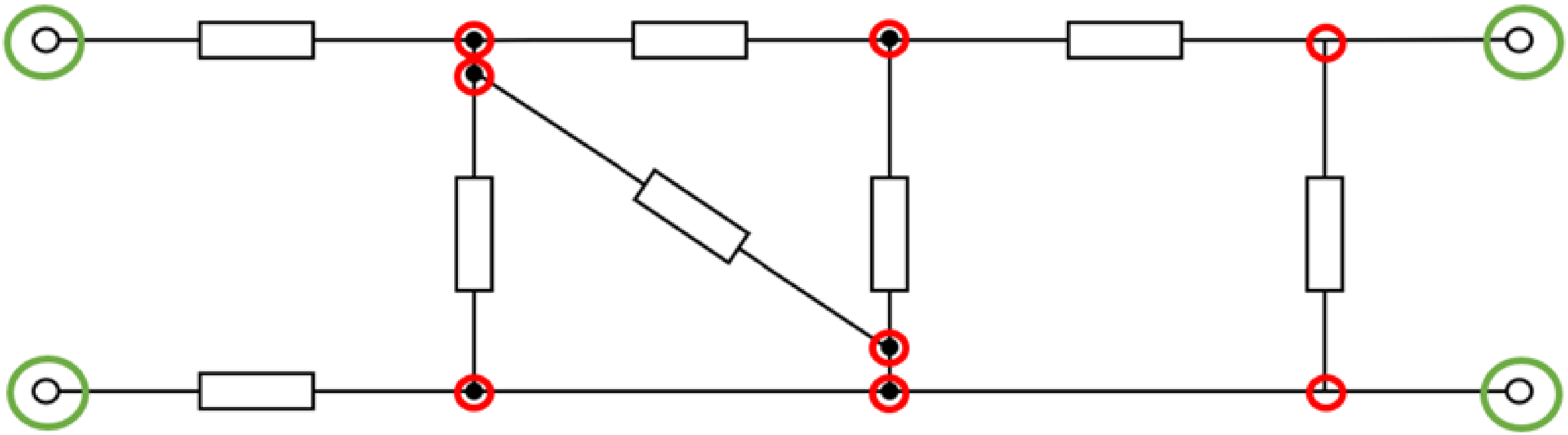
Interconnects

- An **interconnect** is a point of connection between two or more circuit elements. These are show in **red** below. These can be denoted by dots (but not always, as shown in the right two interconnects).



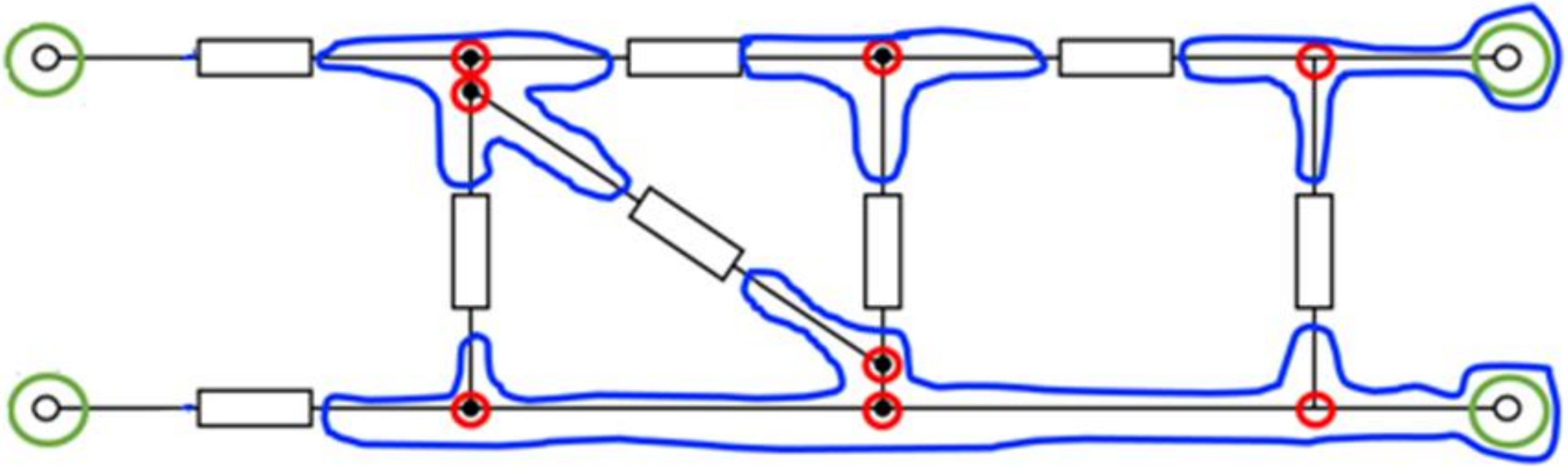
Terminals

- **Terminals** are shown in **green** below. As a terminal is an uncompleted part of a circuit, no current moves in or out of unconnected terminals, but terminals can have non-zero voltage.



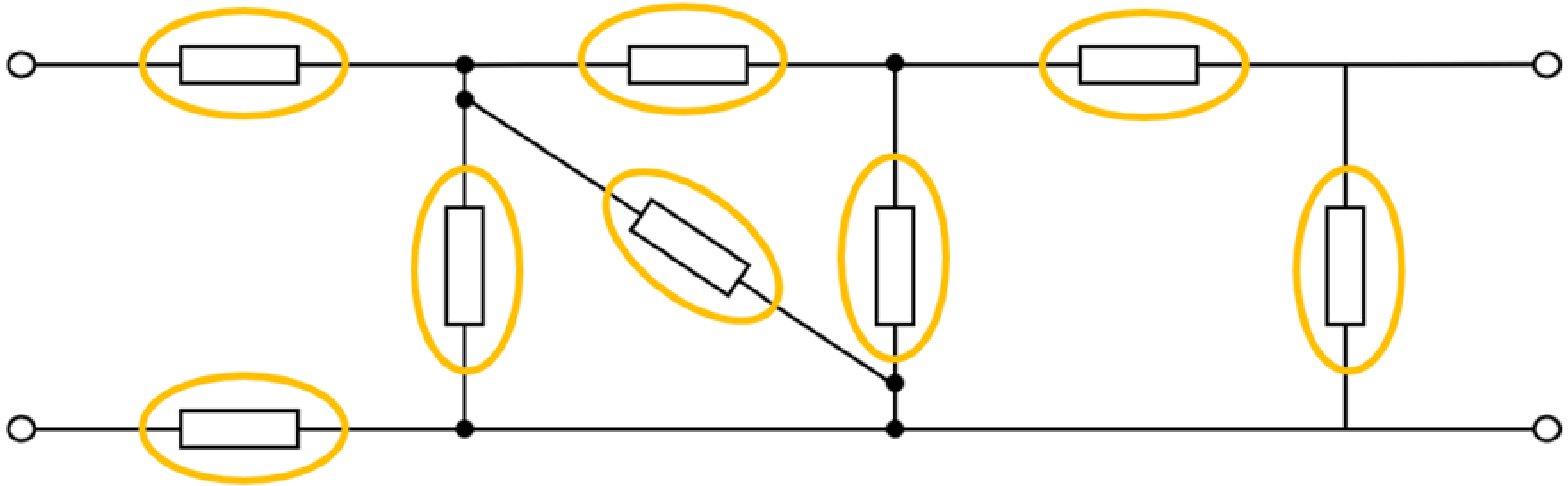
Nodes

- Any interconnects or terminals that are connected by conductors (wires) may be collected together, along with wires, to form a single logical **node**. These are shown in **blue** below.
- Importantly, **all locations in a node have the same voltage**. This is due to the assumption that wires have zero resistance.



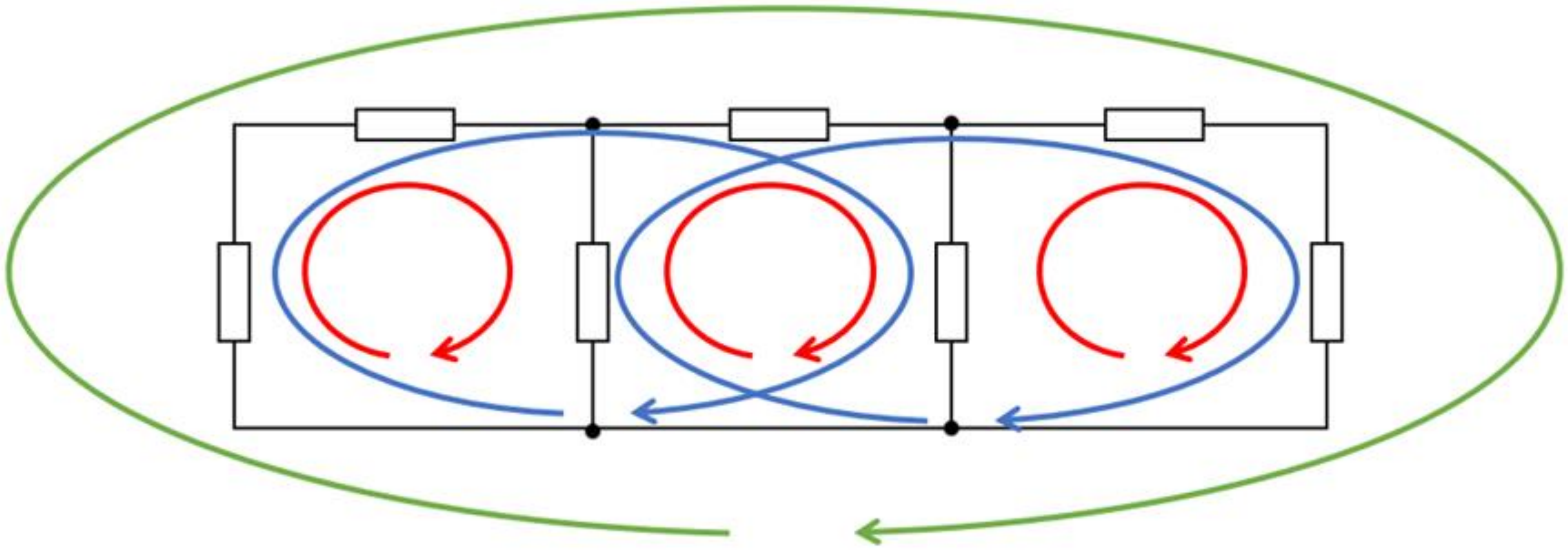
Branches

- A **branch** is any circuit element. Branches are circled in **orange** below



Loops

- A loop is any closed path that starts and ends at the same point through a circuit where no interconnect is encountered more than once.
- Examine the circuit below. Note that there can be many loops, some obvious, some not so obvious (e.g. there are small loops in **red**, medium ones in **blue**, and a large one in **green**).



Kirchhoff’s Laws (KCL, KVL)

- The most fundamental concepts of Electrical engineering are the two laws, Kirchhoff’s current law (KCL) and Kirchhoff’s voltage law (KVL), named after German physicist Gustav Robert Kirchhoff.

- These two laws describe how electricity behaves through a circuit. **Applying these two laws is the basis of circuit analysis**.

Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law (KCL) is a direct consequence of the **law of conservation of charge** – the principle that electric charge cannot be created or destroyed

- the sum of currents (electrical charges) entering a node/interconnect equals the sum of currents (electrical charges) leaving a node/interconnect, which can be written mathematically as:

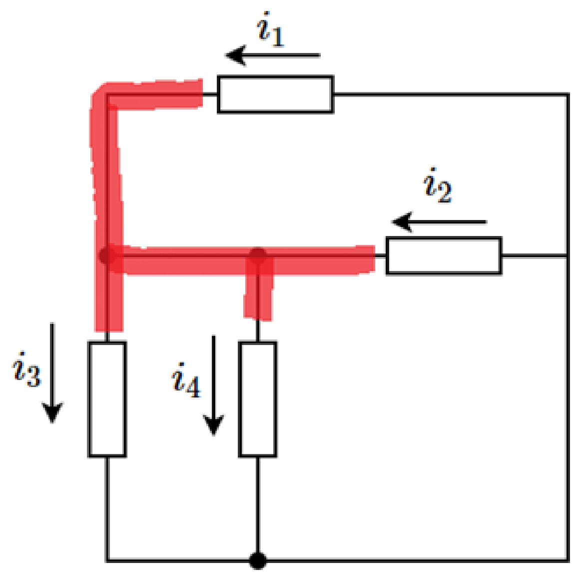
$$\sum i_{in} = \sum i_{out}$$

- Or the mathematical sum of currents leaving (or entering) a node/interconnect is zero:

$$\sum_{n=1}^{m} i_n = 0$$

Kirchhoff’s Current Law (KCL)

- Applying KCL for the red node, we could formulate the following equivalent equations:



Using $\sum i_{in} = \sum i_{out} \longrightarrow i_1 + i_2 = i_3 + i_4$

Using $\sum_{n=1}^{m} i_n = 0$

and considering currents **entering** node as **positive**:

$$0 = i_1 + i_2 + (-i_3) + (-i_4)$$

or considering currents **exiting** the node as **positive**:

$$0 = (-i_1) + (-i_2) + i_3 + i_4$$

Kirchhoff’s Voltage Law (KVL)

- Kirchhoff’s Voltage Law (KVL) is a direct consequence of the **law of conservation of energy** – the principle that energy cannot be created or destroyed in a closed system (loop)

- the **sum of voltages rises** around a closed loop equals the **sum of voltage drops around** the same loop

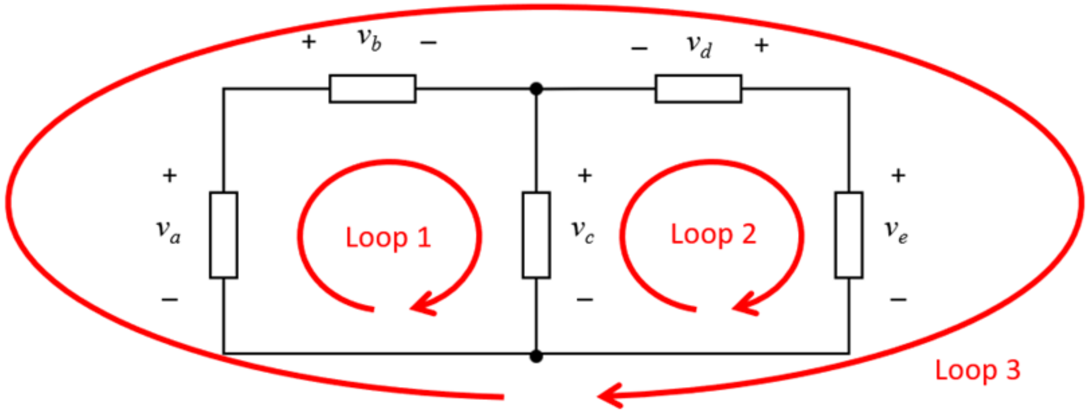
$$\sum v_{rises} = \sum v_{drops}$$

- the mathematical sum of voltage drops (or rises) around a closed loop equals zero:

$$\sum_{n=1}^{m} v_n = 0$$

Kirchhoff's Voltage Law (KVL)

- KVL can be applied around any loop
- For loop 1, we can apply KVL to formulate equations:



Using $\sum v_{rises} = \sum v_{drops} \longrightarrow \quad v_a = v_b + v_c$

v_a is considered a "rise" because for v_a the direction of the loop arrow moves from "-" to "+", while v_b and v_c are considered a "drop" because the loop arrow moves from "+" to "-"

Using $\sum_{n=1}^{m} v_n = 0$

and considering voltage drops as **positive**:

$$0 = (-v_a) + v_b + v_c$$

Or considering voltage drops as **negative**:

$$0 = v_a + (-v_b) + (-v_c)$$

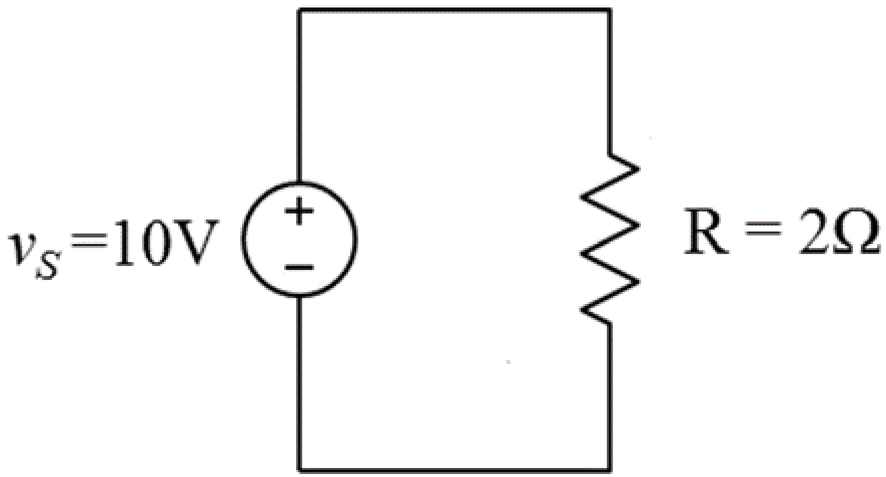
Circuit analysis

- Circuit analysis is the process of finding all (or specific) values of currents and voltages of the elements within a circuit.

- Once we know these values, we are then able to understand how circuits behave.

Basic example

- Consider the example below, where
  - the voltage source could represent an ideal 10V battery, and
  - the resistor could represent an ideal light bulb that converts electrical energy to heat and light
- The battery has a finite amount of energy stored in it, say C = 10000 J
- How long it will be able to run the light bulb for?



Basic example

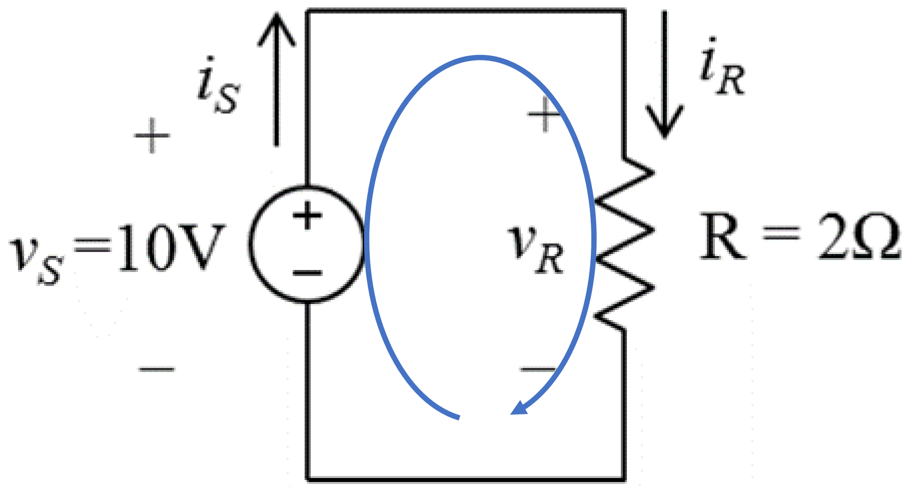
- We need to find out the rate at which the voltage source is supplying energy to the resistor
  - the **power** supplied by the voltage source

- Applying KVL around the (only) loop:

$$0 = -v_s + v_R$$

$$v_R = v_s$$

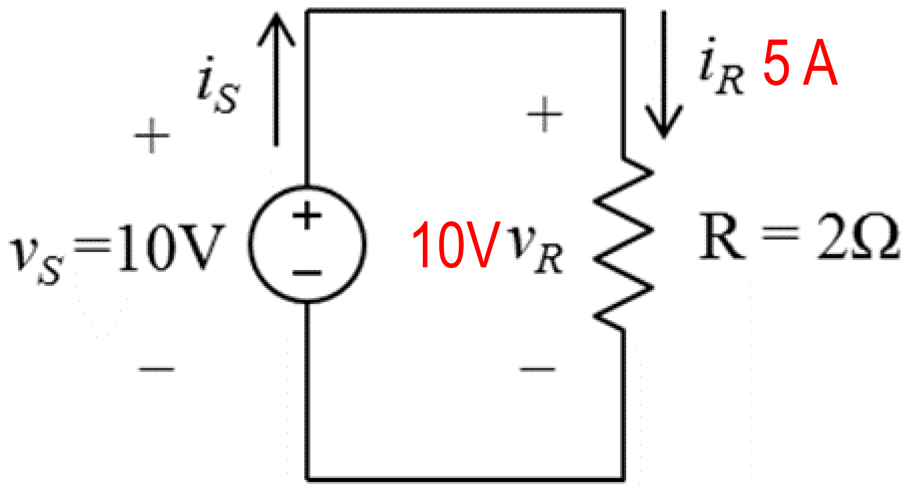
$$v_R = 10V$$



Basic example

- Now that we know v_R, we can apply Ohm's Law across the resistor:

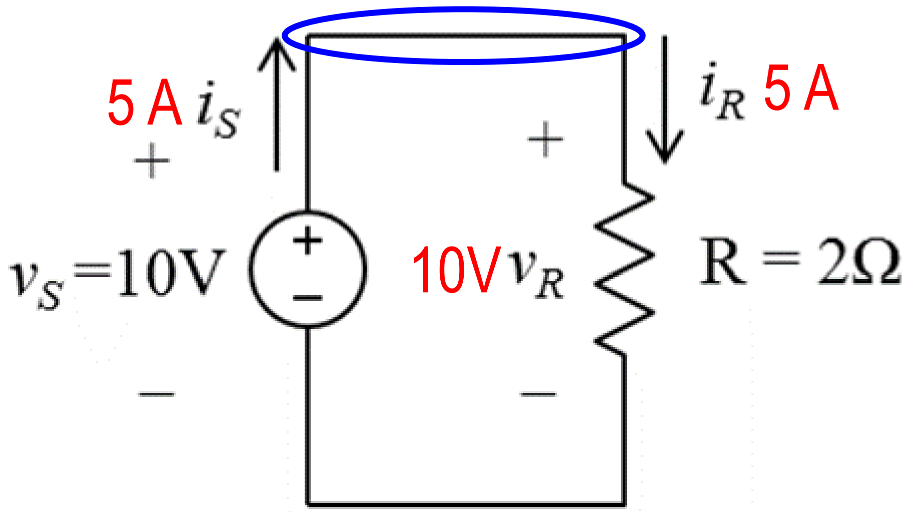
$$v_R = i_R \times R$$
$$10V = i_R \times 2\Omega$$
$$i_R = 5A$$



Basic example

- Applying KCL, and treating the top wire as a “node”, counting current exiting the node as positive:

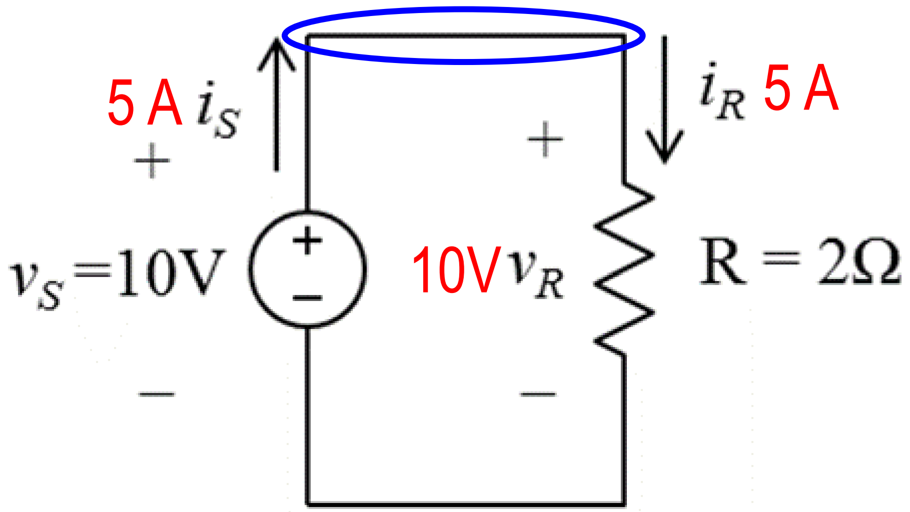
$$0 = -i_s + i_R$$
$$i_s = i_R$$
$$i_s = 5A$$



Basic example

- Work out the power absorbed or supplied by the voltage source p_s:
- Since current i_s exiting the positive terminal of the voltage source, we should (using the passive reference convention) use the value of $-i_s$ to calculate the power absorbed by voltage source:

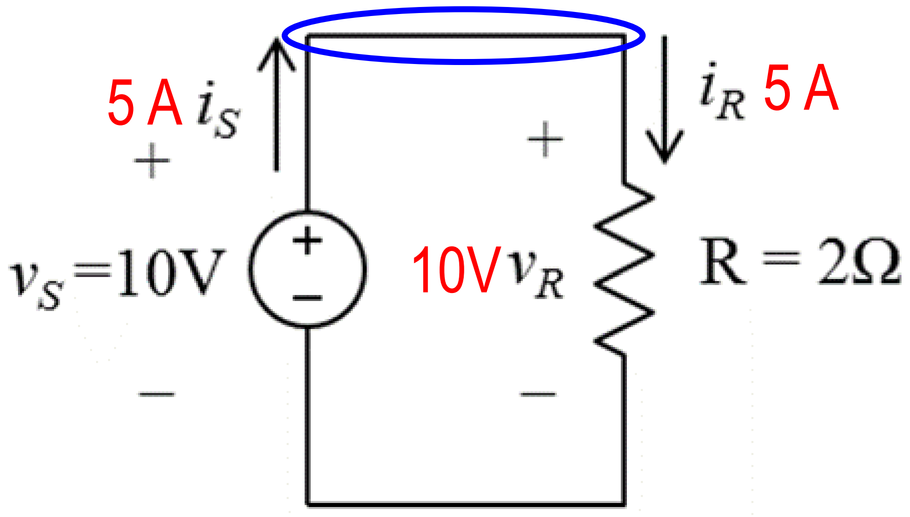
$$\begin{aligned} p_s &= v_s \times (-i_s) \\ &= 10V \times (-5)A \\ &= -50W \end{aligned}$$



Basic example

- Alternatively, use Tellegen's Theorem to find power absorbed by resistor and use $p_s = -p_R$

$$\begin{aligned} p_R &= v_R \times i_R \\ &= 10V \times 5A \\ &= 50W \end{aligned}$$



Basic example

- Now, to answer our initial question, we can divide the energy stored in the battery (C =10000 J) by the rate of power supplied to the circuit p_s = 50 W (= J / sec) to find the time t seconds:

$$
\begin{aligned}
t &= \frac{C}{p_s} \\
&= \frac{10000J}{50W} \\
&= 200sec
\end{aligned}
$$

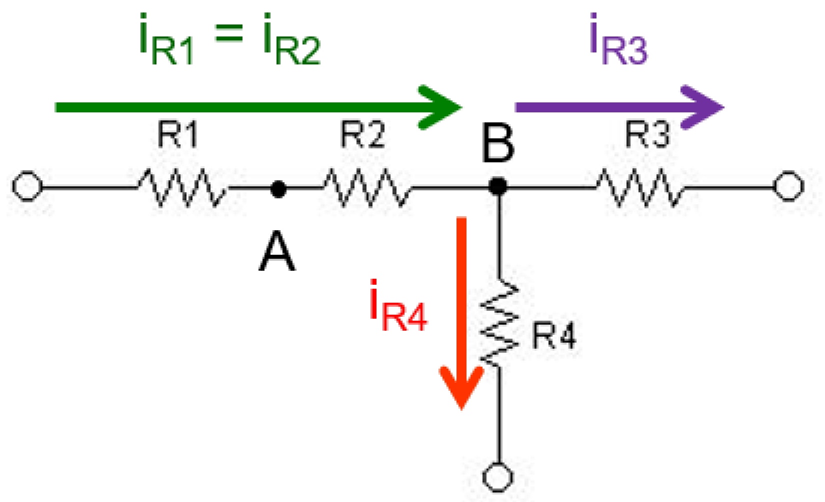
Series and Parallel elements

- In circuits, many elements tend to be arranged in two basic ways – **series** and **parallel**

- It is important to recognise these arrangements as it can simplify the analysis and creation of circuits, as they have special properties (derived from KCL and KVL).

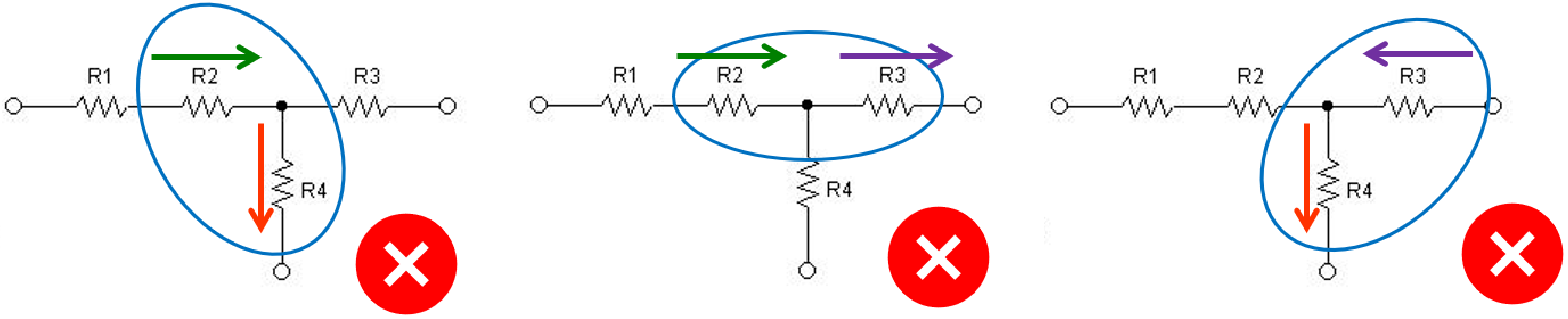
Circuit elements in series

- Two (or more) circuit elements are connected “in series“, if **there is a path through them that does not branch off at any point**
- The application of KCL at the node A between R1 and R2 means that elements in series must share the same value of current. In this case, $i_{R1} = i_{R2}$.



Circuit elements in series

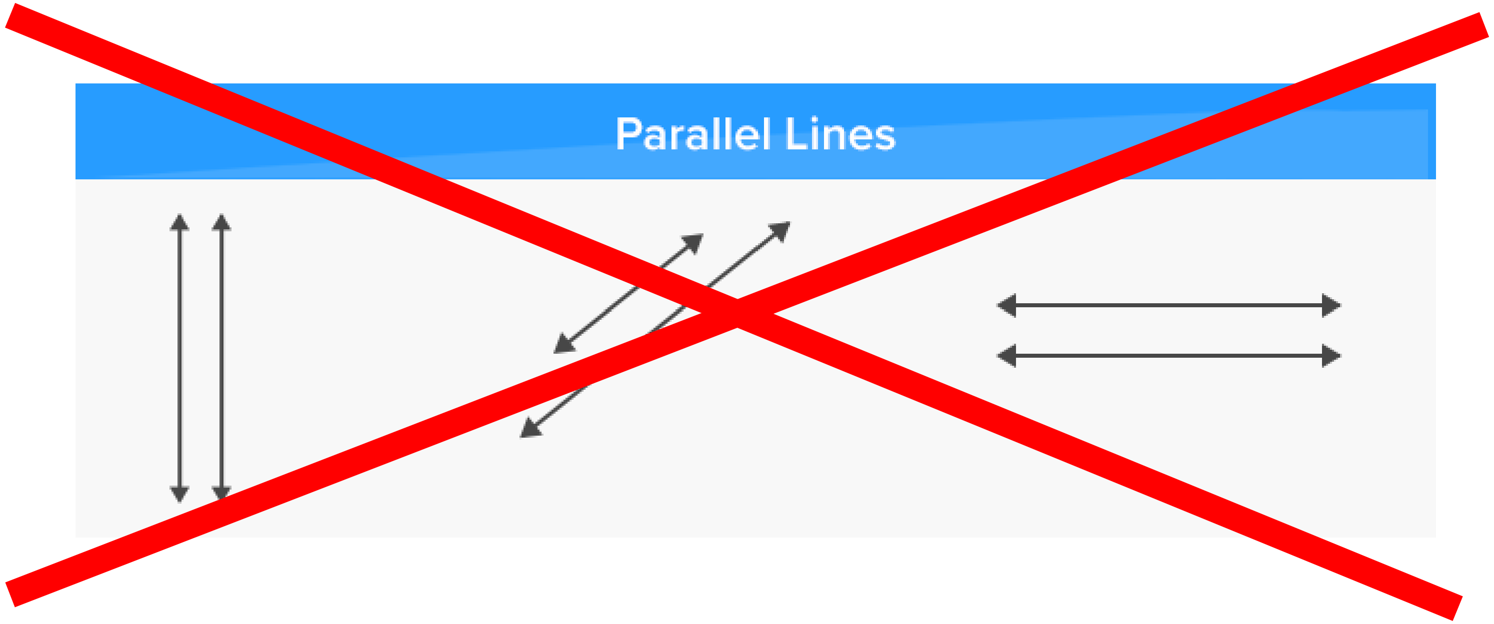
- R3 and R4 are not in series with any other elements, because the (**red** or **purple**) current moving through either of these is not guaranteed by KCL to be the same as any other elements.



- In all the cases, there is a branch at the interconnect between any two elements in which current may take an alternative path

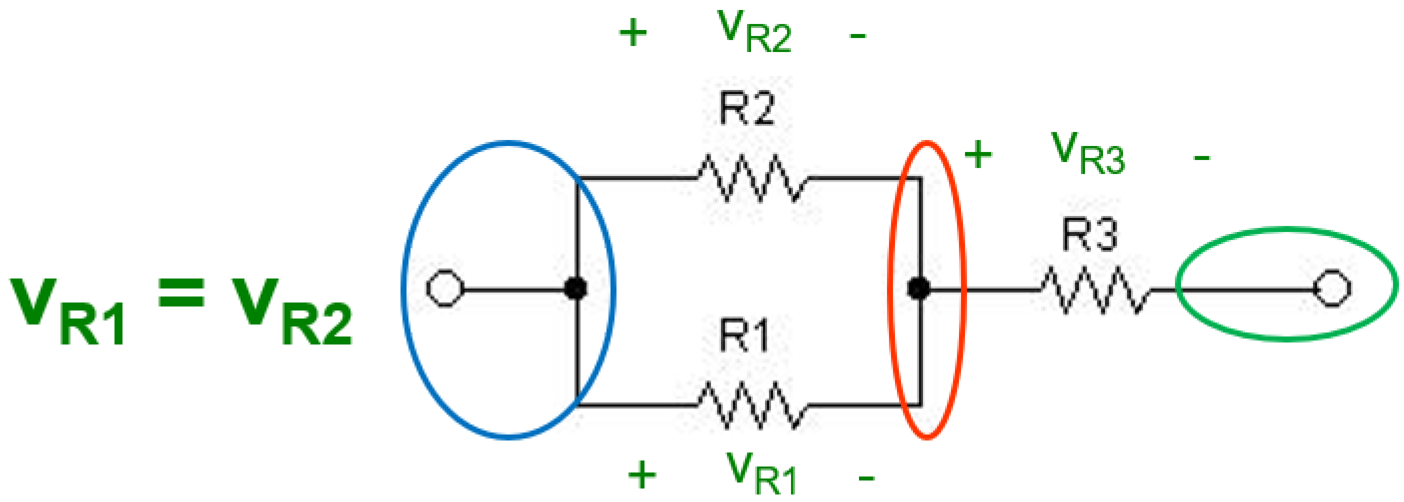
Circuit elements in parallel

⚠ In electrical engineering, the term **parallel does not refer to lines that have the same distance continuously between them**, as you might expect from common usage of the term.



Circuit elements in parallel

- Two (or more) circuit elements are "in parallel", if they **share common nodes at both ends**
  - R1 and R2 are **in parallel** because they have the same nodes at both ends (**blue** and **red**).
  - R3 is **not in parallel with any other elements** because it only shares the **red** node with R1 and R2, and not the **green** node.
- Application of KVL in a loop between two parallel elements means that **elements in parallel share the same value of voltage**. In this case, $v_{R1} = v_{R2}$.



Voltage and current dividers

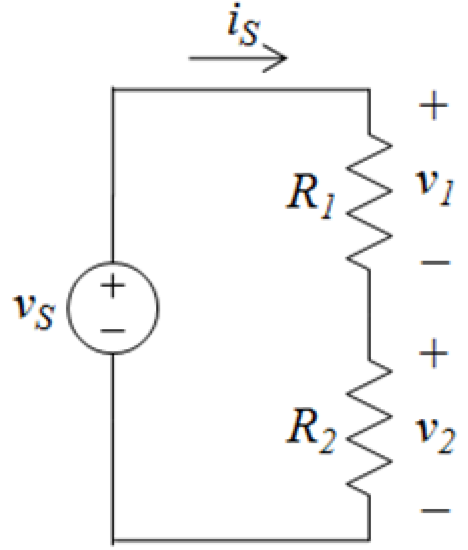
- Some circuit configurations (i.e. the series and parallel arrangements in the previous page) occur quite frequently

- Rather than using a formal application of KCL and KVL every time we come across such a circuit, it is convenient to summarise these steps into simple rules

- Examples are the **voltage divider** and the **current divider**

- Many practical uses
  - Produce a specific voltage or current at a location in a circuit
  - Voltage dividers used in conjunction with sensors to trigger things to happen

Voltage divider



- Consider the circuit, where a voltage source v_s is connected to 2 resistors connected in series with resistance values of R_1 and R_2
- To find voltage across each resistors, use of Ohm's Law:

$$v_1 = i_s R_1 \text{ and } v_2 = i_s R_2 \longrightarrow i_s = \frac{v_1}{R_1} = \frac{v_2}{R_2}$$

- Write a KVL equation around the loop (with voltage drops counted as positive):

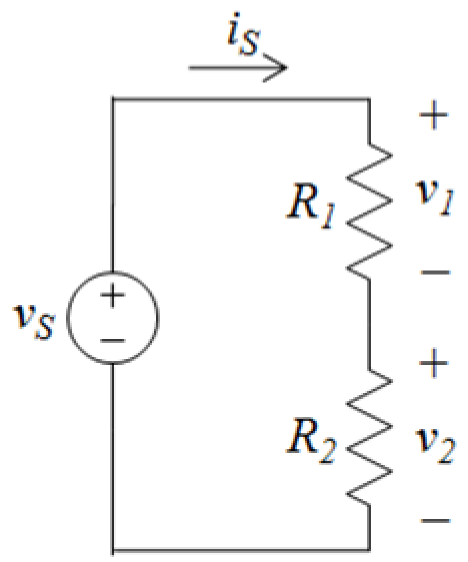
$$\begin{aligned} 0 &= -v_s + v_1 + v_2 \\ v_s &= v_1 + v_2 \\ &= i_s R_1 + i_s R_2 \\ &= i_s (R_1 + R_2) \\ \text{therefore } i_s &= \frac{v_s}{R_1 + R_2} \end{aligned}$$

Voltage divider

- Equating the previous expressions gives:

$$\frac{v_1}{R_1} = \frac{v_s}{R_1 + R_2} \qquad \frac{v_2}{R_1} = \frac{v_s}{R_1 + R_2}$$

$$\boxed{v_1 = v_s\left(\frac{R_1}{R_1 + R_2}\right)} \qquad \boxed{v_2 = v_s\left(\frac{R_2}{R_1 + R_2}\right)}$$



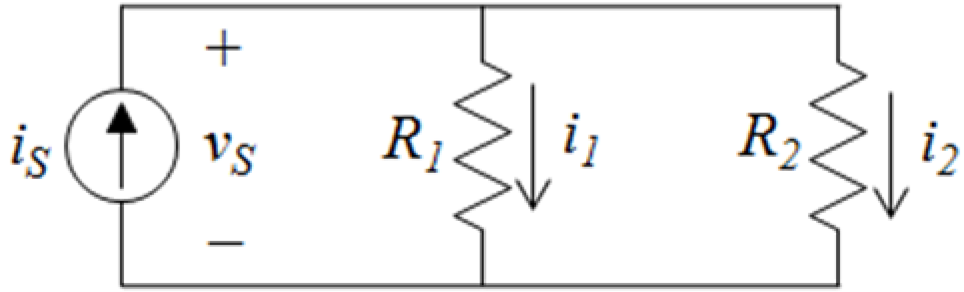
- In general for n resistors in series across voltage v_s:

$$v_i = v_s\left(\frac{R_i}{R_1 + R_2 + \ldots + R_n}\right)$$

⚠ The voltage divider is only applicable to multiple resistors **connected in series**, so when you apply it, you must verify that this is actually the case!

Current divider

- Consider the circuit, where a current source i_s is connected to 2 resistors R_1 and R_2



- To find current through each resistor, make use of Ohm's Law for each resistor and the fact that the resistors are in parallel to find:

$$v_s = i_1 R_1 = i_2 R_2 \longrightarrow i_1 = \frac{v_s}{R_1} \text{ and } i_2 = \frac{v_s}{R_2}$$

- Use KCL at the top node (with positive current exiting the node):

$$0 = -i_s + i_1 + i_2$$

$$i_s = i_1 + i_2$$

Current divider

- Equating the previous expressions gives:

$$i_s = \frac{v_s}{R_1} + \frac{v_s}{R_2}$$

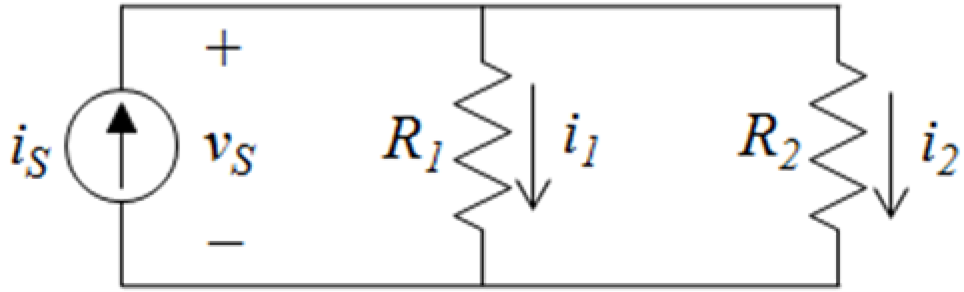
$$i_s = v_s(\frac{1}{R_1} + \frac{1}{R_2})$$

$$v_s = i_s(\frac{R_1 R_2}{R_1 + R_2})$$

- Equating v_s with the first equations from Ohm's law:

$$i_1 R_1 = i_s(\frac{R_1 R_2}{R_1 + R_2}) \qquad i_2 R_2 = i_s(\frac{R_1 R_2}{R_1 + R_2})$$

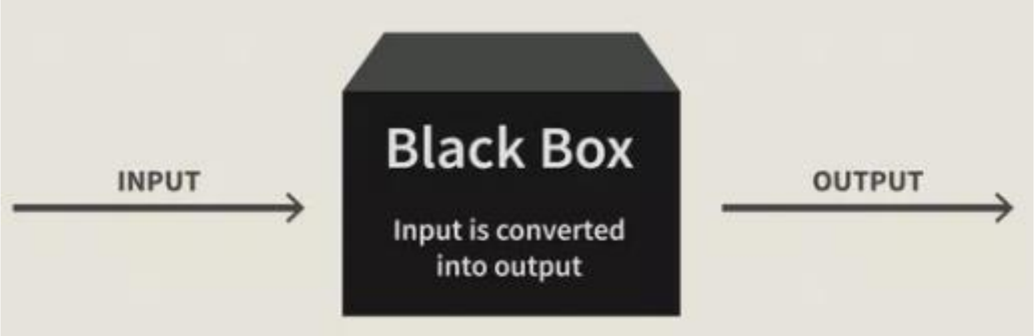
$$\boxed{i_1 = i_s(\frac{R_2}{R_1 + R_2})} \qquad \boxed{i_2 = i_s(\frac{R_1}{R_1 + R_2})}$$



⚠ The current divider **only works for 2 resistors connected in parallel!** In the case of multiple resistors, you would need to resort to finding the voltage across the parallel resistors and using Ohm's Law on each individual resistor.

Circuit equivalence

- Circuits are **systems** of circuit elements connected together that can end up being very complex
- Engineers use the concept of **equivalence** to conceptually replace complex systems with "**black boxes**" that simply describe equivalent behaviour with regards to important inputs and outputs



- In electrical circuits, we typically apply the concept of equivalence in relation to the **behaviour of voltage and current between equivalent circuits**
- Initially, we will do this only for resistors, but next week we will see that we can do this for whole circuits.

Equivalence

- Concept of "equivalence" is one that you have been taught since a very young age, most commonly when using numbers

- Example for LeJon James:
  - “LeJon scored 2+3+1+0+0+0+1 points the last game” or
  - “LeJon scored 7 points in the last game“

- In essence:
  - Both are equivalent and describe the concept of "7" points
  - The first description gives much more detailed information
  - The second description is much more concise

- In engineering, the equivalence helps us to more easily understand complex systems and make them easier to model and compute

Equivalent element combinations

- In electrical circuits, **parallel and series combinations of the same types of elements can often be replaced by a single equivalent element**
- Values of elements of the same type arranged in parallel or series can be combined using one of two methods - **simple addition** or **reciprocal addition**

	Series	**Parallel**
Resistors	Simple addition	Reciprocal addition
Capacitors	Reciprocal addition	Simple addition
Inductors	Simple addition	Reciprocal addition
Voltage sources	Simple addition	N/A*
Current sources	N/A*	Simple addition

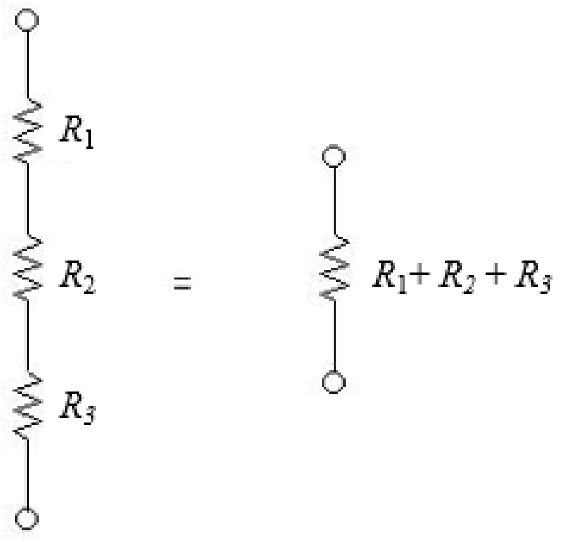
* Only possible in ideal analysis if the values are the same, in which case the combined value is the same value as those being combined.

Simple addition

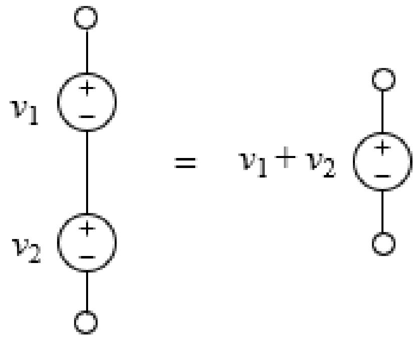
- Given a set of values to "add" together (e.g. R_1, R_2, R_3), simple addition is the **standard mathematical addition** of the values

$$R_{total} = R_1 + R_2 + R_3$$

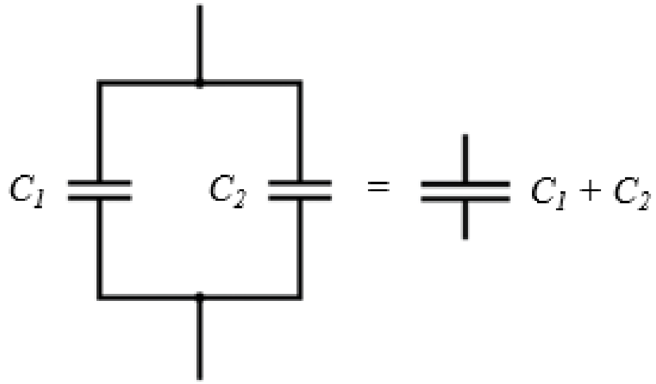
Resistors in series



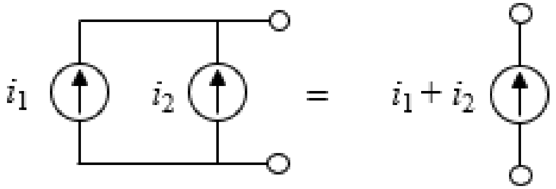
Voltage sources in series



Capacitors in parallel



Current sources in parallel



Reciprocal addition

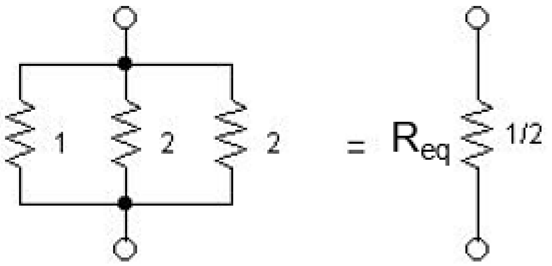
- Given a set of values to "add" together (e.g. R_1, R_2, R_3), reciprocal addition is where the **reciprocal of the total is equal to the sum of the reciprocals of the values**:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{or} \quad R_{total} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

⚠ **Pro-tip**: If there are only two element values (e.g. R_1 and R_2) to add using reciprocal addition, you can use the shortcut:

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

Resistors in parallel



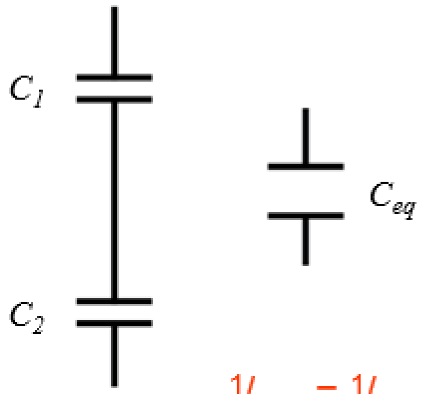
$^1/_{R_{eq}} = {^1/_1} + {^1/_2} + {^1/_2} = 2$

$R_{eq} = {^1/_2}$

$R_{eq} = 1 \,||\, 2 \,||\, 2$

Shorthand for indicating reciprocal addition

Capacitors in series



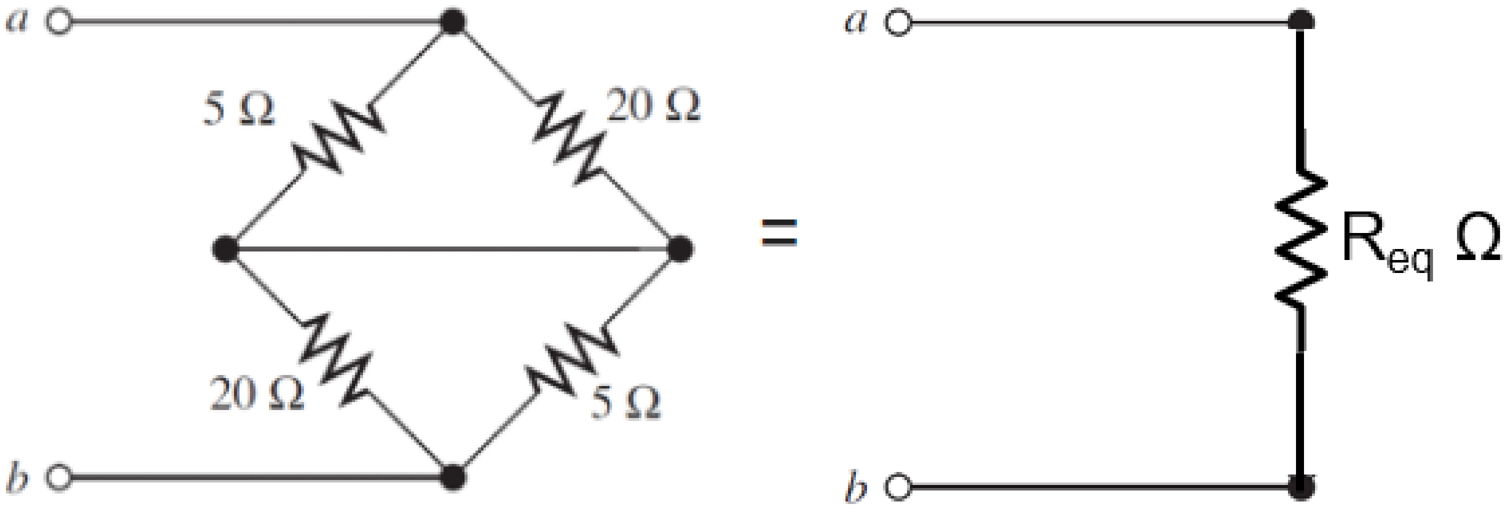
$^1/_{C_{eq}} = {^1/_{C1}} + {^1/_{C2}}$

$C_{eq} = ({^1/_{C1}} + {^1/_{C2}})^{-1}$

$C_{eq} = C_1 \,||\, C_2$

Computing equivalent resistances

- Given any network of resistors (or other types of the same elements), it is possible to reduce them to a single **equivalent resistance** using a simple procedure that involves replacing series and parallel combinations of resistors.
- Consider the circuit below left, with resistors connected to two terminals **a** and **b**. It is possible to replace the entire network of resistors with a single equivalent resistance as shown below right.



Equivalent resistance steps

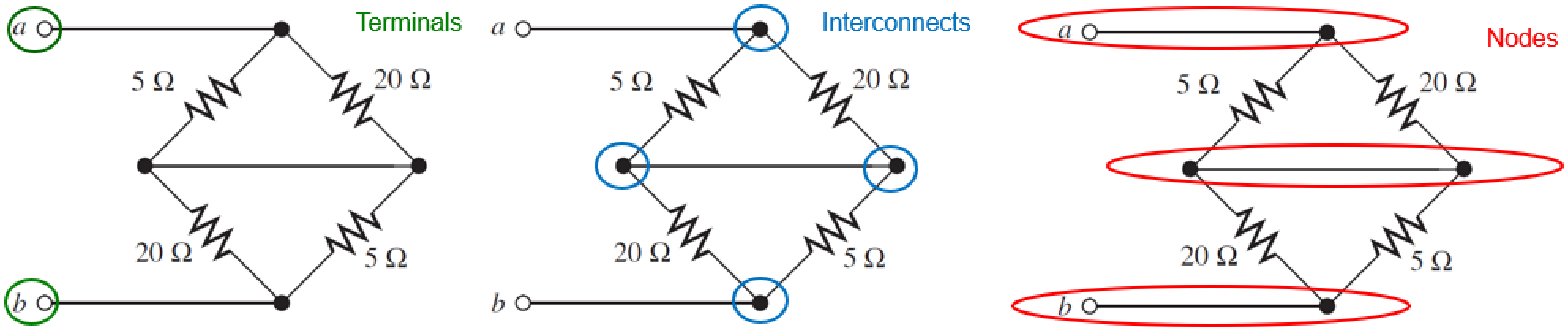
The steps to do this are as follows:

1. Identify nodes
2. Combine parallel resistors
3. Combine series resistors
4. Repeat until only one resistor is left

⚠ This process must be performed **relative to two specified locations (usually terminals)** in a circuit. If there are more than two terminals in a circuit, the equivalent resistance may be different between different pairs of terminals.

Example

1. Identify **nodes**, **not interconnects**



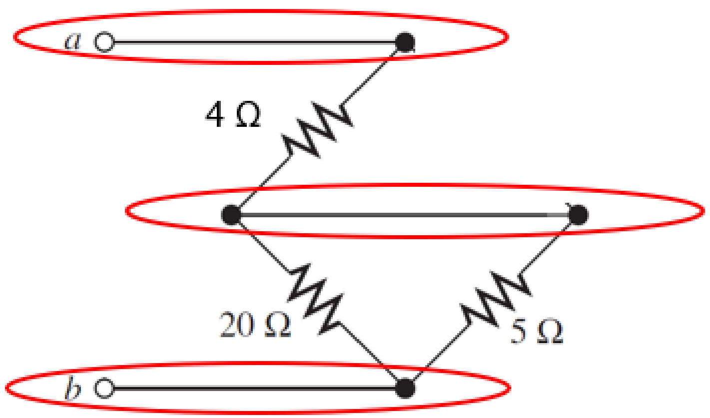
2. Identify the resistors that are arranged in parallel - recall that resistors in parallel share the same nodes at both ends. This means that:
   - the top 5Ω and 20Ω resistors are in parallel as they share nodes **a** and the **centre node** at both ends, and
   - the bottom 20Ω and 5Ω resistors are in parallel as they share nodes **b** and the **centre node** at both ends

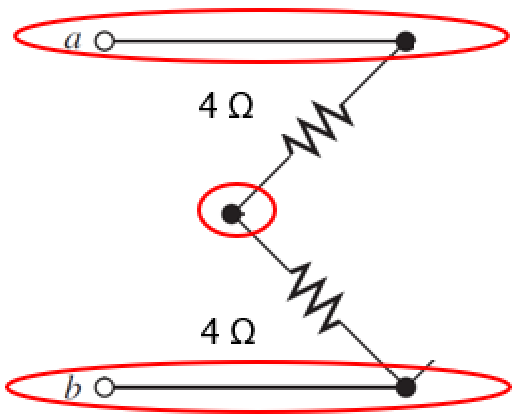
Example

2. Combine parallel resistors

$$5\Omega || 20\Omega = \frac{5\Omega \times 20\Omega}{5\Omega + 20\Omega} = 4\Omega$$

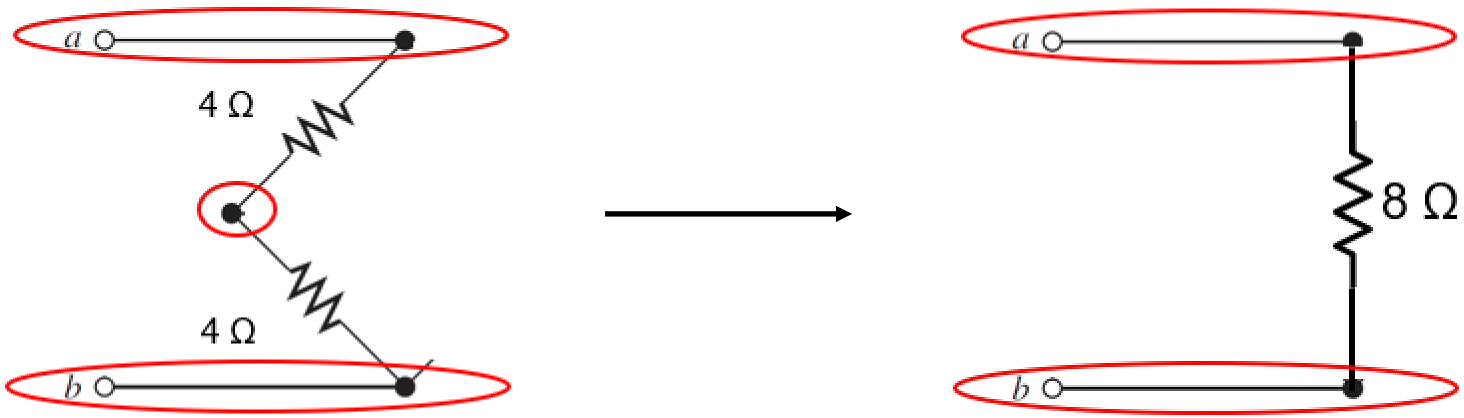
$$20\Omega || 5\Omega = \frac{20\Omega \times 5\Omega}{20\Omega + 5\Omega} = 4\Omega$$





Example

3. Identify and combine series resistors
   - two resultant 4Ω resistors are arranged in **series**, as any current that enters either resistor is forced to move through the other. We can then use simple addition to combine these together

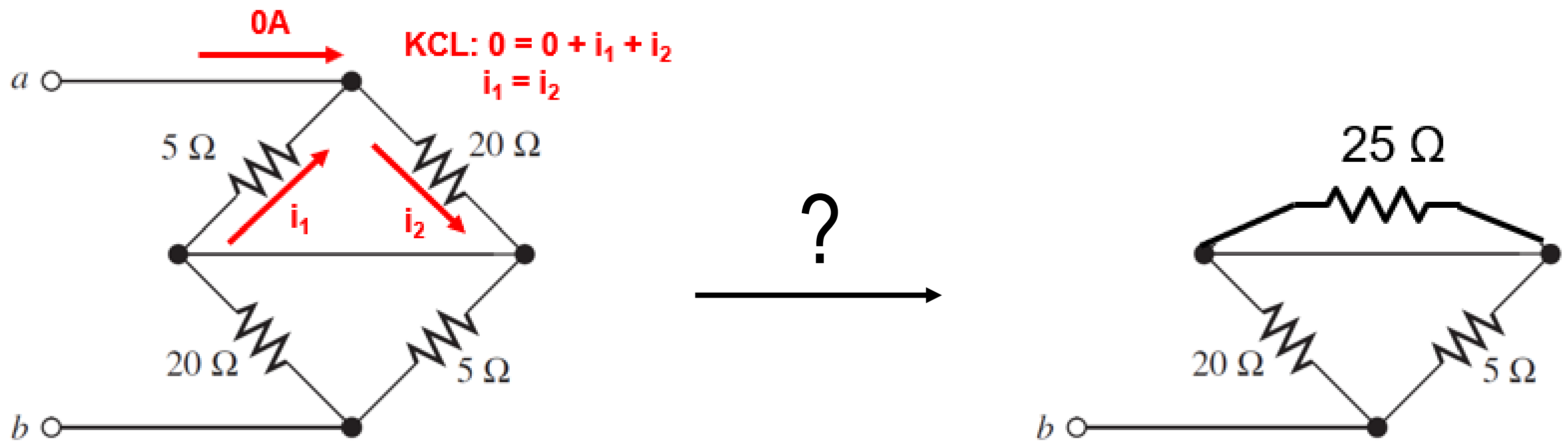


⚠ A key observation to note is that when we combine parallel resistances, the number and locations of nodes in the circuit stays constant. However, **when we combine series resistances, nodes are removed** from the circuit.

- We can now say that the original resistor network has an "equivalent resistance of 8 ohms, measured between terminals a and b".

Inconsistency?

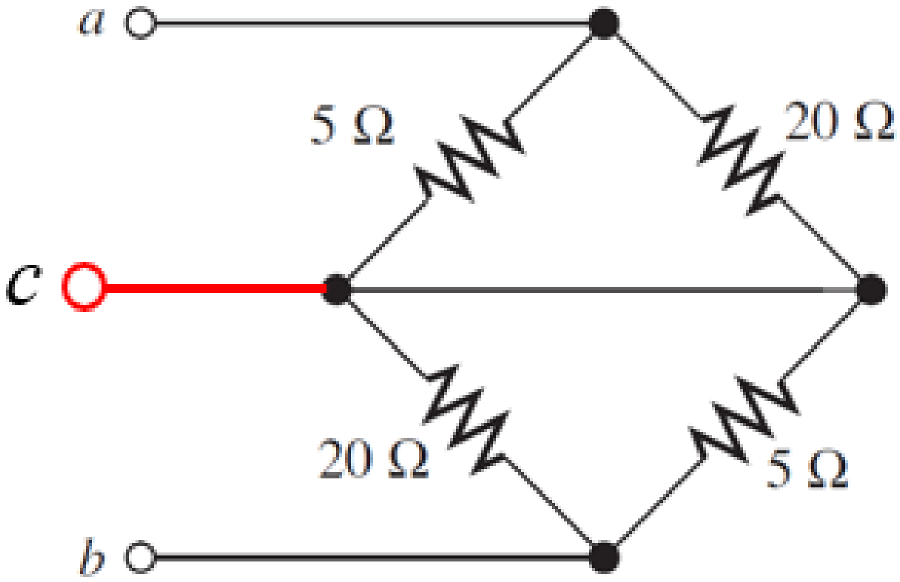
- You may come across situations where you believe that two resistors are in series and should be combined. Example shown below.
  - Given the fact that no current enters/exits terminal a, by KCL $i_1 = i_2$ in all cases for this circuit. Does this mean the top 5 Ω and 20 Ω are actually in series and not parallel?
  - Could we replace the resistors with a single 25 Ω resistance?



- No, combining those resistors in series removes the terminal a.
- Without node a, **we can't find the equivalent resistance between a and b!**

Exercise

- Find the equivalent resistance between terminals a and c ($R_{eq,ac}$), or b and c ($R_{eq,bc}$).



- Spoiler: $R_{eq,ac} = R_{eq,bc} = 4\Omega$

More complex circuit analysis example

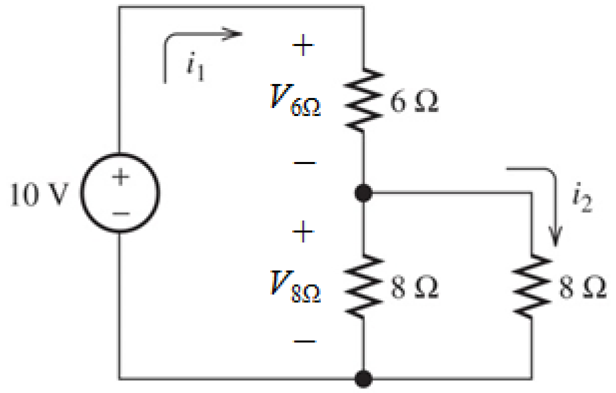
- In this example we will use techniques including Ohm's Law, equivalent resistance, KVL and (optionally) current dividers.

- As a beginner, you would not know "how or where to start" the analysis, don’t worry, practice will help train you to spot patterns!

- Next week, we will introduce a **systematic method of circuit analysis** which can be applied to all circuits, called **Nodal Analysis** which builds upon your understanding of the concepts from this week.

Example

- Consider the circuit, where we want to find the currents i_1 and i_2 and voltages $V_{6\Omega}$ and $V_{8\Omega}$.



- Current i_1 is the total current coming out of the 10V source

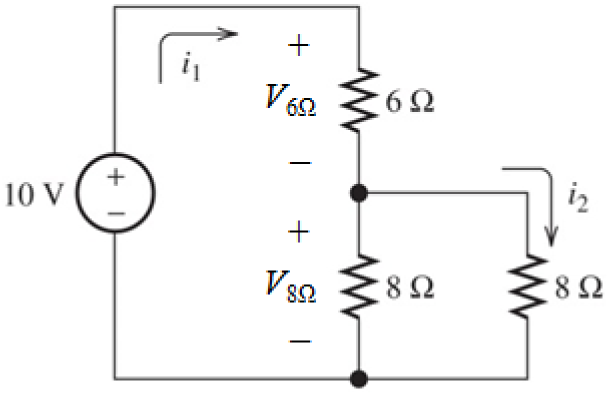
- Recall that Ohm's Law ($v=iR$) only works for a **single resistance**

- Since the 10V source is applied across the entire combination of resistors, this is not directly applicable.

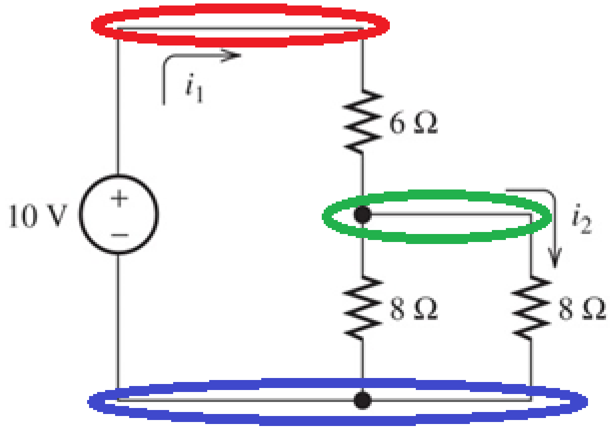
- To use Ohm's law to find i_1, we must **first calculate total equivalent resistance connected to the 10V source**.

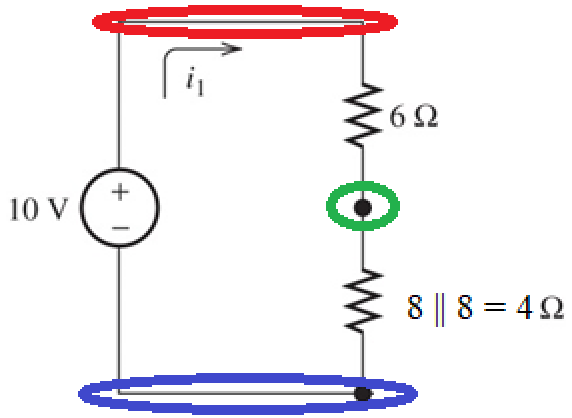
Example

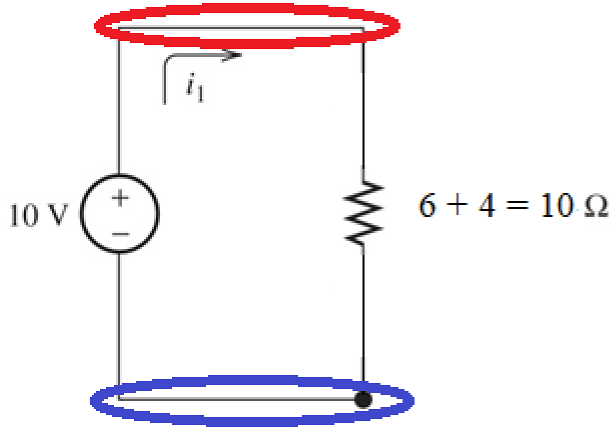
- Identify the nodes (red, green and blue)
- The 8 Ω resistors are in parallel, combine with reciprocal addition to 4 Ω
- The 6 Ω and equivalent 4 Ω are in series, combine to equivalent 10 Ω
- Apply Ohm's Law on equivalent circuit to find i_1



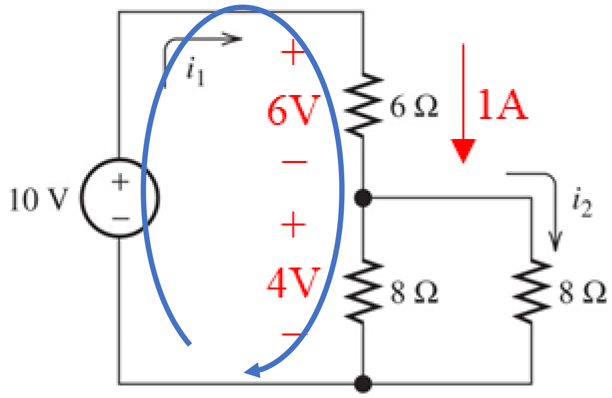
$$i_1 = \frac{V}{R_{eq}} = \frac{10V}{10\Omega} = 1A$$







Example



- We can use i_1 and Ohm's Law to find:

$$V_{6\Omega} = i_1 \times 6\Omega = 6V$$

- Next, find voltage across the left 8 Ω resistors using KVL around blue loop:

$$0 = (-10V) + 6V + V_{8\Omega}$$
$$V_{8\Omega} = 4V$$

- Since both 8 Ω resistors are in parallel, they share same voltage across them so we can find i_1 using Ohm's Law across right 8 Ω resistor:

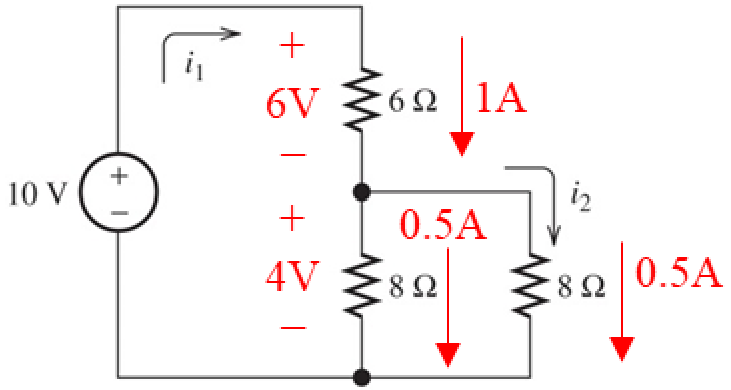
$$i_2 = \frac{4V}{8\Omega} = 0.5A$$

Example

- Alternatively, we could apply a current divider to the two 8 Ω resistors, using 1 A as the source current between the two branches:

$$i_2 = 1A \times \left(\frac{8\Omega}{8\Omega + 8\Omega}\right) = 0.5A$$

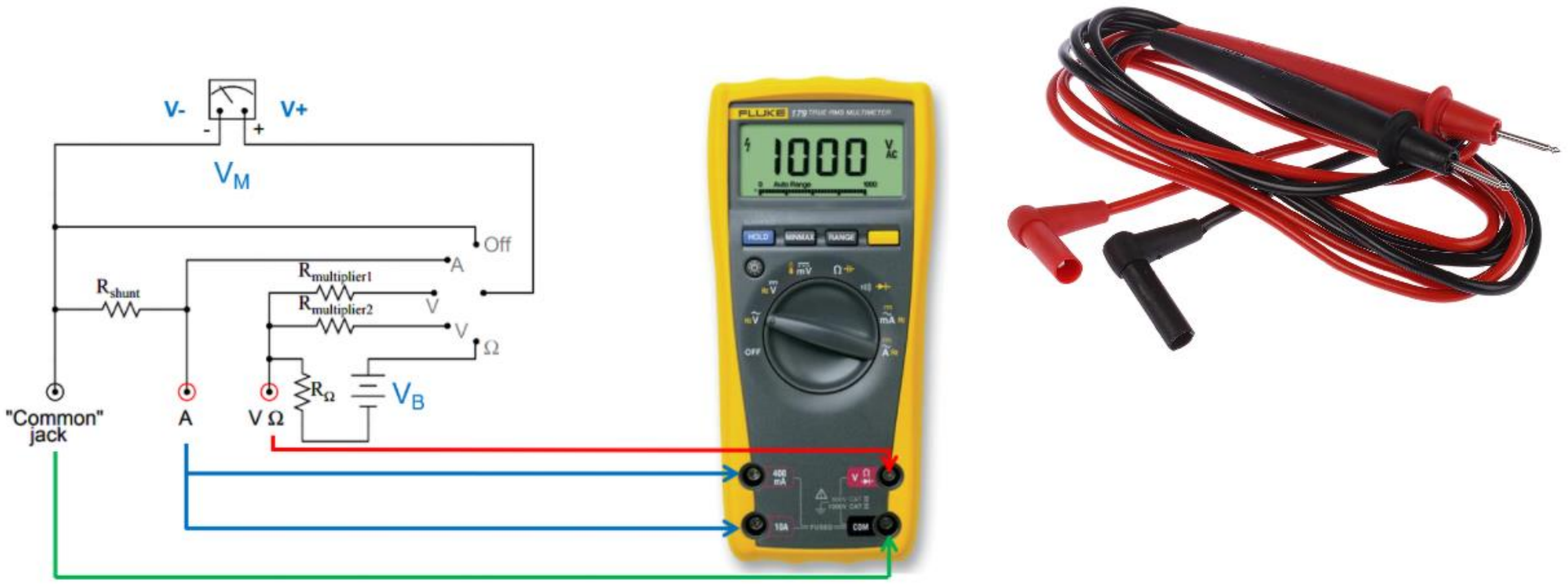
- In either case, the circuit is now fully analysed, with all currents and voltages found:



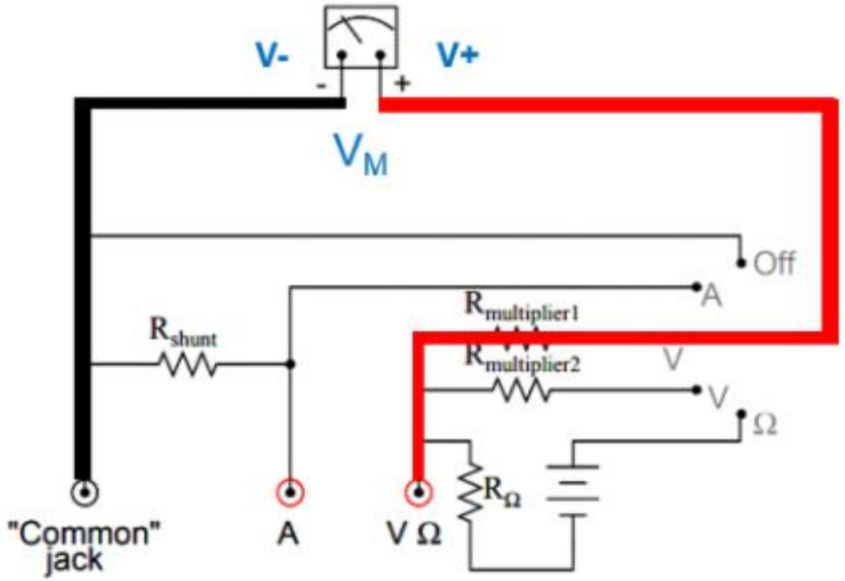
Multimeters

- Given circuits in the physical world, we can use a device called a **multimeter**, shown below, to directly measure values of current, voltage and resistance in circuits.

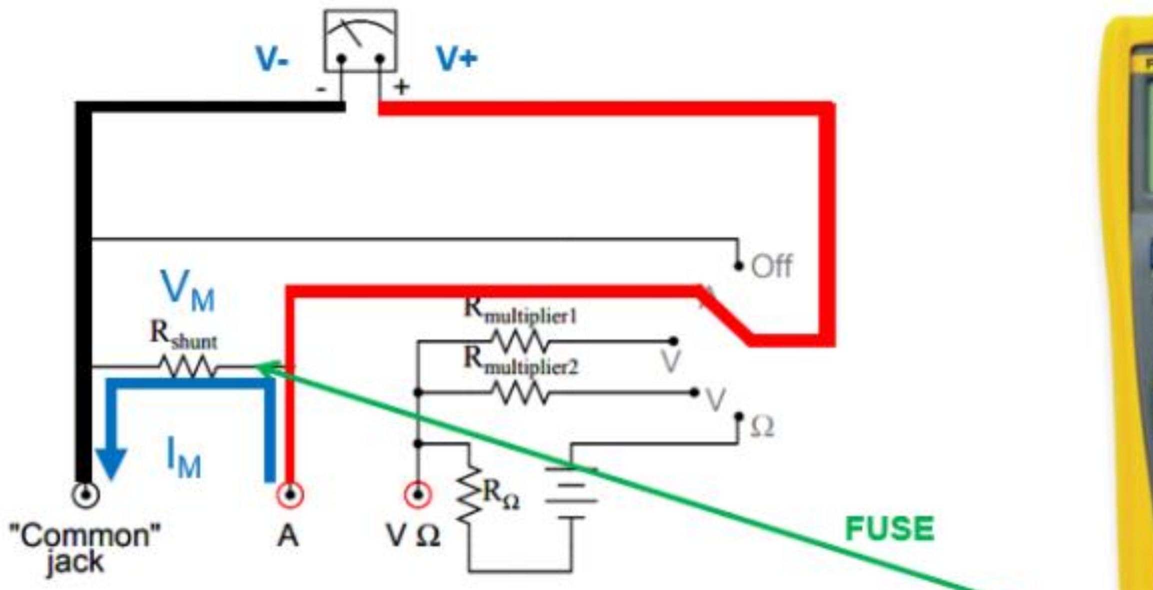
General digital multimeter structure



Measuring voltage



Measuring current

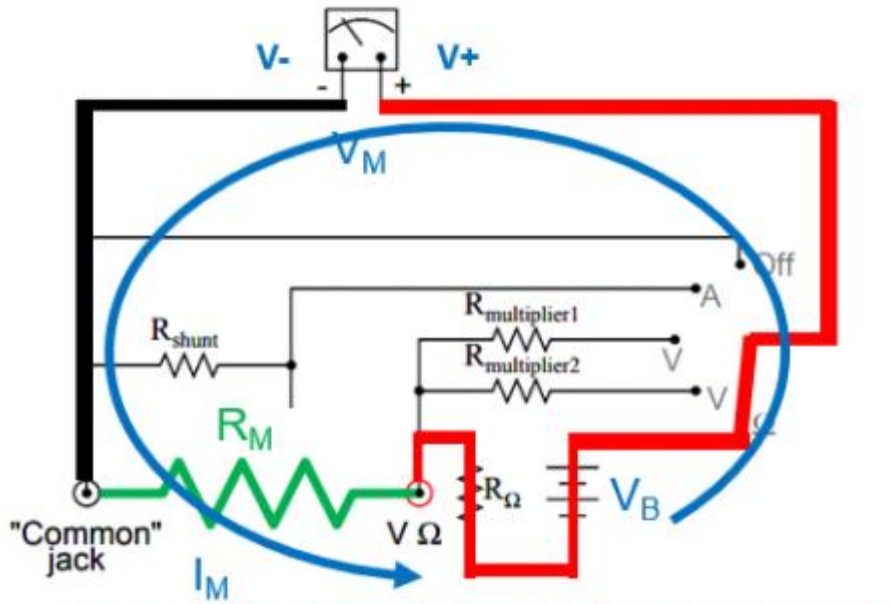


Use Ohm's Law!

$$I_M = V_M / R_{shunt}$$

⚠ If you are trying to measure a small current, and your multimeter isn't responding, it's **likely the fuse has already been blown** previously by someone else (**or you have the probes plugged into the wrong connectors**). In the multimeters pictured, there are "10 A" and "mA" connectors, so we recommend using the "10 A" connectors first as their fuses are unlikely to have failed. The reading may not be as precise, but this is likely to be fine for our applications.

Measuring resistance



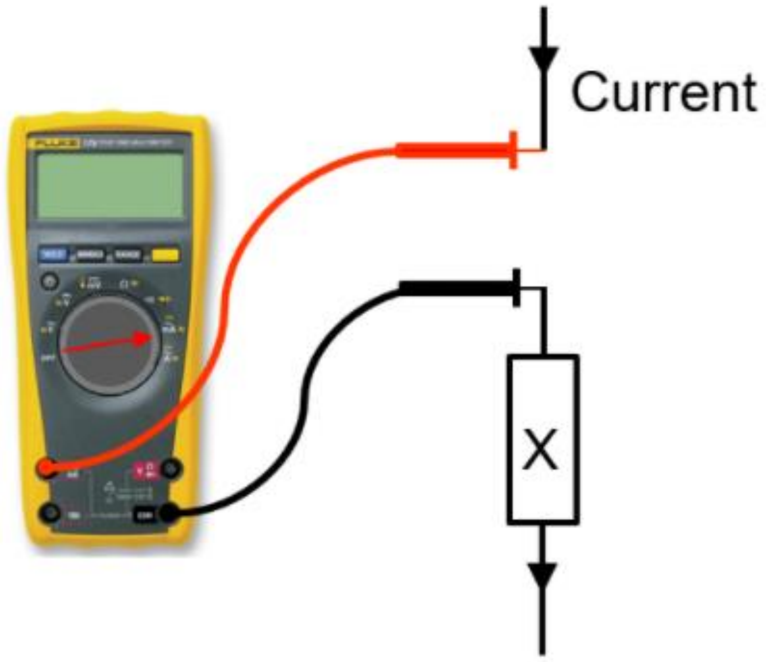
Use Kirchhoff's Voltage Law!

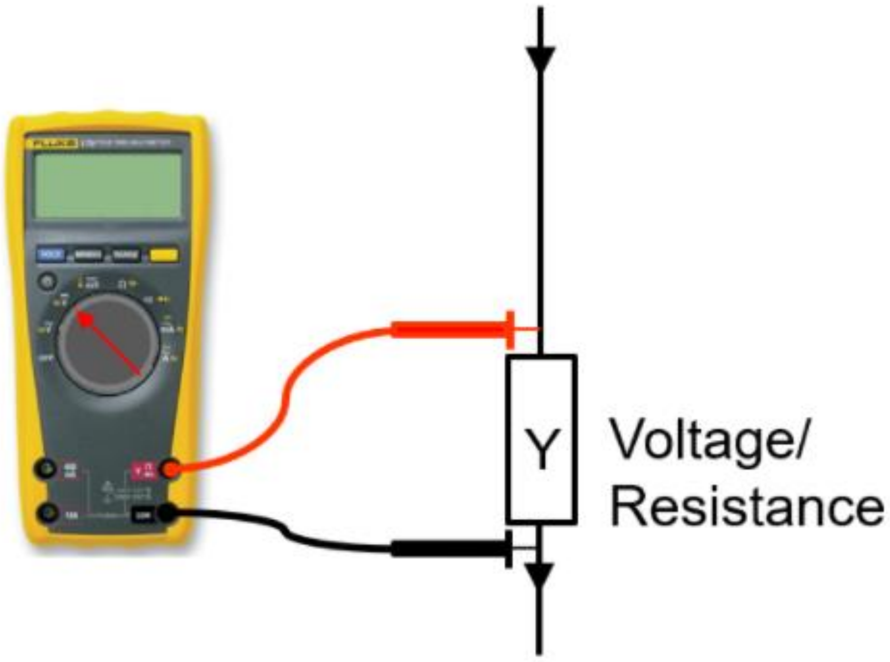
$$V_B = I_M(R_M + R_\Omega) + V_M$$

⚠ A consequence of the way resistance is measured is that **we can only measure resistance when the circuit is off**. If there is a current moving through the resistor from the circuit, then the current through the resistor is not only I_M, and our measured value V_M will not be converted to a true representation of the resistance R_M.

Measuring values in circuits

- The images below show how the multimeter should be used to make measurements of current and voltage/resistance for elements in a circuit.





Measuring values in circuits

1. Make sure the probes are plugged into the correct ports!
2. Make sure the switch is on the correct setting
3. The **red probe** is the "positive" and **black probe** the "negative" terminal
4. Measure:

   current with the multimeter **connected in series** with the element

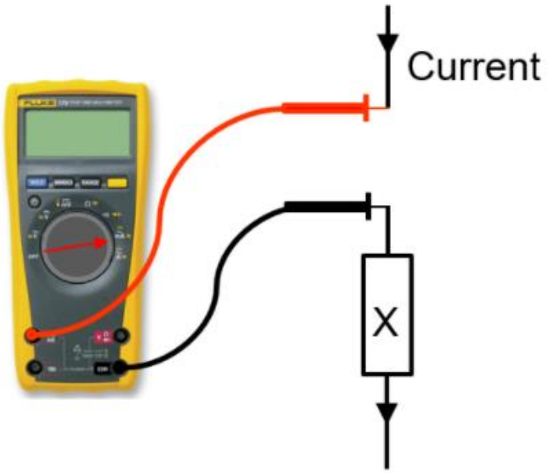
      If true current direction **enters the positive terminal**, the measurement will be displayed as a positive value

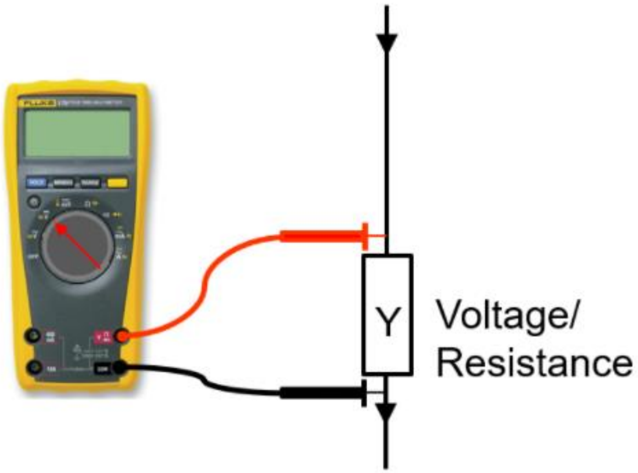
   voltage with the multimeter **connected in parallel** with the element

      If true voltage is **higher at positive terminal**, the measurement value will be displayed as a positive value

   resistance with the multimeter **connected in parallel** with the element

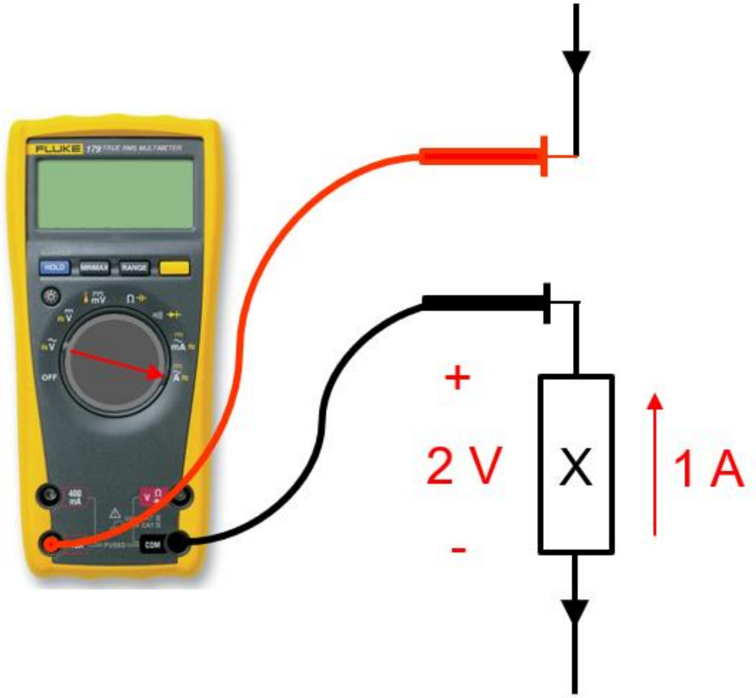
      Resistance measurement is not polarity sensitive and will always be displayed as a positive value





Multimeter measurement examples

- **Question:** The multimeter is configured (probe plugs and switch) to current measurement mode. What value would appear on the multimeter?
- **Answer:** -1 A
- **Question:** If the multimeter was instead configured to voltage measurement mode, what value would appear on the multimeter?
- **Answer:** 0 V



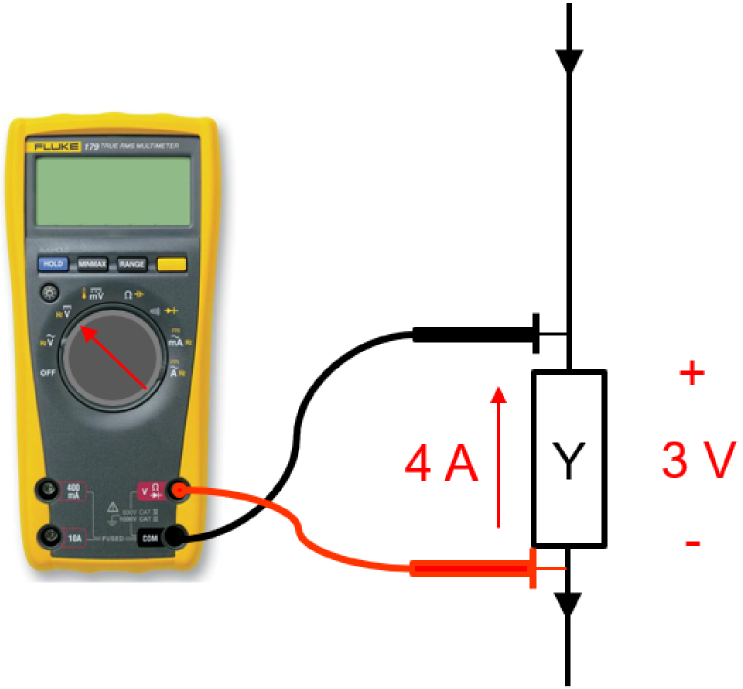
Multimeter measurement examples

- **Question:** The multimeter is configured (probe plugs and switch) to voltage measurement mode. What value would appear on the multimeter?

- **Answer:** −3 V

- **Question:** If the multimeter was instead configured to current measurement mode, what value would appear on the multimeter?

- **Answer:** Error, and you may blow up the multimeter fuse!



⚠ Make sure you don't try to measure current across an element, the multimeter must be connected in series with it.