Spectrum Sensing using Sparse Bayesian Learning

Aiswarya K V, Gundabattula Naga Rama Mangaiah Naidu, Kalaiarasi K, Kavya D and Kirthiga S

***Abstract*—The availability of the radio spectrum is limited. To compromise the increasing demand for high data rate devices, this fixed spectrum need to be used efficiently. The existing challenge is that the larger portion of the licensed spectrum is underutilized. Cognitive Radio is a technology introduced to help to detect the radio spectrum is occupied or not. Spectrum sensing helps to detect the spectrum holes and provides high spectral resolution capability. This is done using sparse techniques such as Compressive sensing and Sparse Bayesian Learning techniques. Compressive Sensing algorithms such as Basis Pursuit and Orthogonal Matching Pursuit are analyzed. Based on the concept of Sparse Bayesian learning, an expectation maximization algorithm is introduced for spectrum sensing and recovery of the original transmitted signal in cognitive radio systems. Performance comparison is done between proposed algorithms and is validated using Register Transfer Level- Software Defined Radio.**

***Index Terms*—Compressive sensing, Expectation Maximization, Relevance Vector Machine, Register Transfer Level (RTL-SDR), Spectrum sensing.**

I. Introduction

SPECTRUM Sensing [1] is the task of obtaining awareness about spectrum usage and the presence of primary users in an area. The existing problem includes demand for high data rate devices and the available spectrum is fixed. So, it is required to find new ways to exploit this available spectrum. Many conventional methods for spectrum are Energy Detection, Matched filter, and cyclostationarity [2]. The signal is detected either by comparing the output of the energy detector against a threshold or using matched filters. These methods have many advantages namely, less computational complexity and the receiver need not know about the primary user's signal. Apart from these advantages, there are many disadvantages to these methods. The major disadvantages to the above mentioned conventional methods maybe sensitivity of threshold value, short detection time, low speed and suitable only for high SNR signals [3]. Based on the energy detection principle, the energy of the received signal is greater than noise. For the Matched filtering method perfect knowledge of primary users is essential. These techniques are not suitable for spread spectrum modified signals. Due to these disadvantages, sparse recovery techniques for spectrum sensing is proposed signal compression process, all samples of the original signal are taken and a large portion of it is instantly discarded. But this process is quite costly.

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So, a new concept for signal compression is proposed. It combines signal reception with compression as a single step which makes it cost-efficient. When compared with the Nyquist Shannon Sampling theory, in Compressive Sensing(CS) the original signal is reconstructed with a minimum number of samples ($f_s < 2f_m$). The concept of the compressed sensing technique is that a sparse signal can be recovered from exactly from a very small number of measurements with high probability by solving a sparsity-constrained optimization problem [4]. Currently basis pursuit (BP), orthogonal matching pursuit (OMP) and Sparse Bayesian learning (SBL) are popular reconstruction algorithms. Basis pursuit (BP) algorithm is an optimization technique used to solve the l_1 minimization problem, which is linear in nature. OMP is an iterative greedy algorithm that selects at each iteration the appropriate basis which is highly correlated with the current residuals. Like the above mentioned recovery algorithms, Bayesian method can also be used for sparse signal recovery. The process of reconstruction in the Bayesian method is similar to the traditional CS algorithm but it is made flexible by adding different prior information. Thus Bayesian methods have better anti-noise performance than BP and OMP algorithms. These sparse recovery techniques and performance comparison on the robustness of each algorithm is presented. Based on Sparse Bayesian Learning technique the sparse signal can be recovered with minimal errors and computational cost is also reduced. Also, the Mean Square Error (MSE) and the probability of detection (Pd) show an improvement in the proposed algorithm. From the analysis in simulation, it is found that using SBL technique sparse signal can be acquired with minimal errors and reduced computational cost. The spectrum sensing simulation analysis is validated using Register Transfer Level - Software Defined Radio (RTLSDR). It is a communication receiver, operating in the frequency range (25-1750) MHz and supporting the bandwidth of 2.4MHz and hence used for real-time signal acquisition for the proposed work [5]. SDR is a technology which is promising in the area of radio communication that digitalizes the radio signals and turns the complicated hardware problems into simple software problems [5].The signal is received using RTL-SDR, which is a radio communication system that can be implemented by means of software instead of hardware components. It is mainly used for down-conversion of frequency signals into an audio frequency band [6-11]. SDR is one of the most easily adaptable and flexible software that tunes any frequency in range according to the hardware.

This paper is organized as follows. Section II introduces the Compressed sensing based Spectrum Sensing and its algorithms are described. In Section III SBL techniques are introduced and discussed. Simulation results and discussions are provided in Section IV followed by conclusions drawn in Section V.

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II. COMPRESSED SENSING BASED SPECTRUM SENSING

In this section, a detailed description of the compressed sensing techniques is presented. Compressed sensing is a signal processing technique used to efficiently acquire and reconstruct a signal, by finding solutions to underdetermined linear systems. The two main reconstruction algorithms BP and OMP recover the sparse signal are discussed below. An underdetermined system is a linear system that has more unknowns than equations and generally has an infinite number of solutions. The system equation is $y=Ax$ where x is the unknown and A is the sensing matrix. Compressive sensing is the signal processing technique which is effectively used to recover the original signal, by finding solutions to the underdetermined linear system. This method helps to save in terms of cost and size. Reconstruction in l_0 norm minimization is highly non-convex and very difficult to solve the optimization problem. So l_1 norm minimization for reconstructing the original signal, which is convex in nature is used and possible via exploiting sparsity .

A. Basis Pursuit

Basis pursuit is a convex optimization algorithm technique which is used to recover the original signal. The main advantages of basis pursuit algorithm are the prior knowledge of signal sparsity is not required, reconstruction problem can be constructed easily because linear programming problem can be solved easily, even under noisy conditions they perform well. Its application is used in image processing, communication networks, and radar systems. To precisely reconstruct x from y, x should be sparse. And the sensing matrix must be constructed in such a way that $y=A^Tx$ should contain the necessary information for proper reconstruction of x. Moreover, the proper algorithm should be selected for implementing CS and hence which gives successful recovery of the given original signal. In compressive sensing technique basis pursuit algorithm is suitable to solve the l1minimization problem since it is a linear programming problem .

Steps to implement BP in SS
x(t) - received time domain signal
T - total time interval
T0 -sampling period for x(t).
After sampling the time domain signal results in the R × 1 vector.
R = T/T0 (R- the number of samples in the given time interval) As the signal is sparse in the frequency domain,
F - DFT sequence
Discrete Fourier Transform (DFT) representation of the given signal is as follows:

$$x_f=F_R\, x_t \tag{1}$$

A random sensing matrix is used to compress the sampled vector to a lower dimension as follows

$$y= \Phi\, x_f \tag{2}$$

Φ - random sensing matrix, with N×R dimension, where N < R.

As the given signal $\bar{x}_f$ is sparse in nature, using CS techniques, $\bar{x}_f$ can be recovered by N measurements as follows

$$\bar{x}_f = \arg\min\| x_f \|_1 \tag{3}$$

The primal-dual interior-point method has been used to solve the linear programming problem. The procedure of BP follows a classical Newton method which is used to recover the original signal.

B. Orthogonal Matching Pursuit

OMP is a greedy compressed sensing recovery algorithm. In compressed sensing, minimizing the l_0 norm is highly non-convex, in order to solve it OMP algorithm is proposed. It is an iterative algorithm that selects the appropriate basis in greedy fashion in order to recover the original signal.
Consider the model,

$$y = \Phi x$$

where,
y - Corrupted signal,
Φ - Sensing matrix, and
x - Original signal

Assume that x is an arbitrary k-sparse signal in R^d where k is sparsity of x, with N measurement vectors i.e $\{x_1, x_2 \ldots x_N\}$..For an $(N\times d)$. Matrix Φ whose rows are the measurement vectors. The matrix Φ is referred to as sensing matrix and its columns are denoted as $\{\phi_1,\phi_2,\ldots\phi_d\}$. Since x has only k non-zero components, the data vector $y=\Phi\, x$ is a linear combination of k columns from Φ. This perspective allows transporting the results on the sparse approximation to the signal recovery problem [9]. To identify the original signal x, the correlation of measurement vector Φ with residue r has to be determined. Based on this recessive process, the original signal x is being reconstructed. In addition to this Least Squares (LS) optimization is performed then to recover the original signal.

The OMP works as follows,
Input
An $N\times d$ sensing matrix Φ
A vector y which is the corrupted signal
The sparsity level k of the original signal x
Output
An estimation of $\hat{x}$ of the original signal x

The steps required to process the input are as follows,
Step (1): Initialization of residual $r_0 = y$, index set $\Lambda_0 = \emptyset$, and iteration counter t = 1.
Step (2): Find the largest coordinate λ_t

In this step, it finds a column of sensing matrix Φ which yields the largest projection on current residual r (vector y in 1st iteration) that is λ_t and no column will be selected twice.

$$\lambda_t = \mathrm{argmax}_j=1,\ldots,d|r_{t-1},\ \phi_j| \tag{4}$$

Break the tie deterministically, if the maximum occurs for multiple indices.
Step (3): Augment the index set

$$\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\} \tag{5}$$

It is a matrix of all the chosen columns and initially Λ_0 is a null matrix.
Step (4): Estimate $\bar{x}$ using Least Square (LS) optimization technique

$$\bar{x}_t=argmin_x||\Lambda\bar{x}\text{-}y||_2 \tag{6}$$

Step (5): Update the residual r

$$r_t = y - \Lambda_t\, \bar{x}_t \tag{7}$$

Step (6): Increment t, and repeat the steps from step (2), if t < k

Step (7): The estimation of $\hat{x}$ for the original signal has non-zero indices at the corresponding columns of Λ_k.

In this section, an explanation of the basis pursuit algorithm and orthogonal matching pursuit algorithm for sparse signal recovery is discussed. The way basis pursuit algorithm works are explained using the classical Newton method. Whereas in OMP a complex problem is resolved into many small optimization problems to find the convex solution. Some applications where the basis pursuit algorithm can be used are also briefed. OMP is easier to implement and efficient when highly sparse but BP is more powerful with high probability to recover a sparse signal.

III. USING SPARSE BAYESIAN LEARNING

Bayesian learning helps to find out the posterior distribution of weights here the weight prior is provided as one of the input. The knowledge about the weight will get modified once new data is observed. Also the computational cost increases as $O(N^3)$ with the increase in the number of training samples. To accommodate the change in weights and reduce the computational complexity SBL is proposed. In SBL predictive mean of the distribution is used to predict the prior weights which are sparse in representation. Relevance Vector Machine (RVM) represents a probabilistic extended linear model with prior on weights that gives sparse solutions. To reduce this computational complexity a new incremental approach namely Expectation Maximization (EM) algorithm is proposed.Based on this concept of SBL an Expectation and Maximization algorithm is introduced for spectrum sensing. The performance accuracy is greatly increased in this algorithm. The enhancement in accuracy leads to the improvement in spectrum utilization of wireless communications. A linear model is a function that outputs some solution and all the input data are linearly combined. If all those data are non-linearly combined, then they are called the Extended Linear Model. In Data Fitting problems the final objective is to identify all the weights. The estimation of the probabilistic distribution of weights needs to be inferred as it provides predictive distributions rather than point estimates. Regularization is a process of including additional information to solve an ill-posed problem. It makes the weights to be small by introducing an extra term to the objective function, that guarantees stable solutions and smoother functions are produced.

The general form of the Extended Linear Model can be formulated as follows:

$$f(x_i) = \sum_{j=1}^{M} \emptyset_j(x_i)\, w_j = \emptyset(x_i)\, w \qquad (8)$$

where $X_i=\{x_i|i=1,\ldots, N\}$ is the training inputs of order Nx1, $\emptyset$ is response matrix (NxM) and w is the weight set(Mx1). The weights w are the unknown parameters of a model. To find this weight set, the training target (y) is needed and is given as follows:

$$y_i = f(x_i) + \epsilon_i \qquad (9)$$

Where ϵ_i is the noise which follows Gaussian distribution $N(0,\sigma^2)$

The negative logarithm of the likelihood function of the weights is used as the target function for estimating w.

$$L = \frac{1}{2}\log(2\pi) + \frac{1}{2}\log(\sigma^2) + \frac{1}{2}||y - \emptyset w||^2 \qquad (10)$$

Since its negative, the log-likelihood function needs to be minimized [13]. The derivative of L is taken with respect to w, equated to zero and hence the value of w is estimated.

$$w = (\emptyset^T \emptyset)^{-1} \emptyset^T y \qquad (11)$$

This is the basic expression of weight given by normal equations. But if number of unknowns is more than the number of measurements i.e. M > N, then infinitely many solutions are produced for w. The term $\emptyset^T \emptyset$ gives a matrix of order M x M, whose rank is less than M so it cannot be inverted. Hence these normal equations need to be regularized by adding a regularization parameter λ given as follows:

$$w = (\emptyset^T \emptyset + \lambda I)^{-1} \emptyset^T y \qquad (12)$$

This value of λ needs to be learned for finding the value of w. In RVM, the regularization parameter λ is replaced by the hyperparameters α *and* β [13]. If uniform priors are defined, then Maximum A Posteriori (MAP) becomes equal to Marginal Likelihood function. So, to find values for hyper parameters the posterior distribution of it is maximized and is given as:

$$p(\log\alpha, \log\beta \,|y, X) \;\propto\; p(y|X, \log\alpha, \log\beta)\; p(\log\alpha, \log\beta)$$

which follows N (0, K). (13)

where $K = \beta I + \emptyset A^{-1} \emptyset^T$. But its logarithm is maximized practically.

The Log-likelihood function is,

$$L = \frac{1}{2}\log(2\pi) + \frac{N}{2}\log\beta - \frac{1}{2}\log|\in| - \frac{1}{2}\sum_{j=i}^{M}\log\alpha_j + \frac{1}{2\beta} y^T(y - \emptyset\mu) \qquad (14)$$

where α *and* β are the hyperparameters , $\in$ is covariance and μ is the mean.

Instead of directly minimizing negative log evidence for the training of RVM, the Expectation Maximization (EM) algorithm is used. EM is generally used for maximization so L is considered to be positive in this algorithm [14].

EM algorithm

Given a Gaussian model, the goal is to optimize the log- likelihood function with respect to the parameters [15], the steps are given as below:

1. Initialize all the parameter values namely mean μ_k variance σ^2 and the hyper parameters $\propto$ *and* β.

2. E-Step: Find mean, covariance and log-likelihood function for present values by using formulas given by:

$$m = \beta \in \varnothing^T y \text{ and } \in = (A + \beta\varnothing^T \varnothing)^{-1} \quad (15)$$

3. M-Step: Find new values for hyperparameters $\propto$ *and* β.

$$\propto_i^{new} = \frac{1}{m_i^2 + \in_{ii}} \quad (16)$$

$$(\beta^{new})^{-1} = \frac{||y - \varnothing m_N||^2 + \beta^{-1}\epsilon_i\gamma_i}{N} \text{ where } \gamma_i = 1 - \alpha_j \in_{jj} \quad (17)$$

In this section Expectation Maximization algorithm using SBL technique has been discussed. Because of the implementation of the probabilistic model in spectrum sensing, the computational complexity is greatly reduced in this algorithm. In addition to reduced computational complexity but the Mean Square Error (MSE) is found to be lower compared to the CS algorithms.

IV. Results And Discussions

The simulations have been done in this section are estimated to analysis the capabilities of the proposed algorithm. The comparison of the three algorithms has been made by the Receiver Operating Characteristics (ROC). It is found that the SBL performs better compared to the Compressive sensing algorithms. Fig. 1. (a) shows the real-time signal received using RTL-SDR. Fig. 1(b) is the Discrete Fourier Transform of the time domain signal. This signal is given as input to all the three recovery algorithms. Fig .2 is the comparison of recovered signal obtained using the OMP, BP and SBL algorithms whereas Fig. 3 shows the reconstruction error of each of the algorithms. ROC is a plot of Probability of false alarm (P_f) against Probability of detection (P_d). Probability of detection (P_d) is analyzed fixing probability of false alarm (P_f) to be zero and this case is considered and compared between algorithms. Fig. 4 shows the ROC plot of the presented algorithms. From the results, SBL has a high P_d value compared to CS algorithms. RTL-SDR is used to acquire real-time signal whose center frequency is 91.1MHz. The specifications of the RTL-SDR(R2832) are tabulated in Table I.

TABLE I
SPECIFICATIONS OF RTL-SDR

S.No	Specification	Value
1.	Frequency Range	25MHz-1.7GHz
2.	Power	25mW
3.	Data Throughput	6.114Mbps
4.	Input Impedance	377 ohms
5.	Antenna Gain	Max 15dB

Standard units are the same as MHz=Mega Hertz, GHz=Giga Hertz, mW= milli Watt, Mbps= Megabits per second, dB=deci Bel.

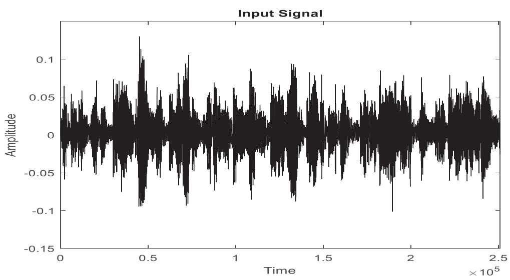

Fig. 1. (a)

Fig. 1. (a) represents the acquired real-time signal from RTL SDR

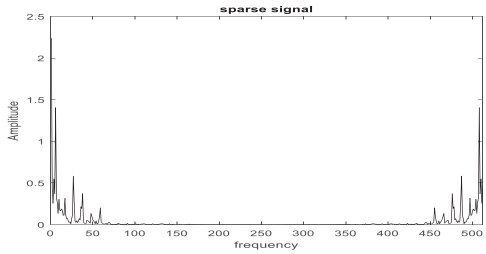

Fig. 1. (b)

Fig. 1. (b) is the Discrete Fourier Transform of the time domain signal

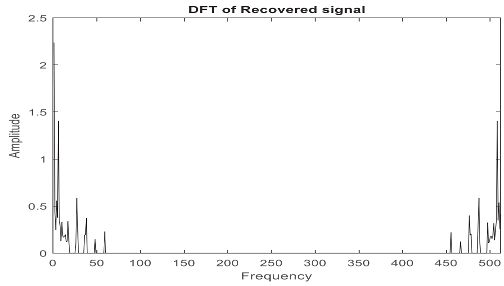

Fig. 2. (a)

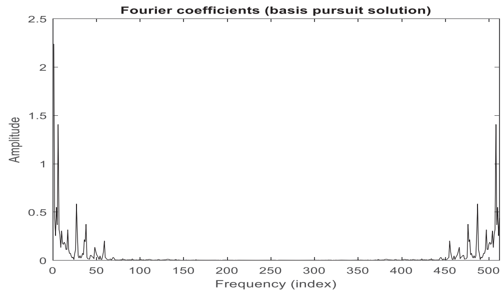

Fig. 2. (b)

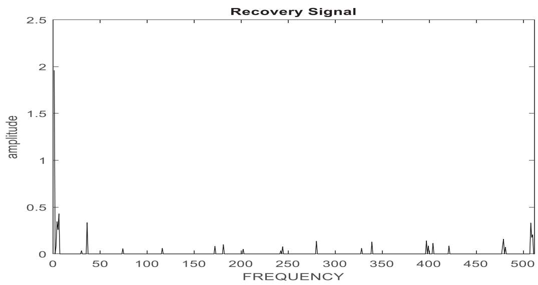

Fig. 2. (c)

Fig. 2(a),(b) and (c). Comparison of recovered Signal

Fig. 2. (a), Fig.2. (b) and Fig.2. (c) represents Discrete Fourier Transfer coefficients of the recovered signal using Orthogonal Matching Pursuit, Basis Pursuit, and Sparse Bayesian Learning Algorithms respectively. Fig 3. (a), Fig 3. (b) and Fig 3. (c) represents reconstruction error using Orthogonal Matching Pursuit, Basis Pursuit and Sparse Bayesian Learning Algorithms with Mean Square Error values of 0.0401,0.0153 and 0.0044 respectively

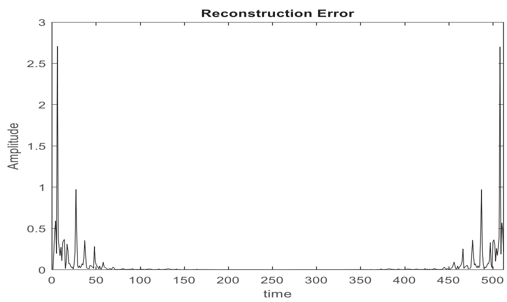

Fig. 3. (a)

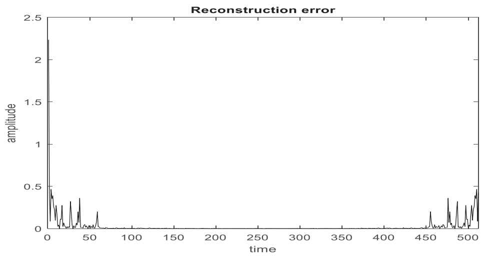

Fig. 3. (b)

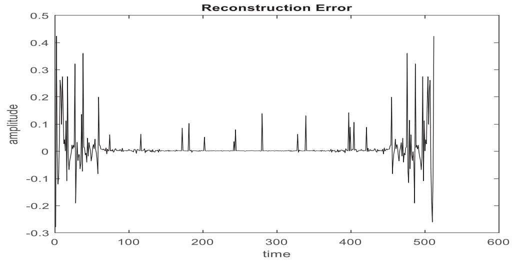

Fig. 3. (c)

Fig. 3(a)(b) and (c).Reconstruction error using orthogonal

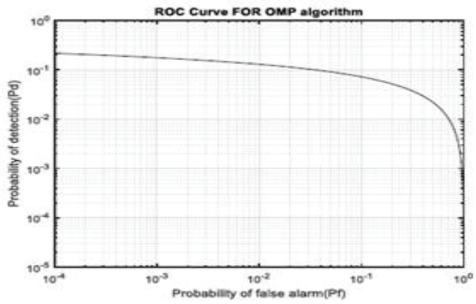

Fig. 4. (a)

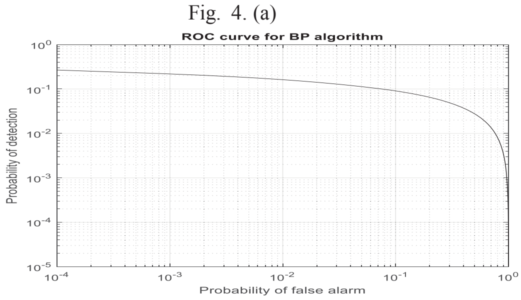

Fig. 4. (b)

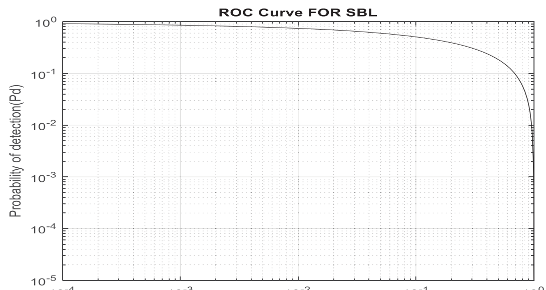

Fig. 4. (c)

Fig. 4(a)(b) and (c) Receiver Operating

Fig. 4. (a), Fig 4. (b) and Fig. 4. (c) represents Receiver Operating Characteristics for Orthogonal Matching Pursuit, Basis Pursuit, and Sparse Bayesian Learning Algorithms with P_d values 0.2162,0.266 and 0.9223 respectively when P_f=0.

V. CONCLUSION

The three algorithms namely Basis pursuit, Orthogonal Matching Pursuit, and Sparse Bayesian learning are compared and tabulated on the basis of their corresponding Receiver Operating Characteristics. Orthogonal Matching Pursuit is known to be fast but not as stable as Basis Pursuit. From the results, it is clear that the Basis Pursuit algorithm has a high performance compared to the Orthogonal Matching Pursuit algorithm. Compared to Compressive sensing algorithm, Sparse Bayesian learning shows improvement in reconstruction error, higher probability detection and hence found to be precise optimal technique for spectrum sensing.

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