

$$\frac{90.5}{116} = 78 + 2 = \textcircled{80}$$

No partial credit will be given for multiple-choice problems, and I will not look at your work for these problems.

For all other problems, YOU MUST SHOW WORK to receive any credit for the problem. A correct answer without supporting work will receive no credit.

No scrap paper will be collected. Work must be shown on the test for non-multiple choice problems.

MULTIPLE CHOICE. CIRCLE THE CORRECT ANSWER.

Use the Leading Coefficient Test to determine the end behavior of the polynomial function.

1) $f(x) = -3x^4 + 3x^3 + 3x^2$

☒ A) falls to the left and falls to the right

B) falls to the left and rises to the right

☐ C) rises to the left and rises to the right

D) rises to the left and falls to the right

MULTIPLE CHOICE. CIRCLE THE CORRECT ANSWER.

For the polynomial, find each real zero and its multiplicity. Determine whether the graph crosses or touches the x-axis at each x-intercept.

2) $f(x) = 3(x - 7)(x - 6)^2$

A) $x = 7$: touches x-axis

$x = 6$: crosses x-axis

$x = 3$: crosses x-axis

☒ B) $x = 7$: crosses x-axis

$x = 6$: touches x-axis

☐ C) $x = -7$: crosses x-axis

$x = -6$: touches x-axis

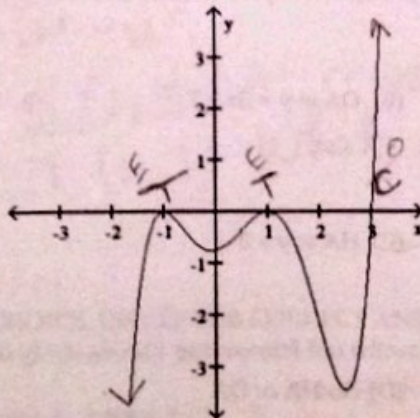
D) $x = -7$: touches x-axis

$x = -6$: crosses x-axis

$x = 3$: crosses x-axis

MULTIPLE CHOICE. CIRCLE THE CORRECT ANSWER.

3) Which of the following polynomial functions might have the graph shown in the illustration below? (Make your decision based on the real zeros and their cross/touch behavior).



A) $f(x) = a(x + 1)^2(x - 1)(x + 3)$

B) $f(x) = a(x + 1)^2(x - 1)(x - 3)^2$

C) $f(x) = a(x + 1)^2(x - 1)^2(x + 3)^2$

☒ D) $f(x) = a(x + 1)^2(x - 1)^2(x - 3)$

MULTIPLE CHOICE. CIRCLE THE CORRECT ANSWER.

4) For the function $f(x) = \frac{x^2 - 81}{x^2 - 9x}$, determine the domain as well as the vertical asymptote(s) and/or hole(s) of its graph.

A) Domain: $\{x \mid x \neq 0, 9\}$
 VA(s): $x = 0$
 Hole(s): $(9, 2)$

B) Domain: $\{x \mid x \neq -3, 3\}$
 VA(s): $x = -3$
 Hole: $(3, 4)$

C) Domain: $\{x \mid x \neq -3, 3\}$
 VA(s): $x = 3$
 Hole: $(-3, -2)$

D) Domain: $\{x \mid x \neq -3, 3\}$
 VA(s): $x = -3, x = 3$
 Hole: None

E) Domain: $\{x \mid x \neq 0, 9\}$
 VA(s): $x = 0, x = 9$
 Hole(s): None

F) Domain: $\{x \mid x \neq 0, 9\}$
 VA(s): None
 Hole(s): $(0, 0), (9, 2)$

MATCHING: Find the Horizontal Asymptote or Oblique Asymptote of each function (if it has one).

5) FUNCTION	HA or OA
(I) $f(x) = \frac{21x + 14}{7x^2 + 1}$	(A) HA at $y = 0$
(II) $f(x) = \frac{21x + 14}{7x + 1}$	(B) OA at $y = 3x + 2$
(III) $f(x) = \frac{21x^3 + 14x^2}{7x^2 + 1}$	(C) HA at $y = 3$
(IV) $f(x) = \frac{21x^4 + 14}{7x^2 + 1}$	(D) No HA or OA

Answers (choose A, B, C, or D):

(I) a (II) c (III) b (IV) d

MULTIPLE CHOICE. CIRCLE THE CORRECT ANSWER.
Solve the inequality. Express the solution using interval notation.

6) $(x - 3)^2(x + 6) < 0$

A) $(-6, \infty)$

B) $(-\infty, -6)$

C) $(-6, 1) \text{ or } (1, \infty)$

D) $(-\infty, -6) \text{ or } (6, \infty)$

7) $\frac{x - 4}{x + 1} \geq 0$

A) $(-1, 4]$

B) $(-\infty, -1] \text{ or } [4, \infty)$

C) $(-\infty, -1) \text{ or } [4, \infty)$

D) $[-1, 4]$

Use either the Factor Theorem or the Remainder Theorem to determine whether $x - c$ is a factor of $f(x)$.
The work that you show must justify your answer.

8) $f(x) = x^3 + 5x^2 - 12x + 14$; $x + 7$

Answer: yes or no

$$\begin{array}{r} x^2 - 2x + 2 \\ x+7 \overline{) x^3 + 5x^2 - 12x + 14} \\ \underline{-(x^3 + 7x^2)} \\ -2x^2 - 12x \\ \underline{+ 2x^2 + 14x} \\ -2x - 14 \\ \underline{+ 2x + 14} \\ 0 \end{array}$$

Remainder = 0
so $x+7$ IS
a factor.

List the potential rational zeros of the polynomial function. Do not find the zeros.

9) $f(x) = 2x^4 - x^2 + 5$

$P = 5$ (1, 5) At most 4 potential zeros.
 $Q = 2$ (1, 2) What are they?

MULTIPLE CHOICE. CIRCLE THE CORRECT ANSWER.

Information is given about a polynomial $f(x)$ whose coefficients are real numbers. Find the remaining zeros of f .

10) Degree 4; zeros: $i, 2 + i$

→ Every imaginary # comes coupled up so it'll be the exact opposites

A) $2 - i$

B) $-i, -2 + i$

C) $-2 + i, 2 - i$

D) $-i, 2 - i$

11) Answer each part of the question to find ALL of the zeros (both real and imaginary) of the function

$$f(x) = x^4 + 3x^3 + 6x^2 + 12x + 8.$$

a) Part (a) is Multiple Choice and may have more than one correct answer.

What possible combinations of real and imaginary zeros can this function have? Circle all correct answers.

A) 4 real; 0 imaginary

B) 3 real; 1 imaginary

C) 2 real; 2 imaginary

D) 1 real; 3 imaginary

E) 0 real; 4 imaginary

b) List all of the possible rational zeros: $\frac{p}{q}$ list: $\pm (1, 2, 4, 8)$ ✓
1, -2

c) Find one real zero: -2 ✓ and the corresponding factor of $f(x)$: $(x+2)$ ✓

$$\begin{array}{r} x+2 \overline{) x^4 + 3x^3 + 6x^2 + 12x + 8} \\ \underline{x^4 + 2x^3} \\ x^3 + 6x^2 + 12x + 8 \\ \underline{x^3 + 2x^2} \\ 4x^2 + 12x + 8 \\ \underline{4x^2 + 8x} \\ 4x + 8 \\ \underline{4x + 8} \\ 0 \end{array}$$

$$x^3 + 1x^2 + 4x + 8$$

d) Use synthetic division to find the depressed function $g(x) = \frac{x^3 + 2x^2 + 4x + 8}{x+2} (-2)$
 $x^3 + x^2 + 4x + 4 (-1)$

e) Solve the depressed equation to find the remaining zeros (both real and imaginary) of $f(x)$:

$$2i, -2i, -2, -1$$

f) Then combine the answers from part (e) with your factor from part (c) to write the fully factored form of $f(x)$. (In "fully factored" form, there should be n total factors. Include both real and imaginary zeros.)

$$(x-2i)(x+2i)(x+1)(x+2) \quad \checkmark$$

For the exponential function $f(x) = 3^{(-x+1)} + 2$, answer each part of the question in the space provided.
You do not have to graph the function.

12)

3^x

x-values

① sub. -1

② ÷ 1

a) What is the Base Function? 3^x

b) What are the three key points of the Base Function? $(0, 1)$; $(1, 3)$; $(-1, \frac{1}{3})$

c) Does the Base Function have a VA or HA? (Circle one.) Equation of the Asymptote: $y=0$
** Remember that VA's have the equation $x = \#$ and HA's have the equation $y = \#$ **

d) What transformations does the Base Function undergo in order to become $f(x) = 3^{(-x+1)} + 2$?
For horizontal shifts and vertical shifts, indicate the NUMBER in the space provided next to the direction.

H: Choose one: RIGHT: _____ or LEFT: 1 or NONE

S: Choose any/all that apply.

VERTICAL STRETCH by a factor of _____ VERTICAL SHRINK by a factor of _____

HORIZONTAL STRETCH by a factor of _____ HORIZONTAL SHRINK by a factor of _____

NO STRETCHES OR SHRINKS of any kind ☒

R: Circle any/all that apply:

Y-AXIS

~~X-AXIS~~

NONE

V: Choose one: UP: 2 or DOWN: _____ or NONE

e) Where do the three key points end up on the final graph of $f(x) = 3^{(-x+1)} + 2$?

$(-1, 1)$; $(0, 5)$; $(1, 3)$ ~~$(-2, 2\frac{1}{3})$~~ $(1, 3)$ $(2, 2\frac{1}{3})$

f) Where will the VA or HA end up on the Final Graph of $f(x)$? $y=2$
** Remember that VA's have the equation $x = \#$ and HA's have the equation $y = \#$ **

MULTIPLE CHOICE. CIRCLE THE CORRECT ANSWER.

Solve the equation.

13) $4^{7-3x} = \frac{1}{16}$

A) $\{-3\}$

B) $\{3\}$

C) $\{\frac{5}{3}\}$

D) No Solution

$7-3x = -2$
 -7 -7
 $-3x = -9$
 $x = 3$

5

All questions on this page are MULTIPLE CHOICE. Circle the correct answer.

Change the logarithmic expression to an equivalent expression involving an exponent.

14) $\log_{1/3}(81) = -4$ *log (Base outside) = inside*

A) $31^{1/3} = 4$

B) $(-4)^{1/3} = 81$

C) $\left(\frac{1}{3}\right)^4 = 81$

D) $\left(\frac{1}{3}\right)^{-4} = 81$

Change the exponential expression to an equivalent expression involving a logarithm.

15) $5^{-3} = \frac{1}{125}$ *log base outside = inside*

A) $\log_5(-3) = \frac{1}{125}$

B) $\log_5\left(\frac{1}{125}\right) = -3$

C) $\log_{-3}\left(\frac{1}{125}\right) = 5$

D) $\log_{1/125}(5) = -3$

Find the exact value of the logarithmic expression.

16) $\log_7\left(\frac{1}{49}\right) = \log_7(49) = \log_7(7^2) \rightarrow 7s \text{ are the same so } -2 \text{ drops}$

A) 2

B) -2

C) 7

D) -7

Find the domain of the function. Write your answer in interval notation.

17) $f(x) = \ln(x - 8)$

A) $(-\infty, -8)$

B) $(-8, \infty)$

C) $(8, \infty)$

D) $(-\infty, 8)$

Solve the equation.

18) $4 \ln(7x) = 12$

A) $\left\{\frac{e^3}{7}\right\}$

B) $\{e^3\}$

C) $\left\{\frac{3}{\ln 7}\right\}$

D) $\{e^{3/7}\}$

$\ln(7x) = 3 \rightarrow e^3 = 7x \rightarrow \frac{e^3}{7} = x$

$\frac{4 \ln(7x)}{4} = \frac{12}{4}$

$\ln(7x) = 3$

All questions on this page (except #21 part a) are MULTIPLE CHOICE. Circle the correct answer.

Expand into a sum and/or difference of logarithms. Express powers as coefficients.

19) $\log_2 \left(\frac{x+3}{x^7} \right)$

A) $7 \log_2 (x) - \log_2 (x+3)$

B) $\log_2 (x+3) + 7 \log_2 (x)$

☒ C) $\log_2 (x+3) - 7 \log_2 (x)$

☒ D) $\log_2 (x) + \log_2 (3) - 7 \log_2 (x)$

Condense into a single logarithm.

20) $2 \log_6 (x) - \log_6 (x+1) + 7 \log_6 (x-5)$: addition (multiply) subtraction means division

A) $\log_6 \left(\frac{2x(x+1)}{7(x-5)} \right)$

B) $\log_6 \left(\frac{14x(x-5)}{x+1} \right)$

C) $\log_6 \left(\frac{x^2(x+1)}{(x-5)^7} \right)$

☒ D) $\log_6 \left(\frac{x^2(x-5)^7}{x+1} \right)$

For #21:

- (a) GUESS the answer to the log below to one decimal place. Explain your guess using nearby logs or exponentials.
 (b) Use the Change of Base property and a calculator to evaluate the answer rounded to three decimal places.

21) $\log_3 29.90$

(a) Guess: 3.02 Explanation: It's between $3^3 = 27$ and $3^4 = 81$ but closer to 3^3 ✓

(b) Actual Answer (choose one):

☒ A) 3.093

B) 1.4757

C) 3.3979

D) 0.323

Solve the equation. Check your answer!!

22) $\log(5x) = \log 4 + \log(x-2)$ → log base is 10. B/c everything is common, logs drop

A) $\{-8, 8\}$

B) $\{-8\}$

C) $\{8\}$

☒ D) No Solution

drop
Addition means mult.

$5x = 4 + (x-2)$

$5x = 4x - 8$

$\frac{-4x}{-4x} \quad \frac{-4x}{-4x}$

$x = -8 \rightarrow$

plug in -8 into x's if positive it's a solution.

-4

