DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

MATHEMATICAL MODELLING OF SUBSTANCE ABUSE BY COMMERCIAL DRIVERS

AMOS KIPNGENO ROTICH
S084-01-2742/2021

A RESEARCH PROJECT SUBMITTED TO THE DEPARTMENT OF MATHEMATICS AND PHYSICAL SCIENCES IN PARTIAL FULFILLMENT OF REQUIREMENT FOR AWARD OF THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND MODELLING PROCESS

SEPTEMBER 2024

Declaration

Declaration by the student

This research proposal is my original work and has not been presented in any university for a degree consideration of any certification.

AMOS KIPNGENO ROTICH **S084-01-2742/2021**

Signature.................................. DATE...

The Supervisor
This research project has been presented for examination purpose with my approval as a university supervisor.

MS AGNES NGETHA

Department on Mathematics and Physical Science

Dedan Kimathi University.

Signature.......................... DATE.........................

Dedication

I dedicate this work to the policymakers and practitioners who are working to address the issue of substance abuse among commercial drivers. It is my hope that this model can provide valuable insights and serve as a guiding tool in reducing such risky behaviors. I also extend this dedication to the families and communities affected by the consequences of substance abuse, with the hope that this research will contribute to creating safer roads and fostering healthier environments.

Your unwavering commitment to finding sustainable solutions to the challenges of substance abuse and its impact on public safety inspires us all. May this research stand as a tribute to your strength, expertise, and the vital role you play in ensuring that substance abuse among commercial drivers is brought under control for the safety of all.

Acknowledgments

First and foremost, I thank Almighty God for granting me good health and peace of mind throughout my research period and my entire life. His grace has been my strength.
Secondly, I extend my sincere gratitude to my supervisors, Mr. Reuben and Ms. Ngetha, for their invaluable guidance and unwavering support in steering my research journey. Their expertise and encouragement have been crucial in the development of this work.
I would also like to acknowledge my fellow students from the Mathematical and Modeling Processes program for their teamwork and insightful discussions during the research and analysis stages. Your collaboration has been a great source of motivation and learning.A heartfelt thanks goes to my family for their constant support, both emotionally and financially, as well as for their prayers throughout my studies.
Lastly, I am deeply thankful to Dedan Kimathi University and its community for providing essential resources, including library access and internet facilities, which enabled me to dive deeper into my research. The support from the university has been instrumental in the success of this work.

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Abstract

In this research, i formulated a model for substance abuse that explains the dynamics of the use and abuse of substances that are perceived as mood changing by commercial drivers. The substance model was analyzed qualitatively and quantitatively. The threshold for the abuse of substance was determined. It was found that the substance free equilibrium point was found to be locally asymptotically stable whenever the reproduction number is less than one and unstable otherwise. The analysis of the contribution of each parameter was performed using sensitivity analysis. The analysis revealed that an increase in the recruitment rate of commercial drivers and the rate at which commercial drivers get into contact and imitate those using the substances, would cause an increase in the reproduction number. Numerical simulations was conducted to see the changes in the population dynamics of susceptible drivers, substance users and substance abusers in the population. The results showed that the contact and imitation rates has an impact on the population of commercial drivers. There are impact on interaction among non-substance users and substance users in the system with time. An increase in the contact or imitation rate increases the population of substance users.

Symbols

μ - mu	Λ - capital lambda
γ - gamma	ρ - rho
β - beta	α - alpha

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Chapter 1

Introduction

1.1 Background of the Study

Substance abuse among commercial drivers represents a critical concern due to its potential to compromise road safety and public health. The nature of the job, which often involves long hours of driving, irregular schedules, and extended periods away from home, creates a conducive environment for substance misuse. Factors such as the need to meet demanding deadlines, cope with job-related stressors, and combat fatigue on the road may contribute to the use of substances as a means of staying awake or managing stress. Moreover, the isolation experienced by drivers during long-haul trips and the lack of access to healthcare services while on the road further exacerbate vulnerabilities to substance abuse. Additionally, the consequences of substance abuse among commercial drivers extend beyond individual health risks to include increased rates of motor vehicle accidents, injuries, fatalities, and economic costs associated with property damage, medical expenses, and legal repercussions. Understanding the complexities of substance abuse among commercial drivers requires an approach that considers the interplay of individual, organizational, and regulatory factors. Epidemiological research provides valuable insights into the prevalence, patterns, and trends of substance use within the commercial driving population, informing targeted interventions and policy development. Some studies shed light on the underlying factors driving substance abuse among drivers, including stress, social influences, and coping mechanisms, emphasizing the need for tailored prevention and intervention strategies. Furthermore, efforts to promote a culture of safety, wellness, and regulatory compliance within the transportation industry are essential for mitigating risks associated with substance abuse. By addressing substance abuse among commercial drivers holistically, stakeholders can work collaboratively to enhance prevention efforts, improve access to resources, and safeguard the well-being of drivers and the communities they serve.

1.2 Problem Statement

Substance abuse among commercial drivers presents a complex challenge with significant implications for road safety and public health. Despite the existence of laws and regulations aimed at curbing drug driving, a considerable number of commercial drivers continue to engage in substance abuse, leading to increased risks of accidents and fatalities. There is a lack of comprehensive mathematical models to analyze the dynamics of substance abuse among commercial drivers and its impact on driving behavior and road safety. Therefore, the primary objective of this project is to develop a mathematical model that can describe the patterns of substance abuse among commercial drivers, identify key factors influencing substance use behaviors, and assess the potential effects on driving performance and accident rates.

1.3 Justification

Cases of driving under the influence of stimulants and mood changing substances are directly associated with road carnage. Commercial vehicles could be deadly for both the driver and other occupants when the driver is driving under the influence of some substance.Therefore Formulating a mathematical model to analyze substance abuse among commercial drivers is important for providing an understanding of the factors influencing driving behaviors under the influence, predicting the impact of substance use on driving performance and road safety outcomes, and guiding the development of evidence-based interventions. Formulating and applying such a model can offer valuable insights into the underlying mechanisms of drug driving behaviors, thereby informing targeted intervention strategies.

1.4 Objectives

General Objective:

To formulate a mathematical model to describe the patterns of substance abuse among commercial drivers.

Specific Objectives

1. Develop a mathematical model
2. To determine the equilibrium points of the mathematical model and investigate their stabilities.
3. Utilize the developed model to analyze the dynamics of substance abuse behaviors among commercial drivers..
4. Identify strategies that effectively reduce substance abuse among commercial drivers.

Chapter 2

Literature Review

This chapter provides reviews of the existing literature on substance abuse and its related issues. It examines various themes, research approaches, and methodologies employed in previous studies, aiming to assess the current state of knowledge and identify effective strategies for understanding and addressing substance abuse problems. Through this review, the chapter highlights key findings, identifies gaps in the research, and suggests directions for future investigation in the field of substance abuse.

Zhao et al. (2014) studied the effects of alcohol on drivers and their performance on straight roads. Using a simulated driving experiment, the researchers collected data on 25 drivers' subjective feelings and driving performance across different blood-alcohol concentration (BAC) levels. The results revealed that alcohol impacted drivers in numerous ways, including their attitude, judgment, vigilance, perception, reaction time, and vehicle control. Higher BAC levels correlated with increased accident rates, as well as significant changes in driving performance metrics such as average speed, speed variability, and lane position variability.

Yunusa et al. (2017) conducted a study to explore the determinants of substance abuse among commercial bus drivers in Kano Metropolis, Kano State, Nigeria. The research highlighted that the use of illicit substances among these drivers is on the rise, which poses significant health risks to both the drivers and their passengers. Despite the increasing trend, there is a lack of empirical data on the factors contributing to this issue. The study selected 196 respondents through a multi-stage cluster sampling technique and collected data using a validated and structured interviewer-administered questionnaire. The analysis revealed that 81.1% of the respondents had abused a substance at some point. The primary factors associated with substance abuse included the desire to relax or sleep after a hard day's work (84.8%), the need to work hard (48%), stress relief (81%), anxiety relief (66.5%), and the pursuit of pleasure (72%). The most commonly abused substances were solution (93.3%), coffee (85.2%), Tramadol (80.6%), local stimulant tea (Gadagi) (78.1%), cola-nut (66.3%), and tobacco (65%). The study concluded that controlling the production and sale of commonly abused substances could reduce substance abuse. Therefore, it recommended that the government implement measures to regulate these substances.

Dini et al. (2019) conducted a systematic review and meta-analysis to assess the prevalence and impact of psychoactive drug consumption among truck drivers. Their study addressed a significant concern within the trucking sector, where the use of psychoactive substances poses risks to both drivers' health and public safety by increasing the likelihood of injuries and traffic accidents. The meta-analysis revealed that 27.6% of truck drivers consumed drugs, with particularly high rates of CNS-stimulant use, including amphetamines (21.3%) and cocaine (2.2%). These substances are often used by drivers as performance enhancers to boost productivity. However, chronic and high-dose consumption of these drugs negatively affects driving skills, compromising road safety. The study highlights the need for further research to enhance the evidence base and inform Occupational Health Professionals and policymakers about this critical issue.

Behnood and Mannering (2017) investigated the effects of drug and alcohol consumption on driver injury severities in single-vehicle crashes. Their study utilized a random parameters logit model to analyze differences in injury severities among unimpaired, alcohol-impaired, and drug-impaired drivers using data from single-vehicle crashes in Cook County, Illinois, over a 9-year period. They considered a wide range of variables potentially affecting driver injury severity, including roadway and environmental conditions, driver attributes, time and location of the crash, and crash-specific factors. The results reveals. Unimpaired drivers were found to be more responsive to variations in lighting, aded significant differences in the determinants of driver injury severities across the different groups of driververse weather, and road conditions. However, they also exhibited more heterogeneity in their behavioral responses to these conditions compared to impaired drivers. Age and gender were identified as important determinants of injury severity, with effects varying significantly across all drivers, particularly among alcohol-impaired drivers. Overall, the study concluded that statistically significant differences exist in driver injury severities among unimpaired, alcohol-impaired, and drug-impaired driver groups. Unimpaired drivers displayed more variability in injury outcomes under adverse weather and road conditions, reflecting their ability to utilize their full knowledge and judgment. In contrast, impaired drivers showed less variability in factors affecting injury severity, indicating a more consistent impact of decision-impairing substances on their driving behavior.

Matonya and Kuznetsov (2021) addressed the global burden of drug abuse by presenting a mathematical model aimed at understanding and controlling drug abuse in Tanzania. The study utilized the next-generation matrix method to compute an epidemic threshold value, establishing conditions for the existence and stability of stationary points within the model. Through numerical simulation, the researchers explored the dynamic behavior of the model and identified the rate of contact between susceptible individuals and drug users, as well as the rate of recovery after rehabilitation, as key factors influencing the dynamics of the drug user population in society. Furthermore, the model was analyzed to study the dynamic behavior of drug abuse when control measures were implemented. The results indicated a significant reduction in the number of drug users, highlighting the importance of early intervention and the establishment of strict laws to address this issue within societies.

Orwa and Nyabadza (2019) addressed the pressing issue of substance abuse, particularly the co-abuse of alcohol and methamphetamine, in the Western Cape province of South Africa, which has exacerbated the drug epidemic in the region. They formulated a mathematical model to understand the dynamics of alcohol and methamphetamine co-abuse and its impact on public health and socio-economic factors. The study demonstrated that the equilibria of the sub-models were locally and globally asymptotically stable when the sub-model threshold parameters were less than unity, indicating stability in certain conditions. The basic reproduction number due to co-abuse was identified as the maximum of the two sub-model reproduction numbers. Sensitivity analysis revealed that the most sensitive parameters in the co-abuse epidemic were the alcohol and methamphetamine recruitment rates, highlighting the importance of addressing these factors in intervention strategies. The prevalence curve indicated a persistent drug problem in the region, underscoring the need for comprehensive social programs to raise awareness of the dangers of multiple substance abuse. The study concluded by emphasizing the importance of promoting educational campaigns in learning institutions, social media, and health institutions to increase awareness and understanding of the risks associated with substance abuse. Additionally, transmission control efforts should focus on enhancing the quitting process and providing support services to drug users during and after treatment to minimize cases of relapse.

Martin et al. (2017) aimed to estimate the relative risks of responsibility for fatal accidents associated with driving under the influence of cannabis or alcohol, along with the prevalence of these influences among drivers and the corresponding attributable risk ratios. Additionally, they sought to estimate these parameters for three other groups of illicit drugs (amphetamines, cocaine, and opiates) and compare the results to a previous study conducted in France between 2001 and 2003. The study analyzed police procedures for fatal accidents in Metropolitan France during 2011, encoding 300 characteristics to create a database of 4,059 drivers. Information on alcohol and illicit drugs was derived from tests for positive and confirmed through blood analysis. Drivers responsible for causing accidents were compared to those involved in accidents for which they were not responsible, serving as a general reference group. The results indicated that the proportion of drivers under the influence of alcohol was estimated at 2.1 percent, while those under the influence of cannabis was estimated at 3.4 % . Drivers under the influence of alcohol were found to be 17.8 times more likely to be responsible for a fatal accident, with an estimated 27.7% of fatal accidents potentially preventable if drivers did not exceed the legal limit for alcohol. Meanwhile, drivers under the influence of cannabis had a 1.65 times increased risk of causing a fatal accident, with an estimated 4.2% of fatal accidents potentially preventable if drivers did not drive under the influence of cannabis.

From the literature above, it is evident that there is need to formulate a mathematical model to describe the patterns of substance abuse among commercial drivers .Developing and applying this model, it aims to offer valuable insights for the development of targeted interventions and policy measures aimed at reducing the risks associated with substance abuse among commercial drivers.

Chapter 3

Research Methodology

3.1 Formulation of the Mathematical Model

3.1.1 Model Assumption

In setting up the model, we consider certain assumptions that would help us concentrate on answering our desired question. The list of some assumptions of the model is given as follows:

- All drivers in susceptible population are enrolled at a constant rate.
- All drivers are born susceptible.
- Drivers interact uniformly within the population.
- Drivers can exit or removed in the system due to death or recovery .

3.1.2 Mathematical Model

The substance model divides the commercial drivers into four compartments depending on their substance use status as follows:

- **S** – Comprises all drivers who are susceptible to substance abuse.
- **D** – All drivers who are substance users who use substance of any form.
- **A** – Comprises all drivers who abuse substances of any form.
- **R** – Drivers who stopped using substances either by abstinence or through rehabilitation.

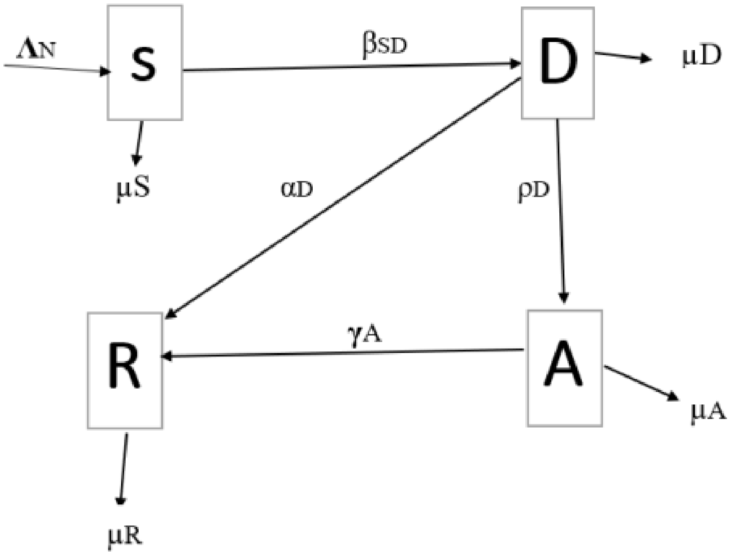

Figure 1: Flow diagram of substance use and abuse.

Parameter $\mathbf{\Lambda}$ represents recruitment rate into the susceptible population **S**. $\boldsymbol{\beta}$ is the contactand imitation rate which is where individuals in **S** start engaging in substance use due to imitation and contact from those who are already using them. $\boldsymbol{\rho}$ is the rate at which substance users abuse substances. $\boldsymbol{\alpha}$ is the recovery rate of substance users. $\boldsymbol{\gamma}$ is the recovery rate of substance abusers. $\boldsymbol{\mu}$ is the death rate.
The total population of the drivers thus becomes:

$$N(t) = S(t) + D(t) + A(t) + R(t)$$

The system of ordinary differential equations obtained from Figure 1 are as follows:

$$(1) \qquad \begin{cases} \frac{dS}{dt} = \Lambda N - (\beta D + \mu)S \\ \frac{dD}{dt} = \beta SD - (\mu + \alpha + \rho)D \\ \frac{dA}{dt} = \rho D - (\mu + \gamma)A \\ \frac{dR}{dt} = \alpha D + \gamma A - \mu R \end{cases}$$

3.1.3 Parameter Interpretation

Symbol	**Description**
t	Time
Λ	Recruitment rate
μ	Death rate
γ	Recovery rate of substance abusers
β	Contact and immitation rate
ρ	Substance abuse rate
α	The recovery rate of substance users

Table 1

3.2 Mathematical Analysis

Analysing the model for the substance abuse by commercial drivers-based on the following sub section to determine all threshold parameter for the substance abuse dynamics and effect of treatment.

3.2.1 Positivity and Boundedness of the Solutions

Positivity

Theorem: The region R given by $R = \{(S, D, A, R) \in \mathbb{R}^4_+ \mid S \geq 0, D \geq 0, A \geq 0, R \geq 0\}$ is positively invariant for all $t \geq 0$.

Proof: Consider the first equation equation in (1):

$$\frac{dS}{dt} = \Lambda - (\beta D + \mu)S$$

Thus,

$$\frac{dS}{dt} \geq -(\beta D + \mu)S$$

Integrating both sides gives:

$$\int \frac{1}{S}\, dS \geq -\int (\beta D + \mu)\, dt$$

$$\ln S \geq -(\beta D + \mu)t + c$$

Exponentiating both sides yields:

$$S \geq e^{-(\beta D+\mu)t+c}$$

At $t = 0$:

$$S = 1 + e^c$$

Hence, $S \geq 0$. Thus, $S(t)$ stays positive.

Consider the second equation:

$$\frac{dD}{dt} = \beta SD - (\mu + \alpha + \rho)D$$

Thus,

$$\frac{dD}{dt} \geq -(\mu + \alpha + \rho)D$$

Integrating both sides gives:

$$\int \frac{1}{D}\, dD \geq -\int (\mu + \alpha + \rho)\, dt$$

$$\ln D \geq -(\mu + \alpha + \rho)t + c$$

Exponentiating both sides yields:

$$D \geq e^{-(\mu+\alpha+\rho)t+c}$$

At $t = 0$:

$$D = 1 + e^c$$

Hence, $D \geq 0$.

Third Equation

Consider the third equation:

$$\frac{dA}{dt} = \rho D - (\mu + \gamma)A$$

Thus,

$$\frac{dA}{dt} \geq -(\mu + \gamma)A$$

Integrating both sides gives:

$$\int \frac{1}{A}\, dA \geq -\int (\mu + \gamma)\, dt$$

$$\ln A \geq -(\mu + \gamma)t + c$$

Exponentiating both sides yields:

$$A \geq e^{-(\mu+\gamma)t+c}$$

At $t = 0$:

$$A = 1 + e^c$$

Hence, $A \geq 0$.

Fourth Equation

Consider the fourth equation:

$$\frac{dR}{dt} = \alpha D + \gamma A - \mu R$$

Thus,

$$\frac{dR}{dt} \geq -\mu R$$

Integrating both sides gives:

$$\int \frac{1}{R}\, dR \geq -\int \mu\, dt$$

$$\ln R \geq -\mu t + c$$

Exponentiating both sides yields:

$$R \geq e^{-\mu t+c}$$

At $t = 0$:

$$R = 1 + e^c$$

Hence, $R \geq 0$.

At $t = \infty$, since the exponential terms decay to 0, the solutions $S(t)$, $D(t)$, $A(t)$, and $R(t)$ will remain non-negative for all time.

Boundedness

Considering $\psi = \{S(t), D(t), R(t), A(t)\}$ are positive for $t \geq 0$. Boundedness refers to the region in which solutions of the model or system are uniformly bounded in the proper subset $\psi \subset \mathbb{R}^4_+$. Considering the total population at any time, t:

$$N(t) = S(t) + D(t) + A(t) + R(t)$$

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dD}{dt} + \frac{dA}{dt} + \frac{dR}{dt}$$

$$\frac{dN}{dt} = (\Lambda - (\beta D - \mu)S) + (\beta SD - (\mu + \alpha + \rho)D) + (\rho D - (\mu + \gamma)A) + (\alpha D + \gamma A - \mu R)$$

By simplification,

$$\frac{dN}{dt} = \Lambda - \mu S - \mu D - \mu A - \mu R$$

There are no substance addiction and recovery in the absence of substance. Hence, $D = 0$, $R = 0$, $A = 0$. It becomes;

$$\frac{dN}{dt} = \Lambda - \mu S$$

If the total population N is equal to the number of Susceptible S, it implies that $N = S$, such that;

$$\frac{dN}{dt} = \Lambda - \mu N$$

Hence,

$$\frac{dN}{dt} + \mu N \leq \Lambda$$

To integrate the above equation, first obtain an integrating factor. The integrating factor is $e^{\int \mu \, dt} = e^{\mu t}$. Then we have

$$\int \frac{d}{dt}[Ne^{\mu t}] \leq \int \Lambda e^{\mu t}$$

$$Ne^{\mu t} \leq \frac{\Lambda}{\mu} e^{\mu t} + C$$

Dividing both sides by $e^{\mu t}$,

$$N \leq \left(\frac{\Lambda}{\mu} + Ce^{-\mu t} \right)$$

Where C is the constant of integration. Taking the limit as $t \to \infty$, we have

$$\lim_{t \to \infty} N(t) \leq \left(\lim_{t \to \infty} \frac{\Lambda}{\mu} + Ce^{-\mu t} \right)$$

Then,

$$N(t) \leq \frac{\Lambda}{\mu}$$

This proves the boundedness of the solution inside $\mathbb{R}$. This implies that all solutions of the system starting in $\mathbb{R}$ remain in $\mathbb{R}$ for all $t \geq 0$. Thus, $\mathbb{R}$ is positively invariant and attracting, and hence it is sufficient to consider the dynamics of the system.

3.2.2 Substance free equilibrium:

The substance-free equilibrium is obtained when the system of differential equations is set to zero. At this point, there are no drug users, addicted individuals, or recovered individuals.

$$(D = 0, R = 0, A = 0)$$

Equating the systems of equations from the model to zero:

$$\begin{aligned}
\Lambda - (\beta D + \mu)S &= 0, \\
\beta S D - (\mu + \alpha + \rho)D &= 0, \\
\rho D - (\mu + \gamma)A &= 0, \\
\alpha D + \gamma A - \mu R &= 0.
\end{aligned}$$

Therefore,

$$\Lambda - \mu S = 0$$

which gives,

$$S^* = \frac{\Lambda}{\mu}$$

Thus, the substance-free equilibrium (SFE) point for the system is $E^* = (S^*, D^*, A^*, R^*) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right)$.

3.2.3 Stability of the Substance-free equilibrium:

Let:

$$f_1(S, D, A, R) = \Lambda N - (\beta D + \mu)S$$

$$f_2(S, D, A, R) = \beta S D - (\mu + \alpha + \rho)D$$

$$f_3(S, D, A, R) = \rho D - (\mu + \gamma)A$$

$$f_4(S, D, A, R) = \alpha D + \gamma A - \mu R$$

The Jacobian matrix J of the system of differential equations is:

$$J(S^*, D^*, A^*, R^*) = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial D} & \frac{\partial f_1}{\partial A} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial D} & \frac{\partial f_2}{\partial A} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial D} & \frac{\partial f_3}{\partial A} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial D} & \frac{\partial f_4}{\partial A} & \frac{\partial f_4}{\partial R} \end{pmatrix}$$

The Jacobian matrix J of the system is given by:

$$J = \begin{pmatrix} -(\beta D+\mu) & -\beta S & 0 & 0 \\ \beta D & -\beta S-(\mu+\alpha+\rho) & 0 & 0 \\ 0 & \rho & -(\mu+\gamma) & 0 \\ 0 & \alpha & \gamma & -\mu \end{pmatrix}$$

Substituting the values at the substance-free equilibrium $(S^*, D^*, A^*, R^*) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right)$, we get:

$$J\left(\frac{\Lambda}{\mu}, 0, 0, 0\right) = \begin{pmatrix} -\mu & -\beta\frac{\Lambda}{\mu} & 0 & 0 \\ 0 & -\beta\frac{\Lambda}{\mu}-(\mu+\alpha+\rho) & 0 & 0 \\ 0 & \rho & -(\mu+\gamma) & 0 \\ 0 & \alpha & \gamma & -\mu \end{pmatrix}$$

The substance-free equilibrium, E^f, is locally asymptotically stable if all the eigenvalues of J are less than 0. We will find these by solving for the eigenvalues. Start with:

$$|J - \lambda I_4| = 0,$$

where

$$|J-\lambda I_4| = \begin{vmatrix} -\mu-\lambda & -\frac{\beta\Lambda}{\mu} & 0 & 0 \\ 0 & -\frac{\beta\Lambda}{\mu}-(\mu+\alpha+\rho)-\lambda & 0 & 0 \\ 0 & \rho & -(\mu+\gamma)-\lambda & 0 \\ 0 & \alpha & \gamma & -\mu-\lambda \end{vmatrix} = 0$$

This simplifies to:

$$(-\mu-\lambda)^2(-\beta\frac{\Lambda}{\mu}-(\mu+\alpha+\rho)-\lambda)(-(\mu+\gamma)-\lambda) = 0.$$

Thus, the eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are:

$$\lambda_1 = -\mu, \quad \lambda_2 = -\beta\frac{\Lambda}{\mu}-(\mu+\alpha+\rho), \quad \lambda_3 = -(\mu+\gamma).$$

Since all eigenvalues $\lambda_1, \lambda_2, \lambda_3$ are negative, the substance-free equilibrium E^f is locally asymptotically stable.

3.2.4 The Basic Reproduction Number R_0

To determine the potential persistence of substance abuse among commercial drivers, we use the basic reproductive number, R_0. This value represents the average number of individuals a substance-abusing driver is expected to influence if introduced into a population entirely composed of susceptible individuals. If $R_0 > 1$, the prevalence of substance abuse is sustained, indicating an ongoing issue within the population unless interventions are implemented. Conversely, if $R_0 < 1$, the prevalence diminishes over time, suggesting that substance abuse may naturally decline without significant external influences.
Reproduction number R_0 is calculated using the method of the next-generation matrix.

Steps to Compute the Basic Reproduction Number R_0:

Step 1: Identify the Infection Compartments
The infection compartments in this system are:

1. D (drug users)
2. A (substance abusers)

Step 2: Construct the Transmission Matrix F
The transmission matrix F represents the rate of new infections produced by each compartment. The relevant new infection term in the equations is βSD. Let $f_1 = \beta SD$, $f_2 = 0$ then:

$$F = \begin{pmatrix} \frac{\partial f_1}{\partial D} & \frac{\partial f_1}{\partial A} \\ \frac{\partial f_2}{\partial D} & \frac{\partial f_2}{\partial A} \end{pmatrix} = \begin{pmatrix} \beta S & 0 \\ 0 & 0 \end{pmatrix}$$

where $S = \frac{\Lambda}{\mu}$. Therefore,

$$F = \begin{pmatrix} \frac{\beta\Lambda}{\mu} & 0 \\ 0 & 0 \end{pmatrix}$$

Step 3: Construct the Transition Matrix V
The transition matrix V represents the rates of transitions out of the infection compartments. Let $f_3 = (\mu + \alpha + \rho)D$, $f_4 = -\rho D + (\mu + \gamma)A$ then:

$$V = \begin{pmatrix} \frac{\partial f_3}{\partial D} & \frac{\partial f_3}{\partial A} \\ \frac{\partial f_4}{\partial D} & \frac{\partial f_4}{\partial A} \end{pmatrix} = \begin{pmatrix} \mu + \alpha + \rho & 0 \\ -\rho & \mu + \gamma \end{pmatrix}$$

This matrix V describes how individuals move between the compartments D (drug users) and A (substance abusers), taking into account the rates of recovery, transitions, and interactions within the population.

Step 4: Calculate the Next-Generation Matrix K
The next-generation matrix K is obtained by multiplying F and V^{-1}. The inverse of V is:

$$V^{-1} = \frac{1}{(\mu + \alpha + \rho)(\mu + \gamma)} \begin{pmatrix} \mu + \gamma & 0 \\ \rho & \mu + \alpha + \rho \end{pmatrix} = \begin{pmatrix} \frac{1}{\mu+\alpha+\rho} & 0 \\ \frac{\rho}{(\mu+\alpha+\rho)(\mu+\gamma)} & \frac{1}{\mu+\gamma} \end{pmatrix}$$

Now, compute K:

$$K = FV^{-1} = \begin{pmatrix} \frac{\beta\Lambda}{\mu} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\mu+\alpha+\rho} & 0 \\ \frac{\rho}{(\mu+\alpha+\rho)(\mu+\gamma)} & \frac{1}{\mu+\gamma} \end{pmatrix} = \begin{pmatrix} \frac{\beta\Lambda}{\mu(\mu+\alpha+\rho)} & 0 \\ 0 & 0 \end{pmatrix}$$

Step 5: Determine the Basic Reproduction Number R_0
The basic reproduction number R_0 is the largest eigenvalue of the next-generation matrix K. Since K is a diagonal matrix, the eigenvalues are simply the entries on the diagonal. Therefore:

$$R_0 = \frac{\beta\Lambda}{\mu(\mu + \alpha + \rho)}$$

3.2.5 Endemic Equilibrium:

The endemic equilibrium is denoted by E^e and defined as a steady-state solution for the model (1). This can occur when there is persistence of substance abuse. Hence $E^e = (S^e, D^e, A^e, R^e)$.

We equate the right side of equations in (1) to zero and solve the resulting steady-state system of equations:

$$\Lambda - (\beta D + \mu)S = 0 \qquad (2)$$
$$\beta SD - (\mu + \alpha + \rho)D = 0 \qquad (3)$$
$$\rho D - (\mu + \gamma)A = 0 \qquad (4)$$
$$\alpha D + \gamma A - \mu R = 0 \qquad (5)$$

From equation (2), solving for D:

$$\Lambda - (\beta D + \mu)S = 0 \implies \Lambda = (\beta D + \mu)S \implies D = \frac{\Lambda - \mu S}{\beta S} \qquad (6)$$

From equation (3), solving for S:

$$\beta SD - (\mu + \alpha + \rho)D = 0 \implies \beta SD = (\mu + \alpha + \rho)D \implies S^e = S = \frac{\mu + \alpha + \rho}{\beta} \qquad (7)$$

From equation (4), solving for A:

$$\rho D - (\mu + \gamma)A = 0 \implies \rho D = (\mu + \gamma)A \implies A^e = A = \frac{\rho D}{\mu + \gamma} \qquad (8)$$

Substituting S from equation (7) into D (6):

$$D = \frac{\Lambda\beta - \mu(\mu + \alpha + \rho)}{\beta(\mu + \alpha + \rho)} \implies D^e = D = \frac{\Lambda\beta - \mu(\mu + \alpha + \rho)}{\beta(\mu + \alpha + \rho)}$$

Substituting D into equation (8):

$$A = \frac{\rho(\Lambda\beta - \mu(\mu + \alpha + \rho))}{\beta(\mu + \alpha + \rho)(\mu + \gamma)} \implies A^e = A = \frac{\rho(\Lambda\beta - \mu(\mu + \alpha + \rho))}{\beta(\mu + \alpha + \rho)(\mu + \gamma)}$$

From equation (5), solving for R:

$$\alpha D + \gamma A - \mu R = 0 \implies R = \frac{\alpha D + \gamma A}{\mu}$$

Substituting the values of D^e and A^e into the equation:

$$R = \frac{\alpha\left(\Lambda\beta - \mu(\mu + \alpha + \rho)\right) + \gamma\rho\left(\Lambda\beta - \mu(\mu + \alpha + \rho)\right)}{\mu\beta(\mu + \alpha + \rho)(\mu + \gamma)}$$

R^e is given by:

$$R^e = R = \frac{(\mu + \gamma)\alpha + \gamma\rho(\Lambda\beta - \mu(\mu + \alpha + \rho))}{\mu\beta(\mu + \alpha + \rho)(\mu + \gamma)}$$

therefore endemic equilibrium is given by ;

$$E^e = \frac{\mu+\alpha+\rho}{\beta}, \frac{\Lambda\beta - \mu(\mu+\alpha+\rho)}{\beta(\mu+\alpha+\rho)}, \frac{\rho(\Lambda\beta - \mu(\mu+\alpha+\rho))}{\beta(\mu+\alpha+\rho)(\mu+\gamma)}, \frac{(\mu+\gamma)\alpha + \gamma\rho(\Lambda\beta - \mu(\mu+\alpha+\rho))}{\mu\beta(\mu+\alpha+\rho)(\mu+\gamma)}$$

3.2.6 Sensitivity Analysis of R_0

The fundamental objective of sensitivity analysis is to quantitatively estimate the importance of each parameter to the spread of substance abuse among commercial drivers. Sensitivity analysis is commonly used to determine the validity of model predictions given parameter values, especially considering errors in data collection and estimation.

The normalized forward sensitivity index of R_0, which depends differentiably on parameter P, is defined by:

$$S_P^{R_0} = \frac{\partial R_0}{\partial P} \cdot \frac{P}{R_0}$$

The basic reproduction number R_0 depends on five parameters: $\Lambda, \beta, \mu, \alpha, \rho$. Specifically,

$$R_0 = \frac{\beta\Lambda}{\mu(\mu+\alpha+\rho)}$$

For the parameter Λ, we calculate the sensitivity index $S_\Lambda^{R_0}$:

$$S_\Lambda^{R_0} = \frac{\partial R_0}{\partial \Lambda} \cdot \frac{\Lambda}{R_0}$$

Applying the quotient rule: Let $U = \beta\Lambda$ and $V = \mu(\mu+\alpha+\rho)$,

$$\frac{\partial R_0}{\partial \Lambda} = \frac{U'V - V'U}{V^2} = \frac{\beta\mu(\mu+\alpha+\rho)}{(\mu(\mu+\alpha+\rho))^2}$$

Therefore,

$$S_\Lambda^{R_0} = \frac{\beta\mu(\mu+\alpha+\rho)}{(\mu(\mu+\alpha+\rho))^2} \cdot \frac{\Lambda}{\frac{\beta\Lambda}{\mu(\mu+\alpha+\rho)}}$$

Simplifying gives us:

$$S_\Lambda^{R_0} = 1$$

This indicates that the sensitivity index $S_\Lambda^{R_0}$ for R_0 with respect to Λ is positive, suggesting that Λ has a significant impact on R_0.

For parameter $P = \beta$:

$$S_\beta^{R_0} = \frac{\partial R_0}{\partial \beta} \cdot \frac{\beta}{R_0}$$

Given:

$$R_0 = \frac{\beta\Lambda}{\mu(\mu+\alpha+\rho)}$$

Let $U = \beta\Lambda$ and $V = \mu(\mu+\alpha+\rho)$,

$$\frac{\partial R_0}{\partial \beta} = \frac{U'V - V'U}{V^2} = \frac{\Lambda\mu(\mu+\alpha+\rho)}{(\mu(\mu+\alpha+\rho))^2}$$

Therefore,

$$S_{\beta}^{R_0} = \frac{\Lambda\mu(\mu+\alpha+\rho)}{(\mu(\mu+\alpha+\rho))^2} \cdot \frac{\beta\mu(\mu+\alpha+\rho)}{\beta\Lambda}$$

Simplifying gives us:

$$S_{\beta}^{R_0} = 1 > 0$$

For parameter $P = \mu$:

$$S_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} \cdot \frac{\mu}{R_0}$$

Given:

$$R_0 = \frac{\beta\Lambda}{\mu(\mu+\alpha+\rho)}$$

Let $U = \beta\Lambda$ and $V = \mu(\mu+\alpha+\rho)$,

$$\frac{\partial R_0}{\partial \mu} = \frac{U'V - V'U}{V^2} = -\frac{\beta\Lambda(\mu+\alpha+\rho)}{\mu^2(\mu+\alpha+\rho)^2}$$

Therefore,

$$S_{\mu}^{R_0} = -\frac{\beta\Lambda(\mu+\alpha+\rho)}{\mu^2(\mu+\alpha+\rho)^2} \cdot \frac{\mu^2(\mu+\alpha+\rho)}{\beta\Lambda}$$

Simplifying gives us:

$$S_{\mu}^{R_0} = -1 < 0$$

For parameter $P = \alpha$:

$$S_{\alpha}^{R_0} = \frac{\partial R_0}{\partial \alpha} \cdot \frac{\alpha}{R_0}$$

Given:

$$R_0 = \frac{\beta\Lambda}{\mu(\mu+\alpha+\rho)}$$

Let $U = \beta\Lambda$ and $V = \mu(\mu+\alpha+\rho)$,

$$\frac{\partial R_0}{\partial \alpha} = \frac{U'V - V'U}{V^2} = -\frac{\beta\Lambda\mu}{\mu^2(\mu+\alpha+\rho)^2}$$

Therefore,

$$S_{\alpha}^{R_0} = -\frac{\beta\Lambda\mu}{\mu^2(\mu+\alpha+\rho)^2} \cdot \frac{\alpha\mu(\mu+\alpha+\rho)}{\beta\Lambda}$$

Simplifying gives us:

$$S_{\alpha}^{R_0} = -\frac{\alpha}{\mu+\alpha+\rho} < 0$$

For parameter $P = \rho$:

$$S_{\rho}^{R_0} = \frac{\partial R_0}{\partial \rho} \cdot \frac{\rho}{R_0}$$

Given:

$$R_0 = \frac{\beta\Lambda}{\mu(\mu+\alpha+\rho)}$$

Let $U = \beta\Lambda$ and $V = \mu(\mu+\alpha+\rho)$,

$$\frac{\partial R_0}{\partial \rho} = \frac{U'V - V'U}{V^2} = -\frac{\beta\Lambda\mu}{\mu^2(\mu+\alpha+\rho)^2}$$

Therefore,

$$S_\rho^{R_0} = -\frac{\beta\Lambda\mu}{\mu^2(\mu+\alpha+\rho)^2} \cdot \frac{\rho\mu(\mu+\alpha+\rho)}{\beta\Lambda}$$

Simplifying gives us:

$$S_\rho^{R_0} = -\frac{\rho}{\mu+\alpha+\rho} < 0$$

Parameter	**Sensitivity index(-ve/+ve)**
Λ	+ve
μ	-ve
β	+ve
ρ	-ve
α	-ve

Table 2

These sensitivity index values indicate whether a change in each parameter β, μ, α, ρ will increase or decrease the value of R_0. A positive index suggests that an increase in the parameter will increase R_0, while a negative index suggests the opposite.

Chapter 4

Numerical analysis and Simulation

Parameter	Parameter values	Sources
Λ	0.027	Kenya birth rate (Kenya National Bureau of Statistics)
μ	0.0164	Centers for Disease Control and Prevention
β	0.00023	Kanyaa, J. K., Osman, S., & Wainaina, M. (2018)
ρ	0.0746	Akande, R. O.,et al. (2023)
α	0.075	Centers for Disease Control and Prevention

With the aim of observing the dynamics of substance abuse behaviours over time, numerical solutions are done using python software. We make use of the parameters in table above in simulation.

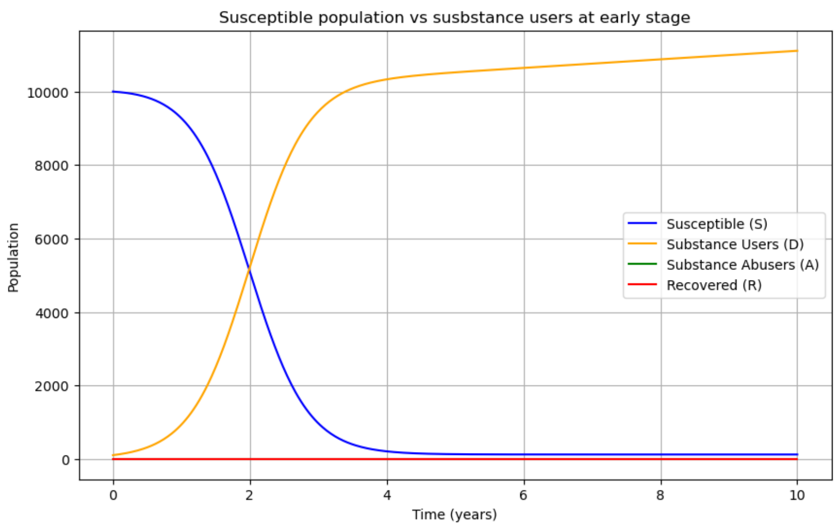

Figure 4.1:

Figure 4.1 is a graphical representation of Susceptible, substance users, substance abusers and Recovered at early stage.This is the time when substance use behaviour is introduced into the susceptible population causing the susceptible population to gradually decreases with time due to a high contact and influence rate (beta) causing a swift movement into the substance use category which results to a sharply increase in the substance users population.

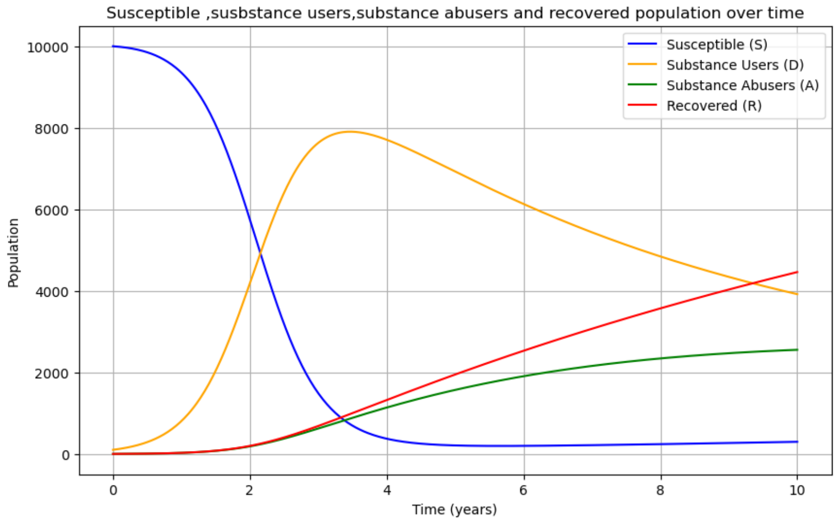

Figure 4.2:

Figure 4.2 provides a visualization of the dynamics between the four populations (Susceptible ,susbstance users,substance abusers and recovered) over time.

Susceptible Population (S)

The susceptible population starts at a high value and decreases gradually over time. Over the first few years, there is a sharp decline in this group, primarily due to a significant portion transitioning into substance use. This rapid decline suggests that during the initial period, many individuals are influenced or pushed towards substance use due to contact and influence from those using the substances. After sometime , the susceptible population begins to stabilize at a much lower level, indicating that most of those who were at risk have either become substance users, moved into the abuser category, or have died due to natural death.

Substance Users (D)

The Substance Users population initially increases, reaching a peak before gradually declining. This rapid increase aligns with the sharp decline in the susceptible population, indicating that a significant number of individuals are transitioning into substance use during this period. However, after reaching its peak, the number of substance users starts to gradually decrease. This decrease is due to various factors such as recovery, transition into substance abuse, or exit from the population due to mortality.

Substance Abusers (A)

The population of substance abusers exhibits a steady and consistent growth throughout the period. Unlike the substance users, whose numbers peak and then decline, substance abusers steadily increase over time. This growth indicates a progression from substance use to abuse, highlighting the long-term risks and challenges associated with substance use.Over time , the population of substance abusers begins to surpass the declining number of substance users, indicating a shift in the population where more individuals are moving from substance use to abuse rather than recovering.

Recovered Population (R)

The recovered population grows slowly but steadily over the entire period. While the initial growth is not noticeable , it becomes more noticeable as the number of substance users begins to decline. By the end of the period, the recovered population surpasses the number of substance users, suggesting that recovery becomes more prevalent as time progresses. This trend highlights the importance of long-term recovery strategies and interventions, which, while slow to take effect, can lead to a significant portion of the population recovering from substance use.

Overall, the graph shows a clear pattern where drivers start from being at risk of substance use, then move on to actually using substances, and eventually either recover or fall into deeper substance abuse. In the early years, there's a quick increase in the number of substance users as more drivers start using. As time goes on, the number of users reaches its highest point and then starts to decrease. Some drivers recover, while others continue to struggle with substance abuse. In the long run, more drivers recover, but the number of those who continue abusing substances still grows, though more slowly. This pattern highlights the complex nature of substance use and the need for urgent interventions at different stages to address it effectively.

4.1 Evaluating the effects of contact and imitation rates (β) on different populations

In this evaluation , we aim to evaluate the impact of contact and imitation rate (β) on the dynamics of Substance within a population. The contact and imitation rate (β) represents the rate at which susceptible individuals come into contact with substance users individuals and subsequently become substance users. β directly influences the transmission of substance use behavior, making it a critical parameter for understanding and controlling substance use and abuse within a population . By examining different values of β, we can gain insights into the effectiveness of potential control measures aimed at reducing substance use and abuse behaviours. We consider four different values of β: 0.00023, 0.00021, 0.00019 and 0.00025.

Effect on susceptible population

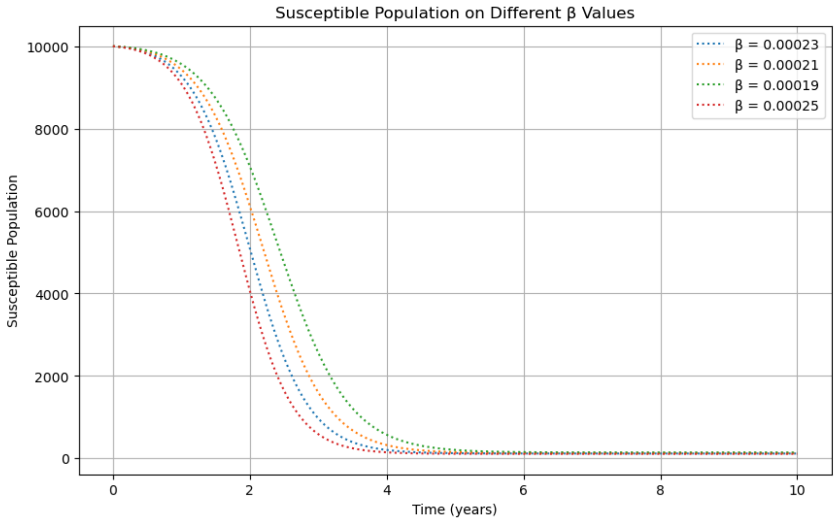

Figure 4.3:

Effect on substance users population

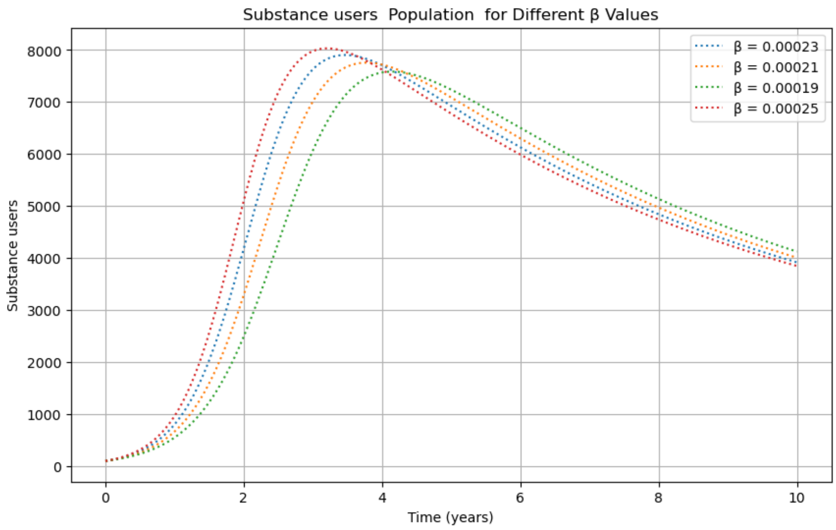

Figure 4.4:

Effect on substance abusers population

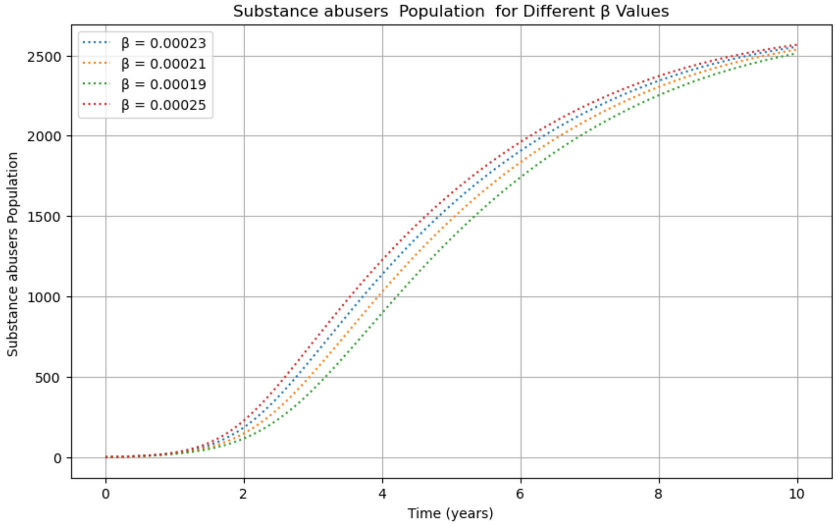

Figure 4.5:

Interpretations

In figure 4.3, the susceptible population decreases over time for all β values. Higherβ values lead to a faster decline in the susceptible population which is because a higher contact and imitation rate means more susceptible individuals are becoming substance users more quickly. The graph shows that with $\beta = 0.00025$, the susceptible population declines the fastest, followed by $\beta = 0.00023$ and $\beta = 0.00021$ and finally 0.00019 .

In figure 4.4, the substance users population initially increases and then starts to decline. Higher β values result in a higher peak of the substance users population. This indicates that with a higher contact and imitation rate , more individuals become substance users before the decline sets in due to death or recovery. The graph shows that the peak of the substance users population is highest for $\beta = 0.00025$ and smallest for $\beta = 0.00019$.

In figure 4.5 ,the substance abusers population increases over time.Higher β values lead to a higher substance abusers population over time. This is because as more individuals become substance users , more individuals transitions to substance abusers. The graph shows that the substance abusers population over time is highest for $\beta = 0.00025$ and smallest for $\beta = 0.00019$.

From the graphs and interpretations above we can see that higher β values lead to a faster spread of substance abuse among commercial drivers, resulting in a higher peak in the population of substance users and a larger number of individuals who progress to substance abuse and for lower β values it results in a slower spread of substance abuse, a lower peak in the population of substance users, and fewer individuals progressing to substance abuse.

These results highlight the importance of controlling the contact and imitation rate (β)

to manage the dynamics of substance use, susceptibility, recovery, and overall health among commercial drivers. Effective interventions that reduce β can lead to a healthier workforce with fewer drivers engaging in substance abuse.

4.2 Discussion

The mathematical model formulated in this research provides crucial insights into the dynamics of substance abuse among commercial drivers, emphasizing the significant role of the contact and imitation rate (β) in influencing the number of substance users and abusers over time. Higher β values lead to a faster spread of substance abuse, resulting in a more rapid decline in the susceptible population, a higher peak in substance users, and a steadily growing population of substance abusers. This underscores the importance of β as a critical parameter in the progression of substance abuse within this population, suggesting that targeted interventions to reduce β could slow the spread of substance use, lower the peak number of substance users, and decrease the long-term population of substance abusers.

Chapter 5

Conclusion and Recomendations

5.1 Conclusion

The substance model was qualitatively and quantitatively analyzed to provide insights into the spread of substance use and abuse among commercial drivers. The threshold numbers for the use and abuse of substances were computed, revealing that substance abuse will continue to spread if the abuse number is greater than one, and will eventually decline if the abuse number is less than one. Sensitivity analysis was conducted to identify the most sensitive parameters influencing the abuse number , thereby guiding decisions on the factors contributing to the spread of substance abuse.A positive sensitivity index indicates that an increase in the parameter leads to an increase in Reproduction number and negative sensitivity index means that an increase in the parameter results in a decrease in Reproduction number.

From sensitivity index in table 2 it shows that an increase in β and Λ will increase the Reproduction number which suggests that a higher influx of individuals into the population or more individuals in contact with those who are using substances will amplify the spread of the substance use and abuse behavior.

Numerical simulations of the model were performed to further explain the dynamics of substance use among commercial drivers, particularly those at risk due to contact and imitation. Figure 4.1 demonstrates that the susceptible population decreases over time as the number of substance users increases . This trend suggests that the population of drivers at risk of substance use grows as more drivers continue to use substances. Additionally, Figure 4.2 shows an increase in the number of substance users as the susceptible population decreases, followed by a subsequent increase in the number of recovered drivers, indicating that as recovery increases, the number of substance users decreases.

An analysis of the imitation and contact rates among drivers was conducted to determine their effect on susceptible population, substance users and abusers. Figure 4.3,4.4 and 4.5 illustrates the effects of these rates, revealing that increased interaction and imitation among drivers lead to a rise in the number of substance users and abusers and a decrease in susceptible population . This confirms that the imitation and interaction among drivers play a significant role in the spread of substance abuse within this population.

Overall, increased imitation and contact rates among drivers are shown to elevate substance use and abuse while reducing the susceptible population, underscoring the significant role of these factors in the dynamics of substance abuse. This study underscores the importance of mathematical modeling in understanding and addressing substance use and abuse dynamics. The model provides valuable insights for policymakers to design effective strategies and interventions to mitigate substance use behaviors effect on the society.

5.2 Recomendations

Recommendations to Curb Substance Use and Abuse by Commercial Drivers

The following recommendations are strategies that can effectively reduce substance abuse among commercial drivers.

1. **Enhanced alcohol testing measures and Monitoring:** Regular substance testing should be conducted for commercial drivers to detect substance use early. Implement random testing policies and ensure that testing protocols are up-to-date and comprehensive.For example alcho blows

2. **Education and Training:** Develop and deliver targeted educational programs about the risks of substance abuse, including the impact on safety and health. Training should also focus on recognizing signs of substance abuse and understanding how to seek help.

3. **Support Programs:** Establish support and intervention programs for drivers struggling with substance abuse. This can include counseling, rehabilitation services, and support groups designed to help drivers recover and reintegrate into their professional roles.

4. **Policy Enforcement:** Strengthen regulations and policies regarding substance use in the transportation industry. Ensure strict enforcement of these policies and establish clear consequences for violations to deter potential offenders.

5. **Engage in Community Outreach:** Collaborate with local communities and organizations to raise awareness about substance abuse issues and promote resources available for drivers and their families.

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Appendices

Python code for substance use dynamics

Python code for graph at early stage

Listing 5.1: Substance Abuse Model

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint

# Define the system of ODEs
def substance_abuse_model(y, t, Lambda, mu, beta, rho, alpha, gamma):
S, D, A, R = y
N = S + D + A + R  # Total population
dS_dt = Lambda*N - (beta * D + mu) * S
dD_dt = beta * S * D - (mu + alpha + rho) * D
dA_dt = rho * D - (mu + gamma) * A
dR_dt = alpha * D + gamma * A - mu * R
return [dS_dt, dD_dt, dA_dt, dR_dt]
# Initial conditions (number of individuals)
S0 = 10000  # Initial susceptible population
D0 = 100       # Initial substance users
A0 = 0     # Initial substance abusers
R0 = 0      # Initial recovered individuals
y0 = [S0, D0, A0, R0]
# Parameter values
Lambda = 0.027  # Recruitment rate
mu = 0.0164      # Death rate
beta = 0.00023      # Substance initiation rate
rho = 0.0     # Substance abuse rate
alpha = 0.0     # Recovery rate of substance users
gamma = 0.75     # Recovery rate of substance abusers
# Time points (in years)
t = np.linspace(0, 5, 50)  # Time from 0 to 50 years, with 500 points



# Solve the ODEs
solution = odeint(substance_abuse_model, y0, t, args=(Lambda, mu,
```

```
 beta, rho, alpha, gamma))
S, D, A, R = solution.T
# Plot all populations together
plt.figure(figsize=(10, 6))
plt.plot(t, S, label='Susceptible (S)', color='blue')
plt.plot(t, D, label='Substance Users (D)', color='orange')
plt.plot(t, A, label='Substance Abusers (A)', color='green')
plt.plot(t, R, label='Recovered (R)', color='red')
plt.xlabel('Time (years)')
plt.ylabel('Population')
plt.title('Dynamics of Substance Abuse Among Commercial Drivers')
plt.legend()
plt.grid()
plt.show()
```

Python code for the graph of the four populations (Susceptible ,susbstance users,substance abusers and recovered) over time.

```
        import numpy as np
        import matplotlib.pyplot as plt
        from scipy.integrate import odeint

        # Define the system of ODEs
 def substance_abuse_model(y, t, Lambda, mu, beta, rho, alpha, gamma):
        S, D, A, R = y
        N = S + D + A + R  # Total population

        dS_dt = Lambda*N - (beta * D + mu) * S
        dD_dt = beta * S * D - (mu + alpha + rho) * D
        dA_dt = rho * D - (mu + gamma) * A
        dR_dt = alpha * D + gamma * A - mu * R

        return [dS_dt, dD_dt, dA_dt, dR_dt]

        # Initial conditions (number of individuals)
        S0 = 10000  # Initial susceptible population
        D0 = 100    # Initial substance users
        A0 = 0      # Initial substance abusers
        R0 = 0      # Initial recovered individuals
        y0 = [S0, D0, A0, R0]

        # Parameter values
        Lambda = 0.027  # Recruitment rate
        mu = 0.0164     # Death rate
        beta = 0.00023  # Substance initiation rate
        rho = 0.076     # Substance abuse rate
        alpha = 0.075   # Recovery rate of substance users
```

```
    gamma = 0.075   # Recovery rate of substance abusers

    # Time points (in years)
t = np.linspace(0, 10, 500)# Time from 0 to 10 years, with 500 points

    # Solve the ODEs
  solution = odeint(substance_abuse_model, y0, t, args=(Lambda, mu,beta,
  rho, alpha, gamma))

    S, D, A, R = solution.T

    # Plot all populations together
    plt.figure(figsize=(10, 6))
    plt.plot(t, S, label='Susceptible (S)', color='blue')
    plt.plot(t, D, label='Substance Users (D)', color='orange')
    plt.plot(t, A, label='Substance Abusers (A)', color='green')
    plt.plot(t, R, label='Recovered (R)', color='red')
    plt.xlabel('Time (years)')
    plt.ylabel('Population')
    plt.title('Susceptible, Substance Users, Substance Abusers, and
    Recovered Population Over Time')
    plt.legend()
    plt.grid()
    plt.show()
```

python code for graph effect of contact and imitation rate on susceptible population

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint

# Define the system of ODEs
def substance_abuse_model(y, t, Lambda, mu, beta, rho,
 alpha, gamma):

S, D, A, R = y
N = S + D + A + R  # Total population

dS_dt = Lambda*N - (beta * D + mu) * S
dD_dt = beta * S * D - (mu + alpha + rho) * D
dA_dt = rho * D - (mu + gamma) * A
dR_dt = alpha * D + gamma * A - mu * R

return [dS_dt, dD_dt, dA_dt, dR_dt]

# Initial conditions (number of individuals)
```

```
S0 = 10000  # Initial susceptible population
D0 = 100    # Initial substance users
A0 = 0      # Initial substance abusers
R0 = 0      # Initial recovered individuals
y0 = [S0, D0, A0, R0]

# Parameter values
Lambda = 0.027  # Recruitment rate
mu = 0.0164     # Death rate
rho = 0.0       # Substance abuse rate
alpha = 0.0     # Recovery rate of substance users
gamma = 0.75    # Recovery rate of substance abusers

# Beta values to test
beta_values = [0.00023, 0.00021, 0.00019, 0.00025]

# Time points (in years)
t = np.linspace(0, 10, 500)

# Plotting
plt.figure(figsize=(10, 6))

for beta in beta_values:
# Solve the ODEs for each beta value
solution = odeint(substance_abuse_model, y0, t, args=
(Lambda, mu, beta, rho, alpha, gamma))

S, D, A, R = solution.T

# Plot Susceptible population (S) with dotted lines
plt.plot(t, S, linestyle=':', label=f'$\beta$ = {beta}')

plt.xlabel('Time (years)')
plt.ylabel('Susceptible Population')
plt.title('Susceptible Population on Different $\beta$ Values')
plt.legend()
plt.grid()
plt.show()
```

python code for graph effect of contact and imitation rate on substance users population

Listing 5.2: Substance Abuse Model in Python

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
```

```
# Define the system of ODEs
def substance_abuse_model(y, t, Lambda, mu, beta, rho,
alpha, gamma):
S, D, A, R = y
N = S + D + A + R  # Total population

dS_dt = Lambda*N - (beta * D + mu) * S
dD_dt = beta * S * D - (mu + alpha + rho) * D
dA_dt = rho * D - (mu + gamma) * A
dR_dt = alpha * D + gamma * A - mu * R

return [dS_dt, dD_dt, dA_dt, dR_dt]

# Initial conditions (number of individuals)
S0 = 10000  # Initial susceptible population
D0 = 100    # Initial substance users
A0 = 0      # Initial substance abusers
R0 = 0      # Initial recovered individuals
y0 = [S0, D0, A0, R0]

# Parameter values
Lambda = 0.027  # Recruitment rate
mu = 0.0164     # Death rate
rho = 0.076     # Substance abuse rate
alpha = 0.075   # Recovery rate of substance users
gamma = 0.075   # Recovery rate of substance abusers

# Beta values to test
beta_values = [0.00023, 0.00021, 0.00019, 0.00025]

# Time points (in years)
t = np.linspace(0, 10, 500)

# Plotting
plt.figure(figsize=(10, 6))

for beta in beta_values:
# Solve the ODEs for each beta value
solution = odeint(substance_abuse_model, y0, t, args=
(Lambda, mu, beta, rho, alpha, gamma))
S, D, A, R = solution.T

# Plot Susceptible population (S)
plt.plot(t, D, linestyle=':', label=f'$\beta$ = {beta}')

plt.xlabel('Time (years)')
plt.ylabel('Substance users')
plt.title('Substance Users Population for
```

```
                 Different $\beta$ Values')

        plt.legend()
        plt.grid()
        plt.show()
```

python code for graph effect of contact and imitation rate on substance abusers population

Listing 5.3: Python code for modeling substance abuse dynamics

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint

# Define the system of ODEs
def substance_abuse_model(y, t, Lambda, mu, beta, rho,
alpha, gamma):
S, D, A, R = y
N = S + D + A + R  # Total population

dS_dt = Lambda*N - (beta * D + mu) * S
dD_dt = beta * S * D - (mu + alpha + rho) * D
dA_dt = rho * D - (mu + gamma) * A
dR_dt = alpha * D + gamma * A - mu * R

return [dS_dt, dD_dt, dA_dt, dR_dt]

# Initial conditions (number of individuals)
S0 = 10000  # Initial susceptible population
D0 = 100    # Initial substance users
A0 = 0      # Initial substance abusers
R0 = 0      # Initial recovered individuals
y0 = [S0, D0, A0, R0]

# Parameter values
Lambda = 0.027  # Recruitment rate
mu = 0.0164     # Death rate
rho = 0.076     # Substance abuse rate
alpha = 0.075   # Recovery rate of substance users
gamma = 0.075   # Recovery rate of substance abusers
# Beta values to test
beta_values = [0.00023, 0.00021, 0.00019, 0.00025]

# Time points (in years)
        t = np.linspace(0, 10, 500)

# Plotting
```

```
plt.figure(figsize=(10, 6))

for beta in beta_values:
# Solve the ODEs for each beta value
solution = odeint(substance_abuse_model, y0, t, args=(Lambda, mu, beta,
           rho, alpha, gamma))
S, D, A, R = solution.T

# Plot Susceptible population (S)
plt.plot(t, A, linestyle=':', label=f'$\beta$ = {beta}')

plt.xlabel('Time (years)')
plt.ylabel('Substance abusers Population ')
plt.title('Substance abusers Population for Different
           $\beta$ Values')
plt.legend()
plt.grid()
plt.show()
```