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The Effect of EV Aggregators with Time-Varying Delays on the Stability of a Load Frequency Control System

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***Abstract*—Participation of EV aggregators in frequency regulation service may cause time-varying delays in load frequency control (LFC) systems. These time-varying delays affect the performance and even cause instability of power systems. In this paper, we investigate a single and multiple time-varying delays-dependent stability of an LFC system with participation of EV aggregators. Based on the Lyapunov theory and linear matrix inequality (LMI) approach, we propose delay-dependent stability criteria for a single and multiple time-varying delays using the Wirtinger-based improved integral inequality. With the stability criteria, we obtain delay margins for LFC with a single and two EV aggregators and investigate the interaction among the delay margins, PI controller gains, and participation ratios of EV aggregators. In addition, we investigate the significance of time-varying delays in fast-response resources by comparing the delay margin obtained in EV aggregator model with that obtained in delayed conventional generators. We discuss an extension to multi-area LFC systems and guidelines on how to improve performance of LFC systems under time-varying delays. We expect that the proposed criteria can help to guide the determination of delay requirements for EV aggregators participating in frequency regulation service.**

***Index Terms*—Electric vehicle, time-varying delay, power system stability, stability criteria, load frequency control**

I. INTRODUCTION

Electric vehicles (EVs) have been paid attention in recent years due to environmental concerns and gradual depletion of fossils resources. With a vehicle-to-grid (V2G) technology [1], EVs are expected to be utilized in a wide range of applications: smoothing of renewable resources [2], [3], resources for ancillary services [4], [5], [6], [7], and so on. Among these applications, provision of frequency regulation service with EVs has attracted attention because battery in EVs can increase or decrease faster power output than conventional generators. This fast response characteristic enhances dynamic performance of load frequency control (LFC) system. In addition, when EV owners participate in frequency regulation service, they gain additional economic benefit from frequency regulation markets. Therefore, the participation of EVs in frequency regulation service will be encouraged in the near future.

For a practical participation in frequency regulation markets, a relatively large number of EVs should be aggregated by a new entity called an *EV aggregator* [9], [10] to meet a minimum regulation capacity. For example, since the ISO-NE sets a minimum regulation capacity to 1 MW, an EV aggregator needs 500 EVs for participation if one EV provides 2kW. In addition to the minimum requirement, EV aggregators should have an automatic generation control (AGC) system which can increase or decrease power output of each EV in a real-time manner. For the AGC system, EV aggregators require their underlying communication networks in which control commands are transferred to EVs. For the communication networks, a dedicated or an open communication network can be selected, but the latter may be preferred due to low installation cost. However, open communication networks may cause time-varying delays in a particular range. These time-varying delays could cause instability of power systems against an expectation that EVs having a fast-response characteristic can improve the LFC performance. Hence, it is important to investigate the stability analysis of load frequency control (LFC) systems when EV aggregators having time-varying delayed responses participate in the LFC systems.

Stability studies on time-delayed LFC systems have recently been attracted due to the extensive use of phasor measurement units (PMUs) and open communication networks. These studies considered the following issues: 1) computing delay margin [11], [12], [13], 2) computing controller gains [14], 3) controller design [15], etc. Computing delay margin is to find an allowable upper-bound delay to guarantee the stable power systems at a given controller gain . Jiang *et al.* [11] investigated the delay-dependent stability criterion of LFC systems with a single time-varying delay and multiple constant time delays. It was shown that a setting of PI gain is affected by the allowable upper-bound delay. Zhang *et al.* [12] improved a delay-dependent stability criterion of LFC systems with multiple constant time delays in terms of accuracy and computation time. Sonmez and Ayasun [13] proposed a method to determine the delay margin of one-area and two-area LFC systems with constant time delays. Computing controller gains is to obtain a set of controller gains to guarantee the stable power systems under given information on a single constant delay [14]. From the viewpoint of controller design, Yu and Tomsovic [15] proposed a controller design method for LFC systems with constant delays.

However, there have been no studies on the stability of LFC systems with EV aggregators which have time-varying delays as well as constant-time delays. In addition, most of the previous studies focused on a single time-varying delay or multiple constant time delays [11], [12], while there may

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exist *multiple time-varying delays* in EV aggregators' domain. Therefore, we need to investigate the stability analysis of LFC systems with multiple EV aggregators each of which has a time-varying delay.

Two types of approaches have been developed to compute the delay margin for stability: frequency- and time-domain. The frequency domain approach is to obtain the exact delay margin, however, it is hard for this approach to be applied to a case of time-varying delays. Zhang *et al.* [12] proposed a frequency domain method to compute the delay margins under multiple constant-time delays. The time domain methods utilizing the Lyapunov theorem and a linear matrix inequality (LMI) approach was used to analyze the stability under time-varying delays, but they yielded conservative delay margins. Hence, many researchers have improved the analysis methods in order to reduce conservatism. For example, Seuret and Gouaisbout [16] proposed a Wirtinger-based improved integral inequality to improve the Jensen's integral inequality. There is a room for improvement in the stability criterion for a single time-varying delay, compared with that used in [11]. In case of multiple time-varying delays, few studies have been done in control society. We also investigated the stability analysis for multiple time-varying delays as part of this work in [17].

In this paper, we investigate single and multiple time-varying delays-dependent stability of an LFC system with EV aggregators. The LFC system is remodeled to include multiple EV aggregators each of which has a time-varying delay characteristic in response to control signals under their underlying communication networks in EV aggregators' domain. We propose an improved stability criterion for a single time-varying delay using the Wirtinger-based improved integral inequality, compared with the criterion used in [11]. We modify the stability criterion for multiple time-varying delays proposed in [17] to apply the LFC system with multiple EV aggregators. With the proposed stability criteria, we obtain the delay margin for LFC with EV aggregators and investigate the interaction between the delay margin, participation ratios of EV aggregators, and PI controller gain. In addition, we investigate the significance of time-varying delays in fast-response resources by comparing the allowable upper-bound delays obtained in EV aggregator model with ones obtained in conventional delayed generators. The proposed criteria are expected to help to guide the determination of delay requirements for EV aggregators participating in frequency regulation service.

In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional real vector space and the set of $n \times n$ matrix space, respectively. For any symmetric matrix P, $P \succ 0$ (or $P \prec 0$) denotes a positive (or negative) definite matrix, and I is an identity matrix with an appropriate dimension. The notation $diag(\cdots)$ represents a block-diagonal matrix. $*$ denotes the symmetric term in a matrix. col denotes the column vector or matrix. e_i is an elementary vector.

II. System Model

We consider that EV aggregators provide frequency regulation services with a large number of EVs in parking lots. During the provision of the frequency regulation service, EV aggregators receive commands for regulation power from a control center and upon reception of the commands, they allocate the received regulation power to each participating EV. The allocation information is transferred to the corresponding EVs with communication delays. Each EV results in a delayed response. The delayed response may affect the dynamic performance of LFC and could even cause instability of power systems.

To investigate the stability of LFC systems when EV aggregators participating in frequency regulation service have a characteristic of time-varying delays, we need a model for the dynamics of LFC systems including EV aggregators. In general, LFC systems are described as a set of non-linear differential equations. For small-signal stability analysis, we can linearize non-linear LFC systems around an equilibrium point and then obtain linear state-space equations. Previous studies on LFC systems obtained linear state-space equations for LFC systems through linearization. We remodel the conventional LFC system by including EV aggregators with time-varying delays. We first describe an EV aggregator model with a single time-varying delay and then an LFC model including multiple EV aggregators having a time-varying delay.

A. EV Aggregator

We model an EV aggregator consisting of a large number of EVs, each of which has a delay component. The dynamic model of an EV battery system is usually described as the following first-order transfer function[1] [18], [19], [20]:

$$G_{EV}(s) = \frac{K_{EV}}{1+sT_{EV}}, \tag{1}$$

where K_{EV} denotes the gain of the EV and T_{EV} denotes the time constant of the EV battery system.

The delay taken for receiving control signals from the EV aggregator is modeled as an exponential transfer function of $e^{-s\tau(t)}$, where $\tau(t)$ denotes a time-varying delay function which models the communication delay from the EV aggregator to the EV and the scheduling delay in the EV aggregator.

For analytical simplicity, we assume that the delays for all EVs are the same as T_{delay} on an average sense and the time constants T_{EV} of all EVs are the same. From the assumption, we obtain an aggregated EV aggregator model consisting of one delay function and one EV dynamics model.

B. LFC Model including EV Aggregators with Delays

Fig. 1 shows a single-area LFC model including multiple EV aggregators with time-varying delays. Except the model of EV aggregators with time-varying delays, other components have been modeled in many LFC-related studies. We assume that all synchronous generators have reheat thermal turbines. We adopt, as an LFC controller, a PI-type controller which has been used for LFC in Germany, Netherlands, Belgium, etc [21]. The output of the PI controller is distributed to the

[1] The battery model can have a complicated element describing state-of-charge of battery, but we assume that EV aggregator does not consider EVs having near full SoC or empty SoC. Thus, we focus on a first-order transfer function.

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reheat thermal generator and N EV aggregators depending on the participation ratios $\alpha_0, \alpha_1, \ldots, \alpha_N$, where α_0 denotes the participation ratio of an aggregated thermal generator and α_i denotes the participation ratio of EV aggregator i for $i = 1, \ldots, N$.

The linearized dynamic model of the single-area LFC model including N EV aggregators with time-varying delays can be expressed as follows:

$$\dot{x}(t) = Ax(t) + B_0u(t) + \sum_{k=1}^{N} B_k u(t - \tau_k(t)) + F\Delta P_d, \quad (2)$$

$$y(t) = Cx(t), \quad (3)$$

where

$$x(t) = [\Delta f \ \Delta P_g \ \Delta P_m \ \Delta X_g \ \Delta P_{EV,1} \ \ldots \ \Delta P_{EV,N}]^T, \quad (4)$$

$$y(t) = ACE, \quad (5)$$

$$B_0 = \begin{bmatrix} 0 & 0 & \frac{F_p\alpha_0}{T_g} & \frac{\alpha_0}{T_g} & 0 & \ldots & 0 \end{bmatrix}^T, \quad (6)$$

$$B_k = \frac{\alpha_k K_{EV,k}}{T_{EV,k}} e_{4+k}, \quad (k = 1, \ldots, N) \quad (7)$$

$$C = \begin{bmatrix} \beta & 0 & 0 & 0 & 0 & \ldots & 0 \end{bmatrix}, \quad (8)$$

$$F = \begin{bmatrix} -\frac{1}{M} & 0 & 0 & 0 & 0 & \ldots & 0 \end{bmatrix}^T, \quad (9)$$

$$A = \quad (10)$$

$$\begin{bmatrix} \frac{-D}{M} & \frac{1}{M} & 0 & 0 & \frac{1}{M} & \cdots & \frac{1}{M} \\ 0 & \frac{-1}{T_c} & \frac{1}{T_c} & 0 & 0 & \cdots & 0 \\ \frac{-F_p}{RT_g} & 0 & \frac{-1}{T_r} & \frac{T_g - F_pT_r}{T_rT_g} & 0 & \cdots & 0 \\ \frac{-1}{RT_g} & 0 & 0 & \frac{-1}{T_g} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{T_{EV,1}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \frac{-1}{T_{EV,N}} \end{bmatrix},$$

and Δf, ΔX_g, ΔP_m, and ΔP_g denote the deviation of the frequency, valve position, mechanical power output, and generator power output, respectively. $\Delta P_{EV,k} (k = 1, \ldots, N)$ is the deviation of the power output in the k-th EV aggregator.

The ACE signal is defined as follows:

$$ACE = \beta \Delta f. \quad (11)$$

A PI-type LFC controller is designed as follows:

$$u(t) = -K_P ACE - K_I \int ACE \, dt. \quad (12)$$

III. Delay-Dependent Stability Criterion

In control society, the stability of linear systems with time-varying delays has been analyzed using Lyapunov theorems. Many stability criteria have been proposed to determine the allowable delay of linear systems. In order to determine the allowable delay of LFC systems including EV aggregators with time-varying delays, we propose Lyapunov theorem-based stability criteria of LFC systems.

In general, using the Lyapunov theorem introduces conservatism in the allowable delay. A few studies have been attempted to reduce the conservatism. As part of efforts to reduce conservatism, the Wirtinger-based improved integral inequality [16] and a reciprocally convex approach [22] were proposed. Using these two approaches, we propose two criteria of linear systems with a single and multiple time-varying delays.

A. Stability Criterion for a Single Time-Varying Delay

We consider the following linear system with a single time-varying delay $\tau(t)$:

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t - \tau(t)), \\ x(t) &= \phi(t), t \in [-\tau^M, 0] \end{aligned} \quad (13)$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $A_0, A_1 \in \mathbb{R}^{n\times n}$ is a constant matrix, the initial condition $\phi(t)$ is a continuously differentiable vector-valued function, $\tau(t)$ is a time-varying differentiable function and satisfy

$$0 \le \tau(t) \le \tau^M, \quad \dot{\tau}(t) < \mu \le 1, \quad (14)$$

where τ^M and μ are constants. We use τ instead of $\tau(t)$.

We propose the following theorem to determine whether the above linear system is stable by using the Wirtinger-based improved integral inequality and the reciprocally convex approach. Before stating the theorem, we define $E_i \in \mathbb{R}^{n\times 5n}$ as a block matrix consisting of 5 $n \times n$ matrices (blocks) and the i-th block is an identity matrix and others are zeros. For an example, $E_2 = [0 \ I \ 0 \ 0 \ 0]$.

Theorem. *The nominal system (13) with (14) is asymptotically stable if there exist symmetric matrices P, S_1, S_2, and R in $\mathbb{R}^{2n\times 2n}, \mathbb{R}^{n\times n}, \mathbb{R}^{n\times n}$, and $\mathbb{R}^{n\times n}$, respectively, and a matrix X in $\mathbb{R}^{2n\times 2n}$ such that the following LMIs are satisfied for τ in $[0, \tau^M]$:*

$$\Psi = \begin{bmatrix} \tilde{R} & X \\ * & \tilde{R} \end{bmatrix} \succ 0, \quad (15)$$

$$\begin{aligned} \Phi(\tau) = {} & 2Y_1^T(\tau)PY_0 + E_1^T(S_1 + S_2)E_1 - (1 - \mu)E_2^T S_1 E_2 \\ & - E_3^T S_2 E_3 + Y_2^T R Y_2 - \frac{\Gamma^T \Psi \Gamma}{(\tau^M)^2} \prec 0, \end{aligned} \quad (16)$$

where

$$\tilde{R} = diag(R, 3R),$$

$$Y_0 = \begin{bmatrix} A_0E_1 + A_1E_2 \\ E_1 - E_3 \end{bmatrix}, Y_1(\tau) = \begin{bmatrix} E_1 \\ \tau E_4 + (\tau^M - \tau)E_5 \end{bmatrix},$$

$$Y_3 = \begin{bmatrix} E_1 - E_2 \\ E_1 + E_2 - 2E_4 \end{bmatrix}, Y_4 = \begin{bmatrix} E_2 - E_3 \\ E_2 + E_3 - 2E_5 \end{bmatrix},$$

$$Y_2 = A_0E_1 + A_1E_2, \Gamma = \begin{bmatrix} Y_3^T & Y_4^T \end{bmatrix}^T.$$

Proof. Please refer to Appendix B.

B. Stability Criterion for Multiple Time-Varying Delays

We extend the above linear system with a single time-varying delay to a linear system with multiple time-varying delays, $\tau_1(t), \ldots, \tau_N(t)$, as follows:

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + \sum_{i=1}^{N} A_i x(t - \tau_i(t)), \\ x(t) &= \phi(t), t \in [-\tau^M, 0] \end{aligned} \quad (17)$$

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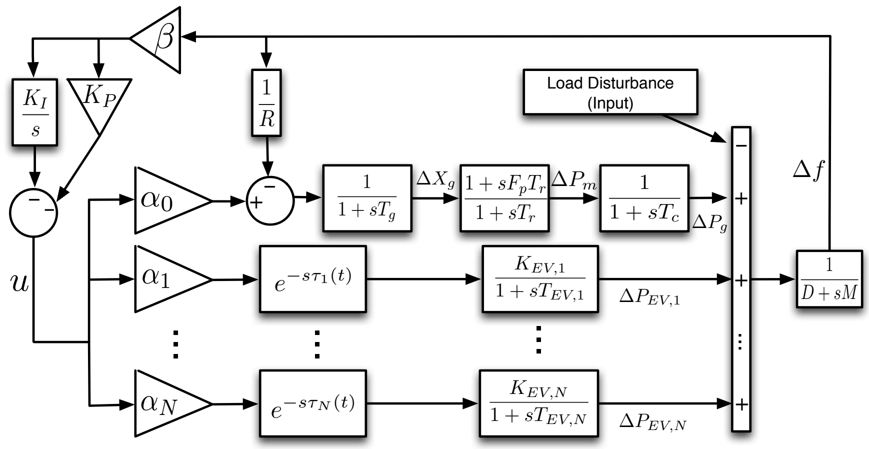

Fig. 1. Modified LFC model including EV aggregators with time-varying delays

where $x(t) \in \mathbb{R}^n$ is the system state vector, $A_i \in \mathbb{R}^{n\times n}$ $(i = 0, \ldots, N)$ are constant matrices, the initial condition $\phi(t)$ is a continuously differentiable vector-valued function, $\tau_i(t)(i = 1, \ldots, N)$ are time-varying differentiable functions and satisfy

$$0 \le \tau_i(t) \le \tau_i^M, \quad \dot{\tau}_i(t) < \mu_i \le 1, \tag{18}$$

where τ_i^M and μ_i $(i = 1, \ldots, N)$ are constants, $\tau^M = \max_{i=1,\ldots,N}(\tau_i^M)$ and without loss of generality, we assume $\tau_1^M \le \tau_2^M \le \ldots \le \tau_N^M$. We use τ_i instead of $\tau_i(t)$ for all i.

The authors proposed the stability criterion for linear systems with multiple time-varying delays in [17]. However, it has a computational problem to directly apply the proposed criterion to our LFC problem. The problem is that stable delay margin may be determined as an unstable one using some semi-definite programming algorithms such as CSDP and SDPT-3. To solve this problem, we modify the proposed criterion to the following theorem. By modifying the coefficients of double integrals from τ_i^M to $1/(\tau_i^M)$, we obtain the following theorem.

Before stating the theorem, we define $a_{i,0}, \ldots, a_{i,4}$ as the ascending ordered values of $0, \tau_{i-1}, \tau_{i-1}^M, \tau_i, \tau_i^M$ for $2 \le i \le N$. For example, $a_{2,0} = 0$, $a_{2,1} = \tau_1$, $a_{2,2} = \tau_2$, $a_{2,3} = \tau_1^M$, and $a_{2,4} = \tau_2^M$ when $i = 2$ and $\tau_1 < \tau_2 < \tau_1^M$. We redefine $E_i \in \mathbb{R}^{n\times(6N-3)n}$ $(i = 1, \ldots, 6N-3)$ as a block matrix consisting of $(6N-3)$ $n \times n$ matrices (blocks) and the i-th block is an identity matrix and others are zeros. For an example of $N = 2$, $E_2 = [0\ I\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$. We define $L(i)$ as $2N + 1 + 4(i-2)$.

Theorem. *The nominal system (17) with (18) is asymptotically stable if there exist a $(N+1)n \times (N+1)n$ real symmetric matrix $P \succ 0$ and $n \times n$ real symmetric matrices $Q_{1i} \succ 0$, $Q_{2i} \succ 0$, and $R_i \succ 0$ $(i = 1, \ldots, N)$, and constant matrices of appropriate dimensions X and $\Psi_{i,l}^{(k)}$ $(2 \le i \le N, 1 \le l \le 6, k = 1, 2, 3)$ such that the following linear matrix inequalities (LMIs) hold for τ_i in $[0, \tau_i^M]$ $(i = 1, 2, \ldots, N)$:*

$$\hat{R}_1 = \begin{bmatrix} R_1 & X \\ * & R_1 \end{bmatrix} \succeq 0, \tag{19}$$

$$\hat{R}_{il}^{(k)} = \begin{bmatrix} \tilde{R}_i & \Psi_{i,l}^{(k)} \\ * & \tilde{R}_i \end{bmatrix} \succeq 0 \begin{pmatrix} k = 1,2,3, l = 1, \ldots, 6 \\ i = 1, \ldots, N \end{pmatrix}, \tag{20}$$

$$\Xi(\tau_1, \ldots, \tau_N) = 2G_0^T P G_2(\tau_1, \ldots, \tau_N) + \Theta - \frac{\Gamma^T \hat{R}_1 \Gamma}{(\tau_1^M)^2} + G_1^T(\sum_{i=1}^N R_i)G_1 - \sum_{i=2}^N \frac{(\Sigma_i^{(k)})^T \hat{R}_i^{(k)} \Sigma_i^{(k)}}{(\tau_i^M)^2} \prec 0, \tag{21}$$

where $\tilde{R}_i = diag\{R_i, 3R_i\}$ *for* $2 \le i \le N$, $\Theta = diag\{\sum_{i=1}^N Q_{1i} + Q_{2i}, -(1-\mu_1)Q_{21}, \ldots, -(1-\mu_N)Q_{2N}, -Q_{11}, \ldots, -Q_{1N}, 0_{4(N-1)}\}$,

$$G_0 = col\{\sum_{i=0}^N A_i E_{i+1}, E_1 - E_{N+1+1}, \ldots, E_1 - E_{N+1+N}\},$$

$$G_1 = \sum_{i=0}^N A_i E_{i+1}, \quad \Gamma = col\{E_1 - E_2, E_2 - E_{N+1+1}\},$$

$$G_2(\tau_1, \ldots, \tau_N) = col\{E_1, G_{2,1}, \frac{1}{2}(G_{2,2} + \sum_{k=1}^4 (a_{2,k} - a_{2,(k-1)})E_{L(2)+k}), \ldots, \frac{1}{2}(G_{2,(N-1)} + \sum_{k=1}^4 (a_{(N-1),k} - a_{(N-1),(k-1)})E_{L(N-1)+k}), \sum_{k=1}^4 (a_{N,k} - a_{N,(k-1)})E_{L(N)+k}\}$$

with for $1 \le i < N$, $G_{2,i} =$

$$\begin{cases} \sum_{k=1}^2 (a_{(i+1),k} - a_{(i+1),(k-1)})E_{L(i+1)+k}, & \text{if } \tau_{i+1} > \tau_i^M \\ \sum_{k=1}^3 (a_{(i+1),k} - a_{(i+1),(k-1)})E_{L(i+1)+k}, & \text{otherwise} \end{cases},$$

for $2 \le i \le N$,

$$\hat{R}_i^{(k)} = \begin{bmatrix} \tilde{R}_i & \Psi_{i,1}^{(k)} & \Psi_{i,2}^{(k)} & \Psi_{i,3}^{(k)} \\ * & \tilde{R}_i & \Psi_{i,4}^{(k)} & \Psi_{i,5}^{(k)} \\ * & * & \tilde{R}_i & \Psi_{i,6}^{(k)} \\ * & * & * & \tilde{R}_i \end{bmatrix} \left(k = 1,2,3 \right),$$

If $\tau_{i-1} < \tau_i \le \tau_{i-1}^M$,

$$\Sigma_i^{(1)} = \begin{bmatrix} E_1 - E_i \\ E_1 + E_i - 2E_{L(i)+1} \\ E_i - E_{i+1}, \\ E_i + E_{i+1} - 2E_{L(i)+2} \\ E_{i+1} - E_{N+i} \\ E_{i+1} + E_{N+i} - 2E_{L(i)+3} \\ E_{N+i} - E_{N+i+1} \\ E_{N+i} + E_{N+i+1} - 2E_{L(i)+4}, \end{bmatrix}. \quad (22)$$

If $\tau_i \le \tau_{i-1} \le \tau_{i-1}^M$,

$$\Sigma_i^{(2)} = \begin{bmatrix} E_1 - E_{i+1} \\ E_1 + E_{i+1} - 2E_{L(i)+1} \\ E_{i+1} - E_i \\ E_{i+1} + E_i - 2E_{L(i)+2} \\ E_i - E_{N+i} \\ E_i + E_{N+i} - 2E_{L(i)+3} \\ E_{N+i} - E_{N+i+1} \\ E_{N+i} + E_{N+i+1} - 2E_{L(i)+4} \end{bmatrix}. \quad (23)$$

If $\tau_{i-1} \le \tau_{i-1}^M < \tau_i \le \tau_i^M$,

$$\Sigma_i^{(3)} = \begin{bmatrix} E_1 - E_i \\ E_1 + E_i - 2E_{L(i)+1} \\ E_i - E_{N+i} \\ E_i + E_{N+i} - 2E_{L(i)+2} \\ E_{N+i} - E_{i+1} \\ E_{N+i} + E_{i+1} - 2E_{L(i)+3} \\ E_{i+1} - E_{N+i+1} \\ E_{i+1} + E_{N+i+1} - 2E_{L(i)+4}\} \end{bmatrix}. \quad (24)$$

In Theorem 2, LMI Eqs. (19), (20), and (21) should be satisfied for all τ_i in $[0, \tau_i^M]$, $i = 1, \ldots, N$. Since the LMI Eqs. (19) and (20) do not depend on $\tau_i\,(i = 1, \ldots, N)$, LMI Eq. (21), $\Xi(\tau_1, \ldots, \tau_N)$, should be carefully treated. $\Xi(\tau_1, \ldots, \tau_N)$ can have several different forms due to $\Sigma_i^{(k)}$, $\hat{R}_i^{(k)}$, and $G_2(\tau_1, \ldots, \tau_N)$ depending on $\tau_i\,(i = 1, \ldots, N)$. Since Σ_i has three different forms: Eqs. (22), (23), and (24) depending on $\tau_{i-1}, \tau_i, \tau_{i-1}^M$, and τ_i^M, we should define $\Xi(\tau_1, \ldots, \tau_N)$ as several different forms depending on conditions of $\tau_{i-1}, \tau_i, \tau_{i-1}^M$, and $\tau_i^M\,(i = 2, \ldots, N)$. Using conditions of $\tau_{i-1}, \tau_i, \tau_{i-1}^M$, and $\tau_i^M\,(i = 2, \ldots, N)$, we need to show that the obtained $\Xi(\tau_1, \ldots, \tau_N)$ according to the conditions is a negative definite matrix. For example of $N = 2$, when $\tau_1 < \tau_2 \le \tau_1^M$, $\Xi(\tau_1, \tau_2)$ is defined as $2G_0^T P G_2(\tau_1, \tau_2) + \Theta - \Gamma^T \hat{R}_1 \Gamma + G_1^T (\sum_{i=1}^2 R_i) G_1 - \frac{\Sigma_2^{(1)} \hat{R}_2^{(1)} \Sigma_2^{(1)}}{(\tau_2^M)^2}$. We should check whether the obtained $\Xi(\tau_1, \tau_2)$ is a negative definite matrix on the region $\tau_1 < \tau_2 \le \tau_1^M$. Note that since $\Xi(\tau_1, \tau_2)$ is linear in τ_1 and τ_2, we can check $\Xi(\tau_1, \tau_2)$ over several points of (τ_1, τ_2) in the region $\tau_1 < \tau_2 \le \tau_1^M$. For other two regions, we can apply the same approach to $\Xi(\tau_1, \tau_2)$.

IV. Case Studies

Case studies are done on LFC systems with a single and multiple EV aggregators each of which has a single time-varying delay. The parameters used in simulations are listed in Table I. The allowable upper-bound delays in EV aggregators are obtained for different values of PI controller gains and participation ratios of EV aggregators through the proposed stability analysis. The allowable upper-bound delays are computed by using MATLAB, Yalmip optimization tool [23], and SDPT 3.0 [24]. In case of a single time-varying delay, the proposed scheme is compared with that used in [11]. In addition, we investigate how delays in resources with a fast-response characteristic more significantly affect the LFC performance than those in resources with a slow-response characteristic in conventional generators.

TABLE I
Parameters of LFC model including EV aggregators

Parameter	Value	Descriptions
M	8.8	Inertia constant
D	1	Load-damping constant
T_g	0.2	Time constant of governor
T_c	0.3	Time constant of turbine
T_r	12	Time constant of reheat
F_p	1/6	Fraction of total turbine power
R	1/11	Speed regulation
β	21	Frequency bias factor
K_{EV}	1	Battery coefficient
T_{EV}	0.1	Time constant of battery

A. A Single EV Aggregator

The allowable upper-bound delay τ^M of a single EV aggregator in the LFC system is obtained at different sets of PI controller. We first investigate how the integral controller is affected by the delay in the single EV aggregator. Fig. 2 shows the allowable upper-bound delay τ^M for varying the integral gain K_I when the participation ratio (α_1) of EV aggregator is set to 0.2 and 0.4. As the K_I parameter value increases, the allowable upper-bound delay τ^M decreases in both of the participation ratios. As the μ increases, the allowable upper-bound delay τ^M decreases due to fast variations of time-varying delays. In other words, as the rate of change in delays becomes large, the allowable upper-bound delay τ^M should be shrunk at the same integration gain. As the participation ratio increases, the allowable upper-bound delay τ^M decreases. From the results, the relation between the delay in the EV aggregator domain and the controller gain becomes more important as the participation of EV aggregators increases.

Next, the allowable upper-bound delay τ^M of the single EV aggregator is calculated when the participation ratio (α_1) of EV aggregator is set to 0.2 and 0.4 and the PI controller is set as follows: $K_P \in \{0.0, 0.1, \ldots, 1.0\}$ and $K_I \in \{0.05, 0.1, 0.15, 0.2, 0.4, 0.6, 0.8\}$. The allowable upper-bound delays are listed in Tables II and III. As the K_I parameter value increases, the underlying power system is more sensitive to the delay in EV aggregators' domain. An increase in K_P has different effects on τ^M depending on K_I. When K_I is set to 0.05 or 0.1, τ^M decreases as the K_P value increases. When the K_I value is set to 0.8, τ^M increases as the K_P value increases. When the K_I value is set to other values, τ^M increases and then decreases as the K_P value increases. As the participation ratio α_1 increases, allowable upper-bound delay becomes smaller. As the K_I and K_P values decrease, the LFC system becomes more robust to delay in EV aggregators' domain. However, the PI controller with these smaller values may result in worse dynamic performance.

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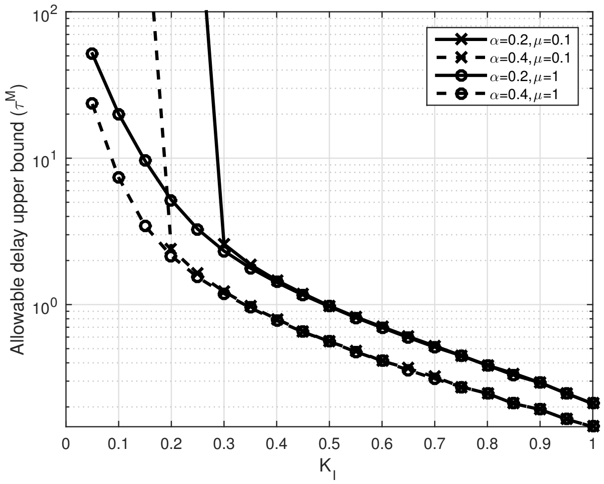

Fig. 2. The allowable upper-bound delay τ_1^M for varying the integral gain K_I

We use Matlab Simulink to evaluate the dynamic performance of LFC systems including EV aggregators with time-varying delays. We simulate the frequency deviation error of LFC system in which a single EV aggregator with $\alpha_1 = 0.2$ has a time-varying delay with an interval $[0, 4s]$ when (K_I, K_P) is set to $(0.1, 0.2)$, $(0.1, 0.4)$, $(0.2, 0.2)$, $(0.2, 0.4)$, $(0.2, 0.6)$, $(0.4, 0.2)$, $(0.4, 0.4)$, $(0.4, 0.6)$, and $(0.6, 0.2)$. The time-varying delay is implemented by using a sine wave function with an amplitude of 2 and a bias of 2. Fig. 3 shows the frequency deviation error of the LFC system when $(0.1, 0.2)$, $(0.1, 0.4)$, $(0.2, 0.2)$, $(0.2, 0.4)$, $(0.2, 0.6)$, $(0.4, 0.4)$, and $(0.4, 0.6)$. When (K_I, K_P) is set to $(0.4, 0.2)$ and $(0.6, 0.2)$, the LFC system becomes unstable. The LFC system is stable in case of $(0.4, 0.4)$ and $(0.2, 0.2)$, but becomes unstable in case of $(0.4, 0.2)$. Thus, a suitable selection of (K_I, K_P) is very important for LCF system stability. As the K_P increases, the settling time decreases. An increase in K_I degrades the settling time and improves the rising time.

We compare the proposed stability criterion and the theorem used in [11] in terms of allowable upper-bound delay. The allowable upper-bound delay obtained from the theorem in [11] is listed in Table IV when the participation ratio is set to 0.2. The allowable upper-bound delay calculated by the proposed scheme is much larger than that by the theorem of [11]. The reason of enhancement is that the proposed approach gives more reduced conservatism than the one used in [11]. Especially, at low values of K_I, the difference among them becomes larger.

B. Single-Delayed Conventional Generators

We investigate how time-varying delays in fast-response resources significantly affect the stability of LFC controller. To this end, we compare the allowable upper-bound delays obtained in LFC system with a single EV aggregator and that with delayed conventional generators. In case of the delayed conventional generators, we replace the singe EV aggregator with a delayed conventional generator. The model for the delayed conventional generator consists of one delay component and the same generator model described in Section II except that it has no governor-free, as shown in Fig 4. The parameters of the replaced generator are the same as Table I. For computing the allowable upper-bound delay, we use the state-space model (28)-(30) shown in Appendix A. The allowable upper-bound delay for the conventional delayed generator is listed in Tables V and VI when the participation ratio is set to 0.2 and 0.4, respectively. In both cases of $\alpha_1 = 0.2$ and 0.4, the conventional delayed generator is less sensitive to delay than the single EV aggregator. From the result, we can infer that when the fast-response resources with communication network delays participate in frequency regulation service without any restriction on the communication network delays, the power system may be more severely affected by the delays in fast-response resources, compared

TABLE II
ALLOWABLE UPPER-BOUND DELAY (τ^M) WHEN $\alpha_1 = 0.2$ (THEOREM 1)

		K_I						
		0.05	0.1	0.15	0.2	0.4	0.6	0.8
K_p	0.0	∞	∞	9.61	5.19	1.41	0.69	0.38
	0.1	∞	∞	10.87	6.34	1.83	0.96	0.58
	0.2	∞	∞	10.57	6.83	2.18	1.19	0.76
	0.3	∞	12.44	9.30	6.71	2.46	1.40	0.93
	0.4	10.93	9.53	7.82	6.18	2.65	1.58	1.08
	0.5	8.34	7.53	6.54	5.52	2.75	1.71	1.19
	0.6	6.66	6.15	5.53	4.88	2.78	1.80	1.30
	0.7	5.49	5.15	4.75	4.32	2.74	1.86	1.37
	0.8	4.65	4.40	4.12	3.83	2.66	1.88	1.42
	0.9	4.01	3.83	3.63	3.42	2.54	1.88	1.46
	1.0	3.51	3.37	3.23	3.07	2.40	1.85	1.47

TABLE III
ALLOWABLE UPPER-BOUND DELAY (τ^M) WHEN $\alpha_1 = 0.4$ (THEOREM 1)

		K_I						
		0.05	0.1	0.15	0.2	0.4	0.6	0.8
K_p	0.0	42.14	7.37	3.45	2.15	0.77	0.41	0.24
	0.1	23.20	7.76	4.14	2.68	1.06	0.60	0.39
	0.2	9.57	6.39	4.15	2.92	1.29	0.77	0.52
	0.3	6.02	4.87	3.73	2.88	1.44	0.91	0.64
	0.4	4.29	3.77	3.19	2.67	1.51	1.01	0.73
	0.5	3.30	3.03	2.71	2.39	1.54	1.08	0.81
	0.6	2.67	2.50	2.32	2.13	1.51	1.11	0.86
	0.7	2.23	2.13	2.00	1.88	1.44	1.12	0.89
	0.8	1.91	1.84	1.75	1.67	1.36	1.10	0.90
	0.9	1.66	1.61	1.55	1.50	1.27	1.07	0.90
	1.0	1.47	1.42	1.39	1.35	1.18	1.02	0.88

TABLE IV
ALLOWABLE UPPER-BOUND DELAY (τ^M) WHEN $\alpha_1 = 0.2$ (BY JIANG)

		K_I						
		0.05	0.1	0.15	0.2	0.4	0.6	0.8
K_p	0.0	∞	∞	5.14	3.56	1.34	0.68	0.38
	0.1	∞	∞	5.85	4.21	1.73	0.95	0.57
	0.2	∞	∞	5.77	4.45	2.04	1.18	0.75
	0.3	∞	6.56	5.19	4.33	2.26	1.38	0.92
	0.4	5.85	5.07	4.46	3.97	2.38	1.53	1.07
	0.5	4.44	4.11	3.81	3.53	2.41	1.64	1.19
	0.6	3.61	3.45	3.27	3.11	2.37	1.72	1.28
	0.7	3.05	2.95	2.85	2.74	2.27	1.74	1.34
	0.8	2.64	2.58	2.51	2.44	2.13	1.73	1.38
	0.9	2.32	2.28	2.24	2.18	1.98	1.69	1.40
	1.0	2.07	2.05	2.01	1.97	1.84	1.62	1.39

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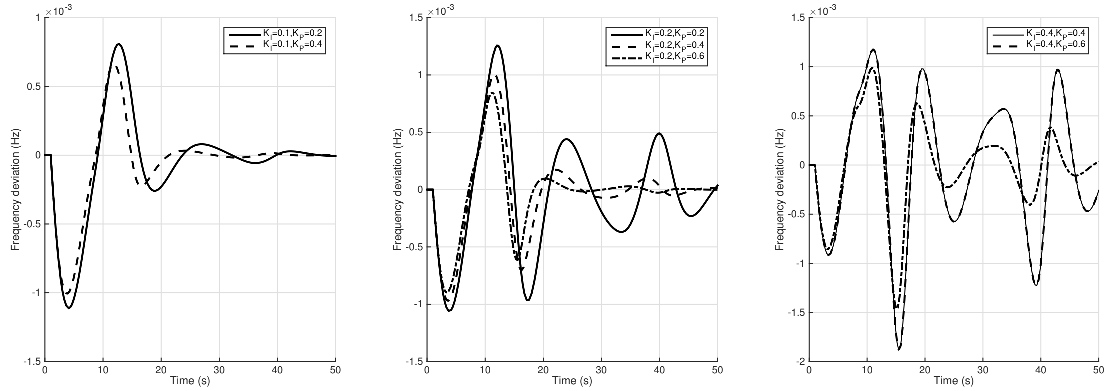

Fig. 3. Frequency deviation when (K_I, K_P) is set to $(0.1, 0.2)$, $(0.1, 0.4)$, $(0.2, 0.2)$, $(0.2, 0.4)$, $(0.2, 0.6)$, $(0.4, 0.4)$, and $(0.4, 0.6)$

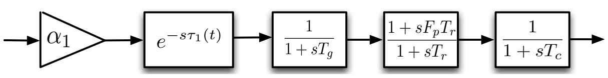

Fig. 4. Model for a conventional generator with a single time-varying delay

TABLE V
ALLOWABLE UPPER-BOUND DELAY (τ^M) ON CONVENTIONAL GENERATORS WHEN $\alpha_1 = 0.2$

		K_I						
		0.05	0.1	0.15	0.2	0.4	0.6	0.8
K_p	0.0	∞	∞	∞	∞	9.50	1.92	0.28
	0.1	∞	∞	∞	∞	12.41	3.41	0.93
	0.2	∞	∞	∞	∞	∞	5.20	1.67
	0.3	∞	∞	∞	∞	∞	6.94	2.60
	0.4	∞	∞	∞	∞	∞	8.49	3.66
	0.5	∞	∞	∞	∞	∞	9.79	4.77
	0.6	∞	∞	∞	∞	∞	10.85	5.83
	0.7	∞	∞	∞	∞	∞	11.74	6.77
	0.8	∞	∞	∞	∞	∞	∞	7.58
	0.9	∞	∞	∞	∞	∞	∞	8.24
	1.0	∞	∞	∞	∞	∞	∞	8.79

with those in the conventional generators.

C. Two EV Aggregators Model

We investigate the allowable upper-bound delays (τ_1^M, τ_2^M) of two EV aggregators in the LFC system at different sets of the PI controller and different combinations of α_1 and α_2. The PI controller is set as follows: $K_P \in \{0.2, 0.4, 0.6, 0.8\}$, $K_I \in \{0.2, 0.4, 0.6\}$. In case of α_1 and α_2, we introduce a ratio r as α_1/α_2 to investigate the effect of each EV aggregator on the stability under the same total participation ratio $\alpha_1+\alpha_2$. We investigate the effect of the participation ratio under the settings: $\alpha_1 + \alpha_2 \in \{0.2, 0.4\}$ and $r \in \{0.1, 0.5, 0.9\}$. In this study, we compute one allowable upper-bound delay τ_2^M when the other τ_1^M is fixed to 1 ms, 100 ms, 1 s, 2 s, 5 s, and 10 s and both μ_1 and μ_2 are set to 1. The τ_2^M value is obtained and listed in Tables VII and VIII. The observations are as follows:

- As the integrator gain K_I increases, the allowable upper-bound delay τ_2^M decreases. This observation is the same as that of the single EV aggregator.
- The change of the allowable upper-bound delay τ_2^M with respect to K_P is different from the observation on K_I. Depending on K_I, the allowable upper-bound delay shows different trends.
  - When K_I is set to 0.2, the allowable upper-bound delay τ_2^M decreases.
  - When K_I is set to 0.4, the bound τ_2^M increases and then decreases.
  - When K_I is set to 0.6, the bound τ_2^M seems to increase, but it decreases when K_P is set to a larger value than 0.8.

  Due the increasing and decreasing trend, the LFC controller is observed to be unstable at low K_P and high K_I such as $(K_P, K_I) = (0.2, 0.6)$ and as K_P increases from 0.2, the LFC controller is achieved to be stable.
- As the allowable upper-bound delay τ_1^M of EV aggregator 1 increases, the other allowable upper-bound delay τ_2^M decreases.
- As the sum of the participation ratios for EV aggregators increases, the allowable upper-bound delay τ_2^M decreases. In other words, the stability region becomes smaller as the $\alpha_1 + \alpha_2$ value increases.
- Under the same value of $\alpha_1 + \alpha_2$, the stability region is more affected by the delay of the EV aggregator which largely contributes to frequency regulation service. Comparing the case of $\alpha_1+\alpha_2 = 0.2$ and $r = 1/9$ ($\alpha_1 = 0.02$ *and* $\alpha_2 = 0.18$) with one of $\alpha_1 + \alpha_2 = 0.2$ and $r = 9$ ($\alpha_1 = 0.18$ *and* $\alpha_2 = 0.02$), the allowable upper-bound delay τ_2^M in $r = 9$ is much larger than

TABLE VI
ALLOWABLE UPPER-BOUND DELAY (τ^M) ON CONVENTIONAL GENERATORS WHEN $\alpha_1 = 0.4$

		K_I						
		0.05	0.1	0.15	0.2	0.4	0.6	0.8
K_p	0.0	123.45	55.68	29.22	16.40	2.92	0.82	0.13
	0.1	126.79	59.28	33.22	18.18	3.90	1.26	0.43
	0.2	122.36	59.29	34.92	19.22	4.83	1.71	0.74
	0.3	115.42	56.76	34.57	19.62	5.62	2.15	1.03
	0.4	107.00	52.90	32.66	18.89	6.23	2.57	1.31
	0.5	97.43	48.13	29.59	16.99	6.64	2.97	1.58
	0.6	86.85	42.53	25.23	14.86	6.88	3.31	1.83
	0.7	75.11	35.76	18.86	13.20	6.96	3.59	2.06
	0.8	61.36	25.52	13.69	11.92	6.93	3.82	2.28
	0.9	39.77	13.53	11.86	10.84	6.80	4.00	2.48
	1.0	12.67	11.48	10.68	9.92	6.61	4.11	2.65

that in $r = 1/9$ because the delay of EV aggregator 1 with a participation ratio of $\alpha_1 = 0.18$ has a larger effect on frequency regulation than that with $\alpha_1 = 0.02$. From the above results, it is shown that there is an interaction between the delays of EV aggregators and their participation ratios.

- In case of $r = 1$, two EV aggregators have the same participation ratio, but have different time-varying delay characteristics. Although the single EV aggregator is not exactly the same as two EV aggregators having the same time-varying delay characteristic if the participation ratio of the single EV aggregator is the same as $\alpha_1 + \alpha_2$ of the two EV aggregators, it is worth to compare the two EV aggregators with the single EV aggregator.
  - In case of the single EV aggregator, the allowable upper-bound delay is obtained as 4.88 when α_1, K_I, and K_P are set to 0.2, 0.2, 0.6, respectively.
  - In case of the two EV aggregators under $K_I = 0.2$, $K_P = 0.6$, $\alpha_1 + \alpha_2 = 0.2$, the LFC system becomes unstable when $\tau_1^M = 5$ or 10 due to a value of 4.88 obtained in the single EV aggregator.

  From the results, the stability region of the two EV aggregators is dominated by one of the single EV aggregator. However, the stability region is affected by the relation between the delays in EV aggregators because they have close interactions as follows:
  - When $\alpha_1 + \alpha_2$, K_I, and K_P are set to 0.4, 0.6, and 0.4, respectively, τ_2^M is obtained as 1.03 at $\tau_1^M = 1$. Compared with an upper bound of 1.01 in the single EV aggregator, since τ_1^M is less than 1.01, τ_2^M has a room for having a larger upper bound delay of 1.03 than 1.01.

From the above results, it is shown that characteristics of time delay in EV aggregators' domain, their participation ratios, and their combinations are important factors for determining the stability of LFC at a given controller gain. Thus, we can infer that these should be considered during the frequency regulation commitment.

V. Discussion

A. Multi-area

For a multi-area LFC system, we simplify all generators in each control area as an equivalent generation unit and multiple EV aggregators as one EV aggregator. As shown in Fig. 5, the dynamic model of a multi-area power system including EV aggregators is described as follows:

$$\dot{x}(t) = Ax(t) + B_0 u(t) + \sum_{k=1}^{N} B_k u(t - \tau_k(t)) + F\Delta P_d,$$
$$y(t) = Cx(t),$$

where

$$x_i(t) = [\Delta f_i \quad \Delta P_{ig} \quad \Delta P_{im} \quad \Delta X_{ig} \quad \Delta P_{EV,i} \quad \Delta P_{tie-i}]^T,$$
$$y_i(t) = ACE_i,$$
$$x(t) = [x_1^T(t) \quad x_2^T(t) \quad \cdots \quad x_N^T(t)]^T,$$
$$y(t) = [y_1(t) \quad y_2(t) \quad \cdots \quad y_N(t)]^T,$$
$$u(t) = [u_1(t) \quad u_2(t) \quad \cdots \quad u_N(t)]^T,$$

$$\Delta P_d(t) = [\Delta P_{d1} \quad \Delta P_{d2} \quad \cdots \quad \Delta P_{dN}]^T,$$
$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix},$$
$$B_0 = \begin{bmatrix} B_{011} & 0 & \cdots & 0 \\ 0 & B_{022} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_{0NN} \end{bmatrix},$$
$$B_i = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & B_{iii} & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix},$$
$$C = diag[C_1 \; C_2 \; \cdots \; C_N], F = diag[F_1 \; F_2 \; \cdots \; F_N],$$
$$B_{0ii} = [0 \quad 0 \quad \frac{F_{ip}\alpha_{i0}}{T_{ig}} \quad \frac{\alpha_{i0}}{T_{ig}} \quad 0 \quad 0]^T,$$
$$B_{iii} = [0 \quad 0 \quad 0 \quad 0 \quad \frac{K_{EV,i}\alpha_{i1}}{T_{EV,i}} \quad 0]^T,$$
$$C_i = [\beta_i \; 0 \; 0 \; 0 \; 0 \; 0]^T, F_i = [\frac{-1}{M_i} \; 0 \; 0 \; 0 \; 0 \; 0]^T,$$
$$A_{ii} = \begin{bmatrix} \frac{-D_i}{M_i} & \frac{1}{M_i} & 0 & 0 & \frac{1}{M_i} & -\frac{1}{M_i} \\ 0 & \frac{-1}{T_{ic}} & \frac{1}{T_{ic}} & 0 & 0 & 0 \\ \frac{-F_{ip}}{R_i T_{ig}} & 0 & \frac{-1}{T_{ir}} & \frac{T_{ig} - F_{ip}T_{ir}}{T_{ir}T_{ig}} & 0 & 0 \\ \frac{-1}{R_i T_{ig}} & 0 & 0 & \frac{-1}{T_{ig}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{T_{EV,i}} & 0 \\ 2\pi \sum_{j=1, j\neq i}^{N} T_{ij} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$A_{ij} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -2\pi T_{ij} & 0 & \cdots & 0 \end{bmatrix},$$
$$T_{ij} = T_{ji},$$

where T_{ij} is the tie-line synchronization coefficient between the i-th and j-th control areas. The ACE signal in the i-th control area is defined as follows:

$$ACE_i = \beta_i \Delta f_i + \Delta P_{tie-i}, \tag{25}$$

where ΔP_{tie-i} is the net exchange of the tie-line power of the i-th control area. Different from the single area, the ACE signal considers the area frequency deviation and the net tie-line power exchange. Note that the sum of the net tie-line

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TABLE VII
ALLOWABLE UPPER-BOUND DELAY (τ_2^M) WHEN $\alpha_1 + \alpha_2$ IS SET TO 0.2

		$\alpha_1+\alpha_2$=0.2, $r=1/9$											
		K_I=0.2				K_I=0.4				K_I=0.6			
	K_P	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
τ_1^M	0.001	7.86	7.16	5.73	4.53	2.51	3.13	3.34	3.20	1.24	1.79	2.06	2.18
	0.1	7.83	7.13	5.68	4.49	2.50	3.11	3.31	3.17	1.33	1.78	2.05	2.17
	1	7.61	6.85	5.41	4.24	2.37	2.93	3.10	2.95	1.23	1.66	1.93	2.02
	2	7.44	6.66	5.25	4.08	2.21	2.75	2.90	2.75	X	X	X	X
	5	6.98	6.17	X	X	X	X	X	X	X	X	X	X
	10	X	X	X	X	X	X	X	X	X	X	X	X

		$\alpha_1+\alpha_2$=0.2, $r=1$											
		K_I=0.2				K_I=0.4				K_I=0.6			
	K_P	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
τ_1^M	0.001	16.62	15.02	11.90	9.36	6.50	8.05	8.35	7.76	3.03	4.48	5.42	5.76
	0.1	16.39	14.76	11.65	9.12	6.18	7.84	8.12	7.53	2.83	4.28	5.20	5.55
	1	14.54	12.85	9.98	7.66	4.25	5.82	6.12	5.64	1.43	2.37	3.15	3.49
	2	13.15	11.53	8.75	6.50	2.41	3.66	4.13	3.78	X	X	X	X
	5	9.82	8.11	X	X	X	X	X	X	X	X	X	X
	10	X	X	X	X	X	X	X	X	X	X	X	X

		$\alpha_1+\alpha_2$=0.2, $r=9$											
		K_I=0.2				K_I=0.4				K_I=0.6			
	K_P	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
τ_1^M	0.001	88.99	79.41	62.12	48.59	40.42	47.42	47.31	42.65	23.18	30.45	34.16	34.55
	0.1	87.13	77.54	60.25	46.71	38.55	45.54	45.43	40.77	21.33	28.57	32.28	32.66
	1	70.73	61.09	44.50	31.80	22.40	29.14	29.13	24.86	4.54	12.17	15.91	16.44
	2	53.91	44.76	30.66	20.50	5.12	12.33	13.03	10.59	X	X	X	X
	5	22.60	15.83	X	X	X	X	X	X	X	X	X	X
	10	X	X	X	X	X	X	X	X	X	X	X	X

TABLE VIII
ALLOWABLE UPPER-BOUND DELAY (τ_2^M) WHEN $\alpha_1 + \alpha_2$ IS SET TO 0.4

		$\alpha_1+\alpha_2$=0.4, $r=1/9$											
		K_I=0.2				K_I=0.4				K_I=0.6			
	K_P	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
τ_1^M	0.001	3.45	3.18	2.54	2.00	1.45	1.73	1.73	1.58	0.87	1.13	1.26	1.25
	0.1	3.43	3.16	2.52	1.98	1.44	1.72	1.72	1.56	0.85	1.13	1.25	1.24
	1	3.23	2.96	2.32	1.78	1.32	1.60	1.59	1.42	X	1.01	1.13	1.11
	2	3.06	2.77	2.13	X	X	X	X	X	X	X	X	X
	5	X	X	X	X	X	X	X	X	X	X	X	X
	10	X	X	X	X	X	X	X	X	X	X	X	X

		$\alpha_1+\alpha_2$=0.4, $r=1$											
		K_I=0.2				K_I=0.4				K_I=0.6			
	K_P	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
τ_1^M	0.001	8.75	7.99	6.23	4.84	3.37	4.34	4.51	4.13	1.74	2.52	3.02	3.16
	0.1	8.53	7.77	6.00	4.62	3.16	4.13	4.30	3.91	1.59	2.34	2.81	2.96
	1	6.55	5.82	4.22	2.91	1.63	2.23	2.28	1.90	X	1.03	1.26	1.23
	2	4.43	3.76	2.27	X	X	X	X	X	X	X	X	X
	5	X	X	X	X	X	X	X	X	X	X	X	X
	10	X	X	X	X	X	X	X	X	X	X	X	X

		$\alpha_1+\alpha_2$=0.4, $r=9$											
		K_I=0.2				K_I=0.4				K_I=0.6			
	K_P	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
τ_1^M	0.001	51.00	44.65	33.39	25.80	24.76	29.19	27.98	23.85	14.96	19.81	21.67	20.85
	0.1	49.12	42.78	31.53	23.97	22.91	27.31	26.11	21.99	13.11	17.92	19.77	18.97
	1	32.89	26.84	16.86	10.21	6.44	11.00	10.20	7.06	X	1.22	3.02	2.82
	2	16.31	11.68	3.98	X	X	X	X	X	X	X	X	X
	5	X	X	X	X	X	X	X	X	X	X	X	X
	10	X	X	X	X	X	X	X	X	X	X	X	X

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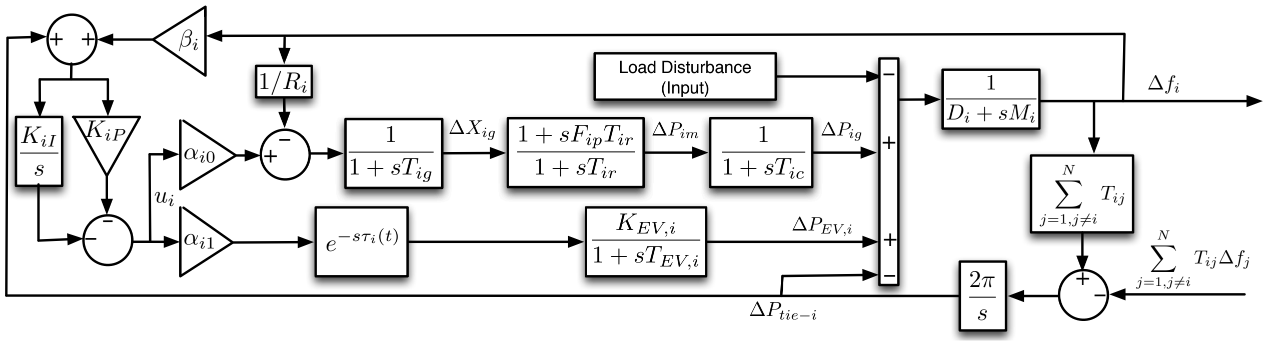

Fig. 5. LFC model of control area i

exchanges over all control areas should be zero as follows:

$$\sum_{i=1}^{N} \Delta P_{tie-i} = 0. \tag{26}$$

In the i-th control area of the multi-area LFC system, a PI-type LFC controller is designed as follows:

$$u_i(t) = -K_{iP}ACE_i - K_{iI}\int ACE_i dt. \tag{27}$$

Using the state-space model and Theorem 2, we can investigate stability of the multi-area LFC system having EV aggregators.

B. Guidelines on Performance Improvement

There are two approaches to improve the system performance under time-varying delays as follows:

- **Selection of suitable controller gains under given information on multiple time varying delays**
  This problem is related to controller synthesis. Under given multiple time-varying delays, controller synthesis is another research area and is beyond the scope of this paper. It will be considered for further study.
- **Improvement on responses from EV aggregators**
  In order to improve the LFC system performance, the responses from EV aggregators may be improved under given controller gains. For example, EV aggregators attempt to reduce delay and/or improve responses using some predictions or some time-delay aware control (We think these schemes are another research problems). Above all, in order to induce this improvement from EV aggregators, some incentive mechanisms are needed. For example, in the North America frequency regulation markets, there have been frequency regulation payments as an incentive mechanism based on frequency regulation performance. For the frequency regulation payments, generators which have a higher performance score can obtain a higher payments.

VI. Conclusion

Participation of EV aggregators in frequency regulation service causes time-varying delays in load frequency control (LFC) systems. The time-varying delays may have significant effects on the performance and may even cause instability in power systems. The delay-dependent stability of an LFC system with participation of EV aggregators with a single and multiple time-varying delays has been investigated. We modified the conventional LFC model by including multiple EV aggregators each of which has a time-varying delay component and modeled it with a PI controller. Based on the Lyapunov stability theory and the linear matrix inequality approach, we proposed an improved delay-dependent stability criterion which yield less conservative results by using a Wirtinger-based improved integral inequality. We obtained the allowable upper-bound delay by utilizing the proposed scheme. It was shown that the proposed scheme can be utilized to obtain a much larger stability region on delay than the conventional scheme.

Through several case studies, it is revealed that an increase in the integrator gain K_I reduces the stability region, that is, (reduces the allowable upper-bound delay) and depending on K_I, the proportional gain K_P gives a different effect on the stability region. At a small value of K_I, the allowable upper-bound delay decreases with an increase in K_P. When K_I has a large value, the allowable upper-bound delay increases and then decreases as K_P increases. As the total participation ratio of EV aggregators each of which has a single time-delay characteristic increases, the stability region is reduced. The stability region can be determined by the single EV aggregator, however, the stability region is affected by the relation between the participation ratios and the allowable upper-bound delays.

In addition, the delay in fast-response resources such as EV aggregators is more significantly sensitive in LFC systems than that in conventional generators. It should be noted that the LFC controller should be carefully designed when a large number of EVs participate in frequency regulation service. The proposed stability criteria are expected to help to guide the determination of delay requirements for EV aggregators participating in frequency regulation service for a given PI controller. We discuss an extension to multi-area LFC systems and guidelines on how to improve performance of LFC systems under time-varying delays.

APPENDIX A

State-space model for a delayed conventional generator

$$A = \begin{bmatrix} \frac{-D}{M} & \frac{1}{M} & 0 & 0 & \frac{1}{M} & 0 & 0 \\ 0 & \frac{-1}{T_c} & \frac{1}{T_c} & 0 & 0 & 0 & 0 \\ \frac{-F_p}{RT_g} & 0 & \frac{-1}{T_r} & \frac{T_g - F_p T_r}{T_r T_g} & 0 & 0 & 0 \\ \frac{-1}{RT_g} & 0 & 0 & \frac{-1}{T_g} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{T_c} & \frac{1}{T_c} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_r} & \frac{T_g - F_p T_r}{T_r T_g} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_g} \end{bmatrix}, \tag{28}$$

$$B_0 = \begin{bmatrix} 0 & 0 & \frac{F_p \alpha_0}{T_g} & \frac{\alpha_0}{T_g} & 0 & 0 & 0 \end{bmatrix}^T, \tag{29}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{F_p \alpha_1}{T_g} & \frac{\alpha_1}{T_g} \end{bmatrix}^T. \tag{30}$$

APPENDIX B
PROOF OF THEOREM 1

The LKF for the nominal system (13) is chosen as

$$V(x_t) = \xi^T(t) P \xi(t) + \int_{t-\tau^M}^{t} x^T(s) S_1 x(s)\, ds + \tag{31}$$
$$\int_{t-\tau}^{t} x^T(s) S_2 x(s)\, ds + \frac{1}{\tau^M} \int_{-\tau^M}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds d\theta,$$

where x_t is a function $x_t : [-\tau^M, 0] \to \mathbb{R}^n : x_t(\theta) = x(t + \theta), \theta \in [-\tau^M, 0]$ and $\xi(t) = \mathrm{col}\{x(t), \int_{t-\tau^M}^{t} x(s)\, ds\}$.

Taking the time derivative of (31) along the trajectory of system yields

$$\dot{V}(x_t) \leq 2\xi^T(t) P \dot{\xi}(t) + x^T(t)(S_1 + S_2)x(t) -$$
$$x^T(t-\tau^M) S_1 x(t-\tau^M) - (1-\mu) x^T(t-\tau) S_2 x(t-\tau) +$$
$$\dot{x}^T(t) R \dot{x}(t) - \frac{1}{\tau^M} \int_{t-\tau^M}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds. \tag{32}$$

We introduce an augmented vector $\chi(t) = col\{x(t), x(t-\tau), x(t-\tau^M), \frac{\int_{t-\tau}^{t} x(s)\, ds}{\tau}, \frac{\int_{t-\tau^M}^{t-\tau} x(s)\, ds}{\tau^M - \tau}\}$.

Using $\chi(t)$, we rewrite $\dot{\xi}(t)$ as follows:

$$\begin{aligned} \dot{\xi}(t) &= col\{A_0 x(t) + A_1 x(t-\tau), x(t) - x(t-\tau^M)\} \\ &= col\{A_0 E_1 + A_1 E_2, E_1 - E_3\} \chi(t) = Y_0 \chi(t). \end{aligned} \tag{33}$$

The $\xi(t)$ can be expressed as follows:

$$\begin{aligned} \xi(t) &= col\{x(t), \int_{t-\tau^M}^{t} x(s)\, ds\} \\ &= col\{E_1, \tau E_4 + (\tau^M - \tau) E_5\} = Y_1(\tau) \chi(t). \end{aligned} \tag{34}$$

According to the definition of matrices Y_0 and $Y_1(\tau)$, the derivative of the LKF can be rewritten as follows:

$$\dot{V}(x_t) \leq \chi^T(t) \Big(2Y_0^T P Y_1(\tau) + E_1^T (S_1 + S_2) E_1 -$$
$$(1-\mu) E_2^T S_2 E_2 - E_3^T S_1 E_3 + Y_2^T R Y_2 \Big) \chi(t) -$$
$$\frac{1}{\tau^M} \int_{t-\tau^M}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds, \tag{35}$$

where $Y_2 := A_0 E_1 + A_1 E_2$.

Applying the Wirtinger's improved inequality lemma [16] to the third line in (35), we can obtain

$$\frac{1}{\tau^M} \int_{t-\tau^M}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds$$
$$= \frac{1}{\tau^M} \Big(\int_{t-\tau}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds + \int_{t-\tau^M}^{t-\tau} \dot{x}^T(s) R \dot{x}(s)\, ds \Big)$$
$$\geq \frac{1}{(\tau^M)^2} \chi(t) \Big(\frac{\tau^M}{\tau} Y_3^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} Y_3 +$$
$$\frac{\tau^M}{\tau^M - \tau} Y_4^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} Y_4 \Big) \chi(t),$$

where $Y_3 := col\{E_1 - E_2, E_1 + E_2 - 2E_4\}$ and $Y_4 := col\{E_2 - E_3, E_2 + E_3 - 2E_5\}$.

From a reciprocally convex approach [22] when $K = 2$, if there exist a matrix X such that

$$\Psi := \begin{bmatrix} \tilde{R} & X \\ 0 & \tilde{R} \end{bmatrix} \succeq 0,$$

where $\tilde{R} := diag\{R, 3R\}$, we obtain the following inequality:

$$-\frac{1}{\tau^M} \int_{t-\tau^M}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds \leq -\frac{1}{(\tau^M)^2} \chi^T(t) \Gamma^T \Psi \Gamma \chi(t), \tag{36}$$

where $\Gamma := col\{Y_3, Y_4\}$.

From (35) and (36), it can be seen that

$$\dot{V}(x_t) \leq \chi^T(t) \Big(2Y_0^T P Y_2(\tau) + E_1^T (S_1 + S_2) E_1 -$$
$$(1-\mu) E_2^T S_2 E_2 - E_3^T S_1 E_3 + Y_2^T R Y_2 -$$
$$\frac{1}{(\tau^M)^2} \Gamma^T \Psi \Gamma \Big) \chi(t) = \chi^T(t) \Phi(\tau) \chi(t). \tag{37}$$

By showing (37) is negative definite, we obtain that the system is asymptotically stable.

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