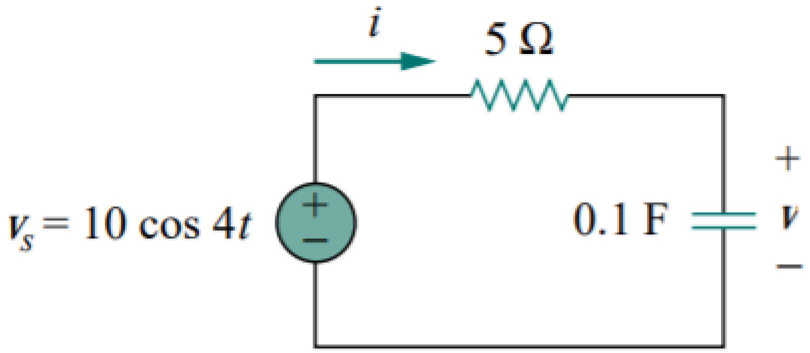

Lecture 13: AC Circuits, Sinusoids and Phasors. Solving problems

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Phasor (1)

The complex number z can also be written in polar or exponential form as

$$z = r\angle\phi = re^{j\phi} \tag{9.14b}$$

where r is the magnitude of z, and ϕ is the phase of z. We notice that z can be represented in three ways:

$$\begin{aligned} z &= x + jy \qquad &&\text{Rectangular form} \\ z &= r\angle\phi \qquad &&\text{Polar form} \\ z &= re^{j\phi} \qquad &&\text{Exponential form} \end{aligned} \tag{9.15}$$

The relationship between the rectangular form and the polar form is shown in Fig. 9.6, where the x axis represents the real part and the y axis represents the imaginary part of a complex number. Given x and y, we can get r and ϕ as

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x} \tag{9.16a}$$

On the other hand, if we know r and ϕ, we can obtain x and y as

$$x = r\cos\phi, \qquad y = r\sin\phi \tag{9.16b}$$

Thus, z may be written as

$$\boxed{z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)} \tag{9.17}$$

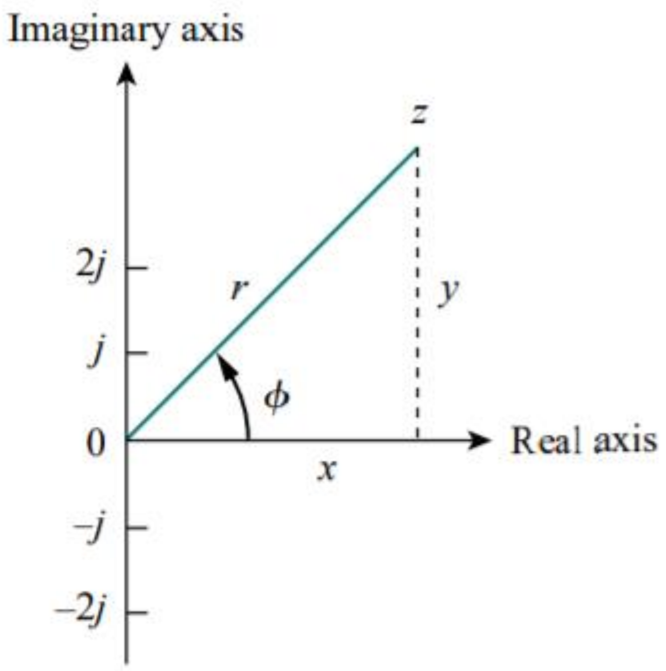


Figure 9.6 Representation of a complex number $z = x + jy = r\angle\phi$.

Phasor (2)

The following operations are important.

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \underline{/\phi_1 + \phi_2}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \underline{/\phi_1 - \phi_2}$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r} \underline{/-\phi}$$

Square Root:

$$\sqrt{z} = \sqrt{r} \underline{/\phi/2}$$

Complex Conjugate:

$$z^* = x - jy = r \underline{/-\phi} \; = re^{-j\phi}$$

Problem 1

Evaluate these complex numbers:

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

(b) $\dfrac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*}$

Solution:

(a) Using polar to rectangular transformation,

$$40\angle 50^\circ = 40(\cos 50^\circ + j\sin 50^\circ) = 25.71 + j30.64$$

$$20\angle -30^\circ = 20[\cos(-30^\circ) + j\sin(-30^\circ)] = 17.32 - j10$$

Adding them up gives

$$40\angle 50^\circ + 20\angle -30^\circ = 43.03 + j20.64 = 47.72\angle 25.63^\circ$$

Taking the square root of this,

$$(40\angle 50^\circ + 20\angle -30^\circ)^{1/2} = 6.91\angle 12.81^\circ$$

(b) Using polar-rectangular transformation, addition, multiplication, and division,

$$\frac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*} = \frac{8.66 - j5 + (3 - j4)}{(2 + j4)(3 + j5)}$$

$$= \frac{11.66 - j9}{-14 + j22} = \frac{14.73\angle -37.66^\circ}{26.08\angle 122.47^\circ}$$

$$= 0.565\angle -160.31^\circ$$

Problem 2

Transform these sinusoids to phasors:

(a) $v = -4\sin(30t + 50^\circ)$

(b) $i = 6\cos(50t - 40^\circ)$

Solution:

(a) Since $-\sin A = \cos(A + 90^\circ)$,

$$v = -4\sin(30t + 50^\circ) = 4\cos(30t + 50^\circ + 90^\circ)$$
$$= 4\cos(30t + 140^\circ)$$

The phasor form of v is

$$\mathbf{V} = 4\underline{/140^\circ}$$

(b) $i = 6\cos(50t - 40^\circ)$ has the phasor

$$\mathbf{I} = 6\underline{/-40^\circ}$$

Problem 3

Find the sinusoids represented by these phasors:

(a) $\mathbf{V} = j8e^{-j20^\circ}$

(b) $\mathbf{I} = -3 + j4$

Solution:

(a) Since $j = 1\angle 90^\circ$,

$$\mathbf{V} = j8\angle -20^\circ = (1\angle 90^\circ)(8\angle -20^\circ)$$

$$= 8\angle 90^\circ - 20^\circ = 8\angle 70^\circ \text{ V}$$

Converting this to the time domain gives

$$v(t) = 8\cos(\omega t + 70^\circ) \text{ V}$$

(b) $\mathbf{I} = -3 + j4 = 5\angle 126.87^\circ$. Transforming this to the time domain gives

$$i(t) = 5\cos(\omega t + 126.87^\circ) \text{ A}$$

Problem 4

Given $i_1(t) = 4\cos(\omega t + 30^\circ)$ and $i_2(t) = 5\sin(\omega t - 20^\circ)$, find their sum.

Solution:

Here is an important use of phasors—for summing sinusoids of the same frequency. Current $i_1(t)$ is in the standard form. Its phasor is

$$\mathbf{I}_1 = 4\underline{/30^\circ}$$

We need to express $i_2(t)$ in cosine form. The rule for converting sine to cosine is to subtract 90°. Hence,

$$i_2 = 5\cos(\omega t - 20^\circ - 90^\circ) = 5\cos(\omega t - 110^\circ)$$

and its phasor is

$$\mathbf{I}_2 = 5\underline{/-110^\circ}$$

If we let $i = i_1 + i_2$, then

$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 &= 4\underline{/30^\circ} + 5\underline{/-110^\circ} \\ &= 3.464 + j2 - 1.71 - j4.698 = 1.754 - j2.698 \\ &= 3.218\underline{/-56.97^\circ}\text{ A}\end{aligned}$$

Transforming this to the time domain, we get

$$i(t) = 3.218\cos(\omega t - 56.97^\circ)\text{ A}$$

Problem 5

The voltage $v = 12\cos(60t + 45^\circ)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Solution:

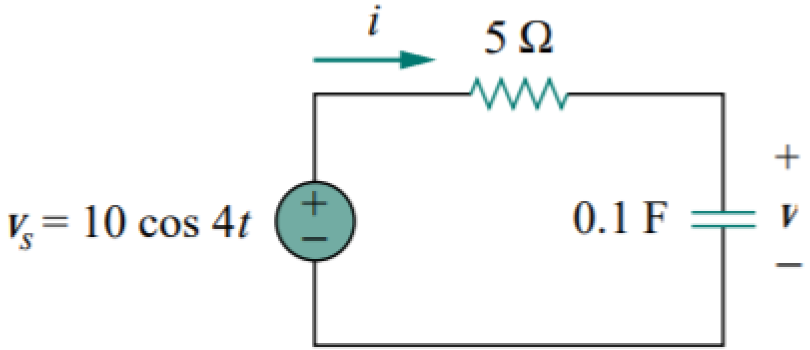
For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$, where $\omega = 60$ rad/s and $\mathbf{V} = 12\underline{/45^\circ}$ V. Hence

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\underline{/45^\circ}}{j60 \times 0.1} = \frac{12\underline{/45^\circ}}{6\underline{/90^\circ}} = 2\underline{/-45^\circ}\text{ A}$$

Converting this to the time domain,

$$i(t) = 2\cos(60t - 45^\circ)\text{ A}$$

Problem 6



Find v(t) and i(t) in the circuit

Solution:

From the voltage source $10\cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10\underline{/0^\circ}\ \text{V}$$

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

Hence the current

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\underline{/0^\circ}}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2}$$
$$= 1.6 + j0.8 = 1.789\underline{/26.57^\circ}\ \text{A}$$

The voltage across the capacitor is

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789\underline{/26.57^\circ}}{j4 \times 0.1}$$
$$= \frac{1.789\underline{/26.57^\circ}}{0.4\underline{/90^\circ}} = 4.47\underline{/-63.43^\circ}\ \text{V}$$

Converting **I** and **V** in Eqs. (9.9.1) and (9.9.2) to the time domain,

$$i(t) = 1.789\cos(4t + 26.57^\circ)\ \text{A}$$
$$v(t) = 4.47\cos(4t - 63.43^\circ)\ \text{V}$$

Notice that $i(t)$ leads $v(t)$ by 90° as expected.