Convenient but Risky Government Bonds*

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Abstract

How does convenience yield interact with sovereign risk and the supply of government bonds? To answer this question, this paper builds a quantitative model of sovereign debt and default, in which convenience yield arises because investors derive non-pecuniary benefits from holding risky government bonds. Convenience yield negatively depends on government bond supply and on haircuts that in turn increase in sovereign risk, reflecting mark-to-market practice on financial markets. Calibrated to Italian data, convenience yield increases the equilibrium debt-to-GDP ratio by around 10% and can rationalize prolonged periods of negative bond spreads even in the presence of default risk. We also provide a decomposition of convenience yield into a collateral valuation and a haircut component. Counterfactual experiments suggest that the elasticity of each component with respect to government bond supply has sizable effects on debt and default dynamics.

Keywords: Sovereign Risk, Convenience Yield, Haircuts, Debt Management

JEL Classification: G12, G15, H63

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1 Introduction

Government bonds play a special role in the financial system of developed economies. One, if not the most important, reason for this is the exceptional degree of liquidity and safety that they provide to investors relative to alternative asset classes. These special attributes make government bonds a key ingredient for the functioning of various financial market segments, such as repo and securities lending markets, where they are posted as collateral by market participants. Investors' willingness to pay a price markup for government bonds due to non-pecuniary benefits that they provide is well documented (see Bansal and Coleman, 1996; Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood et al., 2015; Du et al., 2018). The associated premium is usually referred to as *convenience yield*.

The literature has documented the negative dependence of convenience yield on government bond supply (Krishnamurthy and Vissing-Jorgensen, 2012), its potential to reconcile the high valuations of government debt (Jiang et al., 2022b; Mian et al., 2022) and the implications for government debt management (Greenwood et al., 2015; Jiang et al., 2022a; Gorton and Ordoñez, 2022). However, the focus so far has usually been on the United States and Japan, where sovereign risk is effectively not a concern. In contrast, several European countries facing non-negligible default risk are still able to issue bonds at sizable convenience yields (Jiang et al., 2021).[1] When sovereign risk becomes a non-negligible component of government bond spreads, studying the implications of convenience yield for the conduct of fiscal policy consequently has to take into account potential interactions between convenience yield, sovereign risk, and the supply of government bonds.

This paper contributes to an understanding of this interaction, which is a non-trivial task since all of these factors are jointly and endogenously determined in equilibrium. All else equal, the presence of convenience yield improves borrowing conditions for a government by raising bond prices, which makes it more attractive for the sovereign to issue debt. However, as documented by Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood et al. (2015) for the United States, an increase in *government bond supply* lowers convenience yield by making bonds less scarce and, thus, decreases the collateral valuation and convenience yield, ceteris paribus. *Sovereign risk* exerts another negative effect on convenience yield, ceteris paribus. To cushion against price risk in general and credit risk in particular, collateral pledged in market transactions is subject to haircuts. The higher the haircut, the less collateral can effectively be pledged. Convenience yield thus negatively depends on the haircut, ceteris paribus. Since haircuts are positively related to a security's credit risk, an increase in sovereign risk will adversely affect bond prices not just by raising default risk premia charged by investors, but also by lowering the convenience yield of government bonds.

The supply of public debt and the risk of default are however not exogenously given but a reflection of government behavior, which in turn will respond to changes in borrowing conditions and, therefore, convenience yield. So how do convenience yield and its two components, *collateral valuation* and *haircuts* affect a government's incentives to borrow and default? To illustrate the basic interactions between convenience yield, sovereign risk, and government bond supply, we first propose a simple two-period framework featuring an impatient government issuing defaultable one-period bonds. In the

[1] Risky government bonds are still safer and more liquid than most alternative asset classes available to investors. We will therefore focus on this type of security in this paper and abstract from privately issued debt.

spirit of Krishnamurthy and Vissing-Jorgensen (2012), we assume that investors derive utility from the collateral services of government bonds, which depends negatively on bond supply, valued at market prices, and haircuts. Consistent with mark-to-market practice on repo and securities lending markets, haircuts are positively linked to default risk.[2] Investors' maximization problem yields a pricing schedule for government bonds which is affected by convenience yield in two ways: first, the pricing schedule decreases in debt issuance even absent default risk, since collateral becomes less scarce and convenience yield declines. Second, default risk depresses bond prices and this increase is amplified by mark-to-market haircuts, holding the collateral valuation component constant.

Taken as given this bond pricing schedule and the possibility of default, the government chooses bond issuance to maximize public spending over both periods. Absent convenience yield, optimal debt issuance is determined by the slope of the *debt-issuance Laffer curve* and the government's relative impatience. Convenience yield drives a wedge into this trade-off and we show analytically that a *debt-inelastic* convenience yield affects public policy in the same way as investor (im)patience: it increases debt issuance and default risk. If convenience yield is debt-elastic, the risk-taking effect of convenience yield is dampened and can potentially be even reversed. This observation follows from the fact that a highly debt-elastic convenience yield implies that bond prices are more elastic to debt issuance as well: the curvature of the Laffer curve increases. The point at which the Laffer curve's slope equals relative impatience is therefore lower.

We use the simplified setting to show that not only the elasticity of convenience yield, but also the elasticity of its components with respect to debt issuance is relevant for the government's incentive to issue debt. If the elasticity of collateral valuation with respect to debt issuance is small (large), a higher responsiveness of haircuts with respect to debt issuance (and, thereby, default risk) reduces (increases) the government's debt choice. The potential *complementarity* between collateral scarcity and the haircut effect follows from the fact that collateral valuation depends on the haircut-corrected market value of government bonds:. If debt issuance induces a large increase in haircuts and at the same time collateral scarcity effects are sufficiently strong, bond prices can locally *increase*. This phenomenon has empirical relevance: it was observed for example in 2011 when the negative rating outlook on US government bonds was followed by a decline in bond yields . The government optimally chooses a higher debt issuance in this constellation.

To illustrate the relevance of convenience yield and its components, we embed them in a quantitative model of sovereign debt and default (see Chatterjee and Eyigungor, 2012; Bocola et al., 2019). In the quantitative model, a *risk-averse* government issues *long-term* bonds to investors *without commitment* to smooth *persistent* revenue shocks intertemporally. As in the simple model, convenience yield enters the model through investors' collateral valuation. In addition to default risk and convenience yield, the literature has identified market illiquidity as a potentially important determinant of bond pricing (He and Milbradt, 2014; Pelizzon et al., 2016). It is important to note that government bond markets

[2]In our model, the effective convenience yield that a bond provides depends on the haircut it is subject to on various market segments. When using the term "haircut", we refer to an average haircut over different segments. For example, Drechsler et al. (2016) document that the ECB applied a flatter haircut schedule for its standing facilities than haircuts set by a large central clearing counterparty (CCP). The average haircut encompasses this heterogeneity in a tractable way. For our comparative statics exercises with respect to the steepness of haircut schedules, we do not take a stand on whether it reflects risk management practice of CCPs or central bank collateral policies (see Nyborg, 2017).

are typically characterized by exceptionally high liquidity, such that bid-ask spreads are usually very low during safe times.[3] However, illiquidity discounts increase with credit risk (see Chaumont, 2018; Passadore and Xu, 2020) and therefore can be a distinct channel that might amplify the increase of government bond spreads in times of high default risk. In the quantitative model, we address the concern that interactions between debt, default risk and convenience yield are in fact driven by market illiquidity as a confounding factor by explicitly adding it to the model. To do so, we introduce trading frictions as in Lagos and Rocheteau (2009) into our model. Our interpretation of market illiquidity reflects the notion that investors face trading frictions when selling or buying a security on secondary markets, resulting in illiquidity discounts that are usually measured via bid-ask spreads.

The model is calibrated to Italian data for 2001-2012. Our main variable of interest is the government bond spread, which is the difference between the yield of a government bond and the risk-free interest rate, which we proxy by the EURIBOR. The bond spread reflects credit risk, convenience yield and market illiquidity. We use credit default swap (CDS) spreads as a measure of credit risk since derivatives *(i)* do not provide convenience yield and *(ii)* are not affected by market illiquidity the way bond markets are.[4] Our model permits direct pricing of credit default swaps and the computation of bid-ask spreads, such that we can directly compare model-implied statistics with their empirical counterparts for all variables of interest.

Free model parameters are calibrated to match the mean and volatility of bond spreads, CDS spreads, debt outstanding, and bid-ask spreads over the full sample. Although the model is quite parsimonious, it picks up several important (non-targeted) features of crisis episodes observed in the data. While government bond spreads are negative during safe times in the data, they turn positive in 2008Q4, which prompts us to interpret the time period 2008Q4-2012Q4 as the crisis sub-sample. When applying this filter to model-generated time series, we find that the model-based crisis sub-sample features higher spread volatility and a moderate increase in debt issuance, which is consistent with the data. We can therefore broadly distinguish between two endogenous regimes. In crisis times, default risk is found to dominate the convenience yield component of the bond spread, whereas the opposite is true for safe times, where bond spreads are usually negative.

To further corroborate the validity of our calibrated model, we investigate the role of convenience yield and bond supply for government bond spreads. Specifically, we use the simulated time series to regress the government bond spread on bond supply, while controlling for credit risk and bid-ask spreads. Both data and model-implied regressions reveal a highly significant positive effect of bond supply, which supports and extends the findings of Krishnamurthy and Vissing-Jorgensen (2012) to risky government bonds. Our empirical results complement Jiang et al. (2021) who identify convenience yield in a panel of risky European government bonds. At the same time, we only find a small and insignificant effect of various measures of market illiquidity on bond spreads, supporting our emphasis of convenience yield as second important driver of bond spreads. We also study the impact of convenience yield and

[3] Bid-ask spreads are a common measure of market illiquidity, whose interaction with bond spreads is well established (He and Milbradt, 2014).

[4] These two features make them appropriate measures of default risk in our model, while we do not (need to) assume that CDS spreads are a pure measure of default risk. Other potential risk factors in CDS spreads include counterparty risk, i.e. uncertainty about CDS payoffs in the event of a sovereign default (see Salomao, 2017, for a discussion). In this setting, CDS spreads increase less than one-for-one in expected default risk, since they do not provide perfect insurance against default.

market illiquidity by keeping the haircut schedule at its baseline form, set collateral valuation and market illiquidity to zero. We find that convenience yield explains around 10% of Italy's debt-to-GDP ratio once the model is recalibrated to match average default risk. By contrast, market illiquidity has no visible impact on this margin as well.

In a second experiment, we show that the results from our simple model also carry through to a more elaborate setting. In the baseline calibration, a reduction in the elasticity of the haircut schedule translates into a reduction in average CDS spreads and their volatility. The long-run default rate declines as well. This is due to the disciplining impact of collateral scarcity effects on government borrowing that result in more debt-elastic bond pricing. However, if collateral valuation is constant, the same reduction in the elasticity of haircut schedules translates into a higher and more volatile CDS spread and more frequent default on average. Taking into account the endogeneity of government bond supply and default risk has potentially important implications for the design of haircuts, both on private and public market segments.

Related Literature Our paper also relates to the quantitative sovereign default literature (see Aguiar et al., 2016), which mostly focuses on external public debt and emerging economies. Similar to Bocola et al. (2019), we re-interpret the established sovereign default framework à la Eaton and Gersovitz (1981) and Arellano (2008) as the relationship between a government and lenders regardless of their place of residence and consider an application to a developed economy.[5] In the model, the government receives an exogenous stream of tax revenues, draws utility from public spending and uses bond markets to smooth spending intertemporally.

Our paper is related to recent studies that emphasize the role of convenience yield for optimal debt management. Canzoneri et al. (2016) study optimal Ramsey monetary and fiscal policy when government bonds provide transaction services, leading to a trade-off between tax smoothing and liquidity provision.[6] Jiang et al. (2022a) analyze convenience yield in a model the government trades off insuring bondholders and taxpayers. In order to reap the benefits of convenience yield, the government has to ensure that their bonds have a beta of zero, which comes at the expense of tax smoothing. By contrast, Greenwood et al. (2015) study optimal debt maturity management when short-term government bonds are valued by investors for having money-like properties. While these papers consider the interaction between public debt and non-pecuniary benefits of government bonds from an optimal policy perspective, their focus is on analytical results and – more importantly – they abstract from sovereign risk, market illiquidity and lack of commitment.[7]

Convenience yield on public debt is also discussed in Fisher (2015), who relates changes to the demand for safe and liquid assets to risk premium shocks. Perez (2018) analyzes the role of government bonds as public liquidity, which provides additional repayment incentives in a small open economy model. In contrast to his paper, we assume that investors do not enter the government objective, which

[5]Bocola et al. (2019) show that total government debt, rather than external government debt, matters for default risk, which is captured by our model.

[6]See also Angeletos et al. (2022).

[7]Bonam (2020) studies the implications of convenience yield for fiscal policy in a New Keynesian setting that models fiscal policy via exogenous policy rules and abstracts from sovereign risk.

in our view is a more plausible assumption in light of our application to the Euro area.

Layout The paper is structured as follows. Section 3 introduces our model. Our calibration is presented in Section 4, while Section 5 discusses the interactions between convenience yield, credit risk, and government bond supply. Section 6 concludes.

2 A Simple Model of Default Risk, Convenience Yield, and Haircuts

In this section, we flesh out the basic interactions between sovereign risk, convenience yield and debt issuance in a two-period setting. Convenience yield introduces a wedge into investors' demand for government bonds and the elasticity of this wedge with respect to debt issuance shapes the effect of convenience yield on debt issuance and default risk. Under a standard monotone-hazard rate assumption on public revenues we show that, if convenience yield is *debt-inelastic*, the government issues more bonds than it would do otherwise. Introducing a debt-elastic convenience yield can potentially overturn this result. We decompose convenience yield into a pure collateral valuation component and a haircut component and derive conditions under which the elasticity of both components affects the debt and default behavior of the government.

Environment At time $t = 0$, an impatient government (with discount factor $\beta < 1$) issues bonds at price q to a mass-one continuum of competitive investors who do not discount the future. At $t = 1$, the government draws a revenue shock τ with cumulative density function (cdf) $F(\tau)$ and defaults if government revenues exceed debt outstanding b. The default probability associated with debt choice b is therefore simply given by $F(b)$. In case of default, there is no debt recovery for investors and the government consumes nothing, i.e. sovereign default entails a resource loss. All agents are risk neutral.

Investors draw utility $u(\theta)$ from holding government bonds due to their eligibility as collateral. We assume that the *collateral valuation* function satisfies $u'(\cdot)$, $-u''(\cdot) > 0$ and that $\theta \equiv (1-\kappa)(1-F)b$, the *haircut-adjusted* expected payoff, is relevant for investors' utility from holding government bonds. The presence of collateral valuation $u(\cdot)$ gives rise to a convenience yield $\Lambda(b)$ in the government bond pricing equation, which is given by $q(b) = \big(1-F(b)\big)(1+\Lambda(b))$. The convenience yield $\Lambda(b) = (1-\kappa(b))u'(\theta)$ in turn depends on a pure collateral valuation component $u'(\theta)$ and a haircut component $1-\kappa(b)$. Throughout the analysis, we assume that haircuts are increasing in debt issuance $\frac{\partial \kappa}{\partial b} > 0$. In practice - and in the quantitative model with long-term debt and persistent revenue shocks - haircuts are related to *default risk*. However, in the two-period setting, default risk $F(b)$ is immediately determined by debt issuance b, such that we directly express the dependency of haircuts on the government's policy in terms of the debt choice.

Debt Issuance Taking as given investors' bond pricing condition, the government maximizes

$$\max_b \quad q(b)b - \beta \int_b^{\infty} (\tau - b) dF(\tau) .$$

Throughout this section, we will refer to the amount of resources raised in the first period, $q(b)b$, as the *debt issuance Laffer curve*. The first-order condition requires that the government issues debt up to the point where expected repayment obligations (per unit of debt b) equal the slope of the debt issuance Laffer curve:

$$\beta\big(1-F(b)\big)=q(b)+\frac{\partial q}{\partial b}b \tag{1}$$

Without convenience yield, the bond price simply reflects expected repayment $q=1-F(b)$, and its derivative is given by $\frac{\partial q}{\partial b}=f(b)\equiv\frac{\partial F(b)}{\partial b}$. Let the hazard rate of government revenues τ be denoted by $h(\tau)\equiv\frac{f(\tau)}{1-F(\tau)}$. Plugging this definition together with the bond pricing expression into the first-order condition (1), optimal debt issuance absent convenience yield will be denoted by b^0 and is implicitly defined through

$$h(b^0)b^0=1-\beta\ .$$

The left-hand side $h(b^0)b^0$ can also be interpreted as the elasticity of *default losses* with respect to debt issuance. Under a monotonicity assumption on the hazard rate $h(\cdot)$, the debt choice b^0 is pinned down uniquely.

The Role of Convenience Yield The bond price schedule with convenience yield is given by $q(b)=\big(1-F(b)\big)\big(1+(1-\kappa(b))v'(\theta)\big)$, while its derivative reads $\frac{\partial q}{\partial b}=-(1+\Lambda(b))f(b)+\big(1-F(b)\big)\frac{\partial\Lambda}{\partial b}$. Both objects, the bond price and its derivative, are larger in absolute terms than in the case without convenience yield. This implies that, in the risky borrowing region, *(i)* each additional unit of debt increases public consumption in the first period by more than in the absence of convenience yield and, *(ii)* has a more pronounced negative effect on the bond price. With these two competing effects, the first-order condition for debt issuance is given by

$$\Big((1+\Lambda(b))f(b)-\big(1-F(b)\big)\frac{\partial\Lambda}{\partial b}\Big)b=\big(1-F(b)\big)(1-\beta+\Lambda(b))$$

Expressing this in terms of the hazard rate, we have

$$h(b)b=\frac{1-\beta+\Lambda(b)+b\frac{\partial\Lambda}{\partial b}}{1+\Lambda(b)}\ . \tag{2}$$

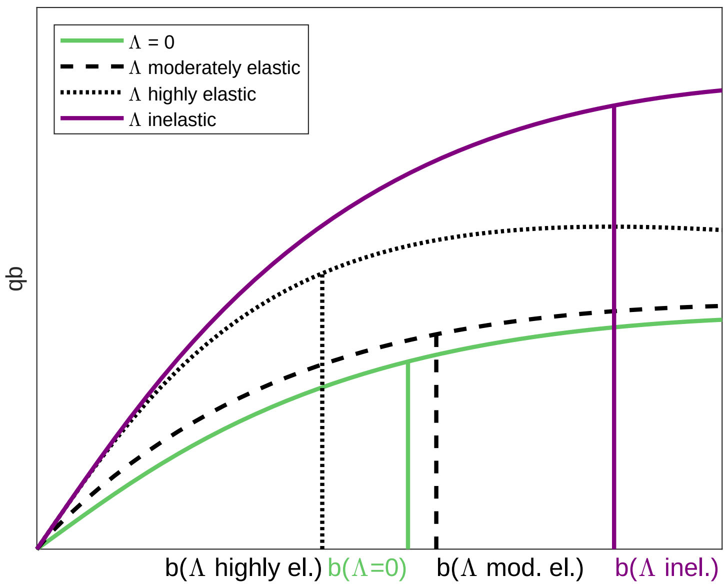
While the LHS is still given by the elasticity of default costs with respect to debt issuance, the RHS now also contains a convenience yield wedge in addition to the relative impatience $(1-\beta)$. Since the sign of marginal convenience yield $\frac{\partial\Lambda}{\partial b}$ is not known in the general case, convenience yield has an ambiguous effect on debt issuance. In the special case of a debt-inelastic constant convenience yield $\widetilde{\Lambda}$, the debt choice (2) reduces to

$$h(\widetilde{b})\widetilde{b}=\frac{1-\beta+\widetilde{\Lambda}}{1+\widetilde{\Lambda}}\ . \tag{3}$$

In this special case, convenience yield makes the government effectively *more impatient* and, thereby, induces more debt issuance and default risk ($\widetilde{b} > b^0$) by the monotone hazard rate assumption on government revenues. This is reflected in the solid purple line in Figure 1, which depicts the government's debt issuance Laffer curve, i.e. the amount of resources $q(b)b$ raised by issuing b units of debt. With a debt-inelastic convenience yield, $q(b)b$ is higher for all b than in the case without convenience yield (solid green line in Figure 1). In each case, the optimal debt levels are indicated by the vertical lines. Consistent with the first-order condition (2), the elasticity of default losses $h(b)b$ with respect to debt issuance equals relative impatience, potentially adjusted for convenience yield.

The intermediate cases with a debt-elastic convenience yield are reflected by the dashed and dotted lines in Figure 1. We consider two cases: the dashed black line corresponds to the case of a relatively *low*, but *moderately elastic* convenience yield. Location and shape of the Laffer curve resemble the case without convenience yield and the debt choice increases slightly. In contrast, the dotted black line reflects a high and *highly elastic* convenience yield. For low levels of default risk, the Laffer curve is almost parallel to the case of inelastic convenience yield. Once default risk becomes larger, it flattens sharply, such that the optimal debt choice is even lower than in the case without any convenience yield.

Figure 1: The Effect of Convenience Yield on Debt Issuance



The Role of Haircuts and Collateral Scarcity Since the responsiveness the convenience yield to debt issuance is pivotal in determining the government's debt choice, the remainder of this section provides a decomposition of $\frac{\partial \Lambda}{\partial b}$ into a collateral scarcity effect and a haircut effect. Specifically, marginal convenience yield $\frac{\partial \Lambda}{\partial b}$ can be expressed as

$$\frac{\partial \Lambda}{\partial b} = \underbrace{(1-\kappa(b))u''(\theta)\frac{\partial \theta}{\partial b}}_{\text{Collateral Scarcity Effect}} + \underbrace{\frac{\partial (1-\kappa(b))}{\partial b}u'(\theta)}_{\text{Haircut Effect}} \tag{4}$$

While the haircut effect is negative by construction, the collateral scarcity effect as an ambiguous sign. Holding haircuts and bond prices constant, for example in a model with risk-free debt such as Mian et al., 2022, more debt issuance makes government bonds less scarce and reduces convenience yield. In the model with default risk, debt issuance can reduce the haircut-corrected market value of government bonds if haircuts are very sensitive to debt issuance and collateral scarcity *increases*. Formally, this is given by

$$\frac{\partial \theta}{\partial b} = \big(1 - F(b)\big)\left((1-\kappa(b))\big(1-h(b)b\big) + \frac{\partial(1-\kappa(b))}{\partial b} b\right) \tag{5}$$

To relate this potential ambiguity to the debt choice, we use the observation that marginal convenience yield $\frac{\partial \Lambda}{\partial b}$ enters the first-order condition (2) only together with debt issuance b. It is therefore possible to express the optimal debt choice with convenience yield in closed form. Denoting the coefficient of relative risk aversion of investors' utility from holding government bonds by $R(\theta)$ and the elasticity of the collateral value $1-\kappa(b)$ with respect to debt issuance by $\varepsilon_{\overline{\kappa}}(b)$, optimal debt issuance can be written as

$$h(b^*)b^* = \frac{1-\beta+\Lambda(b^*)\big(1+\varepsilon_{\overline{\kappa}}(b^*) - R(\theta^*)\big) - R(\theta^*)\varepsilon_{\overline{\kappa}}(b^*)\big)}{1+\Lambda(b^*)\big(1-R(\theta^*)\big)} \,. \tag{6}$$

where $\theta^* = (1-\kappa(b^*))(1-F(b^*))b^*$ is the haircut-corrected debt outstanding, valued at the expected payoff.[8] The elasticity of convenience yield which affects the debt choice in addition to its level can be directly related to the curvature of the collateral valuation function $u(\cdot)$ and the curvature of the haircut function $\kappa(b)$. In isolation, each component of convenience yield provides disciplining incentives to the government. Their interaction term however gives rise to non-monotonic interactions, since the government strategically takes into account collateral scarcity effects on the pricing of its bonds.

[8] We can rewrite marginal convenience yield (4) as

$$b\frac{\partial \Lambda}{\partial b} = b(1-\kappa(b))u''(\theta)\frac{\partial \theta}{\partial b} + b\frac{\partial(1-\kappa(b))}{\partial b}u'(\theta)\,.$$

and plug in (5)

$$b\frac{\partial \Lambda}{\partial b} = b(1-\kappa(b))u''(\theta)\big(1-F(b)\big)\left[(1-\kappa(b))\big(1-h(b)b\big) + \frac{\partial(1-\kappa(b))}{\partial b}b\right] + b\frac{\partial(1-\kappa(b))}{\partial b}u'(\theta)\,.$$

We can factor out $\big(1-\kappa(b)\big)u'(\theta)$ and use the definitions of $\theta = (1-\kappa(b))(1-F(b))b$ and $\varepsilon_{\overline{\kappa}}(b) = \frac{b\frac{\partial(1-\kappa(b))}{\partial b}}{(1-\kappa(b))}$ to write

$$b\frac{\partial \Lambda}{\partial b} = \big(1-\kappa(b)\big)u'(\theta) \times \left\{\frac{u''(\theta)}{u'(\theta)}\theta\Big(\big(1-h(b)b\big)+\varepsilon_{\overline{\kappa}}(b)\Big) + \varepsilon_{\overline{\kappa}}(b)\right\}\,.$$

Together with the definitions of convenience yield $\Lambda(b) = \big(1-\kappa(b)\big)u'(\theta)$ and of the coefficient of relative risk aversion $R(\theta) = -\frac{u''(\theta)}{u'(\theta)}\theta$, we can plug this into the first-order condition (2) to obtain

$$h(b^*)b^* = \frac{1+\Lambda(b^*)+\Lambda(b^*)\times\left\{-R(\theta^*)\Big(\big(1-h(b^*)b^*\big)+\varepsilon_{\overline{\kappa}}(b^*)\Big)+\varepsilon_{\overline{\kappa}}(b^*)\right\}-\beta}{1+\Lambda(b^*)}\,.$$

Re-arranging for $h(b^*)b^*$ yields (6).

Two Special Cases To provide more intuition for this result, it is useful to consider several special cases of (6). For a *debt-inelastic* collateral valuation component, the debt choice (6) simplifies to

$$h(\bar{b})\bar{b} = \frac{1-\beta+\Lambda(\bar{b})\big(1+\varepsilon_{\overline{\kappa}}(\bar{b})\big)}{1+\Lambda(\bar{b})} \,. \tag{7}$$

If the elasticity of the haircut component with respect to debt issuance is sufficiently large (in absolute terms), convenience yield will provide disciplining incentives to the government, counteracting the basic risk-taking effect of convenience yield described in equation (3). In contrast, if the risk-aversion coefficient of $u(\cdot)$ is set to a constant $\widehat{R}$, we obtain:

$$h(\widehat{b})\widehat{b} = \frac{1-\beta+\Lambda(\widehat{b})\big(1-\widehat{R}\big)(1+\varepsilon_{\overline{\kappa}}(\widehat{b}))\big)}{1+\Lambda(\widehat{b})\big(1-\widehat{R}\big)} \,. \tag{8}$$

It is possible to distinguish between two cases: if $\overline{R} < 1$, collateral scarcity and haircuts are *substitutes* in the sense that both components are providing incentives for the government to reduce debt issuance. The debt-inelastic case is nested as the special case $\overline{R} = 0$. In contrast, collateral valuation itself will provide incentives to reduce debt issuance if its curvature is sufficiently high ($\widehat{R} > 1$) - even with constant haircuts ($\varepsilon_{\overline{\kappa}}(b) = 0$ for all b). In this case, there is a *complementarity* between both components and a higher haircut elasticity increases debt issuance and default risk. The reason behind this is that haircuts also affect collateral valuation indirectly through their effect on θ (see equation (5)): if higher haircuts induce a sufficiently large reduction in available collateral θ and if this reduction at the same time induces a sufficiently large collateral scarcity effect ($u''(\theta)$ large in absolute terms), bond prices can even increase locally in response to debt issuance. This situation is consistent with the effects the negative rating outlook had on US borrowing conditions in 2011. The government optimally responds by increasing its debt issuance.

In the following, we will use a more elaborate setting with long-term debt, strategic sovereign default and positive debt recovery to show that convenience yield itself and the non-monotonic interaction of collateral valuation and haircut components have a quantitatively relevant impact on sovereign debt and default risk.

3 Quantitative Model of Convenience Yield and Endogenous Default

Time is discrete and denoted as $t = 0, 1, 2, ..., \infty$. The model features a *government* of an economy that issues bonds to a mass-one continuum of ex-ante homogeneous *investors*. The government receives random tax revenues, cares about the supply of a public good and can trade long-term government bonds with investors. Following the quantitative sovereign default literature, the government cannot commit to future repayment and debt issuance (see Arellano, 2008). Investors value government debt for their pecuniary future benefits as well as their *collateral services*, such that they are willing to pay a premium on government bonds. We follow Gorton and Ordoñez (2022) and refer to this valuation of collateral services as convenience yield.

Each period is divided into two sub-periods. In the first sub-period, investors are subject to uninsur-

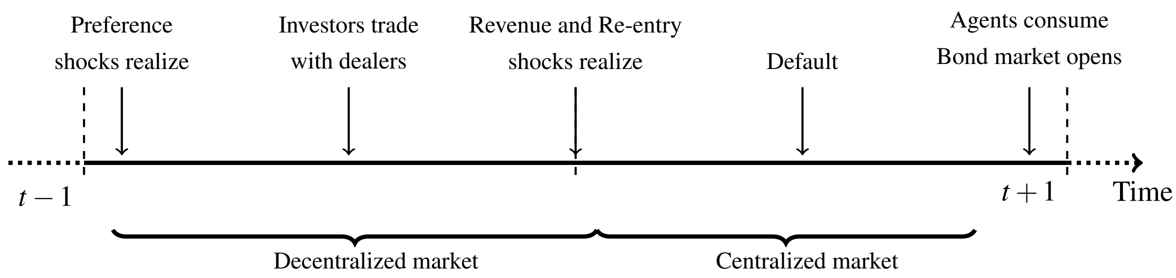


Figure 2: Within-Period Timing

able idiosyncratic shocks to the valuation of collateral service, which introduces benefits from trading government bonds. However, we assume that the government bond market is not open at this stage and investors can only trade bonds with *dealers* on a decentralized over-the-counter market.[9] Restricting their ability to trade with each other will give rise to endogenous bid ask spreads. We assume that the consumption of investors and dealers does not enter the government's objective, either because they reside outside the economy or can trade on international financial markets in a frictionless manner. Given that our model does not distinguish between domestic and external investors, the supply of government bonds in the model corresponds to total debt rather than external debt.[10] Tax revenues realize at the beginning of the second sub-period. All agents consume in the second sub-period, when the competitive and centralized government bond market is open. The timing assumption is summarized in Figure 2.

We begin with the investor problem to derive a pricing schedule for government bonds. We then describe the government's policy problem, taken as given the pricing schedule, which, together with bond market clearing, characterizes the equilibrium of the model.

Investors Investors receive a large, constant endowment e, i.e. they have deep pockets, and discount consumption at the exogenously given, time-invariant rate $r^{rf} > 0$. They maximize their expected lifetime utility

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{c_t+u_i(\theta_t)}{(1+r^{rf})^t}\right]$$

where c_t denotes an investor's consumption and $u_i(\theta_t)$ is the convenience benefit of holding government bonds. The collateral value of government bonds $\theta_t \equiv (1-\kappa_t)m_t b_{t-1}$ consists of three parts. The first part is the stock of government bonds b_{t-1} purchased at the end of last period. The second part is given by the expected bond payoff $m_t \equiv \mathbb{E}_{t-1}[k_t]$, with k_t being the cum-coupon price of a bond in the next centralized market.[11] The third part is the haircut κ_t imposed on government bonds for transactions like repos. The haircut will, consistent with actual practice on repo or securities lending markets, depend on

[9]Dividing a period into two sub-periods that represent a decentralized and a centralized market is in the spirit of Lagos and Wright (2005).

[10]For a detailed discussion, we refer to Bocola et al. (2019).

[11]Since no information about period t tax revenues and government bond payoffs arrives during the first sub-period, we condition the expectations operator on the information set available to investors at the end of the second sub-period of $t-1$.

the credit risk of a security (see further below). Letting the collateral value θ_t depend on the cum-coupon bond price is consistent with mark-to-market practices on repo and securities lending markets.

To give rise to a trading motive in the first sub-period, we assume that investors are subject to i.i.d. preference shocks. With probability $\frac{1}{2}$, investors are a high or low collateral-valuation type, which we denote by $i \in \{L, H\}$. Specifically, investor types differ utility derived from a bond's collateral services, with $u_L(\cdot) < u_H(\cdot)$ and $u_i'(\cdot), -u_i''(\cdot) > 0$, for $i \in \{L, H\}$. Investors cannot directly trade bonds among each other or with the government in the first sub-period. They can however adjust their bond holdings by trading with dealers in exchange for an endogenously determined fee $\phi_{i,t}$. Dealers have access to a competitive inter-dealer market and intermediate between buyers and sellers as in Lagos and Rocheteau (2009). The terms of trade are determined bilaterally between a dealer and an investor via Nash bargaining. The bargaining power of both sides is normalized to $\frac{1}{2}$ in the following. We assume that there are no search frictions, i.e. each buyer/seller is matched to a dealer. In Appendix A, we derive the bond pricing condition consistent with investors' optimality conditions:

$$q_t = \frac{1}{1+r^{rf}} \times \underbrace{m_{t+1}}_{\text{pecuniary benefits}} \times \underbrace{(1+\Lambda_{t+1})}_{\text{convenience yield}}, \tag{9}$$

where

$$\Lambda_{t+1} \equiv \underbrace{(1-\kappa_{t+1})}_{1-\text{haircut}} \times \underbrace{\mathbb{E}_i\left[u_i'(\theta_{t+1}) + \frac{1}{2} \times \left\{u_i'(\widetilde{\theta}_{i,t+1}) - u_i'(\theta_{t+1})\right\}\right]}_{\text{collateral valuation}}, \tag{10}$$

$\theta_{t+1} = (1-\kappa_{t+1})m_{t+1}B_t$ is the collateral value of government bond holdings B_t. The bond price q_t reflects expected pecuniary benefits as well as expected non-pecuniary benefits due to the collateral services provided by a bond. Non-pecuniary benefits in turn are affected by potential trading frictions when dealers have bargaining power vis-à-vis investors. Specifically, the term $u_i'(\widetilde{\theta}_{i,t+1}) - u_i'(\theta_{t+1})$ measures the net marginal gains from trading, with $\widetilde{\theta}_{i,t+1}$ denoting the collateral value of the bond position held by an investor of type i after trading on the decentralized market (see also Appendix A).

Government Each period, the government receives exogenous revenues τy_t, where τ is a constant income tax rate and y_t random domestic income, supplies the public good g_t, issues long-term bonds on the centralized market and decides whether to repay its creditors in order to maximize

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t v(g_t)\right], \quad \beta \in (0,1), v'(\cdot), -v''(\cdot) > 0,$$

subject to the government budget constraint

$$g_t = \tau y_t + h_t \times \left(q_t\left(B_t - (1-\delta)B_{t-1}\right) - \widetilde{\delta} B_{t-1}\right).$$

where B_{t-1} denotes legacy debt at the beginning of the second sub-period of period t, i.e., when the government bond market opens. Budget constraint and bond payoff depend on the government's credit status $h_t \in \{0,1\}$. In the autarky case ($h_t = 0$), the government finances the public good g_t with revenues τy_t only. In the repayment case ($h_t = 1$), it can issue new bonds, but has to make debt payments.

Following Chatterjee and Eyigungor (2012), government bonds are modelled as random-maturity bonds. An outstanding bond matures in the centralized market of the subsequent period with constant probability δ, with $0 < \delta \leq 1$, whereas it does not mature with probability $1-\delta$, pays a fixed coupon χ, and is valued – like newly issued bonds – at (centralized) market price q_t in this case. The per unit bond payoff k_t depends on the default decision $d_t \in \{0,1\}$ of the issuing government, where $d_t = 1$ if the government defaults and $d_t = 0$ if it repays.

When entering a period with a good credit status $h_{t-1} = 1$, a default immediately changes the government's credit status to $h_t = 0$. By contrast, if the government enters a period with $h_{t-1} = 0$, with constant probability ϑ, it is given the offer to repay the constant fraction $\omega \in [0,1]$ of its debt and immediately leave autarky in return. The indicator variable $\xi_t \in \{0,1\}$ denotes whether such an offer is received ($\xi_t = 1$) or not ($\xi_t = 0$). Following Hatchondo et al. (2016), if the government does not accept an offer ($d_t = 1$), it remains in autarky but its bond position is still reduced by $1-\omega$. If the government declines an offer, it will be excluded ($h_t = 0$) until the next period, where, with probability ϑ, it might get a new chance to settle its debt and leave autarky. The law of motion for the credit status can therefore conveniently be written as

$$h_t = \xi_t(1-d_t)(1-h_{t-1}) + (1-d_t)h_{t-1},$$

with the (realized) bond payoff in the centralized market given by

$$\begin{aligned} k_t = &\mathbf{1}\{h_{t-1} = 1 \wedge d_t = 0\} \cdot \left(\widetilde{\delta} + (1-\delta)q_t\right) + \mathbf{1}\{(h_{t-1} = 1 \wedge d_t = 1) \vee (h_{t-1} = 1 \wedge \xi_t = 0)\} \cdot q_t \\ &+ \mathbf{1}\{h_{t-1} = 0 \wedge \xi_t = 1 \wedge d_t = 1\} \cdot \omega q_t + \mathbf{1}\{h_{t-1} = 0 \wedge \xi_t = 1 \wedge d_t = 0\} \cdot \omega(\widetilde{\delta} + (1-\delta)q_t), \end{aligned}$$

where $\widetilde{\delta} \equiv \delta + (1-\delta)\chi$. Bond issuance is therefore given by $B_t - (1-\delta)B_{t-1}$, while $\widetilde{\delta}B_{t-1}$ is the period t debt service. The government is assumed lack the ability to credibly commit to future debt and default policies. Following Bianchi et al. (2018), a default leads to utility costs $\phi(\tau y_t) \geq 0$, which are a function of income, and exclusion from financial markets until debt repayment is settled.

Policy Problem As it is common in the literature, we focus on Markov-perfect equilibria, such that government policy in a given period t only depends on the payoff-relevant aggregate state variables, which consist of the aggregate public debt position B_{t-1}, tax revenues y_t, the government's credit status h_{t-1}, and the offer indicator ξ_t. In the repayment case, assuming bond market clearing ($b = B$), the government faces the following bond pricing schedule for an arbitrary debt choice B', and current income y:

$$q(B',y) = \frac{1}{1+r^{rf}} m^r(B',y)\left(1 + \Lambda^r(B',y)\right). \tag{11}$$

The bond price schedule consists of two terms. The first one, given by the function

$$m^r(B',y) = \mathbb{E}_{y'|y}\left[\ (1-\mathcal{D}(B',y'))\left(\widetilde{\delta}+(1-\delta)\mathcal{Q}^r(B',y')\right)+\mathcal{D}(B',y')\mathcal{Q}^d(B',y')\ \right],$$

captures the expected pecuniary value of the bond in the upcoming centralized market, where $\mathcal{D}(\cdot)$ denotes the default policy function. Bond prices exceed the expected pecuniary value, which is reflected in the *collateral valuation* function

$$\Lambda^r(B',y) = \left(1-\kappa\left(\lambda(B',y),1\right)\right)\times\mathbb{E}_i\left[u_i'\left(\Theta^r\left(B',y,B'\right)\right)+\frac{1}{2}\times\left\{u_i'\left(\Theta^r\left(B',y,\widetilde{b}_i(B')\right)\right)-u_i'\left(\Theta^r\left(B',y,B'\right)\right)\right\}\right].$$

Here, the term

$$\Theta^r\left(B',y,\widetilde{B}\right) = \left(1-\kappa\left(\lambda(B',y),1\right)\right)\times m^r(B',y)\times\widetilde{B},$$

is the effective collateral service for both realizations of the preference shock, which takes into account the adjusted bond positions $\widetilde{b}_i(B')$ held by an i-type investor when entering the centralized market (see Appendix A). The haircut $\kappa(\lambda_t,h_t)$ only depends the government's credit status h_t and the default probability $\lambda_t \equiv \mathbb{E}_{t-1}[d_t]$. The default probability is given by the one-period ahead default policy, weighted by the transition probability of exogenous revenue shocks

$$\lambda(B',y) = \mathbb{E}_{y'|y}\left[\mathcal{D}(B',y')\right].$$

The functions $\mathcal{Q}^r(\cdot)$ and $\mathcal{Q}^d(\cdot)$ determine the equilibrium bond prices in the next period for the repayment and default case, respectively, and are determined in equilibrium. The equilibrium bond price in the repayment case is obtained from evaluating the price schedule at the debt policy function $\mathcal{B}(\cdot)$

$$\mathcal{Q}^r(B,y) = q\left(\mathcal{B}(B,y),y\right). \tag{12}$$

In the default (and autarky) case, debt service is suspended and the debt position is rolled over, such that the associated equilibrium bond price is given as

$$\mathcal{Q}^d(B,y) = \frac{1}{1+r^{rf}}\left(m^d(B,y)(1+\Lambda^d(B,y))\right), \tag{13}$$

where the pecuniary benefits now are given by

$$m^d(B',y) = \vartheta\omega m^r(\omega B',y)+\mathbb{E}_{y'|y}\left[(1-\vartheta)\mathcal{Q}^d(B',y')\right],$$

and the non-pecuniary ones by

$$\Lambda^d(B',y) = (1-\kappa(1,0))\times\mathbb{E}_i\left[u_i'(\Theta^d\left(B',y,B'\right))+\frac{1}{2}\times\left\{u_i'\left(\Theta^d\left(B',y,\widetilde{b}_i(B')\right)\right)-u_i'\left(\Theta^d\left(B',y,B'\right)\right)\right\}\right],$$

with collateral services

$$\Theta^d\left(B', y, \widetilde{B}\right) = (1-\kappa(1,0)) \times m^d(B', y) \times \widetilde{B}.$$

The government's problem can be written recursively. When not in autarky, the government decision problem is given by

$$\mathcal{F}(B,y) = \max_{d\in\{0,1\}} (1-d)\mathcal{F}^r(B,y) + d\mathcal{F}^d(B,y), \tag{14}$$

where the value of repayment $\mathcal{F}^r(B,y)$ satisfies the Bellman equation

$$\mathcal{F}^r(B,y) = \max_{B'} v\left(\tau y + q(B',y)\left(B' - (1-\delta)B\right) - \widetilde{\delta}B\right) + \beta\mathbb{E}_{y'|y}\left[\mathcal{F}(B',y')\right], \tag{15}$$

and the value of default (and autarky) $\mathcal{F}^d(B,y)$ satisfies

$$\mathcal{F}^d(B,y) = v(\tau y) - \phi(\tau y) + \beta\ \mathbb{E}_{y'|y}\left[\vartheta\mathcal{F}(\omega B, y') + (1-\vartheta)\mathcal{F}^d(B,y')\right]. \tag{16}$$

Equilibrium A Markov-perfect equilibrium consists of value, policy and bond price functions $\{\mathcal{F}(\cdot)$, $\mathcal{F}^r(\cdot)$, $\mathcal{F}^d(\cdot)$, $\mathcal{B}(\cdot)$, $\mathcal{D}(\cdot)$, $q(\cdot)$, $\mathcal{Q}^r(\cdot)$, $\mathcal{Q}^d(\cdot)\}$, such that

(i) $\mathcal{F}(B,y)$, $\mathcal{F}^r(B,y)$, and $\mathcal{F}^d(B,y)$ satisfy equation (14) to (16),

(ii) $\mathcal{B}(B,y)$ and $\mathcal{D}(B,y)$ solve equation (14) to (16),

(iii) $q(B,y)$, $\mathcal{Q}^r(B,y)$ and $\mathcal{Q}^d(B,y)$ satisfy equation (11) to (13).

Having defined the model equilibrium, it is now possible to price a derivative security that will serve as the model equivalent of credit default swaps and obtain expressions for model-implied bid-ask spreads. A security reflecting the payoff structure of a CDS can be mapped into our model by removing convenience yield from the pricing condition (11):

$$q^{CDS}(B',y) = \frac{1}{1+r^{rf}}\mathbb{E}_{y'|y}\left[\begin{array}{c}(1-\mathcal{D}(B',y'))\left(\widetilde{\delta} + (1-\delta)\mathcal{Q}^{CDS,r}(B',y')\right)\\ +\mathcal{D}(B',y')\mathcal{Q}^{CDS,d}(B',y'),\end{array}\right] \tag{17}$$

To get the CDS-price that is consistent with equilibrium, the CDS pricing conditions are evaluated at the debt policy arising from trading on markets for bonds that provide collateral services, since these are relevant for the borrower's debt decision:

$$\mathcal{Q}^{CDS,r}(B,y) = q^{CDS}\left(\mathcal{B}(B,y),y\right), \tag{18}$$

where $\mathcal{B}(\cdot)$ denotes the debt policy function for the government. In the default (and autarky) case, the equilibrium CDS price is given as

$$\mathcal{Q}^{CDS,d}(B,y) = \vartheta\omega\mathcal{Q}^{CDS,r}(\omega B,y) + \frac{1}{1+r^{rf}}\mathbb{E}_{y'|y}\left[(1-\vartheta)\mathcal{Q}^{CDS,d}(B,y')\right]. \tag{19}$$

The crucial difference to the bond pricing expressions equation (12) and (13) is the absence of collateral service: the government bond will trade at a higher price than the CDS-price whenever collateral service is positive.

Since market illiquidity is tied to convenience yield in our model, the CDS-price also does not contain illiquidity discounts. Do disentangle convenience yield and market illiquidity, we can however exploit the analytical tractability of the centralized/decentralized market framework: as outlined in Appendix A, the bid-ask spread as a measure of illiquidity is given by

$$ba^h(B,y) = \frac{1}{2}\Big[\frac{u_H(\Theta_H^h(B,y,\widetilde{b}_H(B))) - u_H(\Theta^h(B,y,B))}{\widetilde{b}_H(B) - B} - \frac{u_L(\Theta_L^h(B,y,\widetilde{b}_L(B))) - u_L(\Theta^h(B,y,B))}{\widetilde{b}_L(B) - B}\Big],$$

which in turn depends on the credit status $h \in \{0,1\}$.

4 Calibration

In this section, we present a calibration of our model to Italian data from 2001Q1-2012Q4, such that one period corresponds to one quarter. By choosing this truncation point, we exclude periods with unconventional monetary policy measures of the ECB, since these are difficult to address in our framework. The data are compiled from various sources; a complete list can be found in Table B.1. We solve the model numerically using value function iteration over a discretized state space. The computational algorithm is described in detail in Appendix C.

4.1 Functional Forms and Parameter Choices

Government For the income process, we impose a log-normal AR(1) specification:

$$\log y_t = \rho_y \log y_{t-1} + \sigma_y \varepsilon_{y,t}\ , \quad \varepsilon_{y,t} \sim N(0,1)\ . \tag{20}$$

We estimate the income process parameters using Italian GDP data for 1991Q1-2012Q4. All data is in real terms, seasonally adjusted and de-trended using a linear-quadratic trend. Shocks are discretized as proposed by Tauchen (1986). For the period utility function of the government, we assume a CRRA specification

$$v(g_t) = \frac{(g_t - \underline{g})^{1-\gamma} - 1}{1-\gamma}.$$

Here, $\underline{g}$ denotes a subsistence level of government spending as in Bocola et al. (2019). We set the government's risk aversion to $\gamma = 2$, which is in line with similar models, such as Arellano (2008), Chatterjee and Eyigungor (2012) and Hatchondo et al. (2016). In order to calibrate default risk dynamics, we use the same utility cost function as in Bianchi et al. (2018):

$$\phi(y) = \max\{d_0 + d_1 \log(y), 0\}\ .$$

Bond Structure The maturity parameter δ is chosen such that - without default - debt has an average maturity of 5 years, which corresponds to our empirical specification. Although the average life of Italian bonds slightly exceeds five years during most parts of the sample, using this maturity, has several advantages: the benchmark bonds actually issued by the Italian Treasury have a maturity of 5 years at issuance, bid-ask spreads are computed using the on-the-run 5-year-bond and CDS contracted over 5 years are the most commonly traded credit derivative. Fixing the maturity parameter to on-the-run bonds allows us to easily obtain a plausible value for the coupon parameter. We collect coupon rates of all issues of 5-year-BTP bonds between 2001 and 2012 from the Italian Treasury Department. The (value-weighted) coupon was 4.62, which translates into a quarterly coupon rate of 1.15. The probability for a country in default to receive an debt restructuring offer ϑ and the corresponding recovery rate ω are in the range identified by Cruces and Trebesch (2013).

Investor Preferences Period utility of investors is specified by the CARA functions

$$\begin{aligned}
u_L(\theta_t) &= -\zeta_1 \exp\{-\theta_t\}, \\
u_H(\theta_t) &= -\zeta_1 \exp\{-(\theta_t - \zeta_2)\},
\end{aligned}$$

with $\zeta_1, \zeta_2 > 0$. Bond holdings of high- and low-valuation investor types are then given by

$$\begin{aligned}
\theta_{L,t} &= (1-\kappa_t) m_t B_{t-1} - \frac{1}{2}\zeta_2 \,, \\
\theta_{H,t} &= (1-\kappa_t) m_t B_{t-1} + \frac{1}{2}\zeta_2 \,.
\end{aligned}$$

It follows that the wedge between bond and CDS prices is given by

$$\Lambda_t = (1-\kappa_t)\zeta_1 \exp\{\zeta_2 - (1-\kappa_t) m_t B_{t-1}\},$$

which increases in ζ_1 and ζ_2 and decreases in the amount of available collateral $(1-\kappa_t)m_t B_{t-1}$. The size of convenience yield is then governed by ζ_1, while ζ_2 can be used to control the trading motive between high- and low-valuation investors. The discount (real) quarterly discount rate of investors is proxied by the average 3-month-EURIBOR over quarterly inflation as measured by HCPI growth rates. Consistent with this proxy, we use the EURIBOR-swap rate with corresponding maturity as risk-free reference rate to compute government bond spreads in the data.

Haircut Function For the haircut function $\kappa(\lambda, h)$ we use a simple exponential functional form (see Bindseil, 2014) to capture the negative relationship between the haircut value and default risk

$$\kappa(\lambda, h) = \begin{cases} \min\{\lambda^{\mu}, \overline{\kappa}\}, & \text{if} \quad h = 1 \\ \overline{\kappa}, & \text{if} \quad h = 0 \,, \end{cases} \tag{21}$$

where λ is the risk-neutral default probability. The sensitivity parameter is restricted to $\mu \in (0,1]$, i.e. haircut schedules punish default risk, which is in line with collateral frameworks on the private market

Table 1: Parameterization

Parameter	Value	Source
Government, External Parameters		
Autocorrelation ρ_y	0.937	Estimate of (20)
Variance σ_y^2	8.45e-05	Estimate of (20)
Government risk aversion γ	2	Conventional value
Income tax rate τ	1	Normalization
Financial Markets, External Parameters		
Investor discount rate r^{rf}	0.0013	3-month-EURIBOR minus inflation
Coupon parameter χ	0.0115	Average coupon of 5-year treasury bonds
Maturity parameter δ	0.05	5-year average maturity
Recovery rate ω	0.63	Cruces and Trebesch (2013)
Offer probability ϑ	0.08	Cruces and Trebesch (2013)
Default risk propagation μ	0.2	Bindseil (2014)
Maximum haircut $\overline{\kappa}$	0.4	Nyborg (2017)
Calibrated Parameters		
Government discount Factor β	0.972	Average debt/GDP
Default cost Parameter d_0	22.0313	Average CDS spread
Default cost Parameter d_1	50	Volatility of bond spread
Minimum consumption $\underline{g}$	0.86	Volatility of debt/GDP
Collateral service weight ζ_1	0.05	Average bond spread
Collateral service shifter ζ_2	0.0003	Average bid-ask spread

and implemented by central banks. Here, a larger μ is associated with a high sensitivity of haircuts to default risk, as we show in Figure 3. The solid blue line corresponds to our baseline calibration, where we set $\mu = 0.2$, following Bindseil (2014). Increasing this parameter implies that haircuts are less responsive to default risk, as demonstrated by the dashed green line in Figure 3. We will later vary this parameter to investigate its importance for the model's predictions. We apply a ceiling to haircuts, which is set to equal to the extraordinary haircut applied to distressed Cypriot and Greek government debt (see Nyborg, 2017). In the case of default or autarky ($h_t = 0$), the default probability is zero and the haircut is given by $\overline{\kappa}$. The cap is binding in 1.4% of all periods in which the government is not in autarky. Our quantitative results do not depend critically on $\overline{\kappa}$.

Free Parameters We chose the remaining six parameters $(\beta, d_0, d_1, \underline{g}, \zeta_1, \zeta_2)$ to match key moments of financial market variables observed between 2001Q1-2012Q4. The government's discount factor β, subsistence consumption $\underline{g}$, and the output loss parameters d_0 and d_1 are directly linked to borrowing and default incentives. The subsistence consumption level introduces a right-skewed distribution of default risk and countercyclical fiscal policy. We target the average and volatility of the government debt level and CDS spreads. These mechanics are well-known in quantitative sovereign default models, we refer to Chatterjee and Eyigungor (2012) and Bocola et al. (2019) for a detailed discussion and turn to the parameters that shape convenience yield, illiquidity risk and their interaction with default risk. ζ_1 is chosen to match the average government bonds spread over the total sample, which was 30 basis points

Figure 3: Haircut Function

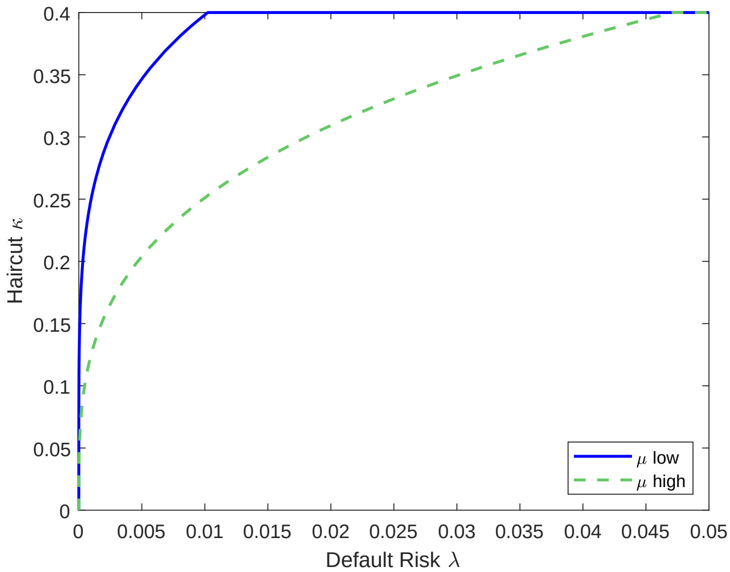
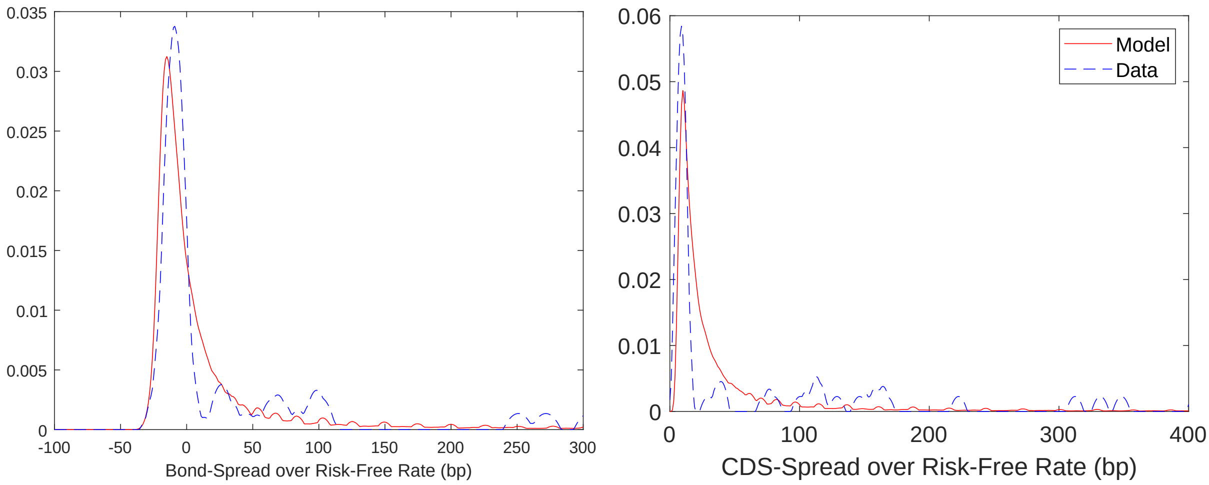


Figure 4: Model Fit: Distribution of Spreads



lower than the CDS spread. The preference shifter ζ_2 is chosen to match the mean bid-ask spread. Our calibration is summarized in Table 1.

4.2 Model Fit

First, we show the most relevant financial market variables in this context, which are the levels and volatilities of government bonds-, CDS-, and bid-ask-spreads. We then demonstrate that the model is capable of generating countercyclical debt issuance and finally provide evidence that the central result regarding the interaction of bond supply and bond spreads of Krishnamurthy and Vissing-Jorgensen (2012) prevails even in the presence of default risk, complementing the findings of Jiang et al. (2021).

Table 2: Model Fit: Financial Market Variables

Variable	Full Sample		Crisis Episodes	
	Data	Model	Data	Model
ave(s_t)	48	46	161	125
ave(cds_t)	78	77	204	162
ave$(\log(Q_t B_t / y_t))$	1.50	1.49	1.58	1.55
ave(ba_t)	11	12	29	11
std(s_t)	114	163	143	226
std(cds_t)	113	182	117	254
std$(\log(Q_t B_t / y_t))$	0.058	0.069	0.032	0.030
std(ba_t)	16	5	17	7

Notes: Spreads are annualized and in basis points. Crisis episodes are all periods with a positive government bond spread. Targeted moments are in color. All model-implied statistics are based on simulations of 50,000 periods after a burn-in period with length of 5,000. We exclude all periods where the government is in financial autarky as well as 40 quarters after re-entering financial markets following an exclusion period.

Financial Markets Results for targeted moments are reported in the left panel of Table 2. The level of debt and all three spreads are matched, whereas the volatility targets show some discrepancy with the data, which is difficult to overcome with our risk-neutral pricing setting. To examine the interaction between convenience yield and credit risk in greater detail, we also report statistics for all periods with a positive bond spread, i.e., in times where the credit risk component of bond spreads dominates the convenience yield component. We refer to this sub-sample as crisis periods. In the our data sample, the bond spread was negative from 2001Q1 until 2008Q4 and turned positive thereafter. In particular, the rise in debt levels associated with the financial crisis is captured by our model. Since our model only features a single exogenous shock as well as risk-neutral bond pricing, the increase of all spreads, bid-ask spreads in particular, is less pronounced than observed in the data.
We also plot kernel density estimations of bond- and CDS spreads obtained from our model against their data counterparts in Figure 4. Both spreads show sizable positive skewness and considerable probability mass at negative bond spreads, consistent with the data.

Debt Management Debt management in developed economies is countercyclical in the sense that the government increases borrowing in response to negative fiscal shocks. Note that such behavior is typically not present in standard models of sovereign debt and default (see Arellano, 2008). The reason for this is that increases in debt severely lower bond prices in bad times due to increased risk of default, which typically incentivizes the government to lower its debt issuance. However, as in Bocola et al. (2019), the inclusion of a minimum consumption level makes the government less responsive to debt-elastic interest rate hikes in such instances, such that countercyclical debt and default risk can arise simultaneously. Mechanically, this works as follows: in bad (low revenue) states, bond prices tend to move downward. Although borrowing is countercyclical and thus collateral becomes more abundant in principle, the increase in nominal debt outstanding is dominated by default risk, especially because

Table 3: Model Fit: Debt Management

Variable	Full Sample		Crisis Episodes	
	Data	Model	Data	Model
$\text{ave}(Q_{t+1}B_t/y_t)$	4.500	4.437	4.847	4.698
$\text{ave}(Q_{t+1}(B_{t+1}-B_t)/y_t)$	0.029	0.007	0.041	0.023
$\text{std}(Q_{t+1}B_t/y_t)$	0.267	0.298	0.158	0.136
$\text{std}(Q_{t+1}(B_{t+1}-B_t)/y_t)$	0.049	0.049	0.051	0.044
$\text{cor}(Q_{t+1}(B_{t+1}-B_t)/y_t, y_t)$	-0.216	-0.532	-0.293	-0.621

Notes: Spreads are annualized and in basis points. Crisis episodes are all periods with a positive government bond spread. All model-implied statistics are based on simulations of 50,000 periods after a burn-in period with length of 5,000. We exclude all periods where the government is in financial autarky as well as 40 quarters after re-entering financial markets following an exclusion period.

the haircut also increases in this case. The total effect results in a negative co-movement between borrowing and bond spreads, which is in line with the data. The model is able to generate sizable negative correlation between government revenues and debt issuance.

Decomposing Government Bond Spreads Finally, we examine how the pricing of different risk factors is reflected in bond prices during times of crisis, corresponding to the high-risk episode of our sample. Therefore, we regress the key endogenous variable from our model, the spread of 5-year Italian government bonds, on potential drivers of the spread, namely credit risk (measured by the CDS spread), market illiquidity (measured by bid-ask spreads or turnover) and convenience yield (measured by the log supply of total or long-term government bonds). Formally, we run the regression

$$s_t = \beta_0 + \beta_1 cds_t + \beta_2 \log\left(\frac{Q_t B_t}{y_t}\right) + \beta_3 ba_t + \varepsilon_t \,. \tag{22}$$

Market illiquidity measures refer to 5-year Italian government bonds, traded on the Milan stock exchange. Results are shown in Table 4. Despite the small sample size, all four regression specifications explain a large share of the spread variance (with an R^2 close to one) and draw a conclusive picture regarding the determinants of government bond spreads. While an increase in credit risk (as measured by CDS-spreads) translates into a bond spread increase almost one for one, bond supply also has a positive effect on bond spreads. The coefficient is highly significant after also controlling for market illiquidity proxies, consistent with the results of Jiang et al. (2021). Market illiquidity seems to have played a minor role as the coefficients on all liquidity measures are insignificant and inconclusive in their sign. While higher bid-ask spreads translate into higher bond spreads, the coefficient on relative turnover switches signs, depending on the regression specification.

We run the same regression on the crisis periods of the simulated time series implied by our model. Since there is only one persistent exogenous shock, tax revenues, and the i.i.d. taste shock, all coefficients are highly significant. Furthermore, there is no additional exogenous driver of bid-ask spreads in

Table 4: Determinants of Government Bond Spreads

	Data				Model
Debt proxy	Total Debt		Long-term Debt		
Illiquidity proxy	Bid-ask	Turnover	Bid-ask	Turnover	
Intercept	-2747	-1977	-1392	-1569	-119
	(254)***	(457)***	(266)***	(155)***	(2.54)***
CDS spread	0.71	0.99	0.86	0.81	0.91
	(0.13)***	(0.10)***	(0.12)***	(0.06)***	(4e-04)***
Debt proxy	1727	1328	1080	1025	63
	(166)***	(333)***	(214)***	(157)***	(1.64)***
Illiquidity proxy	1.36	29.6	0.38	-45	-
	(0.88)	(28.3)	(0.82)	(27)	-
N	16	16	16	16	Large
R^2	0.98	0.97	0.97	0.98	1

Notes: Newey-West standard errors in parenthesis. Significance levels at 5 % (*), 1 % (**) and 0.1 % (***).

the model, and we exclude it from the regression. Consistent with the data, higher bond supply translates into higher bond spreads after controlling for credit risk, i.e., convenience yield declines when bonds become less scarce, which confirms the findings of Krishnamurthy and Vissing-Jorgensen (2012) for a setting with risky government bonds and complements the results of Jiang et al. (2021).

5 Model Mechanics: A Quantitative Exploration

Using our calibration, this section examines the impact of convenience yield on financial market variables and the conduct of public debt management. We proceed in two steps. First, we present a recalibration of the model without convenience yield and without bid-ask spreads, and compare the dynamics of public debt and default risk to the baseline calibration. These experiments suggest that convenience yield has a quantitatively relevant effect on the *debt level*, but only small effects on debt and default *dynamics*. In contrast, market illiquidity has no visible effects at all. This is consistent with the determinants of bond spreads in Table 4. As a second experiment, we separately examine both components of convenience yield: collateral valuation and haircuts. Guided by the analytical results from Section 2, we conduct a comparative statics exercise *(i)* regarding the elasticity of collateral valuation with respect to bond supply and *(ii)* regarding the haircut parameter μ. The results of this exercise are in line with the simple analytical framework: more sensitive haircuts reduce the frequency of default if collateral valuation is *debt-inelastic* and increase the frequency of default if collateral valuation is *debt-elastic* and, thereby, provides disciplining incentives to the government.

5.1 The Role of Convenience Yield and Market Illiquidity

We start by setting $\zeta_1 = 0$, which eliminates convenience yield and - as a by-product - bid-ask spreads. Since this lowers the government's incentive to issue debt and subject itself to default risk, the default cost parameter is reduced to $d_0 = 21.1719$, such that the average CDS-spread matches its target. If d_0 would remain at its higher baseline value, the government would not find it worthwhile to issue bonds in the risky borrowing region absent convenience yield. Put differently, convenience yield induces higher debt issuance. Results under a recalibrated d_0 are displayed in the second column of Table 5. In contrast to the baseline calibration, the government bond spread mechanically increases to the level of the CDS spread. This feature stresses our baseline model's ability to reconcile sovereign risk and very low government bond spreads. Compared to the baseline scenario, the debt-to-GDP ratio decreases by 10% from 1.49 to 1.34. This is not merely driven by the decline in bond prices induced by the absence of convenience yield: the debt-to-GDP ratio at face value (B_t/y_t) is also substantially smaller in this case. The effect on public debt management is quantitatively negligible. The key takeaway from this experiment is that convenience yield increases the debt-GDP ratio once its effect on default risk is controlled for.

In a second step, we recalibrate the model to match the same average debt-to-GDP ratio as in the baseline calibration. This requires the government to be more impatient relative to the baseline case. To counter the positive impact of higher impatience on the incentive to default, the recalibrated default cost parameter d_0 needs to be increased to match our target. The resulting adjustment requires setting $\beta = 0.963$ and $d_0 = 26.6113$. The results are displayed in the third column of Table 5. Compared to the baseline calibration and the case without convenience yield, the effect of convenience yield on financial market variables and government debt management is relatively small.

In a third step, we consider a recalibration of the model with $\zeta_2 = 0$ to isolate the role of bid-ask spreads. In this case, there is no trading motive for investors, such that the valuation equation (10) reduces to

$$\Lambda(B_t, y_t) = (1 - \kappa(\lambda_t, y_t)) \times u'(\theta_{t+1}),$$

which is smaller in size relative to the baseline case. While the discount factor β does not have to be recalibrated at all to match the debt-to-GDP ratio, we reduce $d_0 = 21.9727$ to match average CDS-spreads. The required adjustment of d_0 is very small compared to the case without convenience yield, suggesting already that market illiquidity hardly affects debt and default dynamics. Indeed, as can be seen in the last column of Table 5, the moments for the model without market illiquidity hardly differ from the baseline case. Finally, Panel A of Table 5 highlights another important feature of our model, which standard sovereign default models cannot address by construction. In more than half of all periods where the government is not in autarky, the bond spread is negative.

5.2 The Role of Haircuts and Collateral Scarcity

Up to this point, we discussed how convenience yield improves the quantitative sovereign default model's ability to match the level and joint dynamics of government bond supply, default risk, and bond

Table 5: Convenience Yield and Market Illiquidity: Selected Moments

Variable	Baseline	No Conv. Yield	No Conv. Yield (β recalibrated)	No Illiquidity
Panel A: Financial Markets				
ave(s_t)	46	77	79	46
ave(cds_t)	77	77	79	77
ave$(\log(Q_t B_t/y_t))$	1.49	1.34	1.49	1.49
std(s_t)	163	178	183	161
std(cds_t)	182	178	183	180
std$(\log(Q_t B_t/y_t))$	0.069	0.076	0.063	0.069
Default rate (%)	1.48	1.49	1.45	1.47
Negative Spread (%)	57.1	0	0	55.8
Panel B: Debt Management				
ave(B_t/y_t)	3.985	3.467	4.022	3.936
ave$(Q_{t+1}B_t/y_t)$	4.437	3.846	4.46	4.427
ave$(Q_{t+1}(B_{t+1}-B_t)/y_t)$	0.007	0.007	0.006	0.007
std$(Q_{t+1}B_t/y_t)$	0.298	0.284	0.276	0.298
std$(Q_{t+1}(B_{t+1}-B_t)/y_t)$	0.049	0.047	0.046	0.049
cor$(Q_{t+1}(B_{t+1}-B_t)/y_t, y_t)$	-0.532	-0.532	-0.505	-0.530

Notes: For all three experiments d_0 is recalibrated to match average CDS-spreads. Spreads are annualized and in basis points. All model-implied statistics are based on simulations of 50,000 periods after a burn-in period with length of 5,000. We exclude all periods where the government is in financial autarky as well as 40 quarters after re-entering financial markets following an exclusion period. Default rate (%) and Negative Spread (%) are computed relative to the number of *included* periods.

spreads. In this section, we show how the decomposition of convenience yield into collateral valuation and haircut components affects sovereign risk and, to a small extent, public debt management. Conceptually, we closely follow our simple analytical framework and show that the effects of both components of convenience yield are also present in our full model.

We evaluate the relative importance of these two effects by solving our model for different values of μ and two specifications of the collateral valuation component. The first panel of Table 6 reports the results for the baseline specification of $u(\theta)$. We also recalibrate the model under the assumption of a constant marginal utility of collateral service, such that collateral valuation is given by

$$\Lambda(B_t, y_t) = (1 - \kappa(\lambda_t)) \times \zeta_1 .$$

Note that convenience yield still negatively depends on bond supply through the haircut function. We again recalibrate the model ($\beta = 0.967$, $d_0 = 23.3$, and $\zeta_1 = 0.065$) to ensure that average CDS and government bond spreads and debt-to-GDP ratio remain consistent with the data moments. We report the results for $\mu = 0.2$ (the baseline value), $\mu = 1$ and $\mu = 2$ in Table 6.

The upper panel of Table 6 shows that both the level and volatility of spreads decrease with μ in

Table 6: Haircut and Collateral Valuation Components: Selected Moments

	With scarcity			*Without scarcity*		
Panel A: Financial Markets	$\mu = 0.2$	$\mu = 1$	$\mu = 2$	$\mu = 0.2$	$\mu = 1$	$\mu = 2$
ave(s_t)	47	40	33	49	56	56
ave(cds_t)	77	70	62	79	86	86
std(s_t)	163	156	147	181	189	188
std(cds_t)	182	173	163	186	193	192
Default rate (%)	1.48	1.26	1.22	1.46	1.62	1.65
Panel B: Debt Management						
ave(B_t/y_t)	3.935	3.935	3.935	3.952	3.948	3.945
ave($Q_{t+1}B_t/y_t$)	4.437	4.439	4.453	4.443	4.424	4.422
ave($Q_{t+1}(B_{t+1}-B_t)/y_t$)	0.007	0.006	0.006	0.007	0.007	0.007
std($Q_{t+1}B_t/y_t$)	0.299	0.301	0.302	0.271	0.270	0.269
std($Q_{t+1}(B_{t+1}-B_t)/y_t$)	0.049	0.048	0.048	0.048	0.049	0.048
cor($Q_{t+1}(B_{t+1}-B_t)/y_t, y_t$)	-0.532	-0.521	-0.500	-0.488	-0.510	-0.511

Notes: Effects of varying the haircut parameter μ. A low μ corresponds to a highly elastic haircut schedule (see also Figure 3). Spreads are annualized and in basis points. All model-implied statistics are based on simulations of 50,000 periods after a burn-in period with length of 5,000. We exclude all periods where the government is in financial autarky as well as 40 quarters after re-entering financial markets following an exclusion period.

the baseline case *with scarcity*. The reason for this observation lies in the responsiveness of collateral valuation component to the supply of government bonds. If a highly elastic collateral valuation is combined with a highly elastic haircut schedule, the collateral scarcity effect *dominates* the haircut effect and renders higher debt issuance and default risk optimal to the government. Increasing μ softens this effect.

The impact of μ on level and volatility of spreads is reversed in the case of a *debt inelastic* collateral valuation. Here, less elastic haircut schedules (with a higher μ) are associated with higher default rates and spreads, since the disciplining effects of collateral scarcity is absent. Panel B of Table 6 reveals that haircut elasticities also affect the conduct of public debt management. In the case *without scarcity*, a less elastic convenience yield implies that bond prices are less responsive to a negative revenue shock, holding debt constant. Consequently, fiscal policy is *more countercyclical*. The same argument applies to the case *with scarcity*. However, the effects on public debt management are of a rather small magnitude.

Taken together, our experiments indicate that the *composition* of effective convenience yield matters for sovereign risk and the conduct of fiscal policy. Altering the responsiveness of haircut schedules can change CDS spread level and volatility by up to 15 basis points and default rates by up to 0.25 percentage points. These effects are quantitatively relevant, given that average government bond spreads are around 50 basis points and the default rate around 1.5% in our baseline calibration. The results furthermore suggest that the design of haircut schedules (either on public or private market segments) could benefit from taking into account the nature of collateral demand.

6 Conclusion

In this paper, we have studied how convenience yield, sovereign default risk, and the supply of government bonds interact through the lenses of a quantitative macroeconomic model. Specifically, we have modified an otherwise standard quantitative sovereign default model by allowing for convenience yield as well as endogenous market illiquidity discounts, which both interact with the government's risk of default and debt management. Calibrating the model to Italy, we showed that, despite its parsimonious structure, our model can generate the basic observed properties of sovereign debt, credit risk, sovereign bond spreads, credit default swap spreads and bid-ask spreads. To understand the role of convenience yield in the presence of default risk, we provide a decomposition of convenience yield into individual components. It suggests that the elasticity of collateral valuation and haircut schedules applied to collateral with respect to government bond supply can have sizable effects on debt and default dynamics.

A Investor Problem

This section presents details on the investor problem and the derivation of the bond pricing condition (9).

Maximization Problem The investor decision problem is formulated recursively. An investor's value function in the second sub-period (centralized market) is given as

$$\mathcal{W}_t(\widetilde{b}_t, a_t) = \max_{b_t, c_t} \left\{ c_t + \frac{1}{1+r^{rf}} \mathbb{E}_t \left[\mathcal{V}_{t+1}(b_t) \right] \right\}$$

subject to the budget constraint

$$c_t = e - a_t + k_t \widetilde{b}_t - q_t b_t,$$

where a_t are payments owed to a dealer from trading in the decentralized market in the first sub-period of period t (described below). The probability of becoming an L-type investor is normalized to $\frac{1}{2}$. An investor's value function at the beginning of a period, before idiosyncratic preference shocks are realized, is given by

$$\mathcal{V}_t(b_{t-1}) = \frac{1}{2}\mathcal{V}_{L,t}(b_{t-1}) + \frac{1}{2}\mathcal{V}_{H,t}(b_{t-1}),$$

with the value of being an i-type investor given by

$$\begin{aligned}\mathcal{V}_{i,t}(b_{t-1}) =& u_i\big((1-\kappa_t)m_t\widetilde{b}_{i,t}\big) + \mathbb{E}_{t-1}\left[\mathcal{W}_t(\widetilde{b}_{i,t}, -\widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t})\right] \\ =& u_i\big((1-\kappa_t)m_t\widetilde{b}_{i,t}\big) + m_t(\widetilde{b}_{i,t} - b_t) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t} + \mathbb{E}_{t-1}\left[\mathcal{W}_t(b_t, 0)\right] ,\end{aligned}$$

which holds for an arbitrary b_t. Here $\phi_{i,t}$ is a fee charged by dealers and $\widetilde{q}_t$ is the competitive bond price on the inter-dealer market (both described below). Note that the second equality is due to linearity of the value function $\mathcal{W}_t(\cdot)$ in $\widetilde{b}_t$ and a_t.

Decentralized Market To adjust their bond holdings in response to the i-shock, investors contact dealers. The terms of trade between dealers and investors are determined bilaterally via Nash bargaining. For an i-type investor, the bargaining threat point is

$$\overline{\mathcal{V}}_{i,t}(b_{t-1}) = u_i\big((1-\kappa_t)m_t b_{t-1}\big) + \mathbb{E}_{t-1}\left[\mathcal{W}_t(b_{t-1}, 0)\right],$$

such that the surplus from trading is given by the utility gain from trading $u_i(\widetilde{\theta}_{i,t}) - u_i\big((1-\kappa_t)m_t b_{t-1}\big)$, net of pecuniary benefits, and the fee payed to dealers

$$S_{i,t}(b_{t-1}) = u_i(\widetilde{\theta}_{i,t}) - u_i\big((1-\kappa_t)m_t b_{t-1}\big) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t},$$

In the expression for the surplus, $\widetilde{\theta}_{i,t} \equiv (1-\kappa_t)m_t\widetilde{b}_{i,t}$ is the (haircut-adjusted) value of bond-holdings after adjusting the bond position and $(\widetilde{b}_{i,t}, \phi_{i,t})$ are the terms of trade, which consist of the investor's bond holdings after the meeting $\widetilde{b}_{i,t}$ and the fee charged by the dealer $\phi_{i,t}$. Payments owed to dealers consist of the fee and the desired adjustment of the bond positions,

$$a_{i,t} = \phi_{i,t} + \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) \;,$$

and are settled in the subsequent centralized market. A dealer's surplus in an i-type meeting simply equals $\phi_{i,t}$, which is consumed by the dealer in the centralized market. Dealers do not acquire bonds in the centralized market. The investors' bargaining power is α, the terms of trade solve the generalized Nash bargaining problem

$$\max_{\widetilde{b}_{i,t},\phi_{i,t}} \left[u_i\big((1-\kappa_t)m_t\widetilde{b}_{i,t}\big) - u_i\big((1-\kappa_t)m_t b_{t-1}\big) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1}) - \phi_{i,t}\right]^{\alpha} \phi_{i,t}^{1-\alpha},$$

which leads to the two first-order conditions

$$\widetilde{q}_t = (1-\kappa_t)m_t u_i'(\widetilde{\theta}_{i,t}) + m_t, \tag{A.1}$$

$$\phi_{i,t} = (1-\alpha)\Big(u_i(\widetilde{\theta}_{i,t}) - u_i(\theta_t) + m_t(\widetilde{b}_{i,t} - b_{t-1}) - \widetilde{q}_t(\widetilde{b}_{i,t} - b_{t-1})\Big). \tag{A.2}$$

Note that the dealer fee simply equals the dealer's bargaining power $1-\alpha$ times the total surplus $S_{i,t}(b_{t-1}) + \phi_{i,t}$. As in Lagos and Rocheteau (2009), the dealer fee can be expressed in terms of a meeting-specific bond price $\widetilde{q}_{i,t}$,

$$\phi_{i,t} = (\widetilde{q}_{i,t} - \widetilde{q}_t)(\widetilde{b}_{i,t} - b_{t-1}).$$

Using this relationship, one can derive the expression

$$\widetilde{q}_{i,t} = \alpha\widetilde{q}_t + (1-\alpha)\frac{u_i(\widetilde{\theta}_{i,t}) - u_i(\theta_t) + m_t(\widetilde{b}_{i,t} - b_{t-1})}{\widetilde{b}_{i,t} - b_{t-1}}.$$

The meeting-specific price $\widetilde{q}_{i,t}$ equals a weighted average of the inter-dealer market price $\widetilde{q}_t$ and the total surplus net of payments $\widetilde{q}_{i,t}(\widetilde{b}_{i,t} - b_{t-1})$ divided by the net trading position, which will be useful for deriving bid-ask spreads below. If the investor holds all bargaining power ($\alpha = 1$), the dealer does not charge a mark-up/mark-down, i.e. $\widetilde{q}_{i,t} = \widetilde{q}_t$. If the investor is a net-buyer ($\widetilde{b}_{i,t} > b_{t-1}$), then the ask price of the dealer is $\widetilde{q}_{i,t}$, which exceeds the inter-dealer price $\widetilde{q}_t$ whenever $\alpha < 1$. Similarly, if the investor is a net-seller, $\widetilde{q}_{i,t}$ is the bid price which is below $\widetilde{q}_t$ for $\alpha < 1$. Consistent with the quantitative analysis we normalize the bargaining power to $\alpha = \frac{1}{2}$ in the following. A commonly used measure of the extent to which a market is affected by trading frictions is the bid-ask spread, which is given as

$$\widetilde{q}_{H,t} - \widetilde{q}_{L,t} = \frac{\phi_{H,t}}{\widetilde{b}_{H,t} - b_{t-1}} - \frac{\phi_{L,t}}{\widetilde{b}_{L,t} - b_{t-1}}\;, \tag{A.3}$$

in our model. Market clearing in the competitive inter-dealer market requires

$$\frac{1}{2}\widetilde{b}_{L,t}+\frac{1}{2}\widetilde{b}_{H,t}=B_{t-1},$$

with government bond supply B_{t-1}, such that

$$u_L'\big((1-\kappa_t)m_t\widetilde{b}_{L,t}\big)=u_H'\Big((1-\kappa_t)m_t\times 2\times\Big(B_{t-1}-\frac{1}{2}\widetilde{b}_{L,t}\Big)\Big),$$

determines $\widetilde{b}_{L,t}$ and $\widetilde{b}_{H,t}$ via market clearing.

Centralized Market For the centralized market, the investor first-order condition is

$$-q_t+\frac{1}{1+r^{rf}}\frac{\partial\mathbb{E}_t\left[\mathcal{V}_{t+1}(b_t)\right]}{\partial b_t}=0. \tag{A.4}$$

The value $\mathcal{V}_{t+1}(\cdot)$ can be written as

$$\mathcal{V}_{t+1}(b_t)=\mathbb{E}_i\left[u_i(\theta_t)+S_{i,t+1}(b_t)\right]+\mathbb{E}_t\left[\mathcal{W}_{t+1}(b_t,0)\right],$$

such that

$$\begin{aligned}\frac{\partial\mathcal{V}_{t+1}(b_t)}{\partial b_t}&=\mathbb{E}_i\left[(1-\kappa_{t+1})m_{t+1}u_i'(\theta_{t+1})+\frac{\partial S_{i,t+1}(b_t)}{\partial b_t}\right]+\frac{\partial\mathbb{E}_t\left[\mathcal{W}_{t+1}(b_t,0)\right]}{\partial b_t}\\&=\mathbb{E}_i\left[(1-\kappa_{t+1})m_{t+1}u_i'(\theta_{t+1})+\frac{\partial S_{i,t+1}(b_t)}{\partial b_t}\right]+m_{t+1}\ .\end{aligned}$$

The effect of individual bond holdings on trading frictions is given by

$$\begin{aligned}\frac{\partial S_{i,t+1}(b_t)}{\partial b_t}=&(1-\kappa_{t+1})m_{t+1}\frac{\partial\widetilde{b}_i(b_t)}{\partial b_t}u_i'(\widetilde{\theta}_{i,t+1})-(1-\kappa_{t+1})m_{t+1}u_i'(\theta_{t+1})\\&+m_{t+1}\left(\frac{\partial\widetilde{b}_i(b_t)}{\partial b_t}-1\right)-\widetilde{q}_{t+1}\left(\frac{\partial\widetilde{b}_i(b_t)}{\partial b_t}-1\right)-\frac{\partial\phi_i(b_t)}{\partial b_t}.\end{aligned}$$

Using

$$\frac{\partial\phi_i(b_t)}{\partial b_t}=\frac{1}{2}\left[\begin{array}{c}(1-\kappa_{t+1})m_{t+1}\frac{\partial\widetilde{b}_i(b_t)}{\partial b_t}u_i'(\widetilde{\theta}_{i,t+1})-(1-\kappa_{t+1})m_{t+1}u_i'(\theta_{t+1})\\+m_{t+1}\left(\frac{\partial\widetilde{b}_i(b_t)}{\partial b_t}-1\right)-\widetilde{q}_{t+1}\left(\partial\frac{\widetilde{b}_i(b_t)}{\partial b_t}-1\right)\end{array}\right],$$

together with (A.1), this derivative can be written as

$$\frac{\partial S_{i,t+1}(b_t)}{\partial b_t}=\frac{1}{2}\left[u_i'(\widetilde{\theta}_{i,t+1})-u_i'(\theta_{t+1})\right](1-\kappa_{t+1})m_{t+1},$$

such that

$$\frac{\partial \mathcal{V}_{t+1}(b_t)}{\partial b_t} = \mathbb{E}_i \left[u_i'(\theta_{t+1}) + \frac{1}{2} \times \left\{ u_i'(\widetilde{\theta}_{i,t+1}) - u_i'(\theta_{t+1}) \right\} \right] (1-\kappa_{t+1}) m_{t+1} + m_{t+1}.$$

Combining this condition with the investor first-order condition (A.4) then yields the bond pricing equation (9). As in Lagos and Wright (2005), linearity of investor preferences with respect to consumption and idiosyncratic preference shocks being i.i.d. implies that investors choose the same bond holdings b_t in the centralized market regardless of their current holdings $\widetilde{b}_{i,t-1}$.

B Data Sources

This contains the data sources used in the quantitative analysis.

Debt Debt service is taken from monthly reports of the Italian Department of Treasury. To focus on debt with appropriate maturity, we focus on redemption and coupon payments of bonds with a maturity of at least one year. Including short-term liabilities (treasury bills, called BOT) would introduce a relatively large amount of debt outstanding, that is rolled over potentially multiple times each period. Coupon data are obtained from annual reports on debt issuance published also by the Treasury.

Income Processes GDP data for Italy is obtained from the St Louis Fed database, going from 1961Q1 to 2012Q4. Data is in real terms and seasonally adjusted. Data is logged and de-trended using deviations from a linear-quadratic trend. We subsequently impose an AR(1)-structure and get $(\rho_y, \sigma_y^2) = (0.937, 8.45e-05)$.

Investors To proxy the discount rate of investors, we use 3-month-EURIBOR data (1991Q1-2012Q4) from the Bundesbank and (quarterly) Euro area inflation rates. The real rate is simply obtained by subtracting quarterly inflation from 3-month-EURIBOR. Data on German bonds used to construct the maximum convenience yield is taken from Datastream.

C Solution Algorithm

This section presents details on the numerical solution algorithm used to solve the government problem.

Taste Shocks We solve the model numerically using value function iteration on a discretize state space without interpolation. Let $\mathfrak{B}$ denote the debt grid. Define the expected continuation value as $F(B', y) \equiv \mathbb{E}_{y'|y} \mathcal{F}(B', y')$. As already pointed out by Chatterjee and Eyigungor (2012), convergence problems typically arise in this class of models. We follow Gordon (2018) by introducing Gumbel-distributed taste shocks z to randomize over debt and default choices:

$$\mathcal{F}^r(B, y', z) = \max_{B'} \left\{ v\left(\tau y + q\left(B', y\right) \left(B' - (1-\delta) B\right) - \widetilde{\delta} B \right) + \widetilde{\beta} F(B', y) + \sigma_z z^{B'} \right\} .$$

Table B.1: Data Sources and Ticker

Series	Source	Mnemonic	Frequency
Redemption yield, 5-year benchmark BTP	Bank of Italy	MFN_BMK.M.020.922.0.EUR.205	Monthly average
Turnover, 5-year BTP	Bank of Italy	MFN_QMTS.D.020.926.MKV.EUR.9	Monthly average
Total debt	Bank of Italy	FPI_FP.M-IT.S13.MGD.SBI3.101.112.FAV.EUR.EDP	Monthly
Net issuance, medium and long-term	Bank of Italy	FPI_FP.M.IT.S13.F32.SBI3.103.115.COV.EUR.FPBI	Monthly
Debt service, medium and long-term	Treasury Department	Monthly reports	Monthly
Turnover, total debt	Bank of Italy	MFN_QMTS.D.100010.926.MKV.EUR.9	Monthly average
Bid-ask spreads, 5-year BTP	Treasury Department	Quarterly Bulletins	Monthly
EURIBOR-SWAP, 5-Year	Datastream	ICEIB5Y	Daily
CDS spread Italy, 5-Year	Bocola (2016)		Daily
3-month-EURIBOR	Bundesbank	BBK01.SU0316	Daily
Euro area CPI	St Louis Fed	EA19CPALTT01IXOBQ	Quarterly
Redemption yield Germany, 5-Year	Datastream	TRBD5YT	Daily
GDP, Italy	St Louis Fed	LORSGPORITQ661S	Quarterly

Throughout the algorithm we standardize the value function prior to computing choice probabilities and expected choices. The standardization immediately cancels out when choice probabilities are computed. For the expected value of the maximum choice, consider the well-known expression

$$\mathbb{E}\left(\max_j V(j)+\sigma\varepsilon_j\right)=\sigma\log\left(\sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right) \tag{C.1}$$

where j denotes next period's choice. Dependence on current persistent states is omitted. Expanding both sides by an arbitrary constant $\overline{V}$ yields

$$\overline{V}+\mathbb{E}\left(\max_j V(j)-\overline{V}+\sigma\varepsilon_j\right)=\sigma\log\left(\sum_j \exp\left\{\frac{V(j)-\overline{V}}{\sigma}\right\}\right) \tag{C.2}$$

the right-hand side of (C.2) can be rearranged:

$$\begin{aligned}\overline{V}+\sigma\log\left(\sum_j \exp\left\{\frac{V(j)-\overline{V}}{\sigma}\right\}\right) &= \overline{V}+\sigma\log\left(\exp\left\{-\frac{-\overline{V}}{\sigma}\right\}\sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right)\\ &= \overline{V}+\sigma\log(\exp\left\{-\frac{\overline{V}}{\sigma}\right\})+\sigma\log\left(\sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right)\\ &= \sigma\log\left(\sum_j \exp\left\{\frac{V(j)}{\sigma}\right\}\right)\end{aligned}$$

Plugging back into (C.2) gives

$$\mathbb{E}\left(\max_j V(j)+\sigma\varepsilon_j\right)=\overline{V}+\sigma\log\left(\sum_j \exp\left\{\frac{V(j)-\overline{V}}{\sigma}\right\}\right). \tag{C.3}$$

When $\overline{V}=\max_j V(j)$ is chosen, each term in the exponent on the left hand side is negative and the problem remains numerically tractable. To speed up computation, we use a version of the divide-and-conquer algorithm proposed by Gordon and Qiu (2018) and Gordon (2018), that truncates numerically irrelevant choices based on policy function monotonicity. The improved convergence properties compared to a brute-force algorithm are particularly relevant for the large state space required by our model. Since there are some (numerically small) non-monotonicities at some points of the state-space, we use a guess-and-verify approach. We compute the equilibrium using the divide-and-conquer approach until convergence. After the final step, we use that solution as to compute another iteration. We consider the solution obtained using divide-and-conquer appropriate if the equilibrium objects stay within a close neighborhood of the solution from the previous iteration.

The Algorithm To start off the algorithm, we use the final period of a finite-horizon model to determine default states for iteration 0. Given the results of iteration ι (or initialization), each iteration $\iota+1$ proceeds as follows:

1. For all states (B, y'), evaluate debt choices B' according to

$$\left\{\overline{B}, 0, \frac{1}{2}\overline{B}, \frac{3}{4}\overline{B}, \frac{1}{4}\overline{B}, \frac{7}{8}\overline{B}, \frac{5}{8}\overline{B}, \ldots\right\}$$

   (a) Determine the option value of default F

$$\mathcal{F}^d_{\iota+1}(B, y') = v\left(\tau y' - \underline{g} - \phi(\tau y')\right) + F^d_\iota(B, y') .$$

   (b) Evaluate every numerically relevant debt choice in $\mathfrak{B}$, given current debt over the interval $[B'_-, B'_+]$ (elements of the grid $\mathfrak{B}$) with

$$B'_- = \min\Big(B' | P\left(\mathcal{B}(B_-)\right) > \varepsilon\Big), \quad \text{and} \quad B'_+ \max\Big(B' | P\left(\mathcal{B}(B_+)\right) > \varepsilon\Big) ,$$

   where B_+ and B_- denote the next larger/smaller current debt stock, that has already been evaluated. At the upper bound, the choices are not bounded, while for $\underline{B} = 0$, they are only bounded from above. The value of choosing $B^j \in \mathfrak{B} | B_- < B^j < B_+$ is given by

$$\mathcal{F}^r_{\iota+1}(B^j | B, y') = v\Big(\tau y' - \underline{g} + \mathcal{Q}^r_\iota(B^j, y')\left(B^j - (1-\delta)B\right) - \widetilde{\delta} B\Big) + \widetilde{\beta} F_\iota(B^j, y') .$$

   (c) Compute debt choice and default probabilities using the Type I-extreme value distribution:

$$\Pr(d = 1 | B, y') = \frac{\exp\left(\mathcal{F}^d_{\iota+1}(B, y')/\sigma_z\right)}{\exp\left(\mathcal{F}^d_{\iota+1}(B, y')/\sigma_z\right) + \sum_j \exp\left(\mathcal{F}^r_{\iota+1}(B^j, y')/\sigma_z\right)} ,$$
$$\Pr(B' = B^j | B, y') = \frac{\exp\left(\mathcal{F}^r_{\iota+1}(B^j, y')/\sigma_z\right)}{\exp\left(\mathcal{F}^d_{\iota+1}(B, y')/\sigma_z\right) + \sum_j \exp\left(\mathcal{F}^r_{\iota+1}(B^j, y')/\sigma_z\right)} .$$

   (d) Determine the default policy $\mathcal{D}$ and expected payoffs k^r and k^d w.r.t. z using the probabilities computed in (c):

$$\begin{aligned}
\mathcal{D}_{\iota+1}(B, y') &= \Pr(d = 1 | B, y') , \\
k^r_{\iota+1}(B, y') &= \sum_j \Pr(B^j | B, y')\left[\widetilde{\delta} + (1-\delta)\mathcal{Q}^r_\iota(B^j, y')\right] , \\
k^d_{\iota+1}(B, y') &= \vartheta\omega \left\{ \begin{array}{c} (1 - \mathcal{D}_\iota(\omega B, P(\omega B, y'), y')) \\ \times\left(\widetilde{\delta} + (1-\delta)\mathcal{Q}^r_\iota(\omega B, y')\right) \\ +\mathcal{D}_\iota(\omega B, y')\mathcal{Q}^d_\iota(\omega B, y') \end{array} \right\} + (1-\vartheta)\mathcal{Q}^d_\iota(B, y') .
\end{aligned}$$

   where the bond prices under re-entry $\mathcal{Q}^r_\iota(\omega B, y')$ and $\mathcal{Q}^d_\iota(\omega B, y')$ are interpolated linearly, since ωB is not necessarily part of the debt grid $\mathfrak{B}$.

   (e) Update expected value functions $\mathcal{F}$, default probability λ and pecuniary payoffs m w.r.t. the persistent exogenous state:

$$
\begin{aligned}
&F_{\iota+1}(B,y) = \Pi\mathcal{F}_{\iota+1}(B,y')\ , \quad F^d_{\iota+1}(B,y) = \Pi\mathcal{F}^d_{\iota+1}(B,y'),\\
&m^r_{\iota+1}(B,y) = \Pi k^r_{\iota+1}(B,y')\ , \quad m^d_{\iota+1}(B,y) = \Pi k^d_{\iota+1}(B,y'),\\
&\lambda_{\iota+1}(B,y) = \Pi\mathcal{D}_{\iota+1}(B,y').
\end{aligned}
$$

(f) Compute haircuts κ, haircut weighted collateral Θ, and adjusted bond holdings for both investor types in the repayment case, and combine these elements to get the non-pecuniary part of the payoff, Λ:

$$
\begin{aligned}
\kappa^r_{\iota+1}(B,y) =& \kappa(\lambda_{\iota+1})\ ,\\
\Theta^r_{\iota+1}(B,y) =& (1-\kappa^r_{\iota+1}(B,y))\times m^r_{\iota+1}(B,y')\times B\ ,\\
\Theta^d_{\iota+1}(B,y) =& (1-\overline{\kappa})\times m^d_{\iota+1}(B,y')\times B\ ,\\
\Theta^r_{H,\iota+1}(B,y) =& \Theta^r_{\iota+1}(B,y) - \frac{1}{2}\zeta_2\ ,\\
\Theta^d_{H,\iota+1}(B,y) =& \Theta^d_{\iota+1}(B,y) - \frac{1}{2}\zeta_2\ ,\\
\Theta^r_{L,\iota+1}(B,y) =& \Theta^r_{\iota+1}(B,y) + \frac{1}{2}\zeta_2\ ,\\
\Theta^d_{L,\iota+1}(B,y) =& \Theta^d_{\iota+1}(B,y) + \frac{1}{2}\zeta_2\ ,\\
\Lambda^r_{\iota+1}(B,y) =& (1-\kappa^r_{\iota+1}(B,y))\Big(\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^r_{L,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^r_{\iota+1}(B,y')\big)\big)\\
&+\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^r_{H,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^r_{\iota+1}(B,y')\big)\big)\Big)\\
\Lambda^d_{\iota+1}(B,y) =& (1-\overline{\kappa})\Big(\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^d_{L,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^d_{\iota+1}(B,y')\big)\big)\ ,\\
&+\frac{1}{2}\big(\frac{1}{2}u'\big(\Theta^d_{H,\iota+1}(B,y')\big) - \frac{1}{2}u'\big(\Theta^d_{\iota+1}(B,y')\big)\big)\Big)\ .
\end{aligned}
$$

(g) Update bond prices in repayment and default

$$
\begin{aligned}
&\mathcal{Q}^r_{\iota+1}(B,y) = m^r_{\iota+1}(B,y)\big(1+\Lambda^r_{\iota+1}(B,y)\big)/(1+r^{rf}),\\
&\mathcal{Q}^d_{\iota+1}(B,y) = m^d_{\iota+1}(B,y)\big(1+\Lambda^d_{\iota+1}(B,y)\big)/(1+r^{rf})\ .
\end{aligned}
$$

(h) Apply the cap on bond prices.

2. Compute convergence criteria:

$$
\begin{aligned}
&\Delta\mathcal{F}_{\iota+1} = \max\left\{\Big|\Big|\mathcal{F}_{\iota+1} - \mathcal{F}_{\iota}\Big|\Big|_{\infty}, \Big|\Big|\mathcal{F}^d_{\iota+1} - \mathcal{F}^d_{\iota}\Big|\Big|_{\infty}\right\}\\
&\Delta\mathcal{Q}_{\iota+1} = \max\left\{\Big|\Big|\mathcal{Q}^r_{\iota+1} - \mathcal{Q}^r_{\iota}\Big|\Big|_{\infty}, \Big|\Big|\mathcal{Q}^d_{\iota+1} - \mathcal{Q}^d_{\iota}\Big|\Big|_{\infty}\right\}
\end{aligned}
$$

3. If $\Delta\mathcal{Q}_{\iota+1} < \varepsilon\ \wedge\ \Delta\mathcal{W}_{\iota+1} < \varepsilon$ BREAK, else $\iota = \iota+1$ and go to step 1.

Table C.1: Parameters of Computational Algorithm

Number of grid points for income	$n_y = 201$
Size of income grid	$y \in [-3\sigma_y, +3\sigma_y]$
Number of grid points for debt	$n_B = 301$
Size of debt grid	$B \in [0, 6]$
Taste shock parameter	$\sigma_z = 1$
Maximum default probability	$\overline{\lambda} = 0.75$
Minimum bond spread	$\underline{s} = -105\text{bp}$

Table C.1 summarizes the parameters governing the numerical approximation. Results do not visibly change when grid sizes are increased further. The standard deviation of the taste shock is chosen to ensure stable convergence for all parameter combinations, which is typically achieved in less then 1000 iterations.

Following Chatterjee and Eyigungor (2015), we impose a maximum default probability to rule out extreme debt dilution when the economy is close to default. The maximum default probability is set to 0.75. To rule out implausibly high valuation of collateral services for small amounts of debt outstanding, we also impose a minimum bond spread (or, alternatively, maximum convenience yield). The minimum spread of a 5-year German government bond over the 5-year-interest-rate-swap rate observed in our sample, which was –105bp, serves as lower bound on bond spreads in our calibration. Like the maximum default probability, this constraint does not bind during model simulation and all our numerical results are robust to changes to either constraint.

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