

A.

- ## Tic-Tac-Toe evaluation

examples

$$\text{eval} \left(\begin{array}{c|c|c} \text{blue} & \text{red} & \text{blue} \\ \hline \text{red} & \text{blue} & \text{red} \\ \hline \text{red} & \text{red} & \text{blue} \end{array} \right) = -1 \cdot (0+1) = \underline{-1} \longrightarrow \text{blue is minimizing}$$

$$\text{eval}\left(\begin{array}{|c|c|c|} \hline \text{blue} & \text{red} & \text{blue} \\ \hline \text{red} & \text{blue} & \text{blue} \\ \hline \text{red} & \text{blue} & \text{red} \\ \hline \end{array}\right) = 0 \text{ (draw)}$$

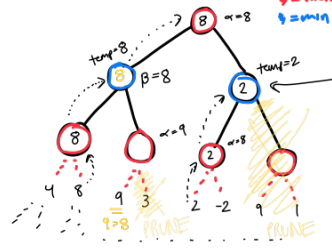
- * $-1 \cdot$ (available squares before placing token)
for the minimizing player

B.

minimax v2 (optimized w/ Alpha-Beta Pruning)

- introduces 2 parameters (α and β), with α representing the "maximization" of that tree node's subtree (β obviously being the minimization of the subtree rooted @ the current node)

~ Alpha and beta are essentially guaranteed safe bets (for max and min players respectively): if the current value of $\alpha = 8$, we shouldn't explore any subtrees where $\beta \leq 8$, because assuming the minimizing plays optimally, they would never choose to go down it. (in other words, when $\alpha \leq \beta$)



after finding a 2 here, we know that the nodes value can only decrease (since blue will minimize). However, it's parent (which maximizes) already has a guarantee of 8. 8 will always trump ≤ 2 ; therefore, there is no purpose of searching the remaining subtree.

code/pseudocode implementation:

```
int minimax(board,  $\alpha$ ,  $\beta$ , maximizing)
    if game is over (board is full or player got 3 in a row)
        return eval(board); // this is the score of this board, which will
                               // be passed up the game tree
    if maximizing
        loop through children game states (recurse on minimax(new_board,  $\alpha$ ,  $\beta$ ,
                               maximizing=false))
         $\alpha = \max(\alpha, \max(\text{children}))$ 
        break if  $\alpha \geq \beta$ 
        return  $\max(\text{children})$ 
    else
         $\beta = \min(\alpha, \min(\text{children}))$ 
        break if  $\alpha \geq \beta$ 
        same as maximizing, except return  $\min(\text{children})$ 
```

ELIMINATED DEPTH VARIABLE
(can simply check if game is over)