

Which of the following converge:

1. $\sum \frac{n^2-1}{5n^5}$

Convergent, using P-Series:

$$\lim_{x \rightarrow \infty} \frac{x^2-1}{5x^5} = \lim_{x \rightarrow \infty} \frac{x^2}{x^5} = \lim_{x \rightarrow \infty} \frac{1}{x^3}, \quad P > 1 \therefore \text{Convergent}$$

2. $\sum \cos\left(\frac{1}{n}\right)$

Divergent, using divergence test:

$$\lim_{x \rightarrow \infty} \cos\left(\frac{1}{x}\right) = \cos(0) = 1; \quad \lim_{x \rightarrow \infty} \neq 0, \quad \therefore \text{Divergent}$$

3. $\sum \sin\left(\frac{1}{n^2}\right)$

Convergent, comparison test:

$$\sin\left(\frac{1}{x^2}\right) \leq \frac{1}{n^2}, \quad \frac{1}{n^2} \text{ is convergent}; \quad \therefore \text{Converges}$$

4. $\sum \frac{n^2+100}{20n^5}$

Convergent, P-Series

$$\lim_{x \rightarrow \infty} \frac{x^2+100}{20x^5} = \lim_{x \rightarrow \infty} \frac{x^2}{x^5} = \lim_{x \rightarrow \infty} \frac{1}{x^3}, \quad P > 1 \therefore \text{Convergent}$$

5. $\sum \frac{\ln(n)}{n}$

Divergent, integral test:

$$\begin{aligned} \int_1^{\infty} \frac{\ln(x)}{x} dx &\Rightarrow \lim_{x \rightarrow \infty} \int_0^n u d(\ln(x)) = \lim_{x \rightarrow \infty} \left(\frac{u^2}{2} \right) \Big|_0^n \\ &= \lim_{x \rightarrow \infty} \frac{n^2}{2} - \frac{0}{2} = \lim_{x \rightarrow \infty} \frac{\ln(x)^2}{2} = \frac{\ln(\infty)^2}{2} = \infty, \quad \therefore \text{Divergent} \end{aligned}$$