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MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set XI, Due Thursday, April 13, 2023

174. Use appropriate substitutions to write down the Maclaurin series for the given binomial: $(1-x)^{1/3}$.

$$\text{Note that: } (1+x)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k$$

$$(1+(-x))^{1/3} = \sum_{k=0}^{\infty} \binom{1/3}{k} (-x)^k = \boxed{\sum_{k=0}^{\infty} \frac{(1/3)_k}{k!} (-1)^k x^k}$$

178. Find the Taylor series of each function with the given center: $\sqrt{x+2}$ at $a=0$.

$$\sqrt{x+2} = (x+2)^{1/2}$$

$$(x+2)^{1/2} = (2+0)^{1/2} \left(1+\frac{x-0}{2+0}\right)^{1/2} = \sqrt{2} \cdot \sqrt{1+\frac{x}{2}}$$

$$\text{Note that: } (1+x)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k$$

$$\sqrt{2} \cdot \sqrt{1+\frac{x}{2}} = \sqrt{2} \cdot \left(1+\frac{x}{2}\right)^{1/2} = \sqrt{2} \sum_{k=0}^{\infty} \binom{1/2}{k} \left(\frac{x}{2}\right)^k$$

$$= \boxed{\sum_{k=0}^{\infty} \left[\sqrt{2} \frac{(1/2)_k}{k!} \left(\frac{x^k}{2^{k-1/2}}\right)\right]}$$

202. Find the Maclaurin series of the function: $f(x) = xe^{2x}$.

$$\text{Note that: } e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$xe^{2x} = x \sum_{k=0}^{\infty} \frac{(2x)^k}{k!} = \boxed{\sum_{k=0}^{\infty} \frac{2^k (x)^{k+1}}{k!}}$$

208. Find the Maclaurin series of $f(x) = \cos^2 x$ using the identity $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$.

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

$$\text{Note that: } \cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2} \cdot \sum_{k=0}^{\infty} (-1)^k \frac{(2x)^{2k}}{(2k)!} = \frac{1}{2} + \sum_{k=0}^{\infty} \frac{(-1)^k}{2} \cdot \frac{(2x)^{2k}}{(2k)!} = \frac{1}{2} + \sum_{k=0}^{\infty} (-1)^k \frac{2^k (x)^{2k}}{(2k)!}$$

$$\text{Solving for a series of } 1/2\text{, Note that: } \sum_{k=1}^{\infty} \frac{1}{2^k} = 1$$

$$\sum_{k=2}^{\infty} \frac{1}{2^k} = 1/2 \sum_{k=2}^{\infty} \frac{1}{2^k} + \sum_{k=0}^{\infty} (-1)^k \frac{2^k (x)^{2k}}{(2k)!} = \boxed{\sum_{k=0}^{\infty} \frac{1}{2^{k+2}} + (-1)^k \frac{2^k (x)^{2k}}{(2k)!}}$$