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MATH F253, Elizabeth S. Allman, Homework 1

**2.1: 4, 7, 9, 12, 14, 17, 26, 30, 32, 38, 40, 46**

4. $\vec{RP}$

$$\text{Given: } R(-3{,}7) \text{ and } P(-1{,}3)$$
$$\vec{RP} = \langle -1-(-3), 3-7 \rangle = \boxed{\text{a.}\, \langle 2,-4 \rangle} = \boxed{\text{b.}\, 2\hat{\mathbf{i}} - 4\hat{\mathbf{j}}}$$

7. $2\vec{PQ} - 2\vec{PR}$

$$\text{Given: } R(-3{,}7), P(-1{,}3), \text{ and } Q(1{,}5)$$
$$2\vec{PQ} - 2\vec{PR} = 2\langle 1-(-1), 5-3 \rangle - 2\langle -3-(-1), 7-3 \rangle$$
$$= 2\langle 2,2 \rangle - 2\langle -2,4 \rangle = \langle 4,4 \rangle - \langle -4,8 \rangle =$$
$$= \boxed{\text{a.}\, \langle 8,-4 \rangle} = \boxed{\text{b.}\, 8\hat{\mathbf{i}} - 4\hat{\mathbf{j}}}$$

9. The unit vector in the direction of $\vec{PQ}$

$$\text{As found: } \vec{PQ} = \langle 2,2 \rangle$$
$$\text{UV of } \vec{PQ} = \frac{\langle 2,2 \rangle}{||\vec{PQ}||}$$
$$\frac{\langle 2,2 \rangle}{\sqrt{2^2+2^2}} = \frac{\langle 2,2 \rangle}{2\sqrt{2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle = \boxed{\frac{\sqrt{2}}{2}\hat{\mathbf{i}} + \frac{\sqrt{2}}{2}\hat{\mathbf{j}}}$$

12. A vector $\mathbf{v}$ has initial point $(-2,5)$ and terminal point $(3,-1)$. Find the unit vector in the direction of $\mathbf{v}$. Express the answer in component form.

$$\text{Given: } \vec{v} = \langle 3-(-2), -1-5 \rangle = \langle 5,-6 \rangle$$
$$\text{UV of } \vec{v} = \frac{\vec{v}}{||\vec{v}||}$$
$$\frac{\langle 5,-6 \rangle}{\sqrt{5^2+(-6)^2}} = \frac{\langle 5,-6 \rangle}{\sqrt{25+36}} = \frac{\langle 5,-6 \rangle}{\sqrt{61}} = \boxed{\left\langle \frac{5}{\sqrt{61}}, -\frac{6}{\sqrt{61}} \right\rangle}$$

14. The vector $\mathbf{v}$ has initial point $P(1,1)$ and terminal point $Q$ that is on the x-axis and left of the initial point. Find the coordinates of terminal point $Q$ such that the magnitude of the vector $\mathbf{v}$ is 10.

$$\text{Given: } P(1,1), Q(q_x,0), \text{ and } ||\vec{P,Q}|| = 10$$
$$\text{Note that: } q_x < 1$$
$$(q_x-1)^2 + (0-1)^2 = 10$$
$$(q_x-1)^2 + 1 = 10$$
$$(q_x-1)^2 = 9$$
$$q_x = \pm\sqrt{9} + 1$$
$$q_x = 1-3 = -2$$
$$\boxed{Q = (-2,0)}$$

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17. Let a be a standard-position vector with terminal point $(-2,-4)$. Let b be a vector with initial point $(1,2)$ and terminal point $(-1,4)$. Find the magnitude of vector $-3a+b-4\mathbf{\hat{i}}+\mathbf{\hat{j}}$.

$$\text{Given: } \vec{a} = \langle -2,-4 \rangle, \vec{b} = \langle -1-1, 4-2 \rangle = \langle -2, 2 \rangle$$
$$-3\langle -2,-4 \rangle + \langle -2,2 \rangle - 4\langle 1,0 \rangle + \langle 0,1 \rangle = \langle 6,12 \rangle + \langle -2,2 \rangle - \langle 4,0 \rangle + \langle 0,1 \rangle = \langle 0,15 \rangle$$
$$||\langle 0,15 \rangle|| = \boxed{15}$$

25. $||\mathbf{v}|| = 7, \mathbf{u} = \langle 3,4 \rangle \leftarrow$ Done accidentally.

$$\vec{v} = 7 * \text{UV of } \vec{u} = 7 * \frac{\vec{u}}{||\vec{u}||}$$
$$\vec{v} = 7 * \frac{\langle 3,4 \rangle}{\sqrt{3^2+4^2}} = 7 * \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$
$$\vec{v} = \boxed{\left\langle \frac{21}{5}, \frac{28}{5} \right\rangle}$$

26. $||\mathbf{v}|| = 3, \mathbf{u} = \langle -2,5 \rangle$

$$\vec{v} = 3 * \text{UV of } \vec{u} = 3 * \frac{\vec{u}}{||\vec{u}||}$$
$$\vec{v} = 3 * \frac{\langle -2,5 \rangle}{\sqrt{(-2)^2+5^2}} = 3 * \left\langle -\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle$$
$$\vec{v} = \boxed{-\left\langle \frac{6}{\sqrt{29}}, \frac{15}{\sqrt{29}} \right\rangle}$$

30. $||\mathbf{u}|| = 6, \theta = 60°$

$$\vec{u} = 6 * \langle \cos(60°), \sin(60°) \rangle$$
$$\vec{u} = 6 * \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle = \boxed{\left\langle 3, 3\sqrt{3} \right\rangle}$$

32. $||\mathbf{u}|| = 8, \theta = \pi$

$$\vec{u} = 8 * \langle \cos(\pi), \sin(\pi) \rangle$$
$$\vec{u} = 8 * \langle -1, 0 \rangle = \boxed{\langle -8, 0 \rangle}$$

38. Consider vectors $a = \langle 2,-4 \rangle, b = \langle -1,2 \rangle$, and $c = 0$. Determine the scalars $\alpha$ and $\beta$ such that $c = \alpha a + \beta b$.

$$\text{Given: } a = \langle 2,-4 \rangle, b = \langle -1,2 \rangle, c = 0$$
$$c = \alpha * a + \beta * b$$
$$0 = \alpha * \langle 2,-4 \rangle + \beta \langle -1,2 \rangle$$
$$\langle 0,0 \rangle = \langle 2\alpha - 1\beta, -4\alpha + 2\beta \rangle$$
$$2\alpha - \beta = 0 \rightarrow 2\alpha = \beta$$
$$-4\alpha + 2\beta = 0 \rightarrow -4\alpha = -2\alpha$$
$$\therefore \boxed{\beta = 2\alpha, \ \alpha \in \mathbb{R}}$$

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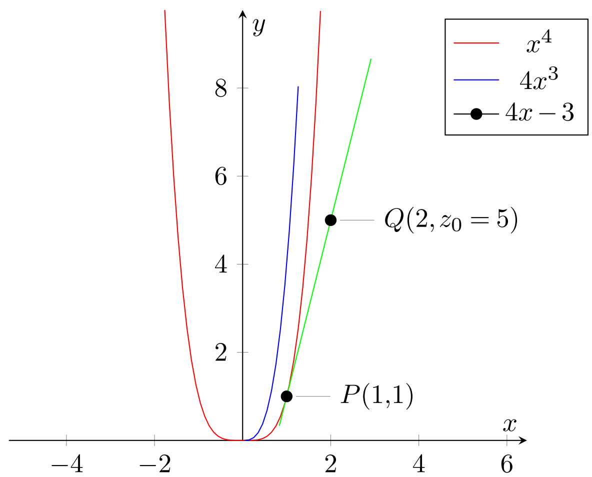
40. Consider the function $f(x) = x^4$ where $x \in \mathbb{R}$.

a. Determine the real number $z_0$ such that point $Q(2, z_0)$ is situated on the line tangent $t$ to the graph of $f$ at point $P(1,1)$.

$$f'(x) = 4x^3$$
$$t(x) = 4(x-1)+1 = 4x-3$$
$$Q = (2, t(2)) = (2, 4(2)-3)$$
$$Q = (2,5)$$
$$z_0 = \boxed{5}$$




b. Determine the unit vector **u** with inital point $P$ and terminal point $Q$.

$$\vec{u} = \frac{\langle 2-1, 5-1 \rangle}{||\vec{u}||}$$
$$\vec{u} = \frac{\langle 1,4 \rangle}{\sqrt{17}} = \boxed{\left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle}$$

46. A baseball player throws a baseball at an angle of 30° with the horizontal. If the initial speed of the ball is 100 mph, find the horizontal and vertical components of the initial velocity vector of the baseball. (Round to two decimal places.)

$$\text{Given: } v_0 = 100\text{mph}, \ \theta = 30°$$
$$\vec{v} = 100\,\langle \cos(30°), \sin(30°) \rangle$$
$$\vec{v} = \left\langle 50\sqrt{3}, 50 \right\rangle = \boxed{\langle 86.60, 50.00 \rangle}$$

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**2.2: 61, 64, 66, 68, 72, 74, 76, 78, 84, 89, 92, 100, 103**

61. Consider a rectangular box with one of the vertices at the origin, as shown in the following figure. If point $A(2,3,5)$ is opposite of the origin, then find:

    a. the coordinates of the other size vertices of the box

    $\boxed{(0,3,5),(0,0,5),(2,0,5),(2,0,0),(2,3,0),(0,3,0)}$

    b. and the length of the diagonal of the box determined by the vertices $O$ and $A$.

    $$||\vec{OA}|| = \sqrt{2^2+3^2+5^2} = \sqrt{4+9+5} = \boxed{\sqrt{18}}$$

64. $(z-2)(z-5)=0$

    $$(z-2)(z-5)=0 \rightarrow z^2-7z+10=0$$

    This is a parabola with zeroes at $x=2$ and $x=5$, and points $(2,0),(3,-2),(4,-2)(5,0)$.

66. $(x-2)^2+(z-1)^2=1$

    This is a circle resting flat on the $xz$ axis with a radius of 1, and points $(3,0,1),(2,0,2),(1,0,1),(2,0,0)$.

72. Center $C(-4,7,2)$ and radius 6.

    $$(x+4)^2+(y-7)^2+(z-2)^2=36$$

74. Diameter $PQ$, where $P(-16,-3,9)$ and $Q(-2,3,5)$

    Given: Diameter (opposite points) $P(-16,-3,9),Q(-2,3,5)$

    $$C = \frac{(-16+(-2),-3+3,9+5)}{2} = \frac{-18,0,14}{2}$$
    $$= (-9,0,7)$$
    $$\therefore (x-(-9))^2+(y-0)^2+(z-7)^2 = ||\vec{PQ}||$$
    $$||\vec{PQ}|| = \sqrt{\left(\frac{-2-(-16)}{2}\right)^2+\left(\frac{3-(-3)}{2}\right)^2+\left(\frac{5-9}{2}\right)^2}$$
    $$= \sqrt{7^2+3^2+2^2} = \sqrt{49+9+4} = \sqrt{62}$$
    $$\boxed{(x+9)^2+y^2+(z-7)^2=\sqrt{62}}$$

76. $x^2+y^2+z^2-6x+8y-10z+25=0$

    $$x(x-6)+y(y+8)+z(z-10)=25$$
    $$(x-3)^2+(y+4)^2+(z-5)^2=25$$
    $$\therefore \boxed{C(3,-4,5)\text{ radius: }5}$$

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78. $P(0,10,5)$ and $Q(1,1,-3)$

$$\vec{PQ} = \langle 1-0, 1-10, -3-5\rangle = \langle 1,-9,-8\rangle$$

a. $\boxed{\langle 1,-9,-8\rangle}$

b. $\boxed{\hat{\mathbf{i}}, -9\hat{\mathbf{j}}, -8\hat{\mathbf{k}}}$

84. $\mathbf{a} = \langle -1,-2,4\rangle, \mathbf{b} = \langle -5,6,7\rangle$

$$\vec{a}+\vec{b} = \langle -1,-2,4\rangle + \langle -5,6,7\rangle = \boxed{\langle -6,4,11\rangle}$$
$$4\vec{a} = 4\langle -1,-2,4\rangle = \boxed{\langle -4,-8,16\rangle}$$
$$-5\vec{a}+3\vec{b} = -5\langle -1,-2,4\rangle + 3\langle -5,6,7\rangle = \boxed{\langle -10,16,1\rangle}$$

87. $\mathbf{u} = 2i+3j+4k, \mathbf{v} = -i+5j+k$ Whoops, did it again.

$$||\vec{u}-\vec{v}|| = ||3i-2j+3k|| = \sqrt{3^2+(-2)^2+3^2} = \boxed{\sqrt{22}}$$
$$||-2\vec{u}|| = \sqrt{(-4)^2+(-6)^2+8^2} = \boxed{\sqrt{116}}$$

89. $\mathbf{u} = \langle 2\cos t, -2\sin t, 3\rangle, \mathbf{v} = \langle 0,0,3\rangle$, where $t \in \mathbb{R}$.

$$||\vec{u}-\vec{v}|| = ||\langle 2\cos t - 0, -2\sin t - 0, 3-3\rangle|| = \sqrt{4\cos^2 t + 4\sin^2 t + 0^2} = 2\sqrt{\sin^2 t + \cos^2 t} = \boxed{2}$$
$$||-2\vec{u}|| = ||\langle -4\cos t, 4\sin t, -6\rangle|| = \sqrt{16\cos^2 t + 16\sin^2 t + 36} = \sqrt{16+36} = \sqrt{56} = \boxed{2\sqrt{13}}$$

92. $\mathbf{a} = \langle 4,-3,6\rangle$

$$\text{UV of } \vec{a} = \frac{\vec{a}}{||\vec{a}||}$$
$$\frac{\vec{a}}{||\vec{a}||} = \frac{\langle 4,-3,6\rangle}{\sqrt{16+9+36}} = \boxed{\left\langle \frac{4}{\sqrt{61}}, -\frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right\rangle}$$

100. $\mathbf{v} = \langle 2,4,1\rangle$, $||\mathbf{u}|| = 15$, $\mathbf{u}$ and $\mathbf{v}$ have the same direction.

$$\vec{u} = 15\frac{\vec{v}}{||\vec{v}||}$$
$$\vec{u} = 15\frac{\langle 2,4,1\rangle}{\sqrt{2^2,4^2,1^1}} = 15\frac{\langle 2,4,1\rangle}{\sqrt{21}} = \boxed{\left\langle \frac{30}{\sqrt{21}}, \frac{60}{\sqrt{21}}, \frac{15}{\sqrt{21}} \right\rangle}$$

103. Determine a vector of magnitude 5 in the direction of vector $\vec{AB}$, where $A(2,1,5)$ and $B(3,4,-7)$.

$$\vec{AB} = \langle 3-2, 4-1, -7-5\rangle = \langle 1,3,-12\rangle$$
$$5\frac{\vec{AB}}{||\vec{AB}||} = 5\frac{\langle 1,3,-12\rangle}{\sqrt{1^2+3^2+(-12)^2}} = 5\frac{\langle 1,3,-12\rangle}{\sqrt{154}} = \boxed{\left\langle \frac{5}{\sqrt{154}}, \frac{15}{\sqrt{154}}, -\frac{60}{\sqrt{154}} \right\rangle}$$

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**2.3: 126, 130, 132, 136, 140, 142, 146, 148, 150, 152, 154, 168, 170, 172**

126. $\mathbf{u} = \langle 4,5,-6 \rangle, \mathbf{v} = \langle 0,-2,-3 \rangle$

$$\vec{u} \cdot \vec{v} = (4*0) + (5*-2) + (-6*-3) = \boxed{8}$$

130. $a = i - j + k, b = j + 3k, c = -i + 2j - 4\hat{\mathbf{k}}$

$$(a \cdot b)c = ((1*0) + (-1*1) + (1*3))(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) = 2(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) = \boxed{\langle -2,4,-8 \rangle}$$

132. $\mathbf{a} = \langle 2,1 \rangle, \mathbf{b} = \langle -1,3 \rangle$

a.
$$\theta = \arccos\left(\frac{a \cdot b}{||a|| * ||b||}\right)$$
$$= \arccos\left(\frac{2}{\sqrt{5} * \sqrt{10}}\right) = \arccos\left(\frac{1}{5\sqrt{2}}\right) = \boxed{1.28 \text{ radians}}$$

b. $\boxed{1.28 * 2 < \pi; acute}$

136. $\mathbf{a} = \langle 0,-1,-3 \rangle, \mathbf{b} = \langle 2,3,-1 \rangle$

$$\theta = \arccos\left(\frac{a \cdot b}{||a|| * ||b||}\right)$$
$$\arccos\left(\frac{(0*2) + (-1*3) + (-3*-1)}{\sqrt{10} * \sqrt{14}}\right) = \arccos\left(\frac{0}{\sqrt{10} * \sqrt{14}}\right) = \arccos(0) = \boxed{\frac{\pi}{2}}$$

140.

142.

146.

148.

150.

152.

154.

168.

170.

172.