Day 7 - 1/26/2024

Vector Valued Functions

$$f \in \mathbb{R}^2 = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

$$f(t) = (f_1(t) + f_2(t)) = \vec{i} f_1(t) + \vec{j} f_2(t`)$$

$$f \in \mathbb{R}^3 = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

$$f(t) = (f_1(t) + f_2(t) + f_3(t)) = \vec{i} f_1(t) + \vec{j} f_2(t) + \vec{k} f_3(t)$$

$$\text{Given } f : [0, 2\pi) \to \mathbb{R}^2$$

$$f(t) = (\cos(t), \sin(t)) = \vec{i}\cos(t) + \vec{j}\sin(t)$$

$$\text{Evaluate } f\left(\frac{3\pi}{4}\right) = \left(\cos\left(\frac{3\pi}{4}\right), \sin\left(\frac{3\pi}{4}\right)\right)$$

$$= \left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \underline{-\vec{i}\frac{\sqrt{2}}{2} + \vec{j}\frac{\sqrt{2}}{2}}$$

Theorem:

$$\overrightarrow{f(t)} = (f_1(t) + f_2(t) + f_3(t))$$

$$\lim_{t \to a} f_1(t) = A_1$$

$$\|f(t) - A\| = \sqrt{(f_1(t) - A_1)^2 + (f_2(t) + A_2)^2 + (f_3(t) + A_3)^2} \to 0$$