Day 5 - 1/24/2024

Volume of a Parallelpiped

To find the area of a parallelpiped with sides $\vec{a}\vec{b}\vec{c}$, use tthe formula $\|\vec{a} \times \vec{b}\| \cdot \mathrm{proj}_{\vec{a} \times \vec{b}} \vec{c}$.

Lines in the plane

Given a line $ax + by + c = 0$ there is a normal vector $\langle a, b \rangle$ and a direction vector $\langle -b, a \rangle$.

Suppose we know two points along the line; $P(q_1, q_2)$ and an arbitrary point O. If we look at a point on the line, (x, y), then a scalar $M = \overrightarrow{OP} + \alpha \overrightarrow{PQ}$.

Parametric equation of a line l:

$$\overrightarrow{OP} + t\overrightarrow{PQ} = M, \text{where } P, Q \in l$$

$$\langle p_1, p_2, p_3 \rangle + t \langle v_1, v_2, v_3 \rangle = \langle x, y, z \rangle$$

$$\begin{aligned} p + tv_1 &= x \\ p + tv_2 &= y \\ p + tv_3 &= z \end{aligned}$$

Suppose $v_1, v_2, v_3 \neq 0$.

$$\frac{x - p_1}{v_1} = \frac{y - p_2}{v_2} = \frac{z - p_3}{v_3} = -t$$

$$\text{Symmetric form}$$

Example: Find the equation for $P(1, 2, 3), \langle 1, 0, 1 \rangle$

$$\text{It's simply } \vec{P} = t\vec{v}$$

Use $P(1, 2, 3), Q(-2, 3, 0)$:

$$\text{Dir. Vec} : \overrightarrow{PQ} = \langle -2-1, 3-2, 0-3 \rangle = \langle -3, 1, -3 \rangle$$
$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-3}{-3}$$

Finding the distance from a line to a point

Given the point M and a point on the line P, we find the point of the line perpendicular to M to be S.

$$MS = PM \cdot \sin\theta$$
$$PQ \cdot MS = PQ \cdot PM \sin\theta$$
$$\|PQ\| \cdot \|MS\| = \frac{\|PQ \times PM\|}{\|PQ\|}$$

If the line is given in the form $\vec{p} + t\vec{v}$, then $\|\vec{p} + t\vec{v} - \vec{m}\|$