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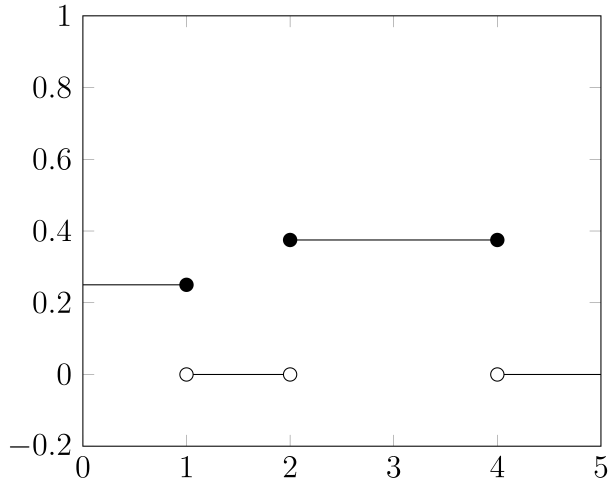
STAT F300 Statistics, Dr. Short
Worksheet 13, Due February 17, 4:15pm

Let X be a continuou random variable ith the pdf given by:

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1/4 & \text{if } 0 < x \le 1 \\ 0 & \text{if } 1 < x \le 2 \\ 3/8 & \text{if } 2 < x \le 4 \\ 0 & \text{if } 4 < x \end{cases}$$

1. Sketch the graph of $f(x)$.



2. Show that $f(x)$ is a pdf.

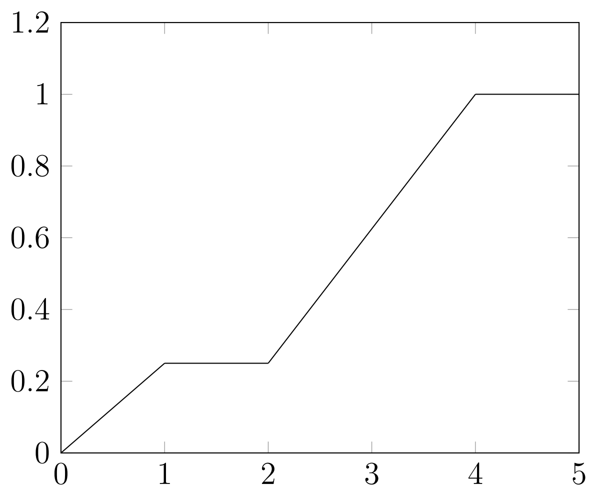
Because the sections are rectangular, we can just sum their width * height:

$$(0 \cdot -\infty) + \left(\frac{1}{4} \cdot 1\right) + (0 \cdot 1) + \left(\frac{3}{8} \cdot 2\right) + (0 \cdot \infty) = 0 + \frac{1}{4} + 0 + \frac{3}{4} + 0 = \frac{4}{4} = 1$$

Because the sum of the “integrals” is 1, $f(x)$ is a pdf.

3. Find $F(x)$, the cdf of X. Sketch its graph. (The sketch can be fairly crude, but be sure to show what happens at $x = 0, 1, 2, 4$. In particular, the cdf, $F(x)$, is continuous because X is a continuous random variable, so there are no jumps in the cdf. It is continuous as well.)

$$F(X) = \int_{-\infty}^{x} f(t)dt$$
$$= \begin{cases} \int_{-\infty}^{0} 0dt \\ \int_{0}^{1} 1/4dt \\ \int_{1}^{2} 0dt \\ \int_{2}^{4} 3/8dt \\ \int_{4}^{\infty} 0dt \end{cases}$$
$$= \begin{cases} 0 & \text{if } 0 < x \\ \frac{x}{4} & \text{if } 0 < x \leq 1 \\ \frac{1}{4} & \text{if } 1 < x \leq 2 \\ \frac{3(x-2)}{8} + \frac{1}{4} & \text{if } 2 < x \leq 4 \\ 1 & \text{if } 4 < x \end{cases}$$



4. Use the cdf to find the following probabilities:

(a) $P(X \leq 2.4)$

$$\begin{aligned} P(X \leq 2.4) &= \frac{3(2.4-2)}{8} + \frac{1}{4} \\ F(2.4) &= \frac{3(0.4)}{8} + 0.25 \\ &= \frac{1.2}{8} + 0.25 = \boxed{0.65} \end{aligned}$$

(b) $P(0.7 \leq X \leq 1.0)$

$$\begin{aligned} P(0.7 \leq X \leq 1.0) &= 1 - F(0.7) \\ &= 1 - \frac{0.7}{4} \\ &= 1 - 0.175 = \boxed{0.825} \end{aligned}$$

(c) $P(x > 2.5)$

$$\begin{aligned} P(X > 2.5) &= 1 - F(2.5) \\ &= 1 - \frac{3(2.5-2)}{8} + 0.25 \\ &= 1 - \frac{3(0.5)}{8} + 0.25 \\ &= 1 - \frac{1.5}{8} + 0.25 \\ &= 1 - 0.1875 + 0.25 = \boxed{4375} \end{aligned}$$

(d) $P(0.3 \leq X \leq 3.5)$

$$\begin{aligned} P(0.3 \leq X \leq 3.5) &= F(3.5) - F(0.3) \\ &= \left(\frac{3(3.5-2)}{8}\right) - \frac{0.3}{4} \\ &= \left(\frac{3(1.5)}{8}\right) - 0.075 \\ &= \left(\frac{4.5}{8}\right) - 0.075 \\ &= 0.5625 - 0.075 = \boxed{0.4875} \end{aligned}$$