Day 2 - 1/18/2024

Unit Vectors

Vectors $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$ both have the same length of $\|\vec{i}\| = \|\vec{j}\| = 1$

Finding the Unit Vector of $\vec{v}$

We can find the unit vector in the direction of $\vec{v}$ by dividing $\vec{v}$ by its magnitude.

Example:

$$\vec{v} = \langle 3, 4 \rangle, \text{Unit vector of } \vec{v} \text{ is:}$$
$$\|\vec{v}\| = \sqrt{3^2 + 4^2} = 5$$
$$\|\vec{v}\| = \frac{\vec{v}}{5} = \frac{\langle 3, 4 \rangle}{5} = \underline{\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle}$$

Vector from Angle α

Given an angle $\alpha = \frac{\pi}{3}$ and a length of the vector $\vec{v}$, $\|\vec{v}\| = 3$, we can use trigonometric identities to determine $\vec{v}$.

The unit vector of $\vec{v}$ will necessarily be defined as $\langle \cos(\alpha), \sin(\alpha) \rangle$, we can then define the vector from before to be $\vec{v} = \langle \|\vec{v}\| \cos(\alpha), \|\vec{v}\| \sin(a) \rangle$. We can solve this to find $\vec{v}$:

$$\vec{v} = \left\langle 3\cos\left(\frac{\pi}{3}\right), 3\sin\left(\frac{\pi}{3}\right) \right\rangle = \underline{\left\langle \frac{3}{2}, \frac{3\sqrt{3}}{2} \right\rangle}$$

Examples of Vector-Valued Quantities

- Newtons (N = kg $\cdot$ $\frac{m}{s^2}$)
- Velocity ($\frac{m}{s}$)
- Acceleration ($\frac{m}{s^2}$)
- Position (m)

3-Dimensional Vectors

3-Dimensional vectors contain an extra z component, making them defined as $\langle x, y, z \rangle$.

xy Planes

An important concept is the xy plane, a 2-dimensional slice along the z axis in a 3-dimensional space.

For any given plane will have the plane $P(x, y, z)$:

$$\begin{aligned} \text{yz plane} &\Leftrightarrow x = 0 \\ \text{xy} &\Leftrightarrow z = 9 \\ \text{zx} &\Leftrightarrow y = 0 \end{aligned}$$

For any given 3 points there is a plane. For example:

The three points $(1, 2, 3)$, $(-4, 4, 3)$, and $(-5.3.3)$:

$$\text{Note that all points have } z = 0, \therefore z = 0$$
$$z = 0 \text{ is the equation for a plane that contains these points}$$

Length

The length of a 3-dimensional vector $\overrightarrow{PQ}$ is defined as:

$$\sqrt{(P_x - Q_x)^2 + (P_y - Q_y)^2 + (P_z - Q_z)^2}$$

Midpoints

The midpoint of a *vector* $\vec{v}$ is defined as:

$$\frac{\vec{v}}{2}$$

The midpoint of any segment PQ is defined as:

$$\left(\frac{P_x + Q_x}{2}, \frac{P_y + Q_y}{2}, \frac{P_z + Q_z}{2}\right)$$