Kaleb Burris
STAT F300 Statistics, Dr. Short
Homework 4, Due Friday, February 10, 23:59

1.

Given 300,000,000 people where 1000 are future terrorists, one is predicted to be a terrorist at a 99% success rate. A person can be correctly identified as not a future terrorist at a 99.9% success rate. What is the probability they are?

$$\begin{aligned}
\#P &= 300{,}000{,}000 \\
T &= \{\text{Future Terrorist}\}; \quad \#T = 1000; \quad P(T) = \frac{1000}{300{,}000{,}000} = \frac{1}{3{,}000{,}000} \approx 0.0000003 \\
F &= \{\text{Flagged as Potential Future Terrorist}\}; \quad \#F = ?; \quad P(F) = \frac{\#F}{300{,}000{,}000} \\
\#F &= \#T(0.99) + \#P(0.001) = 990 + 300{,}000 = 300{,}990 \\
P(F) &= \frac{\#F}{\#P} = \frac{300{,}990}{300{,}000{,}000} = 0.0010033 \\
P(F|T) &= \frac{\#T(0.99)}{\#T} = \frac{990}{1000} = 0.99 \\
P(T|F) &= \frac{P(T)}{P(F)}P(F|T) = \frac{0.0000003}{0.001033}(0.99) = (0.00323)(0.99) = 0.0032 = 0.32\%
\end{aligned}$$

$\boxed{\text{Among those flagged as future terrorists, 0.32\% actually are future terrorists.}}$

This does make me very uneasy; a system that generates a certainty of marking someone in a way that could ruin a person's life. Out of the population, 300,000 were marked as future terrorists which is extreme.

2.

$$P(A \cap B) = P(A)P(B)$$
$$P(A^c \cap B) = (1 - P(A))P(B) = P(B) - P(A)P(B)$$
$$= P(B) - P(A \cap B) \rightarrow P(A^c \cap B) + P(A \cap B) = P(B)$$
$$\therefore P(A^c) \text{ and} P(B) \text{ are independent.}$$

3.

(a) Given a failure for each seam of 0.2, and each seam has at least 1 of 25 rivets failing, what is the probability of a rivet being defective?

$$\begin{aligned} P(R) &= (1 - P(x))^{25} \\ P(S) &= 0.2 = (1 - P(R)) \\ &= 0.2 = 1 - (1 - P(x))^{25} \\ &= -0.8^{\frac{1}{25}} = 1 - P(x) \\ &= -0.991 = 1 - P(x) = \boxed{0.0089} \end{aligned}$$

(b) How small should the probability of a defective rivet be to ensure that only 10% of all seams need reworking?

$$\begin{aligned} P(R) &= (1 - P(x))^{25} \\ P(S) &= 0.1 = (1 - P(R)) \\ &= 0.1 = 1 - (1 - P(x))^{25} \\ &= -0.9^{\frac{1}{25}} = 1 - P(x) \\ &= -0.996 = 1 - P(x) = \boxed{0.0042} \end{aligned}$$

4.