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MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Final Review

Part I

1. Evaluate $\int_1^\infty \frac{1}{1+x^2}dx$

$$\int_1^\infty \frac{1}{1+x^2}dx = \lim_{a\to\infty}\int_1^a \frac{1}{1+x^2}dx$$
$$\lim_{a\to\infty}\int_1^a \frac{1}{1+x^2}dx = \lim_{a\to\infty}\Big[\arctan(x)\Big]_1^a = \lim_{a\to\infty}\arctan(a)-\arctan(1)$$
$$= \frac{\pi}{2}-\frac{\pi}{4} = \boxed{\frac{\pi}{4}}$$

2. Suppose:

   $x = 4-\ln(t)$

   $y = 1+\ln(7t)$

   $1 \le t \le e$

   Compute the arc length.

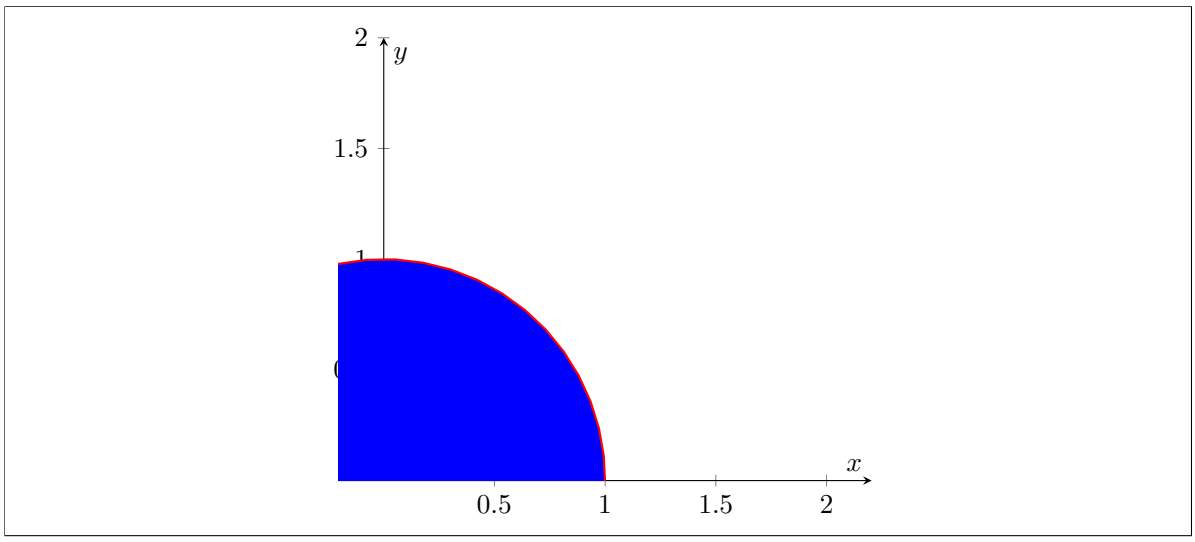
$$\text{Arc length: } \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt$$
$$\frac{dx}{dt} = -\frac{1}{t}$$
$$\frac{dy}{dt} = \frac{7}{7t} = \frac{dt}{t}$$
$$\int_1^e \sqrt{\left(-\frac{1}{t}\right)^2+\left(\frac{1}{t}\right)^2}dt = \int_1^e \sqrt{\frac{1}{t^2}+\frac{1}{t^2}}dt = \int_1^e \sqrt{\frac{2}{t^2}}dt = \sqrt{2}\int_1^e \frac{1}{t}dt$$
$$= \sqrt{2}\Big[\ln(t)\Big]_1^e = \sqrt{2}\Big[\ln(e)-\ln(1)\Big] = \boxed{\sqrt{2}}$$

3. Suppose:

   $x = \cos(3+t)$

   $y = \sin(3+t)$

   What is the area of the region bounded by the graph and the positive x-axis and the positive y-axis?


2
y
1.5
x
0.5
1
1.5
2

Part II