Day 9 - 1/31/202

Derivatives of Vector-valued Functions

$$\frac{d}{dt}\langle r_1, r_2, r_3\rangle = \left\langle \frac{dr_1}{dt}, \frac{dr_2}{dt}, \frac{dr_3}{dt} \right\rangle$$

Integrals of Vector-valued Functions

$$\int_a^b \langle r_1, r_2, r_3\rangle = \left\langle \int_a^b r_1, \int_a^b r_2, \int_a^b r_3 \right\rangle + \vec{c}$$

$$\vec{c} = \langle c_1, c_2, c_3\rangle$$

Quiz tomorrow is on sections 2.4 & 2.5.

Length and Curvature

Given the path $\vec{r}(t)$, we can find the length of a given segment from $\sum \|\vec{v_k}\| \ \Delta t$, where $\lim_{t \to 0}$, $a \leq t \leq b$.

This goes to:

$$\int_a^b \|\vec{v}(t)\| \ dt = \int_a^b \sqrt{v_1^2(t) + v_2^2(t) + v_3^2(t)} dt$$

$$\ell(t) = \int_a^b \|r'(t)\| \ dt$$

<u>Example:</u> Find the length of the helix:

$$r(x) = \langle \cos(t), \sin(t), t\rangle; t \in [0, 2\pi]$$

$$\ell = r'(t) = \langle -\sin(t), \cos(t), 1\rangle$$

$$\ell = \int_0^{2\pi} \sqrt{(-\sin(t))^2 + \cos(t)^2 + 1^2} dt = \int_0^{2\pi} \sqrt{1+1} dt = \int_0^{2\pi} \sqrt{2} dt$$

$$= \sqrt{2}t\Big|_0^{2\pi} = 2\pi\sqrt{2} - 0 = \underline{2\pi\sqrt{2}}$$

Length Parametrization

To derive the position from a given length along the function $\vec{r} : (a, b) \to \mathbb{R}^3$:

$$s(t) \int_a^t \underbrace{\|r'(\tau)\| d\tau}_{\substack{\text{length of the} \\ \text{path at time t}}}$$

<u>Example:</u>

$$\vec{r} = \langle \cos t, \sin t \rangle ; t \in [0, 2\pi]$$

$$s(t) = \int_0^t \|\vec{r}(\tau)\| \; d\tau$$

$$= \int_0^t \|\langle -\sin\tau, \cos\tau \rangle\| \; d\tau = \int_0^t 1 d\tau = t$$

$$s(t) = t \therefore \text{For any time } t, \text{the ending position will be } t$$

$$s \in [0, 2\pi]$$

<u>Example:</u>

$$\vec{r} = \langle t + 3, 2t - 4, 2t \rangle ; 3 \leq t$$

$$s(t) = \int_3^t \|r'(\tau)\| d\tau = \int_3^t \|\langle 1, 2, 2 \rangle\| d\tau$$

$$= \int_3^t \sqrt{1^2 + 2^2 + 2^2} d\tau = \int_3^t 3 d\tau = 3t - 9 = s(t)$$

$$t = \frac{s(t) + 9}{3}$$

$$\vec{r}(s) = \underline{\left\langle \frac{s}{3} + 6, \frac{2s}{3} + 2, \frac{2s}{3} + 6 \right\rangle}$$

Curvature

Curvature is the derivative of your direction; how fast you turn.