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STAT F300 Statistics, Dr. Short
Worksheet 10, Due Friday, February 10, 4:15pm

Let Y denote the number of moving violations that a randomly selected customer of We Gotcha Insurance Company was cited for in the last three years. The pmf of Y is given by

y	0	1	2	3
$p(y)$	0.65	0.23	0.09	0.03

1. Calculate and interpret the expected value of Y.

$$\begin{aligned}\mathbb{E}(Y) &= \sum_{y=0}^{3} y \cdot p(y) \\ &= 0 + 0.23 + 0.18 + 0.09 \\ &= \boxed{0.50}\end{aligned}$$

The expected value of Y is 0.5, meaning that on average a randomly selected customer was cited for 0.5 moving violations in the last three years.

2. Find $\mathbb{E}(Y^2)$, $\text{var}(Y)$ and $\text{sd}(Y)$; interpret $\text{sd}(Y)$.

$\mathbb{E}(Y^2)$:

$$\begin{aligned}\mathbb{E}(Y^2) &= \sum_{y=0}^{3} y^2 \cdot p(y) \\ &= 0 + 0.23 + 0.36 + 0.27 \\ &= \boxed{0.86}\end{aligned}$$

$\text{var}(Y) = \sigma_Y^2 = \mathbb{E}(Y)^2 - (\mu_y)^2 \quad \text{where} \quad \mu_y = \mathbb{E}(Y)$

$$\begin{aligned}\mathbb{E}(Y^2) - (\mu_y)^2 &= 0.86 - (0.5)^2 \\ &= 0.86 - 0.25 \\ &\approx \boxed{0.61}\end{aligned}$$

$\text{sd}(Y)$:

$$\begin{aligned}\text{sd}(Y) &= \sqrt{\text{var}(Y)} \\ &= \sqrt{0.61} \\ &= \boxed{0.78}\end{aligned}$$

The expected number of moving violations is 0.5 with a typical variation of about 0.78 tickets above or below the mean of 0.5.

3. We Gotcha imposes a surcharge of $100Y^2$ dollars on customers with Y many moving violations. Calculate the expected amount of the surcharge. Interpret your answer.

$$\begin{aligned}\mathbb{E}(100Y^2) &= 100 \cdot \mathbb{E}(Y^2) \\ &= 100 \cdot 0.86 \\ &= \boxed{86}\end{aligned}$$

The expected surcharge is \$86. This means that on average, a random customer for the company pays \$86 in moving violations.

4. Find the standard deviation of the surcharge.

Because the tickets and surcharge are related (Every ticket has a surcharge), we can use the standard deviation of Y.

$$\begin{aligned}\text{sd}(100Y^2) &= 100 \cdot \text{sd}(Y) \\ &= 100(0.78) \\ &= \boxed{78}\end{aligned}$$