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MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set I, Due Monday, January 23rd, 2023

1. Suppose $f(x) = x^2 - x$ what is the average value of f on [-1,1]?

$$\overline{f} = \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \quad \text{where} \quad f(x) = x^2 - x$$
$$\Rightarrow \overline{f} = \frac{1}{1-(-1)}\int_{-1}^{1}(x^2-x)\mathrm{d}x = \frac{1}{2}\left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_{-1}^{1}$$
$$= \left(\frac{2x^3 - 3x^2}{12}\right)\Big|_{-1}^{1} = \left(-\frac{1}{12}\right) - \left(-\frac{5}{12}\right) = \frac{4}{12} = \boxed{\frac{1}{3} \approx 0.\overline{333}}$$

2. Evaluate $2+4+6+\cdots+100$.

$$2+4+6+\cdots+100 = \sum_{n=1}^{50} 2n = 2\sum_{n=1}^{50} n$$
$$\text{Using} \sum_{n=1}^{k} n = \frac{k(k+1)}{2} \quad \Longrightarrow \quad \cancel{2}\left(\frac{50(51)}{\cancel{2}}\right) = \boxed{2{,}550}$$

3. Evaluate $\int (7+e^x)^5 e^x \mathrm{d}x$

$$\int (7+e^x)^5 e^x \mathrm{d}x$$
$$\text{let} \quad u = 7+e^x \quad \mathrm{d}u = e^x \mathrm{d}x$$
$$\Rightarrow \int u^5 \mathrm{d}u = \frac{1}{6}u^6 + C = \boxed{\frac{1}{6}(7+e^x)^6 + C}$$

4. Evaluate $\int_{-1}^{1} (3+x)^4 \mathrm{d}x$

$$\int_{-1}^{1} (3+x)^4 \mathrm{d}x$$
$$\text{let} \quad u = 3+x; \quad \mathrm{d}u = \mathrm{d}x$$
$$u_{lower} = 3+(-1) = 2; \quad u_{upper} = 3+1 = 4$$
$$\Rightarrow \int_{-1}^{1} u^4 \mathrm{d}u = \frac{1}{5}u^5\Big|_2^4 = \frac{4^5}{5} - \frac{2^5}{5} = \boxed{198.4}$$