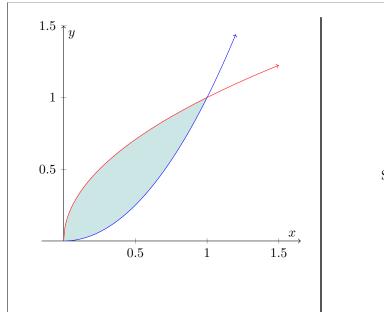
1. Consider the region in the cartesian plane that is bounded by the graphs of  $y = x^2$  and  $y = \sqrt{x}$ . What is the surface area?



$$x^{2} = x^{1/2}$$

$$x^{3/2} = 0$$

$$x\sqrt{x} = 0$$

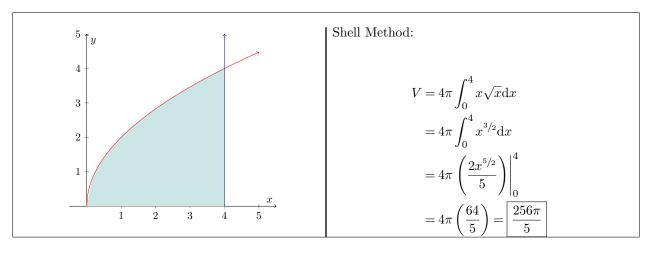
$$x = 0,1$$

Surface Area = 
$$\int_0^1 \left[ \sqrt{x} - x^2 \right] dx$$
  
=  $\frac{3x^{3/2}}{2} - \frac{3x^3}{3} \Big|_0^1$   
=  $\frac{3}{2} - \frac{3}{3} = \boxed{\frac{1}{2}}$ 

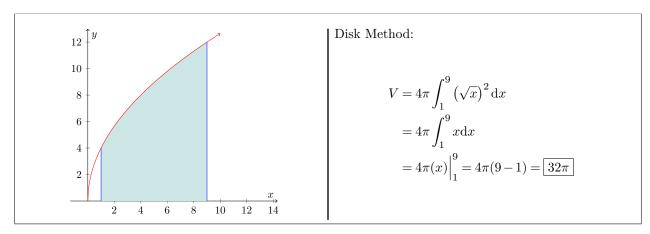
**2.** Consider the graph of  $y = 4x^{3/2}$ . Compute the arc-length on the interval  $1 \le x \le 3$ .

Arc-length = 
$$\int_{a}^{b} \sqrt{1 + f'(x)^{2}} dx$$
  
 $f'(x) = 6x^{1/2}, \quad a = 1, b = 3$   
Arc-length =  $\int_{1}^{3} \sqrt{1 + (6x^{1/2})^{2}} dx$   
=  $\int_{1}^{3} \sqrt{1 + 36x} dx$   
 $\Rightarrow \frac{1}{36} \int_{37}^{109} \sqrt{u} du$   
=  $\frac{1}{36} \left(\frac{2x^{3/2}}{3}\right) \Big|_{37}^{109}$   
=  $\frac{109\sqrt{109} - 37\sqrt{37}}{54}$ 

**3.** Consider the region bounded by  $y = 2\sqrt{x}$ , the x-axis and x = 4. Compute the volume if this is rotated about the y-axis.



**4.** Consider the graph of  $y = 4\sqrt{x}$  where  $1 \le x \le 9$ . Suppose this is rotated about the x-axis. What is the surface area?



5. Evaluate  $\int_1^2 \frac{\mathrm{d}x}{x\sqrt{1-\ln^2(x)}}$ .

$$\int_{1}^{2} \frac{\mathrm{d}x}{x\sqrt{1-\ln^{2}(x)}}$$

$$u = \ln(x) - \mathrm{d}u = \frac{1}{x}\mathrm{d}x$$

$$\int_{lower} = \ln(1) = 0 \int_{upper}^{upper} = \ln(2)$$

$$\int_{0}^{\ln(2)} \frac{\mathrm{d}u}{\sqrt{1-u^{2}}}$$

$$= \arcsin(u) \Big|_{0}^{\ln(2)} = \left[\arcsin(\ln(2)\right)$$