

1. If  $A$  and  $B$  are events with probabilities  $P(A) = 0.7$  and  $P(B) = 0.9$ , is it possible for  $A$  and  $B$  to be mutually exclusive? Why or why not?

No.  $P(A) + P(B) = 1.6$ , since probabilities are portions of 1.0, any sum of probabilities  $> 1.0$  are either impossible or have to contain some elements of each other.

If they were mutually exclusive, then  $P(A \cap B) = \emptyset = P(\emptyset) = 0.0$ .

$$\begin{aligned} \text{The probability of } P(A \cup B) &= (P(A) + P(B)) - P(A \cap B) \leq 1 \\ (0.7 + 0.9) - 0 &= 1.6 \not\leq 1 \end{aligned}$$

2. In a standard 52-card deck of cards,

- (a) How many possible 5-card hands are there?

$$\begin{aligned} T1 &= \text{pick 5 cards} = n_1 = C(52, 5) \\ n &= (n_1) = \left( \frac{52!}{5!(52-5)!} \right) = \left( \frac{52!}{5!(47)!} \right) = \left( \frac{(52)(51)(50)(49)(48)}{120} \right) = \boxed{2,598,960} \end{aligned}$$

- (b) How many hands are there that have 3 of a kind (3 with the same numerical value) and 2 singletons?

$$\begin{aligned} T1 &= \text{pick a \# for the three of a kind} = n_1 = 13 \\ T2 &= \text{pick the three of a kind} = n_2 = C(4, 3) \\ T3 &= \text{pick two \#s for singletons} = n_3 = C(11, 2) \\ T4 &= \text{pick higher singleton} = n_4 = C(4, 1) \\ T5 &= \text{pick lower singleton} = n_5 = C(4, 1) \\ n &= (n_1)(n_2)(n_3)(n_4)(n_5) = (13)\left(\frac{4!}{3!(1!)}\right)\left(\frac{11!}{2!(9!)}\right)(4)(4) \\ &= 13(4)\left(\frac{(11)(10)}{2}\right)(4)(4) = \boxed{45,760} \end{aligned}$$

- (c) What is the probability of getting a hand with 3 of a kind and 2 singletons?

$$\begin{aligned} \# \text{ of possible hands} &: 2,598,960 \\ \# \text{ of special hands} &: 45,760 \\ \text{Probability of special hands} &: \frac{45,760}{2,598,960} \approx \boxed{0.0176} \end{aligned}$$

**3.** Calculate each of the following.

**(a)** Number of “words” consisting of 4 of the letter ‘F’ and 11 of the letter ‘S’. For example, FSSSF SFSFS SSSSS is one such “word”. (Ignore the spaces; they’re just there to make it easier to count the number of letters.)

We need every permutation of the set  $A = \{F, F, F, F\} \cup \{S, S, S, S, S, S, S, S, S, S, S\}$ .

$$T1 = \text{Fill 4 positions with 'F's} = n_1 = C(15, 4) = \binom{15}{4}$$

$$T2 = \text{Fill the remaining with 'S's} = n_2 = C(11, 11) = \binom{11}{11} = 1$$

$$\begin{aligned} n = (n_1)(n_2) &= \left( \frac{15!}{4!(15-4)!} \right) (1) = \frac{15!}{4!(11)!} = \frac{(15)(14)(13)(12)}{4!} \\ &= \frac{32,760}{24} = \boxed{1,365} \end{aligned}$$

**(b)** Number of 4-digit PINs with no repeated digits.

Every combination of 4 numbers from the set  $\{0, 1, 2, 3, \dots, 9\}$ .

$$\begin{aligned} \text{Choose 4 \#s out of 10} &= P(10, 4) = \frac{10!}{(10-4)!} \\ &= \frac{10!}{6!} = (10)(9)(8)(7) = \boxed{5040} \end{aligned}$$

**(c)** Number of ways to seat the 5 VIPs in the 5 seats of the front row at the opera.

Every permutation for 5 to fit into 5.

$$\text{Permute 5 into 5} = P(5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5! = \boxed{120}$$

**(d)** Number of ways to seat the 5 VIPs in the 12 seats of the front row at the opera.

Every permutation for 5 to fit into 12.

$$\text{Permute 5 into 12} = P(5) = \frac{12!}{(12-5)!} = \frac{12!}{7!} = (12)(11)(10)(9)(8) = \boxed{95,040}$$