

1. Of n randomly selected Star Wars fans, X describe themselves as extroverts; whereas of m randomly selected Star Trek fans, Y are extroverts. Let p_1 and p_2 denote the probabilities that a randomly selected Star Wars fan and Star Trek fan, respectively, are extroverts.

- (a) Show that $T = (X/n - Y/m)$ is an unbiased estimator for $p_X - p_Y$.

$$\begin{aligned} p_X - p_Y &= T = (X/n - Y/m) \\ p_1 &= X/n \\ p_2 &= Y/m \\ \therefore p_X - p_Y &= X/n - Y/m \\ \therefore T &\text{ is an unbiased estimator} \end{aligned}$$

- (b) Find $\text{var}(T)$. Find the s.e. of the estimator in (a); i.e. find $\text{sd}(T)$.

$$\begin{aligned} \text{var}(T) &= \text{var}(X/n - Y/m) = \text{var}(X/n) - \text{var}(Y/m) \\ &= \boxed{np_X(1 - p_X) - mp_Y(1 - p_Y)} \\ \text{s.e.}(T) &= \text{sd}(T) = \sqrt{\text{var}(T)} \\ &= \boxed{\sqrt{np_X(1 - p_X) - mp_Y(1 - p_Y)}} \end{aligned}$$

- (c) How would you use the observed values x and y to estimate the standard error of T ?

x and y are the cases of extroverts, and so I would use them to calculate p_X and p_Y as $p_X = x/n$ and $p_Y = y/n$.

- (d) If $n = m = 200$, $x = 127$, and $y = 176$, use the estimator of (a) to obtain an estimate of $p_X - p_Y$.

$$\begin{aligned} T &= X/n - Y/m \\ &= 127/200 - 176/200 = 0.635 - 0.880 = \boxed{-0.245} \end{aligned}$$

- (e) Use the result of part (c) and the data of part (d) to estimate the standard error of the estimator.

$$\begin{aligned} p_X &= X/n = 0.635 \\ p_Y &= Y/m = 0.880 \\ \text{s.e.}(T) &= \sqrt{np_X(1 - p_X) - mp_Y(1 - p_Y)} \\ &= \sqrt{200(0.635)(1 - 0.635) - 200(0.880)(1 - 0.880)} \\ &= \sqrt{46.355 - 21.12} = \sqrt{25.235} \approx \boxed{5.0235} \end{aligned}$$