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Math 307 - Discrete Mathematics
Assignment 1

Homework Problems

Section 1.1

Problem 4

a. $\lceil 0.763 \rceil = \boxed{1}$

b. $2\lceil 0.6 \rceil - \lceil 1.2 \rceil = 1 - 2 = \boxed{-1}$

c. $\lceil 1.1 \rceil + \lceil 3.3 \rceil = 2 + 4 = \boxed{6}$

d. $\lceil \sqrt{3} \rceil - \lfloor \sqrt{3} \rfloor = 2 - 1 = \boxed{1}$

e. $\lceil -73 \rceil - \lfloor -73 \rfloor = -73 - (-73) = -73 + 73 = \boxed{0}$

Problem 8

How many multiples of 10 are there between the following pairs of numbers?

This can be solved for any pair n, m such that $m \leq n$ using the formula:

$$\left\lfloor \frac{n}{10} \right\rfloor - \left\lfloor \frac{m-1}{10} \right\rfloor$$

a. $m = 1, n = 80$

$$\left\lfloor \frac{80}{10} \right\rfloor - \left\lfloor \frac{1-1}{10} \right\rfloor = 8 - 0 = \boxed{8}$$

b. $m = 0, n = 100$

$$\left\lfloor \frac{100}{10} \right\rfloor - \left\lfloor \frac{0-1}{10} \right\rfloor = 10 - (-1) = \boxed{11}$$

$$\text{Note that 0 is a multiple of every integer}$$
$$\text{including 10, so it is included in this count.}$$

c. $m = 9, n = 2967$

$$\left\lfloor \frac{2967}{10} \right\rfloor - \left\lfloor \frac{9-1}{10} \right\rfloor = 296 - 0 = \boxed{296}$$

d. $m = -6, n = 34$

$$\left\lfloor \frac{34}{10} \right\rfloor - \left\lfloor \frac{-6-1}{10} \right\rfloor = 3 - (-1) = \boxed{4}$$

e. $m = 10^4, n = 10^5$

$$\left\lfloor \frac{10^5}{10} \right\rfloor - \left\lfloor \frac{10^4-1}{10} \right\rfloor = 10000 - 999 = \boxed{9001}$$

Problem 18

a. Fact 4 states that between any number n and k there are $\lfloor \frac{n}{k} \rfloor$ multiples of k between 1 and n. Therefore, in the case $k = 1$, there are n positive integers between 1 and n. In effect, the count of numbers between 1 and any positive integer n is simply n. This is obvious as any given count of a collection of size n is the value of n elements.

b. Given fact 4 once more, if $k > n$, then any two given numbers of k and n will result in $\lfloor \frac{n}{k} \rfloor = 0$ multiples of k between 1 and n. This is again obvious as a number greater than another given number cannot evenly divide it into an integer value ≥ 1.

Problem 19

a.

$$\text{Let } x = 0.5, y = 0.5. \text{ Then } \lfloor 0.5 \rfloor + \lfloor 0.5 \rfloor < \lfloor 0.5 + 0.5 \rfloor$$
$$0 + 0 < \lfloor 1 \rfloor \to 0 < 1$$
$$\text{Therefore the inequality holds for } y = 0.5, x = 0.5$$

b.

$$\text{Let } x = 0, y = 0. \text{ Then } \lfloor 0 \rfloor + \lfloor 0 \rfloor = \lfloor 0 + 0 \rfloor$$
$$0 + 0 = \lfloor 0 \rfloor \to 0 = 0$$
$$\text{Therefore the equality holds for } y = 0, x = 0$$

c. All floor operations round down to the nearest integer. Any given pair, x, y must either contain integers or non-integers. If both are integers, then the equation $\lfloor x \rfloor + \lfloor y \rfloor$ simplifies always to $x + y$. In any case for x, y that at least one is a non-integer, the floor operator will reduce that value such that $\lfloor x \rfloor < x$ and/or $\lfloor y \rfloor < y$. Therefore, $\lfloor x \rfloor + \lfloor y \rfloor \leq x + y$.

We know that $\lfloor x + y \rfloor$ is the largest integer less than or equal to $x + y$ and that $\lfloor x \rfloor + \lfloor y \rfloor$ is an integer.

Given these facts, $\lfloor x \rfloor + \lfloor y \rfloor$ must also always be less than or equal to $\lfloor x + y \rfloor$.

Section 1.2

Problem 12

No, two different even integers cannot be relatively prime. We define a given integer n to be even if $2|n$. Any given pair of even integers n, m can be expressed as $n = 2k$ and $m = 2j$ for some integers k, j. The factor 2 in these equations would be a common factor that is not 1, so the two even integers cannot be relatively prime.

Problem 14

a. We can expand the expression lm into the expression adl. We can then very clearly see that both d and adl share the common factor d. Therefore, in any case where l is an integer, ml is evenly divided by d, thus $d \mid ml$.

b. In both cases, we can factor out d, which inherently defines the operations as factors of d.

$$m + n = ad + bd = d(a + b)$$
$$m - n = ad - bd = d(a - b)$$

c. We can see that $17m - 72n = 17ad - 72bd = d(17a - 72b)$ As both a and b are integers, so must $17a - 72b$. Therefore, d divides $17m - 72n$.

Problem 16 b.

The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36. All numbers not sharing these factors are therefore relatively prime (except for sharing 1). Those are: 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, and 35.

Section 1.3

Problem 2

List the elements ({}) in the following sets:

a.

$$\left\{\frac{1}{n} : n = 1, 2, 3, 4\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$$

b.

$$\{n^2 - n : n = 0, 1, 2, 3, 4\} = \{0, 0, 2, 6, 12\}$$

c.

$$\left\{\frac{1}{2^2} : n \in \mathbb{P}, n \text{ is even and } n < 11\right\} = \left\{\frac{1}{4}, \frac{1}{16}, \frac{1}{36}, \frac{1}{64}, \frac{1}{100}\right\}$$

d.

$$\{2 + (-1)^n : n \in \mathbb{N}\} = \{3, 1, 3, 1, ..\} \text{ or simply } \{3, 1\}$$

Problem 6

Repeat Exercise 4 for the following sets (determine the following sets):

a.

$$\{n \in \mathbb{N} : n|12\} = \{1, 2, 3, 4, 6, 12\}$$

b.

$$\{n \in \mathbb{N} : n^2 + 1 = 0\} = \varnothing$$
$$\text{as all values of } n \text{ result in } n^2 + 1 > 0$$

c.

$$\left\{n \in \mathbb{N} : \left\lfloor \frac{n}{3} \right\rfloor = 8\right\} = \{24, 25, 26\}$$

d.

$$\left\{n \in \mathbb{N} : \left\lceil \frac{n}{2} \right\rceil = 8\right\} = \{15, 16\}$$

Problem 8

How many elements are in the following sets? Write ∞ if the set is infinite.

a.

$$\{n \in \mathbb{N} : n^2 = 2\} : \boxed{\varnothing} \text{ as no integer } k \text{ exists such that}$$
$$\sqrt{2} = k \text{ and } k \in \mathbb{N} \text{ as } \sqrt{2} \approx 1.41$$

b.

$$\{n \in \mathbb{Z} : 0 \leq n \leq 73\} : (73 - 0) + 1 = \boxed{74}$$

c.

$$\{n \in \mathbb{Z} : 5 \leq |n| \leq 73\} : (73 - 5) + 1 - (5 - 73) + 1$$
$$= 68 - (-68) + 2 = 136 + 1 = \boxed{138}$$

d.

$$\{n \in \mathbb{Z} : 5 < n < 73\} : 73 - 5 - 1 = \boxed{67}$$

e.

$$\{n \in \mathbb{Z} : n \text{ is even and } |n| \leq 73\} :$$
$$\text{Using the formula defined in worksheet 1 to determine}$$
$$\text{all even numbers between } m \text{ and } n : \left\lfloor \frac{n}{2} \right\rfloor - \left\lfloor \frac{m-1}{2} \right\rfloor$$
$$\left\lfloor \frac{73}{2} \right\rfloor - \left\lfloor \frac{0-1}{2} \right\rfloor = 36 - 0 = 36$$
$$|n| \text{ includes negative even integers as well, thus } 36 * 2 = 72$$
$$\text{Including 0 as an even integer gives a final total of } 72 + 1 = \boxed{73}$$

f.

$$\{x \in \mathbb{Q} : 0 \leq x \leq 73\} : \boxed{\infty}, \text{as there are infinite rational}$$
$$\text{numbers between any two given integers.}$$

g.

$$\left\{x \in \mathbb{Q} : x^2 = 2\right\} : \boxed{0} \text{ as there is no number defined as}$$
$$m \in \mathbb{Z}, n \in \mathbb{Z} \text{ such that } \left(\frac{m}{n}\right)^2 = 2 \text{ or } \frac{m}{n} = \sqrt{2}, \text{because } \sqrt{2} \text{ is irrational.}$$

h.

$$\left\{x \in \mathbb{R} x^2 = 2\right\} : \boxed{2}, \text{we defined before that the number } \sqrt{2} \text{ satisfies this condition.}$$
$$\mathbb{R} \text{ includes irrational numbers whereas } \mathbb{Q} \text{ does not, allowing for } \sqrt{2}.$$
$$\text{Additionally,} -\sqrt{2} \text{ also satisfies this condition as } \left(-\sqrt{2}\right)^2 = 2, \text{making the set be } \left\{-\sqrt{2}, \sqrt{2}\right\}$$