

1. Evaluate $\int_{-1}^1 \frac{4}{x^2} dx$.

$$\begin{aligned}\int_{-1}^1 \frac{4}{x^2} dx &= \int_{-1}^0 \frac{4}{x^2} dx + \int_0^1 \frac{4}{x^2} dx \\ \int_{-1}^0 \frac{4}{x^2} dx &\Rightarrow \lim_{u \rightarrow 0^-} \int_{-1}^u \frac{4}{x^2} dx = \lim_{u \rightarrow 0^-} \left[-\frac{4}{x} \right]_{-1}^u = \lim_{u \rightarrow 0^-} -\left[\frac{4}{u} - \frac{4}{-1} \right] \\ &= -[-\infty + 4] \quad \boxed{\text{Diverges}}\end{aligned}$$

2. Evaluate $\int \frac{dx}{\sqrt{-x^2-2x}}$.

$$\begin{aligned}\int \frac{dx}{\sqrt{-x^2-2x}} &= \int \frac{dx}{\sqrt{1-(x+1)^2}} \\ \text{let } u &= x+1 \quad du = dx \\ \Rightarrow \int \frac{du}{\sqrt{1-u^2}} &= \arcsin(u) + C = \boxed{\arcsin(x+1) + C}\end{aligned}$$

3. Evaluate $\lim_{n \rightarrow \infty} \frac{10n^6+100n}{2(4+n^6)}$.

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{10n^6+100n}{2(4+n^6)} &= \frac{10}{2} \lim_{n \rightarrow \infty} \frac{n^6+10n}{4+n^6} \\ &= \frac{10}{2} \lim_{n \rightarrow \infty} \frac{\frac{n^6}{n^6} + \frac{10n}{n^6}}{\frac{4}{n^6} + \frac{n^6}{n^6}} = \frac{10}{2} \lim_{n \rightarrow \infty} \frac{1 + \frac{10}{n^5}}{\frac{4}{n^6} + 1} \rightarrow \frac{10}{2} \cdot \frac{1+0}{0+1} = \frac{10}{2} = \boxed{5}\end{aligned}$$

4. Evaluate $\lim_{n \rightarrow \infty} \frac{\ln(7+6n^8)}{\ln(1000+n)}$.

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{\ln(6n^8)}{\ln(n)} &\leq \lim_{n \rightarrow \infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} \leq \lim_{n \rightarrow \infty} \frac{\ln(10n^8)}{\ln(n)} \\ &= \lim_{n \rightarrow \infty} \frac{8\ln(n) + \ln(6)}{\ln(n)} \leq \lim_{n \rightarrow \infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} \leq \lim_{n \rightarrow \infty} \frac{8\ln(n) + \ln(10)}{\ln(n)} \\ &= \lim_{n \rightarrow \infty} 8 + \frac{\ln(6)}{\ln(n)} \leq \lim_{n \rightarrow \infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} \leq \lim_{n \rightarrow \infty} 8 + \frac{\ln(10)}{\ln(n)} \\ &= 8 \leq \lim_{n \rightarrow \infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} \leq 8 \quad \therefore \lim_{n \rightarrow \infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} = \boxed{8}\end{aligned}$$

5. Evaluate $\lim_{n \rightarrow \infty} \sqrt[n]{n^{100}+13}$.

$$\begin{aligned}\lim_{n \rightarrow \infty} \sqrt[n]{n^{100}} &\leq \lim_{n \rightarrow \infty} \sqrt[n]{n^{100}+13} \leq \lim_{n \rightarrow \infty} \sqrt[n]{2n^{100}} \\ a = \lim_{n \rightarrow \infty} n^{100/n} &\Rightarrow \ln(a) = \lim_{n \rightarrow \infty} \frac{100 \cdot \ln(n)}{n} \stackrel{(H)}{=} \lim_{n \rightarrow \infty} \frac{\frac{100}{n}}{1} = \frac{0}{1} = 0; \quad \ln(a) = 0 \Rightarrow e^{\ln(a)} = e^0 = 1 \\ b = \lim_{n \rightarrow \infty} 2n^{100/n} &\Rightarrow \ln(b) = \lim_{n \rightarrow \infty} \frac{100 \cdot \ln(2n)}{n} \stackrel{(H)}{=} \lim_{n \rightarrow \infty} \frac{\frac{100}{2n}}{1} = \frac{0}{1} = 0; \quad \ln(b) = 0 \Rightarrow e^{\ln(b)} = e^0 = 1 \\ 1 &\leq \lim_{n \rightarrow \infty} \sqrt[n]{n^{100}+13} \leq 1 \quad \therefore \lim_{n \rightarrow \infty} \sqrt[n]{n^{100}+13} = \boxed{1}\end{aligned}$$