

392. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$

$$\begin{aligned}
 &\text{Using } a^2 - u^2: \quad u = a \sin(\theta) \quad \text{where } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\
 &\Rightarrow x = \sin \theta \quad dx = \cos \theta d\theta \quad \theta = \sin^{-1} x \\
 &x_{lower} = \sin\left(-\frac{1}{2}\right) \quad x_{upper} = \sin\left(\frac{1}{2}\right) \\
 &\int_{\sin(-\frac{1}{2})}^{\sin(\frac{1}{2})} \frac{\cos \theta d\theta}{\sqrt{1 - \sin^2 \theta}} = \int_{\sin(-\frac{1}{2})}^{\sin(\frac{1}{2})} \frac{\cos \theta d\theta}{\sqrt{\cos^2 \theta}} \\
 &= \int_{\sin(-\frac{1}{2})}^{\sin(\frac{1}{2})} \frac{\cos \theta d\theta}{\sqrt{1 - \sin^2 \theta}} = \int_{\sin(-\frac{1}{2})}^{\sin(\frac{1}{2})} \frac{\cos \theta d\theta}{\cos \theta} \\
 &= \int_{\sin(-\frac{1}{2})}^{\sin(\frac{1}{2})} d\theta = \theta \Big|_{\sin(-\frac{1}{2})}^{\sin(\frac{1}{2})} = \sin^{-1} x \Big|_{-\frac{1}{2}}^{\frac{1}{2}} = \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right) \\
 &= \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \boxed{\frac{\pi}{3}}
 \end{aligned}$$

400. $\int \frac{dx}{25+16x^2}$

$$\begin{aligned}
 &\text{Using } a^2 + u^2: \quad u = a \tan \theta \quad \text{where } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\
 &\Rightarrow 4x = 5 \tan \theta \quad dx = \frac{5}{4} \sec^2 \theta d\theta \quad \theta = \tan^{-1}\left(\frac{4x}{5}\right) \\
 &\int \frac{5 \sec^2 \theta d\theta}{4(25 + 25 \tan^2 \theta)} = \int \frac{5 \sec^2 \theta d\theta}{100(1 + \tan^2 \theta)} \\
 &= \frac{1}{20} \int \frac{\sec^2 \theta d\theta}{\sec^2 \theta} = \frac{1}{20} \int d\theta = \frac{\theta}{20} + C = \boxed{\frac{\tan^{-1}\left(\frac{4x}{5}\right)}{20}}
 \end{aligned}$$

424. $\int \frac{e^t}{1+e^t} dt$