Kaleb Burris
STAT F300 Statistics, Dr. Short
Worksheet 15, Due February 27, 4:15pm

1.

(a) Find $p_X(x)$, the marginal pmf for X.

X	$P_X(x)$
1	$0.13 + 0.53 + 0.24 = 0.90$
2	$0.05 + 0.02 + 0.01 = 0.08$
3	$0.02 + 0.00 + 0.00 = 0.02$
	$0.90 + 0.08 + 0.02 = 1.00$

(b) Find $p_Y(y)$, the marginal pmf for Y.

Y	$P_Y(y)$
0	$0.13 + 0.05 + 0.02 = 0.20$
1	$0.53 + 0.02 + 0.00 = 0.55$
2	$0.25 + 0.01 + 0.00 = 0.26$
	$0.20 + 0.55 + 0.26 = 1.00$

(c) Find $P(X = 1 \text{ and } Y = 0)$.

$$P(X = 1 \text{ and } Y = 0) = P\begin{pmatrix} 1 & \text{hikes} \\ 0 & \text{trips to chicago} \end{pmatrix} = \boxed{0.13}$$

(d) Find $P(X = 0 \text{ or } Y = 1)$.

$$\begin{aligned} P(X = 1 \text{ or } Y = 0) &= p_X(1) + p_Y(0) - P(X = 1, Y = 0) \\ &= 0.90 + 0.20 - 0.13 = \boxed{0.97} \end{aligned}$$

(e) Find $P(X + Y = 2)$.

$$P(X + Y = 2) = 0.02 + 0.05 + 0.53 = \boxed{0.60}$$

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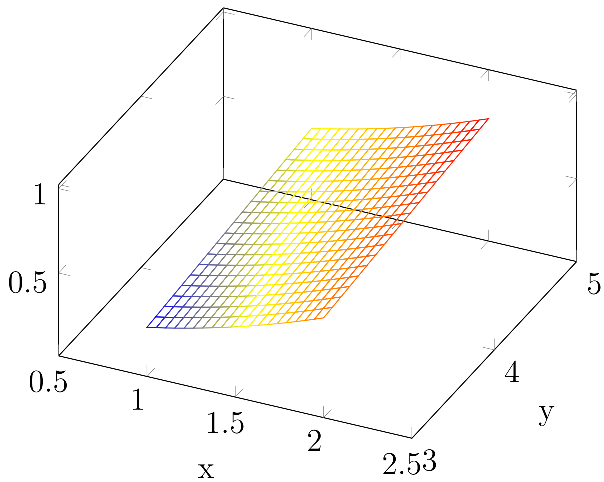
2. Find the joint pdf of X and Y if $f(x,y)$, given below.

$$f(x,y) = \begin{cases} \frac{1}{30}(3x^2+2y) & \text{if } 1 \leq x \leq 2, 3 \leq y \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the support of the joint distribution of X and Y.

Bringing out the tikzplot for this one.



I have no idea but this is a neat plot.

(b) Find $P(3/2 \leq X \leq 2, 4 \leq Y \leq 5)$.

$$\begin{aligned} P(3/2 \leq X \leq 2, 4 \leq Y \leq 5) &= \int_{3/2}^{2} \left[\int_{4}^{5} \left(\frac{1}{30}(3x^2+2y) \right) dy \right] dx \\ &= \int_{3/2}^{2} \left[\int_{4}^{5} \left(\frac{x^2}{30} \right) dy + \int_{4}^{5} \left(\frac{2y}{30} \right) dy \right] dx \\ &= \int_{3/2}^{2} \left[\frac{x^2}{30} + 0.3 \right] dx \\ &= \left[\frac{x^3}{90} + 0.3x \right]_{3/2}^{2} = \left(\frac{2^3}{90} + 0.6 \right) - \left(\frac{(3/2)^3}{90} + 3/2(0.3) \right) \\ &= \left(\frac{8}{90} + 0.6 \right) - \left(\frac{(27/8)}{90} + 0.45 \right) = 0.6\overline{88} - 0.5075 = \boxed{0.1814} \end{aligned}$$

(c) Find $f_X(x)$, the marginal distribution of X. Show that it integrates to 1.

$$
\begin{aligned}
f_X(x) &= \int_{-\infty}^{\infty} \left(\frac{1}{30}(3x^2+2y)\right) dy \\
&= \frac{1}{30}\int_{-\infty}^{\infty} (3x^2+2y)dy \\
&\Rightarrow \frac{1}{30}\left[3x^2y+y^2\right]_3^5 \\
&= \frac{1}{30}\left[\left(15x^2+25\right)-\left(9x^2+9\right)\right] \\
&= \boxed{\frac{1}{30}\left[6x^2+16\right]} \\
\int_{-\infty}^{\infty} f_X(x)dx &\Rightarrow \frac{1}{30}\int_1^2 \left[6x^2+16\right] dx \\
&= \frac{1}{30}\left[2x^3+16x\right]_1^2 = \frac{1}{30}[(16+32)-(2+16)] \\
&= \frac{48-18}{30} = \frac{30}{30} = 1
\end{aligned}
$$

(d) Find $f_Y(y)$, the marginal distribution of Y.

$$
\begin{aligned}
f_Y(y) &= \int_{-\infty}^{\infty} \left(\frac{1}{30}(3x^2+2y)\right) dx \\
&\Rightarrow \frac{1}{30}\int_1^2 (3x^2+2y)dx \\
&= \frac{1}{30}\left[x^3+2yx\right]_1^2 = \frac{1}{30}[(8+4y)-(1+2y)] = \boxed{\frac{2y+7}{30}}
\end{aligned}
$$