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MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set VIII, Due Monday, March 20, 2023

Which of the following converge:

1. $\sum \frac{n^2-1}{5n^5}$

$$\boxed{\text{Convergent, using P-Series:}}$$
$$\lim_{x\to\infty} \frac{x^2-1}{5x^5} = \lim_{x\to\infty} \frac{x^2}{x^5} = \lim_{x\to\infty} \frac{1}{x^3}, \quad P>1 \therefore \text{ Convergent}$$

2. $\sum \cos\left(\frac{1}{n}\right)$

$$\boxed{\text{Divergent, using divergence test:}}$$
$$\lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = \cos(0) = 1; \quad \lim_{x\to\infty} \not=0, \quad \therefore \text{ Divergent}$$

3. $\sum \sin\left(\frac{1}{n^2}\right)$

$$\boxed{\text{Convergent, comparison test:}}$$
$$\sin\left(\frac{1}{x^2}\right) \leq \frac{1}{n^2}, \quad \frac{1}{n^2} \text{ is convergent; } \therefore \text{ Convergent}$$

4. $\sum \frac{n^2+100}{20n^5}$

$$\boxed{\text{Convergent, P-Series}}$$
$$\lim_{x\to\infty} \frac{x^2+100}{20x^5} = \lim_{x\to\infty} \frac{x^2}{x^5} = \lim_{x\to\infty} \frac{1}{x^3}, \quad P>1 \therefore \text{ Convergent}$$

5. $\sum \frac{\ln(n)}{n}$

$$\boxed{\text{Divergent, integral test:}}$$
$$\int_1^\infty \frac{\ln(x)}{x} dx \Rightarrow \lim_{x\to\infty} \int_0^n u d(\ln(x)) = \lim_{x\to\infty} \left(\frac{u^2}{2}\right)\Big|_0^n$$
$$= \lim_{x\to\infty} \frac{n^2}{2} - \frac{0}{2} = \lim_{x\to\infty} \frac{\ln(x)^2}{2} = \frac{\ln(\infty)^2}{2} = \infty, \quad \therefore \text{ Divergent}$$