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STAT F300 Statistics, Dr. Short
Worksheet 7, due Monday 6 February, 4:15pm

**1.** Consider an experiment in which two fair 6-sided dice are tossed, one red, the other white.

$$\begin{aligned} A &= \text{ the the sum is at least 6} \\ B &= \text{ the product is even} \\ C &= \text{ red + 2} \times \text{white = 8} \\ D &= \text{ red is odd} \end{aligned}$$

(a) Are events A and B independent? Be sure to show your work.

Is $P(A \cup B) = P(A)P(B)$?

$$\begin{aligned} \#P &= 6^2 = 36 \\ A &= \{15,51,24,42,33\} \\ B &= \{12,21,22,14,41,16,61,23,32,44,46,64,66\} \\ A \cup B &= \{24,42\} \\ P(A) &= \frac{\#A}{\#P} = \frac{5}{36} = 0.13\overline{8} \\ P(B) &= \frac{\#B}{\#P} = \frac{13}{36} = 0.36\overline{1} \\ P(A \cap B) &= \frac{\#(A \cap B)}{\#P} = \frac{2}{36} = 0.0\overline{5} \\ 0.0\overline{5} &\stackrel{?}{=} \frac{0.13\overline{8}}{0.36\overline{1}} \\ 0.0\overline{5} &\not\approx 0.385 \\ \neq, &\boxed{\therefore A \text{ and } B \text{ are dependant.}} \end{aligned}$$

(b) Find a pair of mutually exclusive events. Why are they mutually exclusive?

$C$ and $D$ are mutually exclusive because the conditions of $C$ require that the red die rolled must be either 2, 4, or 6.

The first die being red:

$$\begin{aligned} C &= \{22, 61\} \\ D &= \{1*, 3*, 5*\} \leftarrow \text{ Using star (*) to represent any digit from 1-6} \\ C \cap D &= \emptyset \quad \boxed{\therefore \text{ C and D are mutually exclusive.}} \end{aligned}$$

(c) Are events $B^C$ and $C$ independent? Explain (i.e. show calculations and explain)

$$
\begin{aligned}
B^C &= \{11,13,31,15,51,33,35,53,55\} \\
P(B^C) &= \frac{\#B^C}{\#P} = \frac{9}{36} = 0.25 \\
B^C \cap C &= \emptyset \\
P(B^C \cap C) &\stackrel{?}{=} P(B^C)P(C) \\
\emptyset &\neq (0.25)(0.08\overline{3}), \boxed{\therefore\ B^C \text{ and } C \text{ dependant.}}
\end{aligned}
$$

(d) Find $P(D|C)$ and $P(D)$. Are $C$ and $D$ independent? Use just the values of $P(D|C)$ and $P(D)$ to answer this question. Explain briefly.

$$
\begin{aligned}
D &= \{11,12\ldots,16,31,32,\ldots,36,51,52,\ldots,56\}, \#D = 18 \\
P(D) &= \frac{\#D}{\#P} = \frac{18}{36} = \boxed{0.5} \\
P(D|C) &= D \cup C = \boxed{\emptyset \therefore \text{ because of the solution to 1. (b), they are also independent.}}
\end{aligned}
$$

**2.** The dogs of the world can be divided into 4 types, according to whether they prefer chasing frisbees or digging in the dirt, and according to whether they do / don't enjoy long car rides. The table below summaries the probabilities that a randomly chosen dog has each combination of characteristics.

Let $F = preferschasingfrisbees$ and $R = likesroadtrips$

(a) Are $F$ and $R$ independent? Be sure to show your calculations.

$$
\begin{aligned}
P(F) &= 0.37 \\
P(R) &= 0.58 \\
P(F \cap R) &= 0.20 \\
P(F \cap R) &\stackrel{?}{=} P(F)P(R) \\
0.2 &\stackrel{?}{=} (0.37)(0.58) \\
0.2 &\neq 0.2146, \boxed{\therefore\ F \text{ and } R \text{ are dependant.}}
\end{aligned}
$$

(b) Are $F^C$ and $R^C$ independent? Be sure to sure your calculations.

$$P(F^C) = 1 - P(F) = 1 - 0.37 = 0.63$$
$$P(R^C) = 1 - P(R) = 1 - 0.58 = 0.42$$
$$P(F^C \cap R^C) = 0.25$$
$$P(F^C \cap R^C) \stackrel{?}{=} P(F^C)P(R^C)$$
$$0.25 \stackrel{?}{=} (0.63)(0.42)$$
$$0.25 \neq 0.2646, \boxed{\therefore F^C \text{ and } R^C \text{ are dependant.}}$$