Part 1: Propagation of Uncertainty

1. Refer to Equation 5, differentiate to find the equation for δx.

$$x = 2M_{p+b}m_b^{-1}y^{\frac{1}{2}}\Delta h^{\frac{1}{2}} \quad (5)$$

$$\delta x = \sqrt{\left(\frac{\partial x}{\partial M_{p+b}}\delta M_{p+b}\right)^2 + \left(\frac{\partial x}{\partial m_b}\delta m_b\right)^2 + \left(\frac{\partial x}{\partial y}\delta y\right)^2 + \left(\frac{\partial x}{\partial \Delta h}\delta \Delta h\right)^2}$$

$$\frac{\partial x}{\partial M_{p+b}} = 2m_b y^{\frac{1}{2}}\Delta h^{\frac{1}{2}}$$

$$\frac{\partial x}{\partial m_b} = 2M_{p+b}y^{\frac{1}{2}}\Delta h^{\frac{1}{2}}$$

$$\frac{\partial x}{\partial y} = M_{p+b}m_b^{-1}y^{-\frac{1}{2}}\Delta h^{\frac{1}{2}}$$

$$\frac{\partial x}{\partial \Delta h} = M_{p+b}m_b^{-1}y^{-\frac{1}{2}}\Delta h^{-\frac{1}{2}}$$

$$\boxed{\delta x = \sqrt{\begin{array}{l}\left(2m_b y^{\frac{1}{2}}\Delta h^{\frac{1}{2}}\delta M_{p+b}\right)^2 + \left(2M_{p+b}y^{\frac{1}{2}}\Delta h^{\frac{1}{2}}\delta m_b\right)^2 \\ + \left(M_{p+b}m_b^{-1}y^{-\frac{1}{2}}\Delta h^{\frac{1}{2}}\delta y\right)^2 + \left(M_{p+b}m_b^{-1}y^{-\frac{1}{2}}\Delta h^{-\frac{1}{2}}\delta \Delta h\right)^2\end{array}}}$$

2. Differentiate to find the equation to propagate the uncertainty δL.

$$L_f = m_{ice}^{-1}(c_w m_h + C_d)(T_h - T_f) + c_w(T_{ice} - T_f)$$

$$\delta c_W = 0$$

$$\frac{\partial L}{\partial m_{ice}} = -m_{ice}^{-2}(c_w m_h + C_d)(T_h - T_f)$$

$$\frac{\partial L}{\partial m_h} = m_{ice}^{-1}(c_w T_{ice} - c_w T_f)$$

$$\frac{\partial L}{\partial C_d} = m_{ice}^{-1}(c_d T_{ice} - C_d T_f)$$

$$\frac{\partial L}{\partial T_{ice}} = c_w$$

$$\frac{\partial L}{\partial T_f} = -c_w$$

$$\boxed{\delta L = \sqrt{\begin{array}{l}\left(-m_{ice}^{-2}(c_w m_h + C_d)(T_h - T_f)\delta m_{ice}\right)^2 + \left(m_{ice}^{-1}(c_w T_{ice} - c_w T_f)\delta m_h\right)^2 \\ + \left(m_{ice}^{-1}(c_d T_{ice} - C_d T_f)\delta C_d\right)^2 + (c_w \delta T_{ice})^2 + (-c_w \delta T_f)^2\end{array}}}$$

Part 2: Caliper

3. Measure the outer dimensions of the 1-2-3 Block with the calipers in cm.

3-side	$7.3\text{cm} \pm 0.05\text{cm}$
2-side	$4.8\text{cm} \pm 0.05\text{cm}$
1-side	$2.2\text{cm} \pm 0.05\text{cm}$

4. Convert your measurements and uncertainties to inches, then compare your values to the accepted values of the outer dimensions of the 1-2-3 block.

   3-Side: $2.87\text{in} \pm 0.02\text{in}$; not within accepted value of 3in. 2-Side: $1.89\text{in} \pm 0.02\text{in}$; not within accepted value of 2in. 1-Side: $0.87\text{in} \pm 0.02\text{in}$; not within accepted value of 1in.

Part 3: DMM

5. Measure the voltage of the batteries. Compare to the accepted value.

Measured Voltage	Accepted Value	Comparison
$1.562\text{V} \pm 0.0005\text{V}$	1.5V	Within accepted voltage.
$3.30\text{V} \pm 0.005\text{V}$	6V	Significantly off of accepted voltage; the battery is probably in need of replacement.

6. Measure each of the resistors. Refer to Figure 6. What is the color code value and tolerance for each of your resistors? Are your measurements within the tolerance of each resistor?

Resistor Color Code	Measured Resistance	Accepted Resistance	Tolerance
OOOS	$33.1\text{K }\Omega \pm 0.01\text{K }\Omega$	$33.0\text{K }\Omega$	$\pm 10\%$
bBYS	$101.2\text{K }\Omega \pm 0.01\text{K }\Omega$	$100.0\text{K }\Omega$	$\pm 10\%$

7. Compare your values to the accepted values.

   All of our measured resistances were well within the accepted tolerance range of 10%, that being: $(34.0 \geq 33.1 \geq 32.0)$ and $(110.0 \geq 101.2 \geq 90.0)$.
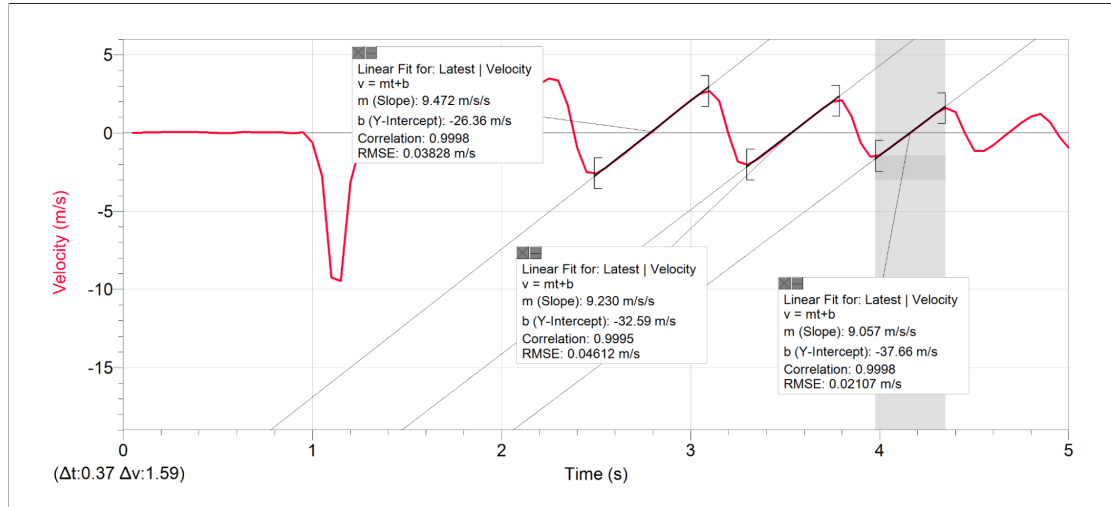
Part 4: LoggerPro Review and Uncertainty Application

8. Measure the mass of the basketball. Report the mass and its uncertainty as $m \pm \delta m$.

   Mass = $599.6\text{g} \pm 0.5$ g = $0.599\text{kg} \pm 0.0005\text{kg}$

9. Analyze the first three positive slopes of the velocity vs. time graph.

   a., b.

Linear Fit for: Latest | Velocity
v = mt+b
m (Slope): 9.472 m/s/s
b (Y-Intercept): -26.36 m/s
Correlation: 0.9998
RMSE: 0.03828 m/s

Linear Fit for: Latest | Velocity
v = mt+b
m (Slope): 9.230 m/s/s
b (Y-Intercept): -32.59 m/s
Correlation: 0.9995
RMSE: 0.04612 m/s

Linear Fit for: Latest | Velocity
v = mt+b
m (Slope): 9.057 m/s/s
b (Y-Intercept): -37.66 m/s
Correlation: 0.9998
RMSE: 0.02107 m/s

(Δt:0.37 Δv:1.59)

   c. Compare to $g = 9.80 m/s^2$ by calculating the percent error for each.

$$E_1 = \frac{|9.80 - 9.472|}{\frac{9.80+9.472}{2}} = \frac{0.33}{9.63} \times 100 = 3.43\%$$

$$E_2 = \frac{|9.80 - 9.230|}{\frac{9.80+9.230}{2}} = \frac{0.57}{9.52} \times 100 = 5.99\%$$

$$E_3 = \frac{|9.80 - 9.057|}{\frac{9.80+9.057}{2}} = \frac{0.74}{9.43} \times 100 = 7.85\%$$

The error found here is within 10%, and gradually increases with each bounce.

This could be due to a torque induced onto the ball when it's been dropped and/or a torque induced when it makes contact with the ground.

There are also energy losses to the ground that make the ball's velocity decrease and thus decrease the amount of data recorded for a bounce.

The range selected for the linear fits also has some variance that could affect the error (although they appear to be selected very accurately).

10. We will need to determine the velocity and its uncertainty uncertainty $(v \pm \delta v)$ from the position vs. time graph. To determine the uncertainty of a measurement from Logger Pro we will need to do a line of best fit.

a., b., c., e.

$$2.215 m/s \pm 0.1583 m/s$$
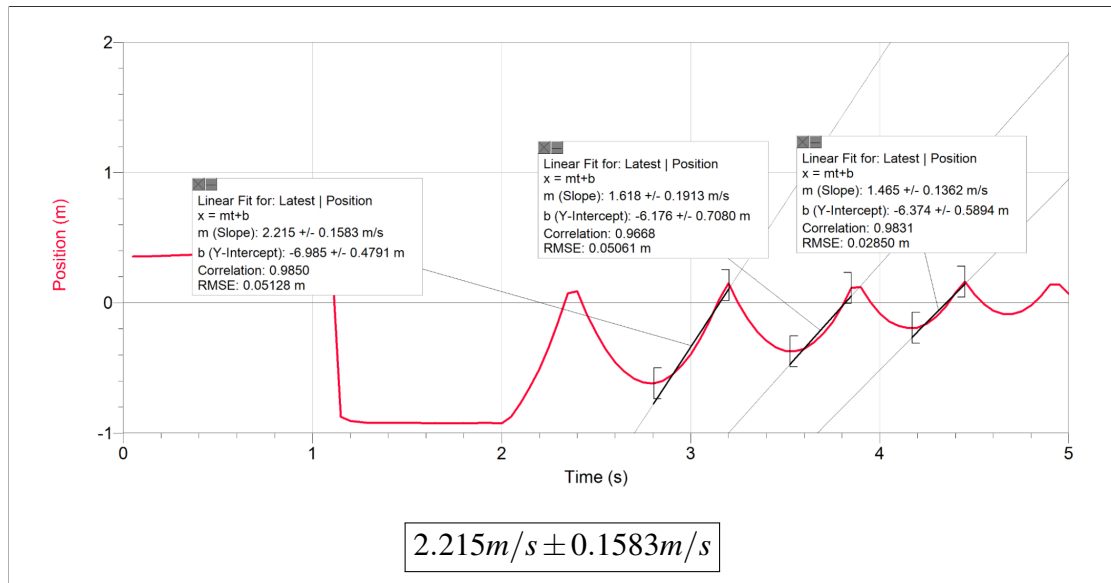
11. Create a calculated column for kinetic energy and graph it.