Kaleb Burris
STAT F300 Statistics, Dr. Short
Worksheet 5, due Monday January 30, 4:15pm

1. If A and B are events with probabilities $P(A) = 0.7$ and $P(B) = 0.9$, is it possible for A and B to be mutually exclusive? Why or why not?

No. $P(A) + P(B) = 1.6$, since probabilities are portions of 1.0, any sum of probabilities > 1.0 are either impossible or have to contain some elements of each other.

If they were mutually exclusive, then $P(A \cap B) = \emptyset = P(\emptyset) = 0.0$.

$$\text{The probability of} \quad P(A \cup B) = (P(A) + P(B)) - P(A \cap B) \quad \leq 1$$
$$(0.7 + 0.9) - 0 = 1.6 \quad \not\leq \quad 1$$

2. In a standard 52-card deck of cards,

(a) How many possible 5-card hands are there?

$$T1 = \text{pick 5 cards} = n_1 = C(52,5)$$
$$n = (n_1) = \left(\frac{52!}{5!(52-5)!}\right) = \left(\frac{52!}{5!(47)!}\right) = \left(\frac{(52)(51)(50)(49)(48)}{120}\right) = \boxed{2{,}598{,}960}$$

(b) How many hands are there that have 3 of a kind (3 with the same numerical value) and 2 singletons?

$$T1 = \text{pick a \# for the three of a kind} = n_1 = 13$$
$$T2 = \text{pick the three of a kind} = n_2 = C(4,3)$$
$$T3 = \text{pick two \#s for singletons} = n_3 = C(11,2)$$
$$T4 = \text{pick higher singleton} = n_4 = C(4,1)$$
$$T5 = \text{pick lower singleton} = n_5 = C(4,1)$$
$$n = (n_1)(n_2)(n_3)(n_4)(n_5) = (13)(\frac{4!}{3!(1!)})(\frac{11!}{2!(9!)})(4)(4)$$
$$= 13(4)(\frac{(11)(10)}{2})(4)(4) = \boxed{45{,}760}$$

(c) What is the probability of getting a hand with 3 of a kind and 2 singletons?

$$\text{\# of possible hands :} \quad 2{,}598{,}960$$
$$\text{\# of special hands :} \quad 45{,}760$$
$$\text{Probability of special hands :} \quad \frac{45{,}760}{2{,}598{,}960} \approx \boxed{0.0176}$$

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3. Calculate each of the following.

(a) Number of "words" consisting of 4 of the letter 'F' and 11 of the letter 'S'. For example, FSSSF SFSFS SSSSS is one such "word". (Ignore the spaces; they're just there to make it easier to count the number of letters.)

We need every permutation of the set $A = \{F,F,F,F\} \cup \{S,S,S,S,S,S,S,S,S,S,S\}$.

$$T1 = \text{Fill 4 positions with 'F's} = n_1 = C(15,4) = \binom{15}{4}$$
$$T2 = \text{Fill the remaining with 'S's} = n_2 = C(11,11) = \binom{11}{11} = 1$$
$$n = (n_1)(n_2) = \left(\frac{15!}{4!(15-4)!}\right)(1) = \frac{15!}{4!(11)!} = \frac{(15)(14)(13)(12)}{4!}$$
$$= \frac{32{,}760}{24} = \boxed{1{,}365}$$

(b) Number of 4-digit PINs with no repeated digits.

Every combination of 4 numbers from the set $\{0,1,2,3\ldots,9\}$.

$$\text{Choose 4 \#s out of } 10 = P(10,4) = \frac{10!}{(10-4)!}$$
$$= \frac{10!}{6!} = (10)(9)(8)(7) = \boxed{5040}$$

(c) Number of ways to seat the 5 VIPs in the 5 seats of the front row at the opera.

Every permutation for 5 to fit into 5.

$$\text{Permute 5 into } 5 = P(5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5! = \boxed{120}$$

(d) umber of ways to seat the 5 VIPs in the 12 seats of the front row at the opera.

Every permutation for 5 to fit into 12.

$$\text{Permute 5 into } 12 = P(5) = \frac{12!}{(12-5)!} = \frac{12!}{7!} = (12)(11)(10)(9)(8) = \boxed{95{,}040}$$