Kaleb Burris
MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set II, Due Thursday, January 24th, 2023; §1.7 # 392, 400, 424

392. $\int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\mathrm{d}x}{\sqrt{1-x^2}}$

$$\text{Using} \quad a^2-u^2: \quad u = a\sin(\theta) \quad \text{where} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$
$$\Rightarrow x = \sin\theta \quad \mathrm{d}x = \cos\theta\,\mathrm{d}\theta \quad \theta = \sin^{-1} x$$
$$x_{lower} = \sin\left(-\frac{1}{2}\right) \quad x_{upper} = \sin\left(\frac{1}{2}\right)$$
$$\int_{\sin\left(-\frac{1}{2}\right)}^{\sin\left(\frac{1}{2}\right)} \frac{\cos\theta\,\mathrm{d}\theta}{\sqrt{1-\sin^2\theta}} = \int_{\sin\left(-\frac{1}{2}\right)}^{\sin\left(\frac{1}{2}\right)} \frac{\cos\theta\,\mathrm{d}\theta}{\sqrt{\cos^2\theta}}$$
$$= \int_{\sin\left(-\frac{1}{2}\right)}^{\sin\left(\frac{1}{2}\right)} \frac{\cos\theta\,\mathrm{d}\theta}{\sqrt{1-\sin^2\theta}} = \int_{\sin\left(-\frac{1}{2}\right)}^{\sin\left(\frac{1}{2}\right)} \frac{\cos\theta\,\mathrm{d}\theta}{\cos\theta}$$
$$= \int_{\sin\left(-\frac{1}{2}\right)}^{\sin\left(\frac{1}{2}\right)} \mathrm{d}\theta = \theta\Big|_{\sin\left(-\frac{1}{2}\right)}^{\sin\left(\frac{1}{2}\right)} = \sin^{-1} x\Big|_{-\frac{1}{2}}^{\frac{1}{2}} = \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)$$
$$= \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \boxed{\frac{\pi}{3}}$$

400. $\int \frac{\mathrm{d}x}{25+16x^2}$

$$\text{Using} \quad a^2+u^2: \quad u = a\tan\theta \quad \text{where} -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$
$$\Rightarrow 4x = 5\tan\theta \quad \mathrm{d}x = \frac{5}{4}\sec^2\theta\,\mathrm{d}\theta \quad \theta = \tan^{-1}\left(\frac{4x}{5}\right)$$
$$\int \frac{5\sec^2\theta\,\mathrm{d}\theta}{4(25+25\tan^2\theta)} = \int \frac{5\sec^2\theta\,\mathrm{d}\theta}{100(1+\tan^2\theta)}$$
$$= \frac{1}{20}\int \frac{\sec^2\theta\,\mathrm{d}\theta}{\sec^2\theta} = \frac{1}{20}\int \mathrm{d}\theta = \frac{\theta}{20} + C = \boxed{\frac{\tan^{-1}\left(\frac{4x}{5}\right)}{20}}$$

424. $\int \frac{e^t}{1+e^t}\,\mathrm{d}t$

$$\text{let} \quad u = 1+e^t \quad \mathrm{d}u = e^t\,\mathrm{d}t$$
$$\int \frac{e^t}{1+e^t}\,\mathrm{d}t \implies \int \frac{\mathrm{d}t}{u} = \ln|u| + C = \boxed{\ln(1+e^t)+C}$$