Kaleb Burris
STAT F300 Statistics, Dr. Short
Worksheet 14, Due February 20, 4:15pm

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Let $X$ be a random variable whose value is the fraction of each can of diced tomatoes that is filled when it leaves the canning factory. The cdf for X is given by:

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^{100} & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x \end{cases}$$

**1.** How do I know that $F(x)$ is a bona fide cdf? What do I need to check? Also, find the pdf of $X$.

You know $F(x)$ is a cdf if the slope of $F(x)$ is never negative as a cdf is the sum of the probabilitie before it. Since those probabilities cannot be zero, the cdf must either be staying the same or increasing. It also much approach 1 as $x \to \infty$ and it must approach 0 as $x \to -\infty$.

For the pdf:

$$\begin{aligned} f(x) &= F'(x) \\ &= \frac{d}{dx}\left[x^{100}\right] \leftarrow \text{Skipping other domains as } \frac{d}{dx} \text{ of a constant is } 0 \\ &= \begin{cases} \boxed{100x^{99}} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

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**2.** Find $\mathbb{E}(X)$, var$(X)$ and sd$(X)$. State, but do not evaluate, the integral whose value is $\mathbb{E}(\log(X))$.

$$\begin{aligned}
\mathbb{E}(X) &= \int_0^1 xf(x)dx \\
&= \int_0^1 x \cdot 100x^{99} \\
&= \int_0^1 100x^{100} \\
&= \left(\frac{100x^{100}}{101}\right)\bigg|_0^1 = \frac{100\cdot(1)^{100}}{101} - \frac{100\cdot(0)^{100}}{101} = \frac{100}{101} \approx \boxed{0.99}
\end{aligned}$$

$$\begin{aligned}
var(X) = \sigma_X^2 &= \int_0^1 \left[(x-\mu_X)^2\right] f(x)dx \\
&= \int_0^1 (x^2 - \mu_X + \mu_X^2)(100x^{99})dx \\
&= \int_0^1 (x^2 - 0.0099)(100x^{99})dx \\
&= \int_0^1 (100x^{101} - 0.99x^{99})dx \\
&= \left(\frac{100x^{102}}{102} - \frac{0.99x^{100}}{100}\right)\bigg|_0^1 = \frac{100\cdot(1)^{102}}{102} - \frac{0.99\cdot(1)^{100}}{100} \\
&= \frac{100}{102} - \frac{99}{100} = 0.98039 - 0.0099 \approx \boxed{0.97049}
\end{aligned}$$

$$\begin{aligned}
sd(X) &= \sqrt{var(X)} \\
&= \sqrt{0.97049} \\
&\approx \boxed{0.985}
\end{aligned}$$

For $\int_a^b f(x) = \mathbb{E}(\log(X))$:

$$\begin{aligned}
\mathbb{E}(\log(X)) &= \int_0^1 (\log(x)f(x))dx \\
&= \boxed{\int_0^1 \left[\log\left(x^{f(x)}\right)\right] dx}
\end{aligned}$$

**3.** There was a disaster. Find $P(X < 0.95)$

$$\begin{aligned} P(X < 0.95) &= F(0.95) \\ &= 0.95^{100} \\ &\approx \boxed{0.0059} \end{aligned}$$

$$1000 \cdot 0.0059 = \boxed{5.9 \text{ cans}}$$

**4.** What is the median of X? What is the 95th percentile of $X$?

Median of $X$:

$$F(a) = 0.5 \qquad = a^{100} = 0.5 = \sqrt[100]{0.5} = \boxed{0.9931}$$

$95^{\text{th}}$ percentile:

$$\begin{aligned} F(a) &= 0.95 \\ &= a^{100} = 0.95 = \sqrt[100]{0.95} \approx \boxed{0.9995} \end{aligned}$$