Kaleb Burris
STAT F300 Statistics, Dr. Short
Homework 3, Due Friday, February 3, 23:59

---

1. Let A and B be events, where $A \subset B$. Show that $P(A) \leq P(B)$. (Hint: B is the disjoint union of $A \text{and} B \cap A^C$. Why? How can this be used to prove the desired result?)

Because $A \subset B, P(B) = P(A) + \text{a value} \geq 0$. The value added must be at least 0 because a probability cannot be negative.

2. If 60% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 80% regularly consume at least one of these two products,

(a) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

$$P(A) = \{\text{People who regularly consume coffee}\} = 0.60$$
$$P(B) = \{\text{People who regularly consume carbonated soda}\} = 0.45$$
$$P(C) = \{\text{People who regularly consume one or the other}\} = 0.80$$

What is $P(A|B)$? What is $P(A \cap B)$?

$$P(C) = P(A \cup B) \Rightarrow 0.80 = P(A \cup B)$$
$$P(A \cap B) = (P(A) + P(B)) - P(A \cup B) = (0.60 + 0.45) - 0.80 = 1.05 - 0.8 = \boxed{0.25}$$

(b) What is the probability that a randomly selected adult consumes one or the other, but not both?

The question is; what is the sum of the probabilities of both $P(A)$ and $P(B)$, but not $P(A \cap B)$?

$$(P(A \cup B)) - P(A \cap B) = 0.80 - 0.25 = \boxed{0.55}$$

3. (Devore §2.3 # 39.) A box in a supply room contains 15 compact fluorescent lightbulbs, of which 5 are rated 13-watt, 6 are rated 18-watt, and 4 are rated 23-watt. Suppose that three of these bulbs are randomly selected.

(a) What is the probability that exactly two of the selected bulbs are rated 23-watt?

$$T1 : \text{Choose 2 32-watt bulbs } = \binom{4}{2} = 6$$

$$T2 : \text{Find the number of pairs of bulbs } = \binom{15}{2} = \frac{15!}{2!(15-2)!} = \frac{15!}{2!(13!)} = \frac{(15)(14)}{2} = \frac{210}{2} = 105$$

$$T3 : \text{Find the probability of two 32-watt bulbs } = \frac{6}{105} \approx \boxed{0.057}$$

(b) What is the probability that all three of the bulbs have the same rating?

$$T1 : \text{Choose 3 32-watt bulbs } = n_1 = \binom{4}{3} = 4$$

$$T2 : \text{Choose 3 18-watt bulbs } = n_2 = \binom{6}{3} = 20$$

$$T3 : \text{Choose 3 13-watt bulbs} = n_3 = \binom{5}{3} = 10$$

$$T4 : \text{Find the number of 3 sets of bulbs } = n_4 = \binom{15}{3} = 455$$

$$T5 : \text{Find the probability of having a set of 3 same-rating bulbs } = \frac{n_1+n_2+n_3}{n_4}$$

$$= \frac{4+20+10}{455} = \frac{34}{455} \approx \boxed{0.0747}$$

(c) What is the probability that one bulb of each type is selected?

$$T1 : \text{Choose 1 32-watt bulb } = n_1 = \binom{4}{1} = 4$$

$$T2 : \text{Choose 1 18-watt bulb } = n_2 = \binom{6}{1} = 4$$

$$T3 : \text{Choose 1 13-watt bulb } = n_3 = \binom{5}{1} = 5$$

$$T4 : \text{Find the number of 3 sets of bulbs } = n_4 = \binom{15}{3} = 455$$

$$T5 : \text{Find the probability of having each type } = \frac{(n_1)(n_2)(n_3)}{n_4} = \frac{(4)(5)(6)}{455}$$

$$= \frac{120}{455} \approx \boxed{0.264}$$

4. The game of Dottie Poker is played with a deck of cards that has 40 cars; there are four suits (clubs, hearts, spades, diamonds) with values Ace, 2, 3, ..., 10. Hands consist of 4 cards.

(a) How many possible hands are there?

$$T1 : \text{Take 4 cards for a hand } = \binom{40}{4} = \frac{40!}{4!(40-4)!} = \frac{40!}{4!(36!)}$$
$$= \frac{(40)(39)(38)(37)}{4!} = \frac{2{,}193{,}360}{24} = \boxed{91{,}390}$$

(b) What is the probability of getting two pairs?

$$T1 : \text{Choose a number } = n_1 = \binom{10}{1} = 10$$
$$T2 : \text{Choose a pair } = n_2 = \binom{4}{2} = 6$$
$$T3 : \text{Choose a second number } = n_3 = \binom{9}{1} = 9$$
$$T4 : \text{Choose a second pairing } = n_4 = \binom{4}{2} = 6$$
$$T5 : \text{Find the number of hands } = n_5 = \binom{40}{4} = 91{,}390$$
$$T4 : \text{Find the probability of a twin-pair } = \frac{(n_1)(n_2)(n_3)(n_4)}{n_5} = \frac{(10)(6)(9)(6)}{91390}$$
$$= \frac{3{,}240}{91{,}390} \approx \boxed{0.0355}$$

(c) What is the probability of getting 3 of one denomination and a singleton?

$$T1 : \text{Choose a number } = n_1 = \binom{10}{1} = 10$$
$$T2 : \text{Choose a three of a kind } = n_2 = \binom{4}{3} = 4$$
$$T3 : \text{Choose a singleton} = n_3 = \binom{36}{1} = 36$$
$$T4 : \text{Find the number of hands } = n_4 = \binom{40}{4} = 91{,}390$$
$$T4 : \text{Find the probability } = \frac{(n_1)(n_2)(n_3)}{n_4} = \frac{(10)(4)(36)}{91{,}390} = \frac{1{,}440}{91{,}390} \approx \boxed{0.0158}$$

5. Dividing 8 players into two groups.

(a) In how many ways can 8 players be divided into teams of size 3 and 5? Explain.

Because the players on either team are complementary, (once one team is made, the other is naturally made), you need only choose 3 players for one team, thus there are $\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!(5)!} = \frac{(8)(7)(6)}{3!} = \frac{336}{24} = \boxed{14}$ possible arrangements of 3 and 5 teams.

(b) In how many ways can 8 players be divided into teams of size 4 and 4? Explain.

The exact same as before, there are $\binom{8}{4} = \frac{8!}{4!(8-4)!} = \frac{8!}{4!(4!)} = \frac{(8)(7)(6)(5)}{4!} = \frac{1{,}680}{24} = \boxed{70}$ arrangements of 4 and 4 teams.