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STAT F300 Statistics, Dr. Short
Worksheet 21, Due March 20, 4:15pm

1. Calculate $\Gamma(5)$, $\Gamma(2)$, $\Gamma(3/2)$ and $\Gamma(7/2)$.

$$\begin{aligned}
\Gamma(5) &= (5-1)! = 4! = \boxed{24} \\
\Gamma(2) &= (2-1)! = \boxed{1} \\
\Gamma(3/2) &\Rightarrow \Gamma(1/2) = \sqrt{\pi}, \quad \Gamma(\alpha) > 1 \text{ then } \Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1) \\
\Gamma(3/2) &= (3/2-1)\Gamma(3/2-1) \\
&= \boxed{\frac{1}{2}\sqrt{\pi}} \\
\Gamma(7/2) &= (7/2-1)\Gamma(7/2-1) = (5/2)\Gamma(5/2) \\
&= (5/2)(5/2-1)\Gamma(5/2-1) = (5/2)(3/2)\Gamma(3/2) \\
&= (15/4)(1/2)\sqrt{\pi} \\
&= \boxed{\frac{15}{8}\sqrt{\pi}}
\end{aligned}$$

2. A sample of asking prices for 2BR/2BA houses in two large U.S. cities were collected. Let $\overline{X_n}$ and $\overline{Y_m}$ be the sample average asking prices for the two cities. All prices are assumed to be independent, within cities and between cities. Let $\mu_X = 268$ and $\mu_Y = 260$ be the average values for the individual houses in the two cities (units are thousands of dollars), and let $\sigma_X = 22$ and $\sigma_Y = 20$ be the standard deviations (also in thousands of dollars).

   (a) Find the expected value and the variance of $\overline{X_n}$ - $\overline{Y_m}$. (Note that the sample sizes, n and m, need not be equal. (Your answer will have both n and m in it.)

$$\begin{aligned}
\mathbb{E}(\overline{X}_n - \overline{Y}_m) &= \mathbb{E}(\overline{X}_n) - \mathbb{E}(\overline{Y}_m) \\
&= 268 - 260 = \boxed{8} \\
\text{var}(\overline{X}_n - \overline{Y}_m) &= \text{var}(\overline{X}_n)/n + (-1)^2\text{var}(\overline{Y}_m)/m \\
&= \boxed{\frac{22}{n} + \frac{20}{m}}
\end{aligned}$$

   (b) If asking prices are normally distributed and the samples are of sizes n = 10 and m = 12, is there enough information to calculate $P(-0.2 \le \overline{X}_n - \overline{Y}_m - 7.0 \le 0.2)$? Why or why not? If yes, estimate the value.

$$\overline{X}_{10} - \overline{Y}_{12} \sim N\left(8, \frac{22}{10} + \frac{20}{12}\right) \approx N(8, 0.1887)$$
$$P(-0.2 \le \overline{X}_n - \overline{Y}_m - 7.0 \le 0.2) = P()$$

(c) If the distribution of the asking prices is unknown, and the samples are of sizes n = 36 and m = 25, is there enough information to calculate $P(-0.2 \leq \overline{X}_n - \overline{Y}_m - 7.0 \leq 0.2)$? Why or why not? If yes, estimate the value.

(d) If the distribution of the asking prices is unknown (not necessarily normal), and the samples are of sizes n = 36 and m = 49, is there enough information to calculate $P(-0.2 \leq \overline{X}_n - \overline{Y}_m - 7.0 \leq 0.2)$? Why or why not? If yes, estimate the value.

3. Let $X_1, \cdots, X_n$ be independent Exponential(θ) random variables.

(a) Show that $\overline{X}$ is biased for θ.

(b) Show that $\overline{X}$ is unbiased for $1/\theta$.