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MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set X, Due Wednesday, April 5, 2023

Find the interval and radius of convergence of:

1. $\sum \frac{kx^k}{5^k}$

$$\sqrt[k]{\left|\frac{kx^k}{5^k}\right|} = \frac{\sqrt[k]{|kx^k|}}{\sqrt[k]{5^k}} = \frac{|x|}{5}$$
$$\frac{|x|}{5} < 1 \Rightarrow |x| < 5 \Rightarrow -5 < x < 5$$
$$\text{Note at } x = 5: \sum \frac{k5^k}{5^k} = \sum k \to \infty, \quad \therefore \sum \frac{k5^k}{5^k} \text{ is Divergent by Divergence test}$$
$$\text{Note at } x = -5: \sum \frac{-k5^k}{5^k} = -\sum k \to -\infty, \quad \therefore \sum \frac{-k5^k}{5^k} \text{ is Divergent by Divergence test}$$
$$\text{Radius} = 5 \quad \text{Interval} = (-5{,}5)$$

2. $\sum \frac{(x-9)^k}{k6^k}$

$$\sqrt[k]{\left|\frac{(x-9)^k}{k6^k}\right|} = \frac{\sqrt[k]{|(x-9)^k|}}{\sqrt[k]{k6^k}} = \frac{|x-9|}{6}$$
$$\frac{|x-9|}{6} < 1 \Rightarrow |x-9| < 6 \Rightarrow -6 < x-9 < 6 \Rightarrow 3 < x < 15$$
$$\text{Note at } x = 3: \sum \frac{(3-9)^k}{k6^k} = \sum \frac{(-6)^k}{k6^k} = \sum \frac{(-1)^k 6^k}{k6^k} = \sum \frac{(-1)^k}{k} \to \text{Converges by OS}$$
$$\text{Note at } x = 15: \sum \frac{(15-9)^k}{k6^k} = \sum \frac{6^k}{k6^k} = \sum \frac{1}{k} \to \text{ Diverges by P-Series}$$
$$\text{Radius} = 6 \quad \text{Interval} = [3{,}15)$$

3. $\sum \frac{(x-1)^k}{k^2 7^k}$

$$\left|\frac{\frac{(x-1)^{k+1}}{(k+1)^2 7^{k+1}}}{\frac{(x-1)^k}{k^2 7^k}}\right| = \frac{|(x-1)^{k+1}|}{|(x-1)^k|} \cdot \frac{(k+1)^2}{k^2} \cdot \frac{7^{k+1}}{7^k} = |(x-1)| \cdot \frac{k^2+2k+1}{k^2} \cdot 7$$
$$\to (x-1) \cdot 1 \cdot 7 = 7|(x-1)|$$
$$7|(x-1)| < 1 \Rightarrow |x-1| < 7 \Rightarrow -7 < x-1 < 7 \Rightarrow -6 < x < 8$$
$$\text{Note at } x = -6: \sum \frac{(-6-1)^k}{k^2 7^k} = \sum \frac{(-7)^k}{k^2 7^k} = \sum \frac{(-1)^k 7^k}{k^2 7^k} = \frac{(-1)^k}{k^2} \to \text{ Converges as OS}$$
$$\text{Note at } x = 8: \sum \frac{(8-1)^k}{k^2 7^k} = \sum \frac{7^k}{k^2 7^k} = \frac{1}{k^2} \to \text{ Converges as P-Series}$$
$$\text{Radius} = 7 \quad \text{Interval} = [-6{,}8]$$

4. $\sum \frac{k^2(x-3)^k}{k!}$

$$\left|\frac{\frac{(k+1)^2(x-3)^{k+1}}{(k+1)!}}{\frac{k^2(x-3)^k}{k!}}\right| = \frac{(k+1)^2}{k^2} \cdot \left|\frac{(x-3)^{k+1}}{(x-3)^k}\right| \cdot \frac{k!}{(k+1)!} = \left(\frac{k+1}{k}\right)^2 \cdot |x-3| \cdot \frac{1}{k} \to 1^2 \cdot |x-3| \cdot 0$$
$$0 < 1, \quad \therefore \text{ Radius} = \infty \quad \text{Interval} = (-\infty, \infty)$$