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Final Review

Part I

1. Evaluate $\int_1^\infty \frac{1}{1+x^2} dx$

$$\int_1^\infty \frac{1}{1+x^2} dx = \lim_{a\to\infty} \int_1^a \frac{1}{1+x^2} dx$$
$$\lim_{a\to\infty} \int_1^a \frac{1}{1+x^2} dx = \lim_{a\to\infty} \Big[\arctan(x)\Big]_1^a = \lim_{a\to\infty} \arctan(a) - \arctan(1)$$
$$= \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$$

2. Suppose:

$x = 4 - \ln(t)$

$y = 1 + \ln(7t)$

$1 \le t \le e$

Compute the arc length.

$$\text{Arc length: } \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$
$$\frac{dx}{dt} = -\frac{1}{t}$$
$$\frac{dy}{dt} = \frac{7}{7t} = \frac{dt}{t}$$
$$\int_1^e \sqrt{\left(-\frac{1}{t}\right)^2 + \left(\frac{1}{t}\right)^2} dt = \int_1^e \sqrt{\frac{1}{t^2} + \frac{1}{t^2}} dt = \int_1^e \sqrt{\frac{2}{t^2}} dt = \sqrt{2} \int_1^e \frac{1}{t} dt$$
$$= \sqrt{2}\Big[\ln(t)\Big]_1^e = \sqrt{2}\Big[\ln(e) - \ln(1)\Big] = \boxed{\sqrt{2}}$$

3. Suppose:

$x = \cos(3+t)$

$y = \sin(3+t)$

What is the area of the region bounded by the graph and the positive x-axis and the positive y-axis?

Part II