Day 1 - 1/17/2024

Vectors in $\mathbb{R}^2$

Defined as a directed segment, $\overrightarrow{AB}$. Similar vectors that are the same when translated onto each other are denoted to be equivalent with $\equiv$.

Vectors with initial point P and terminal point Q are standardized by moving P to the origin and keeing Q relative to P.

For $P = (0, 0)$ and $Q = (3, 3)$, we define $\overrightarrow{AB} = \langle 3, 3 \rangle$, therefore $\overrightarrow{AB} = \langle x_B - x_A, y_B - y_A \rangle$

For any 2 vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$, we can find that $\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{CD} + \overrightarrow{AB}$.

Any vector $\overrightarrow{AA}$ is defined to be directionless and without length.

Vector Multiplication

Scalars - A number.

We define a scalar multiplication with the scalar k to be $2AB = \langle k \cdot (x_B - x_A), k \cdot (y_B - y_A) \rangle$.

Notibly, a scalar multiplication with the scalar k will adjust the length of a vector v to be $k \cdot v$.

kv has length $|k| \; \|v\|$.

Scalars can be negative, reversing a vector $\overrightarrow{AB}$ to be $\overrightarrow{BA}$, inverting its *direction*.

Vector Subtraction

Defined as $v - u = v + (-1)u$

Incredible. Just awesome. Nobody is passing this class.

Example

Define $v - u$.

$$P(1,2)$$
$$Q(2,1)$$
$$A(0,2)$$
$$B(2,2)$$
$$\vec{v} = \overrightarrow{PQ}, \quad \vec{w} = \overrightarrow{AB}$$
$$\vec{u} = 2\vec{v} - \vec{w}$$
$$\vec{v} = \langle 2-1, 1-2 \rangle = \langle 1, -1 \rangle$$
$$\vec{w} = \langle 2-0, 2-2 \rangle = \langle 2, 0 \rangle$$
$$\vec{u} = 2\langle 1, -1 \rangle + -1\langle 2, 0 \rangle$$
$$\vec{u} = \langle 2, -2 \rangle + \langle -2, 0 \rangle$$
$$\vec{u} = \langle 0, -2 \rangle$$

Vector Magnitude

$\vec{v} = \langle a, b \rangle$

How do we find the magnitude/normal/length of $\vec{v}$?

Magnitude of $\vec{v}$: $\|\vec{v}\| = \sqrt{a^2 + b^2}$.

Unit Circle

We define $\vec{i}$ to be $\langle 1, 0 \rangle$ and $\vec{j}$ to be $\langle 0, 1 \rangle$.