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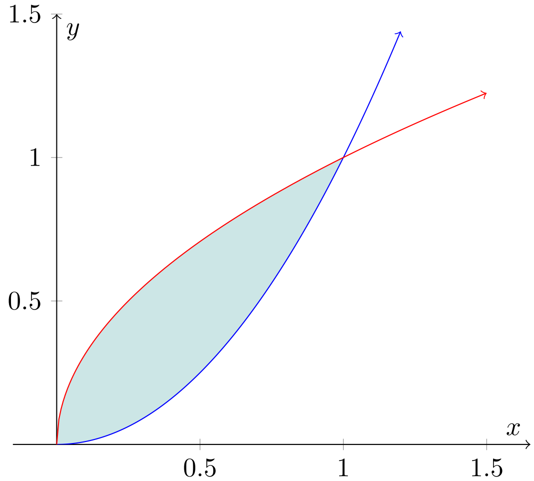
MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Midterm I Review

1. Consider the region in the cartesian plane that is bounded by the graphs of $y = x^2$ and $y = \sqrt{x}$. What is the surface area?



$$\begin{aligned} x^2 &= x^{1/2} \\ x^{3/2} &= 0 \\ x\sqrt{x} &= 0 \\ x &= 0{,}1 \end{aligned}$$

$$\begin{aligned} \text{Surface Area} &= \int_0^1 \left[\sqrt{x} - x^2\right] \mathrm{d}x \\ &= \frac{3x^{3/2}}{2} - \frac{3x^3}{3}\Bigg|_0^1 \\ &= \frac{3}{2} - \frac{3}{3} = \boxed{\frac{1}{2}} \end{aligned}$$

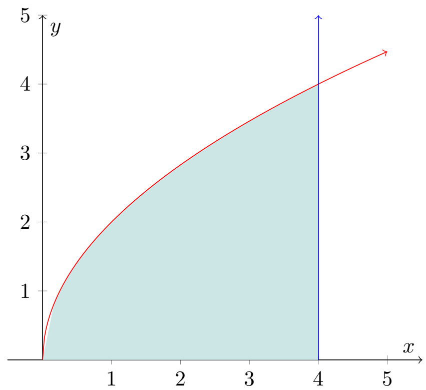
2. Consider the graph of $y = 4x^{3/2}$. Compute the arc-length on the interval $1 \le x \le 3$.

$$\text{Arc-length} = \int_a^b \sqrt{1 + f'(x)^2}\mathrm{d}x$$

$$f'(x) = 6x^{1/2}, \quad a = 1, b = 3$$

$$\begin{aligned} \text{Arc-length} &= \int_1^3 \sqrt{1 + (6x^{1/2})^2}\mathrm{d}x \\ &= \int_1^3 \sqrt{1 + 36x}\mathrm{d}x \\ &= \int_1^3 \sqrt{(6\sqrt{x} - 1)^2}\mathrm{d}x \\ &= \int_1^3 \left(6\sqrt{x} - 1\right) \mathrm{d}x \\ &= \left[4x^{3/2} - x\right]\Big|_1^3 \\ &= (4(3)^{3/2} - 3) - (1 - 1) = \boxed{4\sqrt{27} - 3} \end{aligned}$$

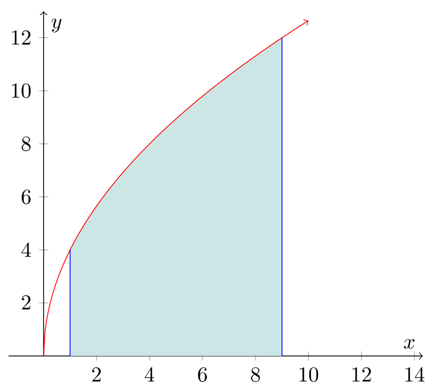
3. Consider the region bounded by $y = 2\sqrt{x}$, the x-axis and $x = 4$. Compute the volume if this is rotated about the y-axis.



Shell Method:

$$\begin{aligned} V &= 4\pi \int_0^4 x\sqrt{x}\mathrm{d}x \\ &= 4\pi \int_0^4 x^{3/2}\mathrm{d}x \\ &= 4\pi \left(\frac{2x^{5/2}}{5}\right)\Bigg|_0^4 \\ &= 4\pi \left(\frac{64}{5}\right) = \boxed{\frac{256\pi}{5}} \end{aligned}$$

4. Consider the graph of $y = 4\sqrt{x}$ where $1 \le x \le 9$. Suppose this is rotated about the x-axis. What is the surface area?



Disk Method:

$$\begin{aligned} V &= 4\pi \int_1^9 \left(\sqrt{x}\right)^2 \mathrm{d}x \\ &= 4\pi \int_1^9 x\mathrm{d}x \\ &= 4\pi(x)\Big|_1^9 = 4\pi(9-1) = \boxed{32\pi} \end{aligned}$$

5. Evaluate $\int_1^2 \frac{\mathrm{d}x}{x\sqrt{1-\ln^2(x)}}$.

$$\int_1^2 \frac{\mathrm{d}x}{x\sqrt{1-\ln^2(x)}}$$

$$u = \ln(x) \qquad -\mathrm{d}u = \frac{1}{x}\mathrm{d}x$$

$$\int_{lower} = \ln(1) = 0 \qquad \int^{upper} = \ln(2)$$

$$\int_0^{\ln(2)} \frac{\mathrm{d}u}{\sqrt{1-u^2}}$$

$$= \arcsin(u)\Big|_0^{\ln(2)} = \boxed{\arcsin(\ln(2))}$$