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STAT F300 Statistics, Dr. Short
Worksheet 1, Due February 15, 4:15pm

The cost, X (in \$), of medical treatment for an individual suffering from a mythical malady (MM) in a mythical country has the pdf,

$$f(x) = k \cdot \exp(-x/500), \quad 0 \le x \le 1000$$

for some positive constant, k.

1. Find the normalizing contant, k.

$$\begin{aligned}
f(x) &= k \cdot e^{(-x/500)} \\
F(x) = 1 &= k \cdot \int_0^{1000} e^{(-x/500)} \mathrm{d}x \\
&= k \cdot \left. \frac{-e^{(-x/500)}}{500} \right|_0^{1000} \\
&= 500k \cdot (-e^{-2} + e^0) \\
&= 500k \cdot (-\frac{1}{e^2} + 1) \\
&= 500k \cdot 0.865 \\
k &= \frac{1}{432.5} = \boxed{0.00231}
\end{aligned}$$

2. Find the probability that the cost for treatment for MM is greater than \$800.

What is F(x) on $800 < x \le 1000$?

$$\begin{aligned}
P(800 < x \le 1000) &= 0.00231 \cdot \int_{800}^{1000} e^{(-x/500)} \mathrm{d}x \\
&= 0.00231 \cdot 500(-e^{(-x/500)} \Big|_{800}^{1000}) \\
&= 0.00231 \cdot 500(-e^{(-2)} - (-e^{(-8/5)})) \\
&= 0.00231 \cdot 500(0.202 - 0.135) \\
&= 0.00231 \cdot 500(0.067) = \boxed{0.0774}
\end{aligned}$$

3. Find the general formula for $P(a \leq X \leq b)$, where $0 \leq a < b \leq 1000$.

$$\begin{aligned}
P(a \leq X \leq b) &= k \cdot \int_a^b e^{-x/500} \mathrm{d}x \\
&= k \cdot 500(-e^{-x/500})\Big|_a^b \\
&= k \cdot 500(e^{-a/500} - e^{-b/500}) \\
&= \boxed{1.155 \cdot (e^{-a/500} - e^{-b/500})}
\end{aligned}$$

4. Find $P(10 \leq X \leq 100)$ and $P(10 < X < 100)$. Why are the answers the same?

$$\begin{aligned}
P(10 \leq X \leq 100) &= 1.155 \cdot (e^{-10/500} - e^{-100/500}) \\
&= 1.155 \cdot (0.98 - 0.819) \\
&= 1.155 \cdot 0.161 = 0.1862
\end{aligned}$$

$$\begin{aligned}
P(10 < X < 100) &= 1.155 \cdot (e^{-10/500} - e^{-100/500}) \\
&= 1.155 \cdot (0.98 - 0.819) \\
&= 1.155 \cdot 0.161 = 0.1862
\end{aligned}$$

They're the same because the inclusion/exclusion of any discrete number on a contininuous graph will result in no net change for the area underneath that graph. The integral for $\int_0^0 f(x)$ will always be the same no matter what $f(x)$ is: 0.