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MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set IX, Due Friday, March 31, 2023

Find the radius and interval of convergence for both of the following:

1 $\sum \frac{x^2}{k3^k}$

$$\sqrt[k]{\left|\frac{x^2}{k3^k}\right|} = \frac{\sqrt[k]{x^2}}{\sqrt[k]{|k3^k|}} = \frac{\sqrt[k]{x^2}}{3}$$

$$\lim_{k\to\infty} \frac{\sqrt[k]{x^2}}{3} < 3 \Rightarrow \lim_{k\to\infty} x^{\frac{2}{k}} < 9 \Rightarrow x^0 < 9 \Rightarrow 1 < 9$$

$$\text{Radius} = \infty \quad \text{Interval} = (-\infty, \infty)$$

2. $\sum \frac{(x-2)^k}{5k^2 4^4}$

$$\left|\frac{\frac{(x-2)^k}{5k^2 4^4}}{\frac{(x-2)^{k+1}}{5(k+1)^2 4^4}}\right| = \left|\frac{(x-2)^k}{5k^2 4^4}\frac{5(k+1)^2 4^4}{(x-2)^{k+1}}\right| = \left|\frac{1}{k^2}\frac{(k+1)^2}{x-2}\right| = \left|\frac{\sqrt{(k+1)^2}}{\sqrt{k^2(x-2)}}\right| = \frac{k+1}{k\sqrt{x-2}}$$

$$\frac{k+1}{k\sqrt{x-2}} \to \frac{1}{\sqrt{x-2}} \Rightarrow \frac{1}{\sqrt{x-2}} < 1 \Rightarrow 1 < \sqrt{x-2} \Rightarrow 1 < x-2 \Rightarrow 3 < x$$

Note: at $x = 3$, $\sum \frac{(3-2)^k}{5k^2 4^4} = \frac{1}{5(4^4)} \sum \frac{1}{k^2}$ the series converges by P-Series

$$\text{Radius} = \infty \quad \text{Interval} = [3, \infty)$$