

Find the interval and radius of convergence of:

1. $\sum \frac{kx^k}{5^k}$

$$\sqrt[k]{\left| \frac{kx^k}{5^k} \right|} = \frac{\sqrt[k]{|kx^k|}}{\sqrt[k]{5^k}} = \frac{|x|}{5}$$

$$\frac{|x|}{5} < 1 \Rightarrow |x| < 5 \Rightarrow -5 < x < 5$$

Note at $x = 5$: $\sum \frac{k5^k}{5^k} = \sum k \rightarrow \infty$, $\therefore \sum \frac{k5^k}{5^k}$ is Divergent by Divergence test

Note at $x = -5$: $\sum \frac{-k5^k}{5^k} = -\sum k \rightarrow -\infty$, $\therefore \sum \frac{-k5^k}{5^k}$ is Divergent by Divergence test

Radius = 5 Interval = $(-5, 5)$

2. $\sum \frac{(x-9)^k}{k6^k}$

$$\sqrt[k]{\left| \frac{(x-9)^k}{k6^k} \right|} = \frac{\sqrt[k]{|(x-9)^k|}}{\sqrt[k]{k6^k}} = \frac{|x-9|}{6}$$

$$\frac{|x-9|}{6} < 1 \Rightarrow |x-9| < 6 \Rightarrow -6 < x-9 < 6 \Rightarrow 3 < x < 15$$

Note at $x = 3$: $\sum \frac{(3-9)^k}{k6^k} = \sum \frac{(-6)^k}{k6^k} = \sum \frac{(-1)^k 6^k}{k6^k} = \sum \frac{(-1)^k}{k} \rightarrow \text{Converges}$

Note at $x = 15$: $\sum \frac{(15-9)^k}{k6^k} = \sum \frac{6^k}{k6^k} = \sum \frac{1}{k} \rightarrow \text{Diverges by P-Series}$

Radius = 6 Interval = $[3, 15)$

3. $\sum \frac{(x-1)^k}{k^2 7^k}$

$$\left| \frac{\frac{(x-1)^{k+1}}{(k+1)^2 7^{k+1}}}{\frac{(x-1)^k}{k^2 7^k}} \right| = \frac{|(x-1)^{k+1}|}{|(x-1)^k|} \cdot \frac{(k+1)^2}{k^2} \cdot \frac{7^k}{7^{k+1}} = |(x-1)| \cdot \frac{k^2 + 2k + 1}{k^2} \cdot 7$$

$$\rightarrow (x-1) \cdot 1 \cdot 7 = 7|(x-1)|$$

$$7|(x-1)| < 1 \Rightarrow |x-1| < 7 \Rightarrow -7 < x-1 < 7 \Rightarrow -6 < x < 8$$

Note at $x = -6$: $\sum \frac{(-6-1)^k}{k^2 7^k} = \sum \frac{(-7)^k}{k^2 7^k} = \sum \frac{(-1)^k 7^k}{k^2 7^k} = \frac{(-1)^k}{k^2} \rightarrow \text{Converges by OS}$

Note at $x = 8$: $\sum \frac{(8-1)^k}{k^2 7^k} = \sum \frac{7^k}{k^2 7^k} = \sum \frac{1}{k^2} \rightarrow \text{Converges as P-Series}$

Radius = 7 Interval = $[-6, 8]$

4. $\sum \frac{k^2(x-3)^k}{k!}$

$$\left| \frac{\frac{(k+1)^2(x-3)^{k+1}}{(k+1)!}}{\frac{k^2(x-3)^k}{k!}} \right| = \frac{(k+1)^2}{k^2} \cdot \left| \frac{(x-3)^{k+1}}{(x-3)^k} \right| \cdot \frac{k!}{(k+1)!} = \left(\frac{k+1}{k} \right)^2 \cdot |x-3| \cdot \frac{1}{k} \rightarrow 1^2 \cdot |x-3| \cdot 0$$

$$0 < 1, \quad \therefore \text{Radius} = \infty \quad \text{Interval} = (-\infty, \infty)$$