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MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Midterm III Review

1. Find if $\sum \frac{1+3^n}{1+4^n}$ converges using ratio or root test.

Using root test:

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{1+3^n}{1+4^n}\right|}$$

$$\lim_{n\to\infty} \sqrt[n]{3^n} \le \lim_{n\to\infty} \sqrt[n]{1+3^n} \le \lim_{n\to\infty} \sqrt[n]{2\cdot 3^n} \Rightarrow 3 \le \sqrt[n]{1+3^n} \le 3$$

$$\therefore \lim_{n\to\infty} \sqrt[n]{1+3^n} = 3$$

$$\lim_{n\to\infty} \sqrt[n]{4^n} \le \lim_{n\to\infty} \sqrt[n]{1+4^n} \le \lim_{n\to\infty} \sqrt[n]{2\cdot 4^n} \Rightarrow 4 \le \sqrt[n]{1+4^n} \le 4$$

$$\therefore \lim_{n\to\infty} \sqrt[n]{1+4^n} = 4$$

$$\therefore \lim_{n\to\infty} \sqrt[n]{\left|\frac{1+3^n}{1+4^n}\right|} = \frac{3}{4} < 1; \quad \boxed{\text{Converges}}$$

2. Find a closed form expression for $\sum \frac{2^k x^{2k}}{k!}$

$$\text{Note that: } e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\frac{2^k x^{2k}}{k!} = \frac{(2x^2)^k}{k!}$$

$$\therefore \sum \frac{2^k x^{2k}}{k!} = \boxed{(2e^2)^x}$$

3.

5. Express $3x\sin(x^3)$ as a Maclaurin series.

$$\text{Note that: } \sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\therefore \quad 3x\sin(x^3) = 3x \sum_{k=0}^{\infty} \frac{(-1)^k (x^3)^{2k+1}}{(2k+1)!} = \boxed{\sum_{k=0}^{\infty} \frac{3(-1)^k x^{6k+4}}{(2k+1)!}}$$