MATH-253: HW1

Due on 1/22/2024

Prof. Oleksandr Bobrovnikov (Sasha), Spring 2024, 1/22/2024

Kaleb Burris

2.1

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a) $\overrightarrow{RP} = \langle -1-(-3), 3-7 \rangle = \underline{\langle 2, -4 \rangle}$

b) $$\overrightarrow{PQ} = \underline{2\vec{i} - 4\vec{j}}$$

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a) $\overrightarrow{PQ} = \langle 2, 2 \rangle, \overrightarrow{PR} = -\overrightarrow{RP} = \langle -2, 4 \rangle$. Therefore:

$$\begin{aligned} 2\overrightarrow{PQ} - 2\overrightarrow{PR} &= 2 \cdot \langle 2, 2 \rangle - 2 \cdot (-2, 4) \\ &= \langle 4, 4 \rangle - (-4, 8) \\ &= \underline{\langle 8, -4 \rangle} \end{aligned}$$

b) $$\underline{8\vec{i} - 4\vec{j}}$$

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$\|\overrightarrow{PQ}\| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

a) $\underline{\langle 1, 1 \rangle}$

b) $\underline{\vec{i} + \vec{j}}$

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$$\vec{v} = \langle 2-(-1), 1-(-3) \rangle = \langle 3, 4 \rangle$$

$$\|\langle v \rangle\| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\underline{\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle}$$

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$$\vec{v} = \langle x-1, 0-1\rangle = \langle x-1, -1\rangle$$
$$\|\vec{v}\| = \sqrt{10} = \sqrt{(x-1)^2 + (-1)^2} = \sqrt{x^2 - 2x + 2}$$
$$10 = x^2 - 2x + 2$$
$$0 = x^2 - 2x - 8$$
$$0 = (x+2) \cdot (x-4)$$
$$x = 2, -4$$
$$x \text{ must be} - 4 : \underline{Q(-2, 0)}$$

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a)
$$\vec{a} = \langle -2, 4\rangle \quad \vec{b} = \langle -2, 2\rangle$$
$$-3\langle -2, 4\rangle + \langle -2, 2\rangle - 4i + j = \langle 6, 12\rangle + \langle -2, 2\rangle + \langle -4, 0\rangle + \langle 0, 1\rangle$$
$$= \langle 0, 15\rangle; \quad \|\langle 0, 15\rangle\| = \underline{15}$$

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$$\|v\| = 3, u = \langle -2, 5\rangle$$
$$\sqrt{(-2)^2 + 5^2} = \sqrt{29}$$
$$3\left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle = \underline{\left\langle \frac{-6}{\sqrt{29}}, \frac{15}{\sqrt{29}} \right\rangle}$$

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$$\vec{u} = 6\left\langle \cos(60^\circ), \sin(60^\circ) \right\rangle = \underline{\left\langle 3, 3\sqrt{3} \right\rangle}$$

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$$\vec{u} = 8\langle -1, 0\rangle = \underline{\langle -8, 0\rangle}$$

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$$\alpha = 0; \beta = 0$$
$$\alpha = 2\beta$$
$$\underline{\alpha \in \mathbb{R}; \beta = 2\alpha}$$

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a)
$$f'(x) = 4x^3; f'(1) = 4$$
$$y = 4x + 1 \Rightarrow y = 4 + 1 = 5$$
$$\underline{Q(2, 5)}$$

b) $\overrightarrow{PQ} = \langle 2 - 1, 5 - 1 \rangle = \langle 1, 4 \rangle; \|u\| = \sqrt{1^2 + 4^2} = \sqrt{17}$
$$\underline{\vec{u} = \left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle}$$

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$$\theta = 30^\circ$$
$$\vec{v} = 100\left\langle \cos\left(30^\circ\right), \sin\left(30^\circ\right) \right\rangle = \left\langle 50, 50\sqrt{3} \right\rangle$$
$$\underline{\text{Horizontal: 50.00 mph, Vertical: 86.60 mph}}$$

2.2

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a) $\underline{(0, 3, 0), (2, 0, 0), (0, 0, 5), (2, 0, 5), (0, 3, 5), (2, 3, 0)}$

b)
$$\sqrt{2^2 + 3^2 + 5^2} = \sqrt{4 + 9 + 25} = \underline{\sqrt{38}}$$

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$$z = 2; z = 5$$
$$\text{The set is 2 planes at } z = 2 \text{ and } z = 5$$

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