MATH-253: HW2

Due on 1/31/2024

Prof. Oleksandr Bobrovnikov (Sasha), Spring 2024, 1/31/2024

Kaleb Burris

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a) $\langle 3, 2, -1 \rangle \times \langle 1, 1, 0 \rangle = i(0+1) - j(0+1) + k(3-2) = \underline{i + j + k}$

b) Image

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$$j \times (k \times j + 2j \times i - 3j \times j + 5i \times k)$$
$$= j \times (-i - 2k - 5j) = \underline{k - 2i}$$

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$$w = \frac{u \times v}{\|u \times v\|}$$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & 6 & 1 \\ 3 & 0 & 1 \end{vmatrix} = i(6-0) - j(2-3) + k(0-18) = 6i + j - 18k$$

$$\|u \times v\| = \sqrt{6^2 + 1^2 + 18^2} = \sqrt{361} = 19$$

$$w = \frac{u \times v}{\|u \times v\|} = \underline{\frac{6i}{19} + \frac{j}{19} - \frac{18k}{19}}$$

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$\vec{u} \times \vec{v}$ is by definition orthogonal to both $\vec{u}$ and $\vec{v}$. If we take the dot product of two vectors to be 0, that also defines them as being orthogonal. Therefore, $(\vec{u} \times \vec{v}) \cdot (\vec{u} + \vec{v}) = 0 + 0$, and $(\vec{u} \times \vec{v}) \cdot (\vec{u} - \vec{v}) = 0 - 0$, making $(\vec{u} \times \vec{v})$ orthogonal to $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$

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$$\sin\alpha = \frac{\|u \times v\|}{\|u\|\|v\|}$$

$$u \times v = \begin{vmatrix} i & j & k \\ -1 & 3 & 1 \\ 1 & -2 & 0 \end{vmatrix} = i(0+2) - j(0-1) + k(2-3) = 2i + j - k$$

$$\|2i + j - k\| = \sqrt{2^2 + 1 + 1} = \sqrt{6}$$

$$\|u\| = \sqrt{1 + 3^2 + 1} = \sqrt{11}; \quad \|v\| = \sqrt{1 + 2^2} = \sqrt{5}$$

$$\sin\alpha = \frac{\sqrt{6}}{\sqrt{11} \cdot \sqrt{5}}$$

$$\alpha = \arcsin\left(\frac{\sqrt{6}}{\sqrt{11} \cdot \sqrt{5}}\right) \approx 19.29^{\circ} \leftarrow \text{Not obtuse!}$$

$$180 - 19.29 = 160.71; \lceil 160.71 \rceil \approx \underline{161^{\circ}}$$

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$$A = \|u \times v\|$$

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = i(1+0) - j(0-1) + k(1-0) = i + j + k$$

$$\|u \times v\| = \sqrt{1 + 1 + 1} = \underline{\sqrt{3}}$$

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a)