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MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set VII, Due Friday, March 3, 2023

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1. Evaluate $\int_{-1}^{1} \frac{4}{x^2}\,dx$.

$$\int_{-1}^{1} \frac{4}{x^2}dx = \int_{-1}^{0} \frac{4}{x^2}dx + \int_{0}^{1} \frac{4}{x^2}dx$$
$$\int_{-1}^{0} \frac{4}{x^2}dx \Rightarrow \lim_{u\to 0^-} \int_{-1}^{u} \frac{4}{x^2}dx = \lim_{u\to 0^-} \left[-\frac{4}{x}\right]_{-1}^{u} = \lim_{u\to 0^-} -\left[\frac{4}{u} - \frac{4}{-1}\right]$$
$$= -[-\infty + 4] \quad \boxed{\text{Diverges}}$$

2. Evaluate $\int \frac{dx}{\sqrt{-x^2-2x}}$.

$$\int \frac{dx}{\sqrt{-x^2-2x}} = \int \frac{dx}{\sqrt{1-(x+1)^2}}$$
$$\text{let } u = x+1 \quad du = dx$$
$$\Rightarrow \int \frac{du}{\sqrt{1-u^2}} = \arcsin(u) + C = \boxed{\arcsin(x+1)+C}$$

3. Evaluate $\lim_{n\to\infty} \frac{10n^6+100n}{2(4+n^6)}$.

$$\lim_{n\to\infty} \frac{10n^6+100n}{2(4+n^6)} = \frac{10}{2}\lim_{n\to\infty} \frac{n^6+10n}{4+n^6}$$
$$= \frac{10}{2}\lim_{n\to\infty} \frac{\frac{n^6}{n^6}+\frac{10n}{n^6}}{\frac{4}{n^6}+\frac{n^6}{n^6}} = \frac{10}{2}\lim_{n\to\infty} \frac{1+\frac{10}{n^5}}{\frac{4}{n^6}+1} \to \frac{10}{2}\cdot\frac{1+0}{0+1} = \frac{10}{2} = \boxed{5}$$

4. Evaluate $\lim_{n\to\infty} \frac{\ln(7+6n^8)}{\ln(1000+n)}$.

$$\lim_{n\to\infty} \frac{\ln(6n^8)}{\ln(n)} \le \lim_{n\to\infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} \le \lim_{n\to\infty} \frac{\ln(10n^8)}{\ln(n)}$$
$$= \lim_{n\to\infty} \frac{8\ln(n)+\ln(6)}{\ln(n)} \le \lim_{n\to\infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} \le \lim_{n\to\infty} \frac{8\ln(n)+\ln(10)}{\ln(n)}$$
$$= \lim_{n\to\infty} 8+\frac{\ln(6)}{\ln(n)} \le \lim_{n\to\infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} \le \lim_{n\to\infty} 8+\frac{\ln(10)}{\ln(n)}$$
$$= 8 \le \lim_{n\to\infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} \le 8 \quad \therefore \lim_{n\to\infty} \frac{\ln(7+6n^8)}{\ln(1000+n)} = \boxed{8}$$

5. Evaluate $\lim_{n\to\infty} \sqrt[n]{n^{100}+13}$.

$$\lim_{n\to\infty} \sqrt[n]{n^{100}} \le \lim_{n\to\infty} \sqrt[n]{n^{100}+13} \le \lim_{n\to\infty} \sqrt[n]{2n^{100}}$$
$$a = \lim_{n\to\infty} n^{100/n} \Rightarrow \ln(a) = \lim_{n\to\infty} \frac{100\cdot\ln(n)}{n} \overset{(H)}{=} \lim_{n\to\infty} \frac{\frac{100}{n}}{1} = \frac{0}{1} = 0\ln(a) = 0 \Rightarrow e^{\ln(a)} = e^0 = 1$$
$$b = \lim_{n\to\infty} 2n^{100/n} \Rightarrow \ln(b) = \lim_{n\to\infty} \frac{100\cdot\ln(2n)}{n} \overset{(H)}{=} \lim_{n\to\infty} \frac{\frac{100}{2n}}{1} = \frac{0}{1} = 0\ln(b) = 0 \Rightarrow e^{\ln(a)} = e^0 = 1$$
$$1 \le \lim_{n\to\infty} \sqrt[n]{n^{100}+13} \le 1 \quad \therefore \lim_{n\to\infty} \sqrt[n]{n^{100}+13} = \boxed{1}$$