# Day 15 - 2/12/2024

## Multivariate Functions

Suppose

$$f : (a, b) \to \mathbb{R}; c \in (a, b)$$

$$\lim_{x \to c} \varepsilon > 0, \exists \delta > 0, 0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

$$g : E \to \mathbb{R}; E \subset \mathbb{R}^2; c \in E$$

$$\lim(x \to c) f(x) = L$$

$$\forall \varepsilon > 0, \exists \delta > 0, 0 < \|x - c\| < \delta \Rightarrow |g(x) - L| < \varepsilon$$

$$\lim_{x \to c^-} f(x) = L$$

$$\lim_{x \to c^+} f(x) = L$$

### Limits

Examples:

1.
$$\lim_{(x,y) \to (-1,0)} \left(2x^2 + 4x + 3xy + 2y^2 + e^y\right)$$
$$= \left(2(-1)^2 + 4(-1) + 3(-1)(0) + 2(0)^2 + e^0\right)$$
$$= 2 - 4 + 1 = -1$$

2.
$$\lim_{(x,y) \to (0,0)} \frac{x + 1}{x^2 + y^2}; \text{Use polar coordinates:}$$
$$(x, y) \to 0 \Leftrightarrow \|\langle x, y \rangle - \langle 0, 0 \rangle\| \to 0$$
$$\sqrt{x^2 + y^2} = r$$
$$\lim_{(x,y) \to (0,0)} \frac{x + 1}{x^2 + y^2} = \lim_{r \to 0} \frac{\overbrace{r \cos \theta + 1}^{1}}{\underbrace{r^2}_{0}} = +\infty$$

3.
$$\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}} = \lim_{r\to 0} \frac{r(\cos\theta+\sin\theta)}{r} = \lim_{r\to 0} \cos\theta + \sin\theta$$

$$\theta \in \mathbb{R}; \sin\theta + \cos\theta > 0$$

$$\text{limit does not exist}$$

4.
$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

$$\text{Approach along x = y : } \lim_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$$

$$\text{Approach along x = 0 : } \lim_{y\to 0} \frac{0}{y^2} = 0$$

$$\text{Approach along x = y\^{}2 : } \lim_{y\to 0} \frac{y^3}{y^4+y^2} = \lim_{y\to 0} \frac{y}{y^2+1} = 0$$