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STAT F300 Statistics, Dr. Short
Worksheet 16, Due March 3, 4:15pm

1.

(a) Find$p_{Y|X}(y|1)$, and show that this is a pmf.

$$\sum_x p_X(1) = 0.13 + 0.05 + 0.02 = 0.20$$

$p_{Y|X}(y|1) =$

Y	$p_{Y\|X}(y\|1)$
0	$0.13/0.20 = 0.65$
1	$0.05/0.20 = 0.25$
2	$0.02/0.20 = 0.10$
	$0.65 + 0.25 + 0.10 = 1.00$

$\sum_Y p_{Y|X}(y|1) = 1.00 \quad \therefore p_{Y|X}(y|1)$ is a pmf

(b) Find $\mathbb{E}(Y|1)$. Interpret this value.

$$\begin{aligned}\mathbb{E}(Y|1) = \mathbb{E}(Y|X=1) &= \sum_Y y \cdot p_{x|y}(y|1) \\ &= 0(0.65) + 1(0.25) + 2(0.10) = 0.25 + 0.2 = \boxed{0.45}\end{aligned}$$

Given that we went on a single hike with our dog on any given, the expected number of trips to Chicago that day is 0.45.

(c) Are X and Y independent? Explain.

$$\sum_X p_X(1) = 0.20; \quad \sum_Y (0) = 0.13 + 0.53 + 0.24 = 0.90$$

$$\begin{aligned}p(X=1, Y=0) &\stackrel{?}{=} \sum_X p_X(1) \cdot \sum_Y (0) \\ 0.13 &\stackrel{?}{=} 0.2 \cdot 0.90 \\ 0.13 &\neq 0.18 \quad \therefore \text{ X and Y are dependent}\end{aligned}$$

2.

(a) Find $f_{X|Y}(x|y)$ when $y > 5$ or $y < 3$.

$$f_{X|Y}(x|y) = \frac{p(x,y)}{p(y)}(3 > y, y > 5) = \frac{\frac{1}{30}x^2}{0} \quad \boxed{\text{always undefined}}$$

(b) Find $f_{X|Y}(x|y)$ when $3 < y < 5$. Show that this is a pdf.

$$\begin{aligned} f_{X|Y}(x|y) &= \frac{p(x,y)}{p(y)}(3 < y < 5) \\ &= \begin{cases} \frac{\frac{1}{30}\left[3x^2+2\left(\frac{7}{15}+\frac{1}{15}y\right)\right]}{\frac{7}{15}+\frac{1}{15}y} & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

(c) Find $\mathbb{E}(X|4)$.

$$\begin{aligned} \mathbb{E}(X|4) = \mathbb{E}(X|Y=4) &= \int_1^2 f(x,4)dx = \int_1^2 \left[\frac{1}{30}(x^2+2(4))\right]dx = \int_1^2 \left[\frac{1}{30}(x^2+8)\right]dx \\ &= \frac{1}{30}\left[\frac{x^3}{3}+x\right]_1^2 = \frac{1}{30}\left[\left(\frac{2^3}{3}+2\right)-\left(\frac{1^3}{3}+1\right)\right] \\ &= \frac{1}{30}\left[\left(\frac{8}{3}+2\right)-\left(\frac{1}{3}+1\right)\right] = \frac{1}{30}\left[\frac{7}{3}+1\right] = \frac{1}{30}\left[\frac{9}{3}\right] = \frac{9}{90} \\ &= \boxed{\frac{1}{10} = 0.10} \end{aligned}$$

(d) Are X and Y independent? Explain.

$$\begin{aligned} f_{X|Y}(x|y) &\stackrel{?}{=} f_X(x) \cdot f_Y(y) \\ \frac{1}{30}(3x^2+2y) &\stackrel{?}{=} \left(\frac{1}{5}x^2+\frac{8}{15}\right)\left(\frac{7}{30}+\frac{1}{15}y\right) \\ \frac{1}{30}(3x^2+2y) &\neq \left(\frac{7}{150}x^2+\frac{1}{15}x^2y+\frac{8}{225}y+\frac{8}{225}\right) \\ &\neq \therefore X \text{ and } Y \text{ are dependent} \end{aligned}$$