

1. If A and B are events with probabilities $P(A) = 0.7$ and $P(B) = 0.9$, is it possible for A and B to be mutually exclusive? Why or why not?

No. $P(A) + P(B) = 1.6$, since probabilities are portions of 1.0, any sum of probabilities > 1.0 are either impossible or have to contain some elements of each other.

If they were mutually exclusive, then $P(A \cap B) = \emptyset = P(\emptyset) = 0.0$.

$$\begin{aligned} \text{The probability of } P(A \cup B) &= (P(A) + P(B)) - P(A \cap B) \leq 1 \\ (0.7 + 0.9) - 0 &= 1.6 \not\leq 1 \end{aligned}$$

2. In a standard 52 card deck of cards,

- (a) How many possible 5-card hands are there?

$$\begin{aligned} T1 &= \text{pick 5 cards} = n_1 = C(52, 5) \\ n &= (n_1) = \left(\frac{52!}{5!(52-5)!} \right) = \left(\frac{52!}{5!(47)!} \right) = \left(\frac{(52)(51)(50)(49)(48)}{120} \right) = \boxed{2,598,960} \end{aligned}$$

- (b) How many hands are there that have 3 of a kind (3 with the same numerical value) and 2 singletons?

$$\begin{aligned} T1 &= \text{pick a \# for the three of a kind} = n_1 = 13 \\ T2 &= \text{pick the three of a kind} = n_2 = C(4, 3) \\ T3 &= \text{pick two \#s for singletons} = n_3 = C(11, 2) \\ T4 &= \text{pick higher singleton} = n_4 = C(4, 1) \\ T5 &= \text{pick lower singleton} = n_5 = C(4, 1) \\ n &= (n_1)(n_2)(n_3)(n_4)(n_5) = (13)\left(\frac{4!}{3!(1!)}\right)\left(\frac{11!}{2!(9!)}\right)(4)(4) \\ &= 13(4)\left(\frac{(11)(10)}{2}\right)(4)(4) = \boxed{45,760} \end{aligned}$$

- (c) What is the probability of getting a hand with 3 of a kind and 2 singletons?

$$\begin{aligned} \# \text{ of possible hands} &: 2,598,960 \\ \# \text{ of special hands} &: 45,760 \\ \text{Probability of special hands} &: \frac{45,760}{2,598,960} \approx \boxed{0.0176} \end{aligned}$$

3. Calculate each of the following.

(a) Number of “words” consisting of 4 of the letter ‘F’ and 11 of the letter ‘S’. For example, FSSSF SFSFS SSSSS is one such “word”. (Ignore the spaces; they’re just there to make it easier to count the number of letters.)

We need every permutation of the set $A = \{F, F, F, F\} \cup \{S, S, S, S, S, S, S, S, S, S, S\}$.

$T1 = \text{Fill 4 positions with ‘F’s} = n_1 = C(15, 4)$

$T2 = \text{Fill the remaining with ‘S’s} = n_2 = C(11, 11)$