Day 6 - 1/25/2024

Equation of a Plane

$$\text{Given point } P(p_1, p_2, p_3) \text{ on the plane :}$$
$$a(x - p_1) + b(y - p_1) + c(z - p_3) = 0$$
$$ax + by + cz - \underbrace{ap_1 - bp_2 - cp_3}_{d} = 0$$
$$\underbrace{ax + by + cz + d = 0}_{\text{Final Form}}$$
$$\text{In this form, } \langle a, b, c \rangle \text{ is the normal vector from } P$$

$$\text{Given } Q \in \text{plane :}$$
$$d = \frac{\overrightarrow{QP} \cdot \vec{n}}{\|\vec{n}\|}$$

Plane A is spanned by $\vec{c} = \langle 1, 0, 4 \rangle$ and $\vec{b} = \langle 2, 1, 0 \rangle$, and A passes through $(1, 1, 1)$.

Find the equation of A, and find the distance between A and $P(0, 0, 0)$.

$$\text{1. Find } \vec{u} = \vec{b} \times \vec{c} = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 0 \end{vmatrix}$$
$$= i(0 - 3) - j(0 - 6) + k(1 - 0) = \langle -3, 6, 1 \rangle$$
$$\underline{-3(x - 1) + 6(y - 1) + 1(z - 1) = 0}$$
$$\text{2. Find the distance between } A \text{ and } P(0, 0, 0)$$
$$\frac{-3(0 - 1) + 6(0 - 1) + (0 - 1)}{\sqrt{(-3)^2 + 6^2 + 1^2}} = \frac{3 - 36 - 1}{\sqrt{9 - 6 - 1}} = \underline{\frac{-4}{\sqrt{46}}}1$$