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Examples:

- $x = t^2 + 1, \quad y = 2t^2, \quad x \geq 1$

  Here we can express x as a function of t:

$$\begin{aligned}
x - 1 &= t^2 \\
t &= \pm\sqrt{x-1} \\
y &= 2(\pm\sqrt{x-1})^2 \\
&= 2(x-1) \\
&= 2x - 2 \\
\text{Note that: } x \geq 1, &\text{ and if } t = 0, x = 1 \\
x &= \cos t \\
y &= \sin t \\
x^2 &= \cos^2 t \\
y^2 &= \sin^2 t \\
x^2 + y^2 &= 1
\end{aligned}$$

- Suppose:

$$\begin{aligned}
x &= a + bt \\
y &= c + dt \\
t &= \frac{x-a}{b} \\
\text{Then:} & \\
y &= c + d\frac{x-a}{b} \\
&= c + \frac{d}{b}x - \frac{da}{t} \\
&= \frac{d}{b}x + \left(c - \frac{da}{t}\right) \\
x &= a(t - \sin t) \\
y &= a(1 - \cos t)
\end{aligned}$$

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The Calculus of parameterized curves

Remember that:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \tag{1}$$

Therefore, for the derivative of a circle:

$$\begin{aligned} x &= \cos t \\ y &= \sin t \\ \frac{dy}{dx} &= \frac{\cos t}{-\sin t} = -\cot t \end{aligned} \tag{2}$$

At different values of t:

$$\begin{aligned} t = \pi/2 : & \frac{\cos(\pi/2)}{-\sin(\pi/2)} = \frac{0}{-1} = 0 \\ t = 0 : & \frac{\cos(0)}{-\sin(0)} \quad = \frac{1}{0} \quad = \text{Undefined} \\ t = \pi : & \frac{\cos(\pi)}{-\sin(\pi)} \quad = \frac{-1}{0} = \text{Undefined} \end{aligned}$$