Kaleb Burris
STAT F300 Statistics, Dr. Short
Worksheet 18, Due March 6, 4:15pm

1.

(a) Find $\mathbb{E}(X)$, $\mathbb{E}(X^2)$, and $\text{var}(X)$.

$$\begin{aligned}
\mathbb{E}(X) &= \sum_x \sum_x xp(x,y) \\
&= 1(0.20) + 2(0.55) + 3(0.25) \\
&= 0.20 + 1.10 + 0.75 = \boxed{2.05} \\
\mathbb{E}(X^2) &= \sum_y \sum_x x^2 p(x,y) \\
&= 1^2(0.20) + 2^2(0.55) + 3^2(0.25) \\
&= 0.20 + 2.20 + 2.25 = \boxed{4.65} \\
\text{var}(X) &= \mathbb{E}(X^2) - \mathbb{E}(X)^2 \\
&= 4.65 - (2.05)^2 = 4.65 - 4.2025 = \boxed{0.4475}
\end{aligned}$$

(b) Find $\text{var}(Y)$.

$$\begin{aligned}
\text{var}(X) &= \mathbb{E}(Y^2) - \mathbb{E}(Y)^2 \\
&= \sum_Y y^2 p_Y(2) - \left(\sum_Y y p_Y(Y)\right)^2 \\
&= 0.12 - (0.16)^2 = 0.12 - 0.0256 = \boxed{0.0944}
\end{aligned}$$

(c) Find $\mathbb{E}(XY)$.

$$\begin{aligned}
\mathbb{E}(XY) &= \sum_X \sum_Y xyp(x,y) \\
&= (1)(1)(0.05) + (1)(2)(0.02) + (2)(1)(0.02) + (3)(1)(0.01) \\
&= 0.05 + 0.04 + 0.04 + 0.03 = \boxed{0.16}
\end{aligned}$$

(d) Find $\text{cov}(X,Y)$, $\text{cor}(X,Y)$.

$$\begin{aligned}
\text{cov}(X,Y) &= \mathbb{E}[(X-\mu_X)(Y-\mu_Y)] \\
&\Rightarrow \mathbb{E}(XY) - \mu_X\mu_Y \\
&= 0.16 - (2.05)(0.16) = \boxed{-0.168} \\
\rho_{XY} &= \frac{\text{cov}(XY)}{\text{sd}(X)\text{sd}(Y)} \\
&= \frac{-0.168}{\sqrt{2.05}\sqrt{0.16}} \\
&= -\frac{0.168}{(1.4318)(0.40)} \approx \boxed{-0.2933}
\end{aligned}$$

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2.

(a) Find $\mathbb{E}(X)$, $\mathbb{E}(X^2)$, and $\text{var}(X)$.

$$\begin{aligned}
\mathbb{E}(X) &= \int_{-\infty}^{\infty} x f_X(x) dx \\
&= \int_1^2 x\left(\frac{1}{5}x^2 + \frac{8}{15}\right) dx = \left[\frac{x^3}{15} + \frac{8x^2}{30}\right]_1^2 = \left[\left(\frac{2^3}{15} + \frac{8(2)^2}{30}\right) - \left(\frac{1^3}{15} + \frac{8(1)^2}{30}\right)\right] \\
&= \frac{7}{15} + \frac{24}{30} = \boxed{\frac{38}{30} = 1.2\overline{66}} \\
\mathbb{E}(X^2) &= \int_{-\infty}^{\infty} x^2 f_X(x) dx \\
&= \int_1^2 x^2\left(\frac{1}{5}x^2 + \frac{8}{15}\right) dx = \left[\frac{x^4}{20} + \frac{8x^3}{45}\right]_1^2 \overset{\text{wolfram alpha}}{\approx} \boxed{2.4844} \\
\text{var}(X) &= \mathbb{E}(X^2) - \mathbb{E}(X)^2 \\
&= 2.4844 - (1.2666)^2 = 2.4844 - 1.6044 = \boxed{0.88}
\end{aligned}$$

(b) Find $\text{var}(Y)$.

$$\begin{aligned}
\text{var}(Y) &= \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2 \\
&= 16.689 - (4.0444)^2 = 16.689 - 16.3572 = \boxed{0.3318}
\end{aligned}$$

(c) Find $\mathbb{E}(XY)$.

$$\begin{aligned}
\mathbb{E}(XY) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf(x,y)dxdy \\
&= \int_3^5\int_1^2 xy\left(\frac{1}{30}(3x^2+2y)\right) dxdy \\
&= \frac{1}{30}\int_3^5 \left[x^3 + 2yx\right]_1^2 dy = \frac{1}{30}\int_3^5 [7+3y]dy = \frac{1}{30}\left[7y + \frac{3y^2}{2}\right]_3^5 \\
&= \frac{1}{30}\left[\left(7(5) + \frac{3(5)^2}{2}\right) - \left(7(3) + \frac{3(3)^2}{2}\right)\right] = \frac{1}{30}\left[21 + \frac{54}{2}\right] = \boxed{\frac{48}{30} = 1.6}
\end{aligned}$$

(d) Find $\text{cov}(X,Y)$, $\text{cor}(X,Y)$.

$$\begin{aligned}
\text{cov}(X,Y) &= \mathbb{E}[(X-\mu_X)(Y-\mu_Y)] \\
&\Rightarrow \mathbb{E}(XY) - \mu_X\mu_Y = 1.6 - (1.2667)(4.0444) \approx \boxed{-3.523} \\
\rho_{XY} &= \frac{\text{cov}(XY)}{\text{sd}(X)\text{sd}(Y)} \\
&= \frac{-3.523}{\sqrt{0.88}\sqrt{0.3318}} \\
&= -\frac{3.523}{(0.9381)(0.5760)} \approx \boxed{-6.5197}
\end{aligned}$$