

1. Suppose $f(x) = x^2 - x$ what is the average value of f on $[-1, 1]$?

$$\begin{aligned}\bar{f} &= \frac{1}{b-a} \int_a^b f(x) dx \quad \text{where } f(x) = x^2 - x \\ \Rightarrow \bar{f} &= \frac{1}{1 - (-1)} \int_{-1}^1 (x^2 - x) dx = \frac{1}{2} \left(\frac{x^3}{3} - \frac{x^2}{2} \right) \Big|_{-1}^1 \\ &= \left(\frac{2x^3 - 3x^2}{12} \right) \Big|_{-1}^1 = \left(-\frac{1}{12} \right) - \left(-\frac{5}{12} \right) = \frac{4}{12} = \boxed{\frac{1}{3} \approx 0.333}\end{aligned}$$

2. Evaluate $2 + 4 + 6 + \cdots + 100$.

$$\begin{aligned}2 + 4 + 6 + \cdots + 100 &= \sum_{n=1}^{50} 2n = 2 \sum_{n=1}^{50} n \\ \text{Using } \sum_{n=1}^k n &= \frac{k(k+1)}{2} \quad \Rightarrow \quad 2 \left(\frac{50(51)}{2} \right) = \boxed{2,550}\end{aligned}$$

3. Evaluate $\int (7 + e^x)^5 e^x dx$

$$\begin{aligned}&\int (7 + e^x)^5 e^x dx \\ \text{let } u &= 7 + e^x \quad du = e^x dx \\ \Rightarrow \int u^5 du &= \frac{1}{6} u^6 + C = \boxed{\frac{1}{6} (7 + e^x)^6 + C}\end{aligned}$$

4. Evaluate $\int_{-1}^1 (3 + x)^4 dx$

$$\begin{aligned}&\int_{-1}^1 (3 + x)^4 dx \\ \text{let } u &= 3 + x; \quad du = dx \\ u_{\text{lower}} &= 3 + (-1) = 2; \quad u_{\text{upper}} = 3 + 1 = 4 \\ \Rightarrow \int_{-1}^1 u^4 du &= \frac{1}{5} u^5 \Big|_2^4 = \frac{4^5}{5} - \frac{2^5}{5} = \boxed{198.4}\end{aligned}$$