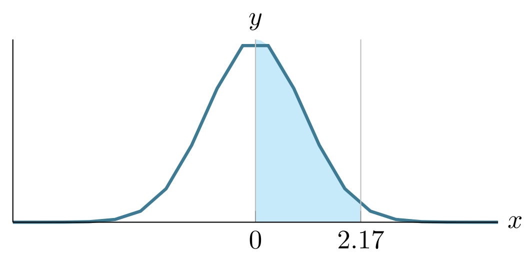
Kaleb Burris
STAT F300 Statistics, Dr. Short
Homework 6, Due Friday, March 3, 23:59

1. Let Z be a standard normal random variable and calculate the following probabilities, drawing pictures for each (such as in the lecture notes).

(a) $P(0 \leq Z \leq 2.17)$

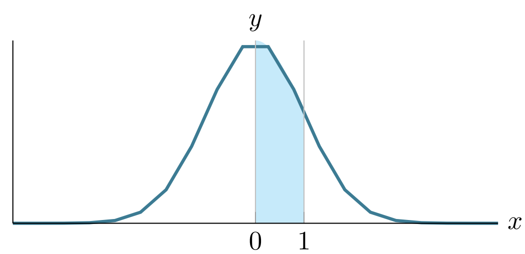
$$Z \sim N(\mu = 0, \sigma = 1)$$

$$\begin{aligned} &P(0 \leq Z \leq 2.17) \\ &= P\left(Z \leq \frac{2.17 - \mu}{\sigma}\right) - P\left(Z \leq \frac{0 - \mu}{\sigma}\right) \\ &= \Phi\left(\frac{2.17}{1}\right) - \Phi\left(\frac{0}{1}\right) \\ &= 0.985 - 0.50 = \boxed{0.485} \end{aligned}$$



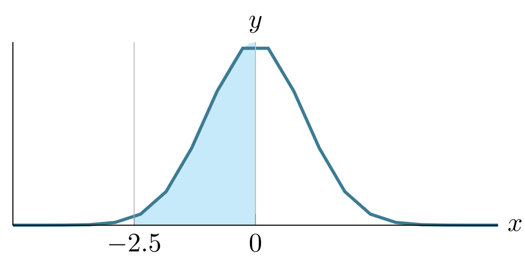
(b) $P(0 \leq Z \leq 1)$

$$\begin{aligned} &P(0 \leq Z \leq 1) \\ &= P\left(Z \leq \frac{1 - \mu}{\sigma}\right) - P\left(Z \leq \frac{0 - \mu}{\sigma}\right) \\ &= \Phi\left(\frac{1}{1}\right) - \Phi\left(\frac{0}{1}\right) \\ &= 0.841 - 0.50 = \boxed{0.341} \end{aligned}$$



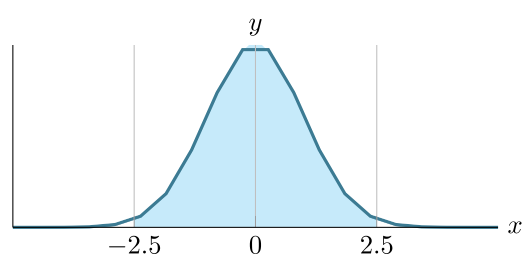
(c) $P(-2.50 \leq Z \leq 0)$

$$\begin{aligned} &P(-2.50 \leq Z \leq 0) \\ &= P\left(Z \leq \frac{0 - \mu}{\sigma}\right) - P\left(Z \leq \frac{-2.50 - \mu}{\sigma}\right) \\ &= \Phi\left(\frac{0}{1}\right) - \Phi\left(\frac{-2.50}{1}\right) \\ &= 0.50 - 0.00621 = \boxed{0.49379} \end{aligned}$$



(d) $P(-2.50 \leq Z \leq 2.50)$

$$\begin{aligned} &P(-2.50 \leq Z \leq 2.50) \\ &= P\left(Z \leq \frac{2.50 - \mu}{\sigma}\right) - P\left(Z \leq \frac{-2.50 - \mu}{\sigma}\right) \\ &= \Phi\left(\frac{2.50}{1}\right) - \Phi\left(\frac{-2.50}{1}\right) \\ &= 0.99379 - 0.00621 = \boxed{0.98758} \end{aligned}$$

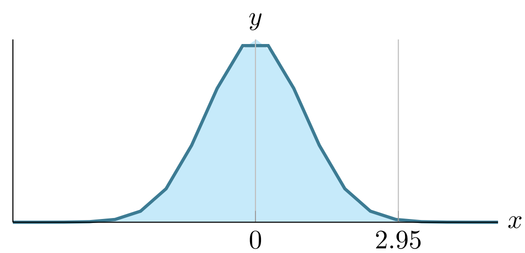


2. In each case, determine the value of the constant c that makes the probability statement correct. For each of these, sketch and label a $N(0,1)$ pdf to illustrate; these also makes it much much easier to get an idea of what c might be.

(a) $\Phi(c) = 0.9838$

$$\Phi(c) = 0.9838 \Rightarrow P(Z \leq c) = 0.9838$$

Using a look-up table, $c \approx \boxed{2.945}$

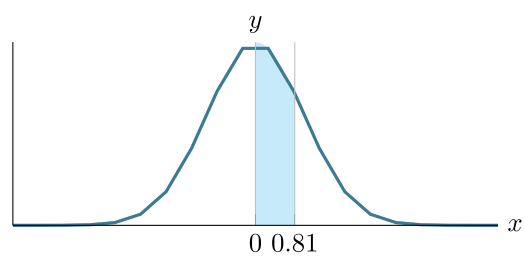


(b) $P(0 \leq Z \leq c) = 0.291$.

$$P(0 \leq Z \leq c) = 0.291 \Rightarrow \Phi(c) - 0.5 = 0.291$$

$$0.291 + 0.5 = \boxed{0.791}$$

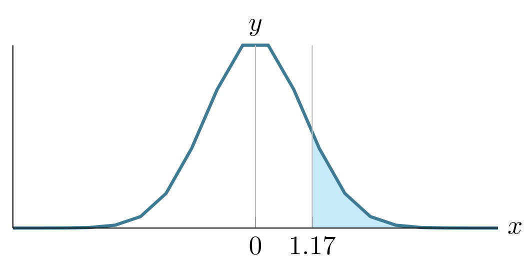
$$\Phi(0.791) = 0.81$$



(c) $P(c \leq Z) = 0.121$.

$$P(c \leq Z) = 0.121 = 1 - \Phi(c) = 0.121$$

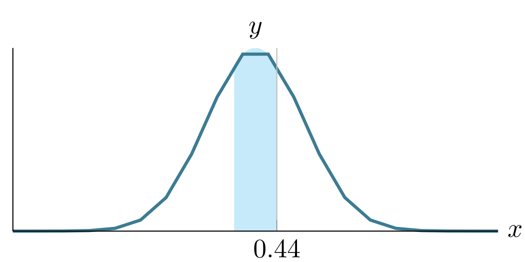
$$\Phi(c) = 0.879; \quad c = \boxed{1.17}$$



(d) $P(-c \leq Z \leq c) = 0.668$.

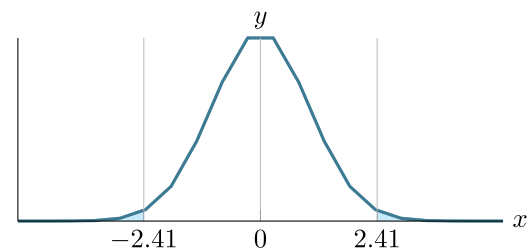
$$P(-c \leq Z \leq c) = 0.668 \Rightarrow 2\Phi(c) = 0.668$$

$$\Phi(c) = 0.334, \quad c = \boxed{\pm 0.44}$$



(e) $P(c \leq |Z|) = 0.016$.

$$P(c \leq |Z|) = 0.016 = p(c \leq Z) + p(-c \geq Z)$$
$$\Rightarrow 2\Phi(c) = 0.016$$
$$\Phi(c) = 0.008; \quad c = \boxed{\pm 2.405}$$



3. The maximum speed of a certain type of moped has a normal distribution with mean value 46.8 km/h and standard deviation 1.75 km/h.

(a) What is the probability that the maximum speed of a randomly chosen moped is at most 50 km/h?

$$X \sim N(\mu = 46.8, \sigma 1.75)$$

$$P(Z \leq 50) = \Phi\left(\frac{50 - 46.8}{1.75}\right)$$
$$= \Phi(1.83) = \boxed{0.96638}$$

(b) What is the probability that maximum speed differs from the mean value by at most 1.5 standard deviations?

$$P(\mu - (\sigma \cdot 1.5) \leq Z \leq \mu + (\sigma \cdot 1.5)) = P\left(\frac{2.625}{1.75}\right) - P\left(\frac{-2.625}{1.75}\right)$$
$$= P(1.5) - P(-1.5) = 0.93319 - 0.06681 = \boxed{0.86638}$$

4. There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3.0 cm and standard deviation 0.1 cm. The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation 0.02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?

$$P(2.9 \leq m_1 \leq 3.1) = \Phi\left(\frac{3.1 - 3.0}{0.1}\right) - \Phi\left(\frac{2.9 - 3.0}{0.1}\right)$$
$$= \Phi(1) - \Phi(-1) = 0.84134 - 0.15866 = 0.68268$$
$$P(2.9 \leq m_2 \leq 3.1) = \Phi\left(\frac{3.1 - 3.04}{0.02}\right) - \Phi\left(\frac{2.9 - 3.04}{0.02}\right)$$
$$= \Phi(3) - \Phi(-7) = 0.99865 - (\sim 0) = 0.99865$$

Machine 2 is more likely to produce acceptable corks, $\sim 99.87\%$ compared to Machine 1's $\sim 68.27\%$

Kaleb Burris

5. The weight distribution of parcels sent in a certain manner is normal with mean value 12 pounds and standard deviation 3.5 pounds. The parcel service wishes to establish a weight value c beyond which there will be a surcharge. What value of c is such that 99% of all parcels are at least 1 pound under the surcharge weight?

$$X \sim N(\mu = 12, \sigma = 3.5)$$

$$\begin{aligned}
P(c \leq Z) &= 0.99 \\
\Rightarrow P\left(Z \geq \frac{c-12}{3.5}\right) &= 0.99 \\
\Phi\left(\frac{c-12}{3.5}\right) &= 0.99 \\
\Phi(a) &= 0.99 \Rightarrow \Phi(2.305) \approx 0.99 \\
2.305 &= \frac{c-12}{3.5}
\end{aligned}$$

$$c = 8.0675 = 12 = \boxed{20.0675 \text{ pounds}}$$