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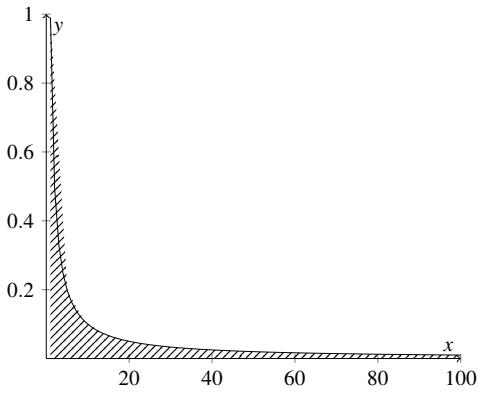
MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set IV, Due February 2, 2023: §2.3 # 122, 124, 130

122. $y = \frac{1}{x}, \quad x = 1, \quad \text{and} \quad x = 100$



$$\begin{aligned} V &= 2\pi \int_1^{100} \left(\frac{1}{x}\right) \mathrm{d}x \\ &= 2\pi \ln(x)\Big|_1^{100} \\ &= 2\pi(\ln(100) - \ln(1)) \\ &= \boxed{2\pi \ln(100)} \end{aligned}$$

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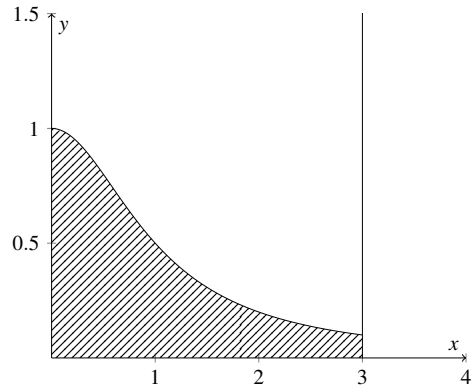
124. $y = \frac{1}{1+x^2}, \quad x = 0, \quad \text{and} \quad x = 3$



$$\begin{aligned} V &= 2\pi \int_0^3 \left(\frac{1}{1+x^2}\right) \mathrm{d}x \\ &= 2\pi \tan^{-1}(x)\Big|_1^3 \\ &= \boxed{2\pi[\tan^{-1}(3) - \tan^{-1}(1)]} \end{aligned}$$

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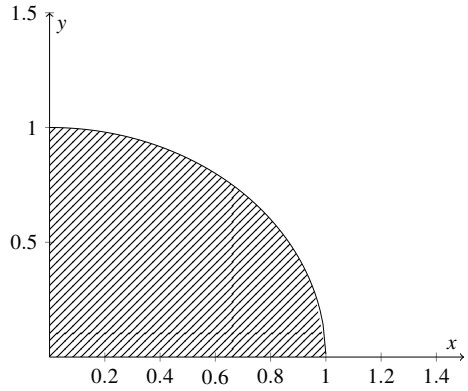
130. $y = \sqrt{1-x^2}, \quad x = 0, \quad \text{and} \quad x = 1$



$$V = 2\pi \int_0^1 \left(\sqrt{1-x^2}\right) \mathrm{d}x$$

$$\text{Using} \quad a^2 - u^2 \quad \rightarrow \quad u = a\sin\theta, \quad \mathrm{d}u = a\cos\theta \mathrm{d}\theta$$

$$x = \sin\theta, \quad \mathrm{d}x = \cos\theta \mathrm{d}\theta \quad \theta = \sin^{-1} x$$

$$\Rightarrow V = 2\pi \int \left(\sqrt{1-\sin^2\theta}\cos\theta\right) \mathrm{d}\theta$$

$$= 2\pi \int \left(\sqrt{\cos^2\theta}\cos\theta\right) \mathrm{d}\theta = 2\pi \int (\cos^2\theta) \mathrm{d}\theta$$

$$= 2\pi \int \left(\frac{1+\cos\theta}{2}\right) \mathrm{d}\theta \implies 2\pi \left(\frac{\theta + \sin\theta}{2}\right) \implies 2\pi \left(\frac{\sin^{-1}x + \sin\sin^{-1}x}{2}\right)\Big|_0^1$$

$$= 2\pi \left(\frac{\frac{\pi}{2} + 1 - 0}{2}\right) = 2\pi \left(\frac{\frac{2\pi}{2}}{2}\right) = \boxed{\frac{\pi}{4}}$$