Kaleb Burris
MATH F252, Dr. J. Gimbel, University of Alaska Fairbanks
Homework Set VI, Due Friday, February 17, 2023: §3.2 # 76, 86 §3.3 # 134, 138, 146

76. $\int \sin^7(2x)\cos(2x)dx$

$$\text{Let} \quad u=\sin(2x) \quad \frac{du}{2}=\cos(2x)dx$$
$$\Rightarrow \frac{1}{2}\int u^7 du = \frac{1}{2}\left(\frac{u^8}{8}\right)+C = \boxed{\frac{\sin^8(2x)}{16}+C}$$

86. $\int \sqrt{\sin x}\cos^3 x dx$

$$\int \sqrt{\sin x}(1-\sin^2 x)\cos x dx = \int (\sqrt{\sin x}-\sin^{5/2})\cos x dx$$
$$\text{Let} \quad u=\sin x \quad du=\cos x dx$$
$$\Rightarrow \int \left(\sqrt{u}-u^{5/2}\right)du = \frac{2u^{3/2}}{3}-\frac{2u^{7/2}}{7}+C = \boxed{\frac{2(\sin x)^{3/2}}{3}-\frac{2(\sin x)^{7/2}}{7}+C}$$

134. $\int \frac{dx}{\sqrt{4-x^2}}$

$$\text{Using} \quad a^2-u^2 \Rightarrow u=a\sin\theta; \quad x=2\sin\theta \quad dx=2\cos\theta d\theta$$
$$\Rightarrow \int \frac{2\cos\theta d\theta}{\sqrt{4-4\sin^2\theta}} \int \frac{2\cos\theta d\theta}{\sqrt{4(1-\sin^2\theta)}} = \int \frac{\cos\theta d\theta}{\sqrt{1-\sin^2\theta}} = \int \frac{\cos\theta d\theta}{\sqrt{\cos^2\theta}} = \int \frac{\cos\theta d\theta}{\cos\theta} = \int d\theta$$
$$=\theta+C = \boxed{\sin^{-1}\left(\frac{x}{2}\right)+C}$$

138. $\int \frac{x^2dx}{\sqrt{1-x^2}}$

$$\text{Using} \quad a^2-u^2 \Rightarrow u=a\sin\theta; \quad x=\sin\theta \quad dx=\cos\theta d\theta$$
$$\int \frac{\sin^2\theta\cos\theta d\theta}{\sqrt{1-\sin^2\theta}} = \int \frac{\sin^2\theta\cos\theta d\theta}{\sqrt{\cos^2\theta}} = \int \frac{\sin^2\theta\cos\theta d\theta}{\cos\theta} = \int \sin^2\theta d\theta \Rightarrow \int \frac{1-\cos 2\theta}{2}d\theta$$
$$=\frac{2\theta-\sin 2\theta}{4}+C = \boxed{\frac{\sin^{-1}(x)-\sin(2\sin^{-1}(x))}{4}+C}$$

146. $\frac{dx}{(1+x^2)^{3/2}}$

$$\text{Using} \quad a^2+u^2 \Rightarrow u=a\tan\theta; \quad x=\tan\theta \quad dx=\sec^2\theta d\theta$$
$$\int \frac{\sec^2\theta d\theta}{(1+\tan^2\theta)^{3/2}} = \int \frac{\sec^2\theta d\theta}{(\sec^2\theta)^{3/2}} = \int \frac{\sec^2\theta d\theta}{\sec^3\theta} = \int \sec^{-1}\theta d\theta = \int \cos\theta d\theta$$
$$=\sin\theta+C = \boxed{\sin(\tan^{-1}(x))+C}$$