Technische Mechanik - Uebungen 1

1) i.

$$\boldsymbol{r_P}(t) = \begin{pmatrix} \frac{6L}{5} + \frac{L}{4}\cos\pi t \\ \frac{6L}{5} + \frac{L}{4}\sin\pi t \end{pmatrix}$$

$$\boldsymbol{v_P}(t) = \frac{d\left(r_{P(t)}\right)}{dt} \qquad = \begin{pmatrix} \frac{-\pi L\sin(\pi t)}{4} \\ \frac{\pi L\cos(\pi t)}{4} \end{pmatrix}$$

ii.

$$\begin{aligned} v_p &= \sqrt{\left(\frac{-\pi L\sin(\pi t)}{4}\right)^2 + \left(\frac{\pi L\cos(\pi t)}{4}\right)^2} \\ &= \sqrt{\frac{\pi^2 L^2\sin^2(\pi t)}{16} + \frac{\pi^2 L^2\cos^2(\pi t)}{16}} \\ &= \frac{\pi^2 L^2}{16} \end{aligned}$$

iii. Circle with radius $\frac{L}{4}$ and center $\left(6\frac{L}{5}, 6\frac{L}{5}\right)$.

2)

$$\begin{aligned} x(t) &= at \\ t &= \frac{x}{a} \\ y(t) &= b - \frac{a^2}{b}t^2 \\ y(x) &= b - \frac{a^2}{b}\frac{x^2}{a^2} = b - \frac{x^2}{b} \\ y(t=0) &= b \therefore \neg D \\ y(x=a) &= b - \frac{a^2}{b} \\ y(x=b) &= b - \frac{b^2}{b} = 0 \therefore \text{The answer is B} \end{aligned}$$

3) Polar:

$$\begin{aligned} \rho(t) &= L_0 + at \\ \varphi(t) &= \Omega t \\ \frac{d(\varphi(t))}{dt} &= \Omega \\ \boldsymbol{r} &= \rho e_\rho \\ \boldsymbol{v_A} &= \frac{d(\boldsymbol{r})}{dt} = \frac{d(L_0 + at)}{dt}e_\rho + (L_0 + at)\frac{d\left(e_\rho\right)}{dt} \\ &= ae_\rho + (L_0 + at)\dot{\varphi}e_\varphi \\ &= ae_\rho + (L_0 + at)\Omega e_\varphi \end{aligned}$$

Cartesian:

$$
\begin{aligned}
x &= \rho\cos(\varphi) = (L_0 + at)\cos(\Omega t) \\
y &= \rho\sin(\varphi) = (L_0 + at)\sin(\Omega t) \\
\boldsymbol{r} &= xe_x + ye_y \\
\boldsymbol{v_A} &= (-(L_0 + at)\Omega\sin(\Omega t) + a\cos(\Omega t))e_x + ((L_0 + at)\Omega\cos(\Omega t) + a\sin(\Omega t))e_x
\end{aligned}
$$

4) Geschwindigkeit:

$$
\begin{aligned}
\boldsymbol{r} &= \frac{\sqrt{3}}{3}(1 - \cos\mu t)e_\rho + (3 - \cos\mu t)e_z \\
\dot{e_\rho} &= \dot{\varphi}e_\varphi = \sqrt{3}\mu e_\varphi \\
\dot{r} &= \frac{\sqrt{3}}{3}\mu\sin(\mu t)e_\rho + \frac{\sqrt{3}}{3}(1 - \cos\mu t)\dot{e_\rho} + \mu\sin(\mu t)e_z \\
&= \frac{\sqrt{3}}{3}\mu\sin(\mu t)e_\rho + \frac{\sqrt{3}}{3}(1 - \cos\mu t)\sqrt{3}\mu e_\varphi + \mu\sin(\mu t)e_z \\
&= \frac{\sqrt{3}}{3}\mu\sin(\mu t)e_\rho + (1 - \cos\mu t)\mu e_\varphi + \mu\sin(\mu t)e_z
\end{aligned}
$$

Schnelligkeit:

$$
\begin{aligned}
|\dot{r}| &= \sqrt{\left(\frac{\sqrt{3}}{3}\mu\sin(\mu t)\right)^2 + ((1 - \cos\mu t)\mu)^2 + (\mu\sin(\mu t))^2} \\
&= \sqrt{\frac{1}{3}\mu^2\sin^2(\mu t) + \mu^2(1 - 2\cos(\mu t) + \cos^2(\mu t)) + \mu^2\sin^2(\mu t)} \\
&= \sqrt{\mu^2\left(\frac{1}{3}\sin^2(\mu t) + 1 - 2\cos(\mu t) + \cos^2(\mu t) + \sin^2(\mu t)\right)} \\
&= \mu\sqrt{\frac{1}{3}\sin^2(\mu t) + 2 - 2\cos(\mu t)}
\end{aligned}
$$

Um die Geschwindigkeit in Kartesische Koordinaten zu finden, man kann einfach die einheits Vektoren durch die Kartesische Aequivelanten ersetzen, so dass man nicht nochmal ableiten muss.

Die Schnelligkeit soll gleich sein.

5)

$$
\begin{aligned}
v(t_1) &= \frac{dx}{dt}(t_1)e_x + \frac{dy}{dt}(t_1)e_y \\
\frac{d\left(a(x(t))^2\right)}{dt} &= 2ax(t)\dot{x(t)} \\
&= 1e_x + \left(2ax(t)\dot{x(t)}\right)e_y = 1e_x + 2ae_y \\
|v(t_1)| &= \sqrt{1^2 + (2a)^2} \qquad = \sqrt{1 + 4a^2}
\end{aligned}
$$