Technische Mechanik

Contents

Kinematiks - how a model is currently at motion
Statics - Which conditions (forces & moments) are needed to keep a system at rest
Dynamics - Which conditions are needed to create movement in a system in a certain way

$\vec{a} \times (b + c) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
κ - Set of all points in a body
Time derivative - $\dot{x} = \frac{dy}{dt}$

Coordinate Systems

Orthogonale Koordinatensystemen:

$$\text{Cartesian: } e_x \times e_y = e_z$$
$$\text{Cylindrical: } e_\rho \times e_\varphi = e_z$$

Position Vectors:

$$\text{Cartesian: } \boldsymbol{r} = x e_x + y e_y + z e_z$$
$$\text{Cylindrical: } \boldsymbol{r} = \rho e_\rho + z e_z$$

There is no separate e_φ component in a cylindrical position vector, as it's already accounted for by the e_ρ unit vector. The following derivatives are useful for calcutions in the cylindrical co-ordinate system:

$$\text{Wegen des Einheitskreises: } e_\rho = \cos(\varphi) e_x + \sin(\varphi) e_y$$
$$\text{Intuitiv: } e_\varphi = \frac{d(e_\rho)}{d\varphi} = -\sin(\varphi) e_x + \cos(\varphi) e_y$$
$$\frac{d(e_\varphi)}{d\varphi} = -\cos(\varphi) e_x - \sin(\varphi) e_y = -e_\rho$$
$$\frac{d(e_\rho)}{d\varphi} = e_\varphi$$

The time derivatives can be found by deriving the cartesian formulae with respect to time and doing some substitution:

$$\frac{d(e_\varphi)}{dt} = -\dot{\varphi} e_\rho$$
$$\frac{d(e_\rho)}{dt} = \dot{\varphi} e_\varphi$$

Thus the velocity formula in the cylindrical co-ordinate system:

$$\vec{v} = \dot{\rho} e_\rho + \rho \dot{\varphi} e_\varphi + \dot{z} e_z$$

Rigid bodies

A body in which deformation is negligible. There are no ideal rigid bodies in real life.

Let P, Q be points in a rigid body

$$\forall P, Q \in \mathbb{R}^3, |\boldsymbol{r_Q} - \boldsymbol{r_P}| = \text{Constant}$$

Satz der Projizierten Geschwindigkeiten

The velocities of any two points in a rigid body projected on the vector between them is always the same. This means the body is not getting shorter or longer (deforming).
Useful for determining the velocities of points on rigid bodies with relation to each other.

$$|r_Q - r_P| = \text{Konst } \forall P, Q \in \mathbb{R}^3 \to \overrightarrow{v_Q} \cdot e = \overrightarrow{v_P} \cdot e$$
$$\text{wo } e = \frac{r_Q - r_P}{|r_Q - r_P|}$$

$\overrightarrow{v_A} \cdot e = v'_A$ = Velocity of A projected onto the vector alongside a rigid body.

Translation - for all points $P, \overrightarrow{v_P}$ is equal

Movement across a plane

- All velocities are parallel to a certain plane
- All points along a normal to the plane have the same velocity
- It is either a translation or a rotation at any point in time

Rotation

If at least two points in a rigid body do not have the same velocity, it is currently rotating. The momentary, static center of rotation is the intersection of lines perpendicular to the velocities of two points. The points rotate around the center with the **same angular velocity** ω.

Considering a point with vector $\overrightarrow{r_P}$ from the center of rotation, rotating with angular velocity $\omega = \frac{d\Theta}{dt}$. Its velocity vector can be determined as:

$$\overrightarrow{v_P} = (\omega e_z) \times \overrightarrow{r_P}$$

The unit vector e_z is simply needed so the resulting direction is perpendicular to $\overrightarrow{r_P}$.

Polbahn - The path traced by the momentary center of rotation of a rigid body.

Movement in space

In 3D space, simultaneous translation & rotation is possible due to the extra dimension.

Starrkörperformel

The following extremely useful formula can be used to link the unique angular velocity vector to the velocity of any two points in a rotating body:[1]

$$\overrightarrow{v_P} = \overrightarrow{v_B} + \vec{\omega} \times \overrightarrow{r_{BP}}$$

The following properties of movement in space are constant and called "Invariants":

1. $I_1 = \vec{\omega}$ - The angular velocity is the same regardless of the reference point
2. $I_2 = \vec{\omega} \cdot v_P \forall P \in \kappa$ - The component of the velocity of a point in the direction of the rotation axis is the same for all points in the body

TODO: Test these in a simulation :)

Schraubung - The combination of a rotation with a translation in the direction of the rotation axis

Types of movement in space:

1. Translation: $\vec{\omega} = 0$
2. Rotation: $\vec{\omega} \neq 0 \land I_2 = 0$
3. Schraubung: $I_2 \neq 0$

TODO in lernphase: Understand Rechteck Beispiel in script

Degrees of freedom

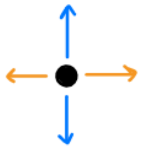
The minimum number of coordinates to clearly determine the state of a system.

Considering a system with several bodies. For a sum of degrees of freedom of n, and b restricted degrees of freedom due to connections, the resulting degrees of freedom of the whole system is:

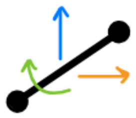
$$f = n - b$$

[1] Derivation available in the 5th Powerpoint of Dr. P Tiso

2D (Ebene)



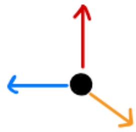
Punkt $f = \underline{2}$



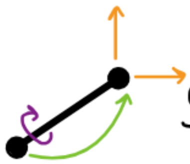
Stab $f = \underline{2 \cdot 2 - 1 = 3}$

Starrer Körper $f = \underline{3}$

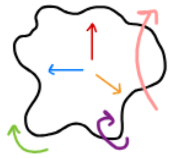
3D



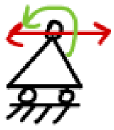
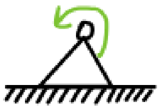
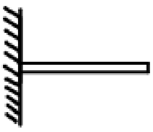
Punkt $f = \underline{3}$

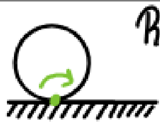


Stab $f = \underline{2 \cdot 3 - 1 = 5}$



Starrer Körper $f = \underline{6}$

b von	2D
 Auflager (beidseitig)	1
 Gelenk (Festlager)	2
 Einspannung	3

b von	2D
 Gelenk (2 SKs verbunden)	2
 Rollen ohne gleiten	2
 Gelenk (n SK verbunden)	$(n-1) \cdot 2$