Derivative of $a^x, a \in \mathbb{R}$:

$$\begin{aligned}
\lim_{h \to 0} \frac{a^{x+h} - a^x}{h} &= \lim_{h \to 0} \frac{a^x a^h - a^x}{h} \\
&= a^x \lim_{h \to 0} \frac{a^h - 1}{h} \\
&= a^x ? \text{ would need l'hopitals rule, nvm}
\end{aligned}$$

Instead:

$$\begin{gathered}
a^x = \left(e^{\ln a}\right)^x = e^{x \ln a} \\
\text{Ketten regel:} \\
\frac{d(a^x)}{dx} = e^{x \ln a} \ln a = a^x \ln a
\end{gathered}$$

Ableitung von $f(x) = \tan(x)$:

$$\begin{aligned}
\tan(x) &= \frac{\sin(x)}{\cos(x)} \\
\frac{d\left(\sin(x) \cdot \frac{1}{\cos(x)}\right)}{dx} &= \frac{d(\sin(x))}{dx} \cdot \frac{1}{\cos(x)} + \sin(x) \cdot \frac{d\left(\cos(x)^{-1}\right)}{dx} \\
&= \frac{\cos(x)}{\cos(x)} + \sin(x) \cdot \left(-\cos^{-2}(x) \cdot -\sin(x)\right) \\
&= 1 + \sin(x) \cdot \frac{\sin(x)}{\cos^2(x)} \\
&= 1 + \tan^2(x)
\end{aligned}$$

$$\begin{aligned}
\sin^2(x) + \cos^2(x) &= 1 \\
\tan^2(x) + 1 &= \frac{1}{\cos^2(x)} = \sec^2(x)
\end{aligned}$$

Ableitung von $f(x) = \ln(x)$ durch Umkehrregel:

$$\begin{aligned}
f(x) &= \ln(x) \\
f^{-1}(x) &= e^x \\
f'(x) &= \frac{1}{f^{(-1)'}(f(x))} \\
f^{-1}(f(x)) &= e^{\ln(x)} \\
\frac{d\left(e^{\ln(x)}\right)}{d(f(x))} &= e^{\ln(x)} = x \\
f'(x) &= \frac{1}{x}
\end{aligned}$$

Ableitung von $f(x) = \arcsin(x)$ durch Umkehrregel:

$$f(x) = \arcsin(x)$$

$$f'(x) = \frac{1}{\frac{d(\sin(\arcsin(x)))}{d(\arcsin(x))}}$$

IMPORTANT: We shouldn't simply change $\sin(\arcsin(x))$ to x, because x cannot be treated as a constant here, it is a function of $\arcsin(x)$ which we are differentiating with respect to.

$$= \frac{1}{\cos(\arcsin(x))}$$

$$= \frac{1}{\sqrt{1 - \sin^2(\arcsin(x))}}$$

$$= \frac{1}{\sqrt{1 - x^2}}$$