Lineare Algebra - Serie 1

1.

$$\boldsymbol{Ax} = \boldsymbol{b}$$

$$\begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

Gaussische Elimination:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{cccc|c} 1 & 1 & 2 & 2 & b_1 \\ 1 & 2 & 3 & 4 & b_2 \\ 1 & 3 & 6 & 10 & b_3 \\ 1 & 4 & 10 & 20 & b_4 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{cccc|c} 1 & 1 & 2 & 2 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & 2 & 4 & 8 & b_3 - b_1 \\ 0 & 3 & 8 & 18 & b_4 - b_1 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{pmatrix} \left(\begin{array}{cccc|c} 1 & 1 & 2 & 2 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & 0 & 2 & 4 & b_3 - b_1 - 2(b_2 - b_1) \\ 0 & 0 & 5 & 12 & b_4 - b_1 - 3(b_2 - b_1) \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & \frac{5}{2} & 1 \end{pmatrix} \left(\begin{array}{cccc|c} 1 & 1 & 2 & 2 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & 0 & 2 & 4 & b_3 - b_1 - 2(b_2 - b_1) \\ 0 & 0 & 0 & 2 & b_4 - b_1 - 3(b_2 - b_1) - \frac{5}{2}(b_3 - b_1 - 2(b_2 - b_1)) \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 2 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & 0 & 2 & 4 & b_3 - 3b_1 - 2b_2 \\ 0 & 0 & 0 & 2 & b_4 - b_1 - 3b_2 - 3b_1 - \frac{5}{2}b_3 + \frac{5}{2}b_1 + 5b_2 - 5b_1 \end{array}\right)$$

TODO: Attempt to reach identity matrix, the protokol matrix should be the inverse

$$2x_4 = b_4 - 6.5b_1 + 2b_2 - \frac{5}{2}b_3$$

$$2x_3 = b_3 - 3b_1 - 2b_2 - 4\left(b_4 - 6.5b_1 + 2b_2 - \frac{5}{2}b_3\right)$$

$$= 23b_1 - 10b_2 + 11b_3 - 4b_4$$

$$x_2 = b_2 - b_1 - \frac{1}{2}(23b_1 - 10b_2 + 11b_3 - 4b_4) - \left(b_4 - 6.5b_1 + 2b_2 - \frac{5}{2}b_3\right)$$

$$= -6b_1 + 4b_2 - 3b_3 + b_4$$

$$x_1 = b_1 - (-6b_1 + 4b_2 - 3b_3 + b_4) - (23b_1 - 10b_2 + 11b_3 - 4b_4) - \left(b_4 - 6.5b_1 + 2b_2 - \frac{5}{2}b_3\right)$$

$$= -9.5b_1 + 4b_2 - 5.5b_3 + 2b_4$$

$$\boldsymbol{B} = \begin{pmatrix} -9.5 & 4 & -5.5 & 2 \\ -6 & 4 & -3 & 1 \\ \frac{23}{2} & -5 & \frac{11}{2} & -2 \\ -3.25 & 1 & -\frac{5}{4} & \frac{1}{2} \end{pmatrix}$$

$$\boldsymbol{Bb} = \boldsymbol{x}$$

$$\begin{pmatrix} -9.5 & 4 & -5.5 & 2 \\ -6 & 4 & -3 & 1 \\ \frac{23}{2} & -5 & \frac{11}{2} & -2 \\ -3.25 & 1 & -\frac{5}{4} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -4.5 \\ 2 \\ 3.5 \\ -1.75 \end{pmatrix}$$

There is an arithmetic error somewhere in the above calculations, which would take too much time to find in relation to how much it would teach me.

8.
$$\left(\begin{array}{cc|c} -\frac{1}{2} & -1 & 1 \\ 2 & 2 & 2 \end{array}\right)$$

$$\left(\begin{array}{cc|c} -\frac{1}{2} & -1 & 1 \\ 0 & -2 & 6 \end{array}\right)$$

$$x_2 = -3$$

$$x_1 = -2(x_2 + 1) = -2(-2) = 4$$

9.
$$\left(\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 17 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 23 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\text{Answer} = \begin{pmatrix} 23 \\ -7 \\ 3 \end{pmatrix}$$