Finden Sie die Singularwerte $\sigma_1 \geq \sigma_2 \geq \sigma_3$ von der Matrix $A = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$.

$\sigma_1 =$

$\sigma_2 =$

$\sigma_3 =$

$$A^T = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \qquad A^TA = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Error

$$\times(1-2\lambda) \begin{pmatrix} \frac{1}{2}-\lambda & 0 & \frac{1}{2} \\ 0 & 1-\lambda & 0 \\ \frac{1}{2} & 0 & \frac{1}{2}-\lambda \end{pmatrix} \to \begin{pmatrix} \frac{1}{2}-\lambda & 0 & \frac{1}{2} \\ 0 & 1-\lambda & 0 \\ \frac{1}{2}-\lambda & 0 & (\frac{1}{2}-\lambda)(1-2\lambda) \end{pmatrix}$$

$$\left(\frac{1}{2}-\lambda\right)(1-2\lambda) - \frac{1}{2}$$

$$= \frac{1}{2} - \lambda - \lambda + 2\lambda^2 - \frac{1}{2}$$

$$= 2\lambda^2 - 2\lambda$$

$$2\lambda = 2$$

$$\det \begin{pmatrix} \frac{1}{2}-\lambda & 0 & \frac{1}{2} \\ 0 & 1-\lambda & 0 \\ 0 & 0 & (\frac{1}{2}-\lambda)(1-2\lambda) - \frac{1}{2} \end{pmatrix} = \left(\frac{1}{2}-\lambda\right)(1-\lambda)(2\lambda^2-2\lambda) = 0$$

$\lambda = \cancel{\frac{1}{2}}$ or 1 AM2 $\quad \therefore \sigma = 1, 1, \cancel{\frac{1}{\sqrt{2}}}\ 0$

Berechnen Sie die Singulärwertzerlegung von

$$\overset{2\times 3}{A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}$$

$$\overset{3\times 2}{A^T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}} \qquad \overset{3\times 3}{A^T A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}}$$

$$\overset{2\times 2}{A A^T = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}}$$

$$\lambda(A A^T):\ (2-\lambda)^2 - 1 = 0$$

$\lambda = 1$ or 3

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} v_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} v_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$v_1 = \begin{pmatrix} \alpha \\ -\alpha \end{pmatrix} \Big| \alpha \in \mathbb{R} \qquad v_2 = \begin{pmatrix} \beta \\ \beta \end{pmatrix} \Big| \beta \in \mathbb{R}$$

$$V = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad V^T = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{pmatrix} \begin{matrix} \\ -\frac{I}{(1-\lambda)} \\ -\frac{(1-\lambda)}{\lambda^2-3\lambda+1} II \end{matrix} \rightarrow \begin{pmatrix} 1-\lambda & 1 & 0 \\ 0 & \frac{\lambda^2-3\lambda+1}{(1-\lambda)} & 1 \\ 0 & 1 & 1-\lambda \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda & 1 & 0 \\ 0 & \frac{\lambda^2-3\lambda+1}{1-\lambda} & 1 \\ 0 & 0 & 1-\lambda-\frac{(1-\lambda)}{\lambda^2-3\lambda+1} \end{pmatrix}$$

$$\frac{(2-\lambda)(1-\lambda)-1}{(1-\lambda)} = \frac{2-2\lambda-\lambda+\lambda^2-1}{1-\lambda} = \frac{\lambda^2-3\lambda+1}{(1-\lambda)}$$

$$-(1-\lambda) + (1-\lambda)\big((1-\lambda)(2-\lambda)-1\big)$$
$$= (1-\lambda)\big(-1+(1+-3\lambda+\lambda^2)\big)$$
$$= (1-\lambda)(\lambda^2-3\lambda) = 0$$

$\lambda = 1, 3, 0$

$\lambda = 0$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} u_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$u_1 = \begin{pmatrix} -\alpha \\ \alpha \\ -\alpha \end{pmatrix} \Big| \alpha \in \mathbb{R}$$

$$u_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \qquad u_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\therefore U = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 2 \\ -1 & 1 & 1 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \sqrt{1} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Die Singulärwert-Zerlegung der Matrix

$$A = \begin{bmatrix} -\frac{12}{5} & -\frac{14}{5} & -\frac{2\cdot\sqrt{3}}{\sqrt{5}} \\ -\frac{7}{5} & \frac{21}{5} & -\frac{7\cdot\sqrt{3}}{\sqrt{5}} \end{bmatrix}$$

ist gegeben durch

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 7 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{5} & \frac{3}{5} & -\frac{\sqrt{3}}{\sqrt{5}} \\ \frac{3}{5} & \frac{7}{10} & \frac{\sqrt{3}}{2\cdot\sqrt{5}} \\ \frac{\sqrt{3}}{\sqrt{5}} & -\frac{\sqrt{3}}{2\cdot\sqrt{5}} & -\frac{1}{2} \end{bmatrix}$$

Berechnen Sie die Psuedoinverse $A^\dagger =$

$$\begin{bmatrix} \square & \square \\ \square & \square \\ \square & \square \end{bmatrix}$$

$$A = U \Sigma V^H$$

$$A^+ = V \Sigma^+ U^H$$

$$= \begin{pmatrix} -\frac{1}{5} & \frac{3}{5} & -\frac{\sqrt{3}}{\sqrt{5}} \\ \frac{3}{5} & \frac{7}{10} & \frac{\sqrt{3}}{2\sqrt{5}} \\ \frac{\sqrt{3}}{\sqrt{5}} & -\frac{\sqrt{3}}{2\sqrt{5}} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{7} & 0 \\ 0 & \frac{1}{4} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-3}{20} & \frac{-1}{35} \\ \frac{-7}{40} & \frac{3}{35} \\ \frac{\sqrt{15}}{40} & \frac{\sqrt{15}}{35} \end{pmatrix}$$