

Lineare Algebra - Serie 1

1.

$$Ax = b$$

$$\begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

Gaussische Elimination:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 & 2 & | & b_1 \\ 1 & 2 & 3 & 4 & | & b_2 \\ 1 & 3 & 6 & 10 & | & b_3 \\ 1 & 4 & 10 & 20 & | & b_4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 & 2 & | & b_1 \\ 0 & 1 & 1 & 2 & | & b_2 - b_1 \\ 0 & 2 & 4 & 8 & | & b_3 - b_1 \\ 0 & 3 & 8 & 18 & | & b_4 - b_1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 & 2 & | & b_1 \\ 0 & 1 & 1 & 2 & | & b_2 - b_1 \\ 0 & 0 & 2 & 4 & | & b_3 - b_1 - 2(b_2 - b_1) \\ 0 & 0 & 5 & 12 & | & b_4 - b_1 - 3(b_2 - b_1) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & \frac{5}{2} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 & 2 & | & b_1 \\ 0 & 1 & 1 & 2 & | & b_2 - b_1 \\ 0 & 0 & 2 & 4 & | & b_3 - b_1 - 2(b_2 - b_1) \\ 0 & 0 & 0 & 2 & | & b_4 - b_1 - 3(b_2 - b_1) - \frac{5}{2}(b_3 - b_1 - 2(b_2 - b_1)) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 2 & 2 & | & b_1 \\ 0 & 1 & 1 & 2 & | & b_2 - b_1 \\ 0 & 0 & 2 & 4 & | & b_3 - 3b_1 - 2b_2 \\ 0 & 0 & 0 & 2 & | & b_4 - b_1 - 3b_2 - 3b_1 - \frac{5}{2}b_3 + \frac{5}{2}b_1 + 5b_2 - 5b_1 \end{pmatrix}$$

TODO: Attempt to reach identity matrix, the protokol matrix should be the inverse

$$2x_4 = b_4 - 6.5b_1 + 2b_2 - \frac{5}{2}b_3$$

$$\begin{aligned} 2x_3 &= b_3 - 3b_1 - 2b_2 - 4\left(b_4 - 6.5b_1 + 2b_2 - \frac{5}{2}b_3\right) \\ &= 23b_1 - 10b_2 + 11b_3 - 4b_4 \end{aligned}$$

$$\begin{aligned} x_2 &= b_2 - b_1 - \frac{1}{2}(23b_1 - 10b_2 + 11b_3 - 4b_4) - \left(b_4 - 6.5b_1 + 2b_2 - \frac{5}{2}b_3\right) \\ &= -6b_1 + 4b_2 - 3b_3 + b_4 \end{aligned}$$

$$\begin{aligned} x_1 &= b_1 - (-6b_1 + 4b_2 - 3b_3 + b_4) - (23b_1 - 10b_2 + 11b_3 - 4b_4) - \left(b_4 - 6.5b_1 + 2b_2 - \frac{5}{2}b_3\right) \\ &= -9.5b_1 + 4b_2 - 5.5b_3 + 2b_4 \end{aligned}$$

$$B = \begin{pmatrix} -9.5 & 4 & -5.5 & 2 \\ -6 & 4 & -3 & 1 \\ \frac{23}{2} & -5 & \frac{11}{2} & -2 \\ -3.25 & 1 & -\frac{5}{4} & \frac{1}{2} \end{pmatrix}$$

$$Bb = x$$

$$\begin{pmatrix} -9.5 & 4 & -5.5 & 2 \\ -6 & 4 & -3 & 1 \\ \frac{23}{2} & -5 & \frac{11}{2} & -2 \\ -3.25 & 1 & -\frac{5}{4} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -4.5 \\ 2 \\ 3.5 \\ -1.75 \end{pmatrix}$$

There is an arithmetic error somewhere in the above calculations, which would take too much time to find in relation to how much it would teach me.