Technische Mechanik

Kinematiks - how a model is currently at motion
Statics - Which conditions (forces & moments) are needed to keep a system at rest
Dynamics - Which conditions are needed to create movement in a system in a certain way

Time derivative - $\dot{x} = \frac{dy}{dt}$

Starre Koerper

A body in which deformation is negligible. There are no ideal rigid bodies in real life.

Let P, Q be points in a rigid body

$$\forall P, Q \in \mathbb{R}^3, \left|\boldsymbol{r_Q} - \boldsymbol{r_P}\right| = \text{Constant}$$

Satz der Projizierten Geschwindigkeiten: The velocities of any two points in a rigid body projected on the vector between them is always the same. This means the body is not getting shorter or longer (deforming).
Useful for determining the velocities of points on rigid bodies with relation to each other.

$$\left|r_Q - r_P\right| = \text{Konst } \forall P, Q \in \mathbb{R}^3 \rightarrow v_Q \cdot e = v_P \cdot e$$
$$\text{wo } e = \frac{r_Q - r_P}{\left|r_Q - r_P\right|}$$

v'_A = Velocity of A projected onto the vector alongside a rigid body.

Coordinate Systems

Orthogonale Koordinatensystemen:

$$\text{Cartesian: } e_x \times e_y = e_z$$
$$\text{Cylindrical: } e_\rho \times e_\varphi = e_z$$

TODO: Think about how Pythagoras makes sense geometrically in all coordinate systems

Position Vectors:

$$\text{Cartesian: } \boldsymbol{r} = xe_x + ye_y + ze_z$$
$$\text{Cylindrical: } \boldsymbol{r} = \rho e_\rho + ze_z$$

There is no separate e_φ component in the cylindrical position vectors, as it's already accounted for by the e_ρ unit vector.

$$\text{Wegen des Einheitskreises: } e_\rho = \cos(\varphi)e_x + \sin(\varphi)e_y$$

This is important to remember when taking time derivatives in the cylindrical / spherical coordinate systems.

$$\text{Intuitiv: } e_\varphi = \frac{d\left(e_\rho\right)}{d\varphi} = -\sin(\varphi)e_x + \cos(\varphi)e_y$$
$$\frac{d\left(e_\varphi\right)}{d\varphi} = -\cos(\varphi)e_x - \sin(\varphi)e_y = -e_\rho$$
$$\frac{d\left(e_\rho\right)}{d\varphi} = e_\varphi$$

TODO: How to derive the derivatives with respect to t?

$$\frac{d\left(e_\varphi\right)}{dt} = -\dot{\varphi} e_\rho$$

$$\frac{d\left(e_\rho\right)}{dt} = \dot{\varphi} e_\varphi$$

Example of how to use the above derivatives:

$$r = \begin{pmatrix} \rho \\ \varphi \\ z \end{pmatrix} = \begin{pmatrix} \cos(t) \\ t \\ \sin(t) \end{pmatrix} = \cos(t) e_\rho + \sin(t) e_z$$

$$t = \varphi, \therefore$$

$$\dot{r} = \begin{pmatrix} \frac{d\left(\cos(t) e_\rho\right)}{dt} \\ \frac{d(\sin(t) e_z)}{dt} \end{pmatrix} = -\sin(t) e_\rho + \cos(t) \dot{e_\rho} + \cos(t) e_z + \sin(t) \dot{e_z}$$

$$= -\sin(t) e_\rho + \cos(t) e_\varphi + \cos(t) e_z$$

$$|\dot{r}| = \sqrt{(-\sin(t))^2 + (\cos(t))^2 + \cos^2(t)}$$

$$= \sqrt{1 + \cos^2(t)}$$