Bridging Test Answers

1. Considering the equation of a straight line, $y = mx + c$, the answer requires a positive gradient (m) and positive y intercept (c).

   Therefore the answer must be b, $y = \frac{1}{10}x + \frac{3}{2}$.

2. $$a, b \in \mathbb{R}_{>0}$$

   a)

   $$\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$$

   We can see this is impossible by testing it with $a, b = 1$, which yields $\frac{1}{2} \neq 2$.

   However, for the purposes of practicing typst:

   $$\begin{aligned} \frac{1}{a+b} &= \frac{b}{ab} + \frac{a}{ba} \\ \frac{1}{a+b} &= \frac{a+b}{ab} \\ 1 &= \frac{(a+b)^2}{ab} \\ &= \frac{a^2 + 2ab + b^2}{ab} \\ a^2 + ab + b^2 &= 0 \end{aligned}$$

   To satisfy the above equation, $ab < 0$, which is impossible considering they are both positive reals. Therefore a is **not** a valid equation.

   b)

   $$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$

   Once again, this is impossible when tested with $a, b = 1$, resulting in $\sqrt{2} = 2$. Mathematical proof:

   $$\begin{aligned} a + b &= \left(\sqrt{a} + \sqrt{b}\right)^2 \\ &= a + \sqrt{ab} + b \\ ab &= 0 \end{aligned}$$

   Either a or b must be equal to zero to satisfy the above equation and b is therefore **not** a valid answer.

   c)

   $$\begin{aligned} (a+b)(c+d) &= ac + bd \\ &= (a+b)(c+d) - ad - bd \\ 0 &= ad + bd \\ &= a + b \end{aligned}$$

Either a and b are both 0 or one of them must be negative, therefore *c* is **not** a valid answer.

d)

$$
\begin{aligned}
\ln(a+b) &= \ln(a) + \ln(b) \\
&= \ln(ab) \\
a+b &= ab
\end{aligned}
$$

This is not true for the entire set in which a and b exist, therefore the answer is *e) Keine.*

3. $f(x) = x - 2$ is a horizontal straight line with y intercept −2 and gradient 1. It equals 3 when $x = \begin{cases} 5 \\ -1 \end{cases}$ due to the surrounding $||$. When plotting a sketch of the graph, we can see that $y \leq 3$ between $-1 \leq x \leq 5$.

   Therefore the answer is *e) Keine der obigen Antworten ist richtig.*

4.
$$
\begin{aligned}
&\text{for } a, b > 0 \\
\ln(a^4 b^2) - \ln(a^2 b^{-2}) &= \ln\left(\frac{a^4 b^2}{a^2 b^{-2}}\right) \\
&= \ln(a^2 b^4)
\end{aligned}
$$

   The answer is *d)* $\ln(a^2 b^4)$.

5.
$$
\begin{aligned}
\ln(e) &= 1 \\
\ln\left(\frac{1}{e}\right) &= \ln(e^{-1}) = -1 \\
e^2 &< 3^2 \\
1 &< e \\
-1 &< \frac{1}{e} < 1
\end{aligned}
$$

   Therefore the correct answer is *c)* $\ln\left(\frac{1}{e}\right) < \frac{1}{e} < \ln(e) < e < e^2 < 9 = -1 < \frac{1}{e} < 1 < e < e^2 < 9$

6. The answer is *a)* $g(x) = (x-2)^3$.

7. Let the vertices of triangle be called A, B and C, where A and C are the bottom corners.

   Since $\angle CAB$ is $\frac{\pi}{3}$ and it is an isoceles triangle, $\angle ACB$ and therefore also $\angle ABC$ are $\frac{\pi}{3}$. Hence, side AC is also 1 unit long.

$$
\sin(\angle x) = \frac{\text{opposite}}{\text{hypotenuse}}
$$

   According to Pythagoras' theorem, $1^2 = \left(\frac{1}{2}\right)^2 + \text{opposite}^2$.

$$
\begin{aligned}
\text{opposite} &= \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \\
\sin\left(\frac{\pi}{3}\right) &= \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}
\end{aligned}
$$

   The answer is *d).*

8.
$$\begin{aligned} n &\in \mathbb{N} \\ \sin(\pi n) &= 0 \\ \cos(\pi n) &= \begin{cases} 1 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd} \end{cases} \end{aligned}$$

Therefore the answer is *b)* $\cos(2021\pi) < \sin(2021\pi) < \cos(2020\pi) = -1 < 0 < 1$.

9.
$$\sin^2(x) + \cos^2(x) = 1$$

Therefore the answer is *b)*.

10. The period of $\sin(\theta)$ is 2π.
$$\begin{aligned} 2\pi &= 2x \\ x &= \pi \end{aligned}$$

The answer is *d)*.

11.
$$\begin{aligned} f'(x) &= \frac{3}{4}x^{-\frac{1}{4}} \\ g'(x) &= \frac{4}{5}x^{-\frac{1}{5}} \\ f''(x) &= -\frac{3}{16}x^{-\frac{5}{4}} \\ g''(x) &= -\frac{4}{25}x^{-\frac{6}{5}} \end{aligned}$$

$$\begin{aligned} f''(1) &= -0.1875 \\ g''(1) &= -0.16 \end{aligned}$$

At $x = 1$ the second derivative of both is negative, hence they are both concave downards. The derivative is decreasing at a greater rate for $f(x)$, therefore the answer is *c)*.

12.
$$\frac{2n^3 - 1}{10n^3 + n + 21} = \frac{2n^3\left(1 - \frac{1}{2n^3}\right)}{2n^3\left(5 + \frac{1}{2n^2} + \frac{21}{2n^3}\right)}$$
$$\lim_{n\to\infty} \frac{1 - \frac{1}{2n^3}}{5 + \frac{1}{2n^2} + \frac{21}{2n^3}} = \frac{1}{5}$$

The answer is *d)* $\frac{1}{5}$.

13.
$$\sum_{k=0}^{n} \frac{(-1)^k}{2^k} = \sum_{k=0}^{n} \left(-\frac{1}{2}\right)^k$$
$$1, -0.5, 0.25, -0.125, 0.0625, -0.03125,$$

It is an arithmetic series with first term 1 and common ratio $-\frac{1}{2}$.

$$S_\infty = \frac{a}{1-r} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$$

Therefore the answer is *b)* $\frac{2}{3}$.

14. $$\frac{\sqrt{2+h}-\sqrt{2}}{h} \times \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}} = \frac{h}{h\sqrt{2+h}+h\sqrt{2}}$$
$$= \frac{1}{\sqrt{2+h}+\sqrt{2}}$$
$$\lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

Therefore the answer is *b)* $\frac{1}{2\sqrt{2}}$.

15. The answer is *e)*.

16. According to the product rule, $(f \cdot g)'(x) = f'(x)g(x) + g'(x)f(x)$
$$2 \times 3 + 4 \times 1 = 10$$

The answer is *d)* 10.

17. Chain rule:
$$\frac{df(x)}{du} = \frac{df(x)}{du}\frac{du}{dx}$$
$$\begin{aligned} f(x) &= e^{2x} \\ u &= 2x \\ f(x) &= e^{u} \\ f'(x) &= 2e^{u} = 2e^{2x} \end{aligned}$$

The answer is *c)* $2e^{2x}$.

18. $$0 \leq \sin x \leq 1$$
$$\begin{aligned} u &= \sin x \\ \frac{du}{dx} &= \cos x \\ f(x) &= \ln(u) \\ f'(x) = \frac{1}{u}\cos x &= \frac{\cos x}{\sin x} \end{aligned}$$

The answer is *b)* $\frac{\cos x}{\sin x}$.

19. $$\frac{d\left(\frac{e^x+e^{-x}}{2}\right)}{dx} = \frac{1}{2}\frac{d(e^x+e^{-x})}{dx} = \frac{e^x-e^{-x}}{2} = \sinh(x)$$

The answer is *a)* $\sinh(x)$.

20. $$\begin{aligned} u &= 3x \\ f(x) &= -\cos(u) \\ \frac{df(x)}{dx} &= 3\sin(u) = 3\sin(3x) \\ \sin\left(\frac{3\pi}{2}\right) &= -1 \end{aligned}$$

Therefore the tangent at $\frac{\pi}{2}$ is $3 \times -1 = -3$. The answer is *a)*.

21. At 5, the function crosses the x axis. As the gradient remains negative throughout this interval, $f(4) > 0$ and $f(6) < 0$. The answer is *d)*.

22.

$$\int_0^2 3x^2\,dx = \left[x^3\right]_0^2$$
$$= 2^3 - 0^3 = 8$$

23.

$$\int_0^1 e^{-2x}\,dx = \left[-\frac{e^{-2x}}{2}\right]_0^1$$
$$= \left(-\frac{e^{-2}}{2}\right) - \left(-\frac{e^0}{2}\right)$$
$$= -\frac{1}{2e^2} + \frac{1}{2}$$
$$= \frac{1}{2} - \frac{1}{2e^2}$$

The answer is *e)* $\frac{1}{2} - \frac{1}{2e^2}$.

24. a)

$$\int_0^{\frac{\pi}{2}} \cos(2x) = \left[-2\sin(2x)\right]_0^{\frac{\pi}{2}}$$
$$= (-2\sin(\pi)) - (-2\sin(0))$$
$$= 0 + 0 = 0$$

b)

$$\int_0^{\frac{\pi}{2}} \cos^2(x) = \left[-2\sin(x)\cos(x)\right]_0^{\frac{\pi}{2}}$$

c)

$$\int_0^{\frac{\pi}{2}} \sin(2x) = \left[2\cos(2x)\right]_0^{\frac{\pi}{2}}$$

d)

$$\int_0^{\frac{\pi}{2}} \sin^2(x) = \left[2\sin(x)\cos(x)\right]_0^{\frac{\pi}{2}}$$

The answer is *a)* $\int_0^{\frac{\pi}{2}} \cos(2x)$.

25.
$$
\begin{aligned}
A &= \int_0^1 \cos(x)dx = [\sin(x)]_0^1 \\
&= (\sin(1)) - (\sin(0)) = \sin(1) \\
B &= \int_0^{-1} \cos(x)dx = [\sin(x)]_0^{-1} \\
&= (\sin(-1)) - (\sin(0)) = \sin(-1) = -\sin(1) \\
\frac{A}{B} &= -\frac{\sin(1)}{\sin(1)} = -1
\end{aligned}
$$

The answer is *b)* -1.

26.
$$
\begin{aligned}
f(x) &= mx \\
F(b) &= \int_0^b f(x)dx = \frac{m}{2}x^2 + c
\end{aligned}
$$

After the point *c*, the area remains constant. The area under the graph increases in a linear fashion. Before, it is a quadratic function. Therefore the answer is *c)*.

27.
$$
\begin{aligned}
\int_0^1 xdx &= \left[\frac{x^2}{2}\right]_0^1 \\
&= \left(\frac{1}{2}\right) - 0 = \frac{1}{2} \text{ Total area} = \frac{1}{2} \times 2 = 1
\end{aligned}
$$

The answer is *b)* 1.

28. According to the Fundamental Theorem of Calculus, $f'(x) = \sin(x)$. The answer is *d)*.

29. *c)* $\vec{w}$.

30.
$$
\begin{aligned}
1(x-1) + 2(y-1) + 3(z-1) &= 0 \\
x - 1 + 2y - 2 + 3z - 3 &= 0 \\
x + 2y + 3z &= 6
\end{aligned}
$$

The answer is *b)* $x + 2y + 3z = 6$.

31.

$$\vec{v} \times \vec{w} = \begin{pmatrix} -5 \\ 7 \\ 9 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha \\ \beta \end{pmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 7 & 9 \\ 1 & 1 & \lambda \end{vmatrix} = \begin{pmatrix} 7\lambda - 9 \\ \cdots \\ \cdots \end{pmatrix}$$

$$7\lambda - 9 = 0$$

$$\lambda = \frac{9}{7}$$

$$\vec{w} \times \vec{v} = \begin{pmatrix} 1 \\ 1 \\ \lambda \end{pmatrix} \times \begin{pmatrix} -5 \\ 7 \\ 9 \end{pmatrix} = \begin{pmatrix} 0 \\ \gamma \\ \delta \end{pmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & \lambda \\ -5 & 7 & 9 \end{vmatrix} = \begin{pmatrix} 9 - 7\lambda \\ \cdots \\ \cdots \end{pmatrix}$$

$$9 - 7\lambda = 0$$

$$\lambda = \frac{9}{7}$$

The answer is *b)* $\frac{9}{7}$.

32.

$$\overrightarrow{OP} = t\vec{v} = \begin{pmatrix} 2.5t \\ 1.25t \end{pmatrix}$$

$$\left|\overrightarrow{OP}\right| = \sqrt{5} = \sqrt{(2.5t)^2 + (1.25t)^2}$$

$$5 = 6.25t^2 + 1.5625t^2$$

$$\frac{5}{7.8125} = t^2$$

$$t = 0.8$$

$$\overrightarrow{OP} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\overrightarrow{Pu} = -\vec{u} + \overrightarrow{OP} = \begin{pmatrix} 1 \\ 1 - b \end{pmatrix}$$

$$\overrightarrow{Pu} \cdot \vec{v} = 0$$

$$\begin{pmatrix} 1 \\ 1 - b \end{pmatrix} \cdot \begin{pmatrix} 2.5 \\ 1.25 \end{pmatrix} = 2.5 + 1.25 - 1.25b = 0$$

$$\frac{3.75}{1.25} = b$$

$$b = 3$$

The answer is *b)* 3.