

## Surface / Volume Integration nudge factors:

Cylindrical coordinate system:  $r$

Spherical coordinate system:  $r^2 \sin(\vartheta)$  (where  $\vartheta$  is the angle from the z axis)

## Das Coulomb'sche Gesetz

$$\vec{F}_2 = \frac{Q_1 Q_2 \vec{e}_{12}}{4\pi\epsilon_0 |\vec{r}_{12}|^2}$$

## Das elektrostatische Feld

Zum Beispiel: Elektrische, magnetisch

- Distance is from centre of point charge
- A positive test charge of 1C is used to plot electric fields - this means a positive charge has arrows away from it
- Unit vector is always  $\frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \vec{e}_{12}$
- Electric field lines cannot be closed loops with a constant direction - this would lead to perpetual motion (see electric potential closed integral)

*Feld* - die eigenschaften eines Raums, die eine Wirkung auf andere Ladungsstoffen haben

*Homogene Feld* - Field with the same strength and direction in all points. In reality it exists only when zooming in on a small area. Antonym: *inhomogene / ortsabhaengige Feld*.

$$\vec{E} = \frac{\vec{F}}{Q_2}$$

Um die Kraft in einer gewisse Punkt im Raum zu finden, man muss die Ladung mit dem Feld an diesem Punkt multiplizieren.

Any point in a field always has a unique direction. The electric field at the point exactly between two like charges is 0 as they cancel each other out.

*Quantitative* - Numbers based

*Qualitative* - Interpretation based

A cluster of like charges behaves like a point charge with the sum of their charges on the macroscopic scale. Opposite charge clusters (ex. Dipole) lead to no field around them on a large scale.

## Das elektrostatische Potential

$$\vec{F} = \vec{E}Q$$

$$W_e = - \int_{P_0}^{P_1} \vec{F} \cdot d\vec{s} = -Q \int_{P_0}^{P_1} \vec{E} \cdot d\vec{s}$$

The work done as a positive charge moves towards another positive is negative, as external energy is needed in order to overcome the repulsive force.

$\oint$  - Closed integral, when a line begins and ends at the same point in space.

Provided the speed as the charge moves through the field tends towards 0 (to minimize the arising electromagnetic field):

$$\oint_C \vec{E} \cdot d\vec{s} = 0$$

Electrostatic potential is the work needed per unit of charge to move it between two points.

$$\varphi = \frac{W_e}{Q}$$

A reference potential (ground) must always be defined, often the Earth's surface / an infinitely far away point is taken as  $\varphi_e = 0$ . In a circuit, the negative terminal is often used.

Taking  $r_1$  as an infinitely far away point with potential 0, the electrostatic potential in the space surrounding a point charge  $Q$  as a scalar is:<sup>1</sup>

$$\varphi(r_2) = \frac{Q}{4\pi\epsilon_0 r_2}$$

The change in electrostatic potential does not depend on the path taken through the field, only the start and end point.

$$W_e = -Q \int_{P_0}^{P_1} \vec{E} \cdot d\vec{s} = Q [\varphi_{e(P_1)} - \varphi_{e(P_0)}]$$

*Voltage (U)* - Difference between two potentials with the same reference potential.

$$U_{12} = \varphi(P_1) - \varphi(P_2) = \int_{P_1}^{P_2} \vec{E} \cdot d\vec{s}$$

## Elektrische Fluss (Flux)

TODO: Reread Anhang C to understand where the name Fluss comes from

*Elektrische Flussdichte (aka elektrische Erregung)*: How the electric field interacts with a material at a point in space. TODO: Expand after learning about permittivity.

$$\vec{D} = \epsilon_0 \vec{E} = \vec{e}_r \frac{Q}{4\pi r^2}$$

*Elektrische Fluss (Ψ)* - Total flux density flowing through a surface, Considering a charge  $Q$  inside a sphere with radius  $r$ :

$$r = \text{constant}$$

$$\Psi_D := \oiint \vec{D} \cdot d\vec{A}$$

Nudge factor needed for spherical coordinate system

$$= \int_0^{2\pi} \int_0^\pi r^2 \sin(\vartheta) \epsilon_0 \overrightarrow{E(r, \vartheta, \varphi)} \cdot d\vartheta d\varphi$$

$$= \frac{\epsilon_0 Q r^2}{4\pi \epsilon_0 r^2} \int_0^{2\pi} \int_0^\pi \sin(\vartheta) \vec{e}_r \cdot d\vartheta d\varphi$$

$\vec{e}_r \cdot d\vartheta d\varphi$  is 1, as they are always parallel.

$$= \frac{Q}{4\pi} \int_0^{2\pi} (-\cos(\pi)) - (-\cos(0)) d\varphi = \frac{Q}{4\pi} \int_0^{2\pi} 2 d\varphi$$

$$= \frac{Q}{4\pi} ((4\pi) - (0)) = \frac{4\pi Q}{4\pi} = Q$$

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<sup>1</sup>Derivation in Elektrotechnik, Albach 1.8.1

## Gauss'sche Gesetz

The above derived relationship is known as Gauss's law:

$$\Psi_E := \oiint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\Psi_D := \oiint \vec{D} \cdot d\vec{A} = Q$$

The total electric flux through an arbitrary closed surface is equal to the charge enclosed inside, regardless of the charges position / area of the surface.

This law is one of Maxwell's equations and can be used in reverse with infinitely small Gaussian surfaces to calculate the electric field around certain charge distributions, for example, the electric field at the surface of any point on a charged plane.

## Leitenden Koerper

The electric lines are always perpendicular to the surface of a conductor, as any tangential component of the field redistributes charges to prevent this.

The electric field inside a conductor is always 0, as the free charges repel each other and arrange themselves on the surface, of which the superposed electric fields at any point in the conductor are 0. The same applies if the conductor is brought into an external electric field.

The negative charge on the surface of a conductor arises from a **surplus of electrons**. Positive charge arises due to lack of electrons - a **surplus of holes**. Protons do not move throughout the conductor.

*Influenzierten Ladungen* - The separation of charges influenced by an electric field.