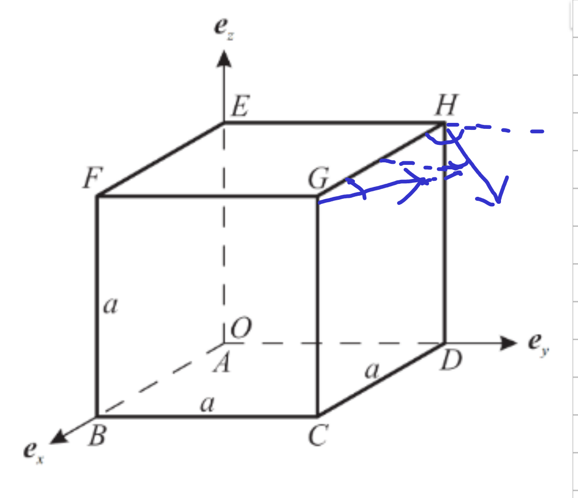
Technische Mechanik - Serie 3

3) i. $v_G' = v_H'$

$$\vec{v}_G = \begin{pmatrix} -v \\ v \\ 0 \end{pmatrix} \quad \vec{v}_H = \begin{pmatrix} v_{Hx} \\ 0 \\ -v \end{pmatrix} \quad \vec{r}_{GH} = \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix}$$



$$\vec{v}_G \cdot \vec{r}_{GH} = \vec{v}_H \cdot \vec{r}_{GH}$$

$$-va = v_{Hx} a, \quad v_{Hx} = -v$$

ii) $\vec{v}_H = \vec{v}_G + \vec{\omega} \times \vec{r}_{GH}$

$$\left\| \begin{pmatrix} e_x & e_y & e_z \\ \omega_x & \omega_y & \omega_z \\ -a & 0 & 0 \end{pmatrix} \right\| = \begin{pmatrix} 0 \\ a\omega_y \\ -a\omega_z \end{pmatrix}$$

$$\vec{\omega} \times \vec{r}_{GH} = \begin{pmatrix} 0 \\ -v \\ -v \end{pmatrix} \quad \therefore \quad \vec{\omega} = \begin{pmatrix} 0 \\ \frac{-v}{a} \\ \frac{a}{v} \end{pmatrix} = I_1$$

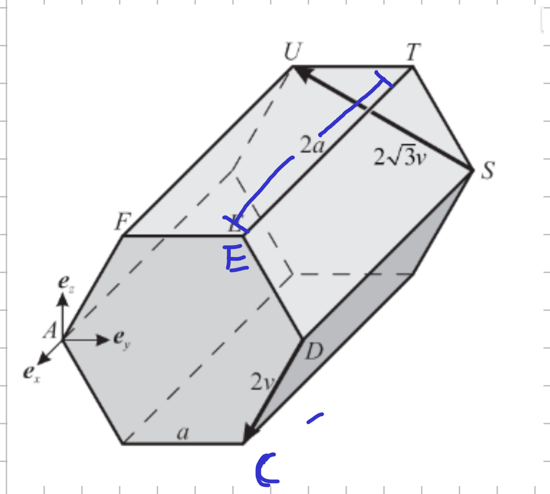
Error! Should be $\frac{v}{a}$

$$I_1 \quad \left\| \begin{pmatrix} e_x & e_y & e_z \\ 0 & \frac{-v}{a} & \frac{a}{v} \\ 0 & -a & -a \end{pmatrix} \right\| = \begin{pmatrix} -v + \frac{a^2}{v} \\ 0 \\ 0 \end{pmatrix}$$

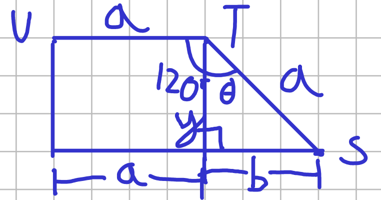
$$\vec{v}_A = v_H + \vec{\omega} \times \vec{r}_{HA} = \begin{pmatrix} -v \\ 0 \\ -v \end{pmatrix} + \begin{pmatrix} -v + \frac{a^2}{v} \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -2v + \frac{a^2}{v} \\ 0 \\ -v \end{pmatrix}$$

4. i)



$$\vec{r}_{SU} = \begin{pmatrix} 0 \\ \frac{-3a}{2} \\ \frac{\sqrt{3}a}{2} \end{pmatrix}$$



$$\theta = 120 - 90 = 30°$$

$$b = a\sin(30) = \frac{a}{2}$$

$$a + b = \frac{3a}{2}$$

$$y = a\cos 30 = \frac{\sqrt{3}a}{2}$$

$$\vec{r}_{DE} = \begin{pmatrix} 0 \\ \frac{-3a}{2} \\ \frac{\sqrt{3}a}{2} \end{pmatrix} \qquad |r_{DE}| = \frac{\sqrt{12}\,a}{2} = \sqrt{3}a$$

$$\vec{r}_{DC} = \begin{pmatrix} 0 \\ \frac{-3a}{2} \\ \frac{-\sqrt{3}a}{2} \end{pmatrix}$$

$$\vec{V}_E' = \vec{V}_D'$$

$$\vec{V}_E \cdot \vec{e}_{DE} = \vec{V}_D \cdot \vec{e}_{DE}$$

$$\begin{pmatrix} V_{Ex} \\ V_{Ey} \\ 0 \end{pmatrix} \vec{e}_{DE} = \frac{8v}{12a^2} \begin{pmatrix} 0 \\ \frac{-3a}{2} \\ \frac{-\sqrt{3}a}{2} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \frac{-3a}{2} \\ \frac{\sqrt{3}a}{2} \end{pmatrix} = \frac{9\cancel{a^2}}{\cancel{4}} \times \frac{\cancel{8}^{2}v}{6\cancel{12a^2}} + \frac{\cancel{3}a^2}{\cancel{4}} \times \frac{\cancel{8}^{2}v}{\cancel{6}\cancel{12a^2}_{2}}$$

$$= \frac{9v}{6} + \frac{v}{2}$$

$$|r_{DC}| = \sqrt{\frac{9a^2}{4} + \frac{3a^2}{4}} = \frac{\sqrt{12}\,a}{2}$$

$$\frac{2 \times 2v}{\sqrt{12}a} = \frac{4v}{\sqrt{12}a} \quad \therefore \vec{V}_D = \frac{4v}{\sqrt{12}a} \vec{r}_{DC}$$

$$\frac{-3\cancel{a}}{\cancel{2}} \times \frac{\cancel{2}}{\sqrt{12}\cancel{a}} \times V_{Ey} = \frac{9v}{6} + \frac{v}{2}$$

$$V_{Ey} = \frac{\cancel{12}^{2}v}{\cancel{6}_{3}} \times \frac{\sqrt{12}}{-\cancel{3}} = \frac{2\sqrt{12}v}{-3} = \frac{-4\cancel{\sqrt{3}}v}{\sqrt{3}} \; ?$$

5 i) $\vec{v}_C = \vec{v}_A + \vec{\omega} \times \vec{r}_{AC}$

$$\begin{pmatrix} 0 \\ -2v \\ 0 \end{pmatrix} = \begin{pmatrix} 2\omega_y R \\ -\omega_x 2R \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} e_x & e_y & e_z \\ \omega_x & \omega_y & \omega_z \\ 2R & 0 & R \end{vmatrix}$$

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{AB}$

$$\begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_y R \\ \omega_x R - \omega_z 2R \\ -\omega_y 2R \end{pmatrix}$$

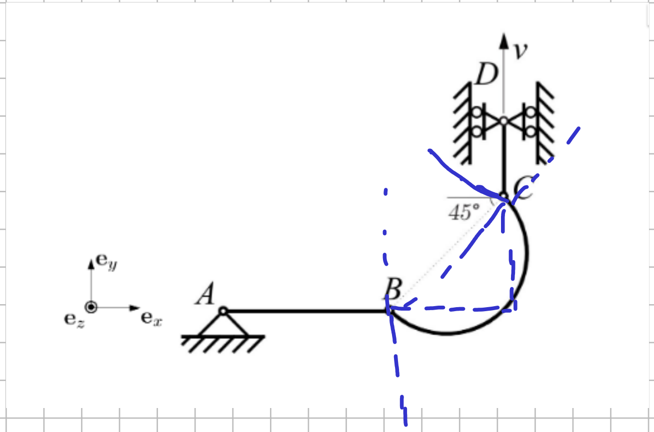
$-v = v - \omega_z 2R$

$\omega_z = \frac{2v}{2R} = \frac{v}{R}$

$$\therefore \omega = \begin{pmatrix} \frac{v}{R} \\ 0 \\ \frac{v}{R} \end{pmatrix}$$

$$\text{Rotationsachse}(\lambda) = \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ R \end{pmatrix}$$

6)



$\vec{e}_{CD} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\vec{e}_{BC} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

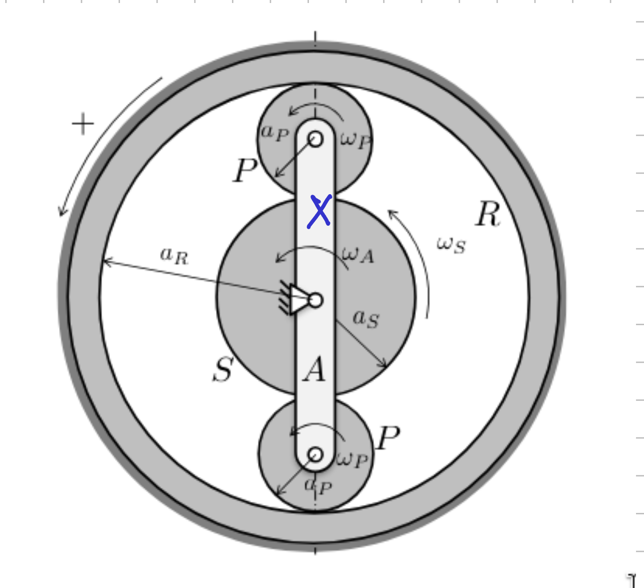
$v_{Dy} = v_{Cy} = V$

$|v_B| = |v_C| = \quad V = \sqrt{V^2 + V_{Cx}}$

$V_{Cx} = 0$ or $-2V$

Translation $\begin{pmatrix} 0 \\ V \end{pmatrix}$ Rotation

7 i)



$$V_x = \omega_S a_S \;,\; \omega_S = \frac{V_x}{a_S}$$

$$V_x = -\omega_p 2a_p \;/\; \omega_p = \frac{V_x}{-2a_p}$$

$$\frac{\omega_p}{\omega_S} = \frac{\cancel{V_x} a_S}{-2a_p \cancel{V_x}} = \frac{a_S}{-2a_p}$$

$\therefore$ a

ii) $V_p = \omega_A(a_s + a_p)$

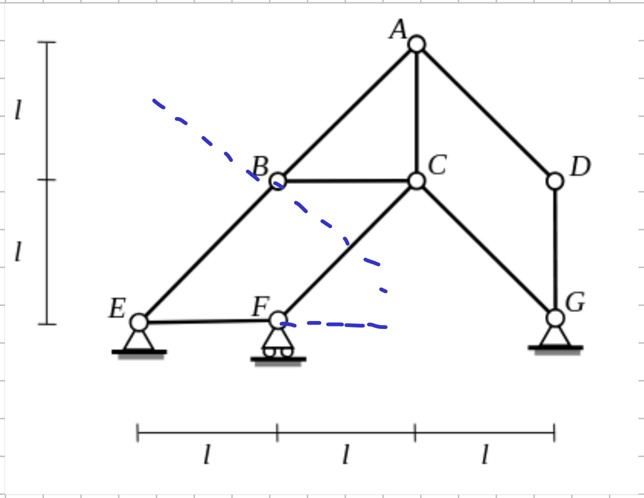
Dynamic $\therefore$ Error

$$V_x = V_p + \omega_p a_p = \omega_A(a_s + a_p) + \omega_p a_p = -\omega_p 2a_p$$

$$\omega_p(-2a_p - a_p) = \omega_A(a_s + a_p)$$

$$\frac{\omega_p}{\omega_A} = \frac{a_s + a_p}{-3a_p}$$

8)



ΔABC = rigid $\therefore$ ACDG rigid

Prediction: 0 DOF

$n = 9 \times 3 = 27$

$b = 2\times2 + \underset{5}{1} + 4 + \underset{15}{6} + 4 + 2 + 2 + 2 + 2 = 27$

$\therefore$ 27 − 27 = 0 DOF