Lineare Algebra

LGS - Lineare Gleichung System - linear system of equations

Vektoren

Lineare kombination - Summe von skalierten Vektoren
Basis - the set of base vectors $e_1 ... e_n$ that define space R^n

Vektoren werden immer als Spalten in diesem Kurs gezeichnet.
Standard vector notation:

$$\boldsymbol{a_1} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$$

Matrix multiplication comes from the motivation for an efficient way of representing LGSs.

Geometry of an LGS (Beispiel 1.1.0.8)

An LGS can be viewed geometrically (2D/3D) in multiple different ways:

1. A linear combination of vectors (the columns of the matrix), where we are solving for the set of scalars where the superposition of the vectors is equal to the RHS.
2. Alternatively it can be viewed as a set of line / plane equations, where each row is the normal vector to the plane (unsure if the coefficients are meaningful in ax + by=c)

Superposition:

The solution of a LGS is finding the scalars which make the linear combination of n vectors $\in R_n$ equal to the RHS vector. It is utterly NOT the same as finding the points of intersection with a plane.

In this example, one of the LHS vectors is a linear combination of the other two. This results in the LGS only being able to express vectors in a single plane rather than the entire 3D space (it doesn't contain a 3rd component).

Infinite solutions - if the RHS vector lays in the plane expressed by $\boldsymbol{a_{1-3}}$, any point in the positive / negative direction of the solution vector lays in the plane.
No solutions - the vector does not lay perfectly on the plane, the LHS vectors lack a component (not necessarily base unit vector) in its direction.

Line / Plane equations:

The solution is the point at which the lines / planes represented by the horizontal equations intersect. There are many possible arrangements which we can visualize, especially in $3D$ space.

Unique solution - Common point of intersection of n non parallel lines / planes.
Infinite solutions - Sheaf of planes or if all lines are the same.
No solution - Not all lines / planes meet at a common point, which is more likely the more equations are introduced into the system. Examples: Parallel lines, trianglular prism from 3 planes.

Gaussische Eliminationsverfahren

Pivot - element on the diagonal of a matrix that has a non 0 coefficient

Rang / rank - number of non 0 pivots, TODO: Intuitive meaning

Kombatilitaetsbedingungen - Empty rows at the bottom of the matrix (0 coefficients in one of the equations). If their result is not 0 then there are no solutions for the system. If their result is 0 and the number of equations $\leq$ the number of variables, there are infinite solutions.
Intuition: When thinking of the LGS as superposition, each LHS vector has a 0 component in this

dimension, meaning that $\forall x \in \mathbb{R}$ scalar in the Lineare Kombination satisfies the system. Viewing the system with insufficient equations as a system of planes, two planes will intersect along an entire line. In 2D, there would just be a single line, which of course has solutions along its entirety.

Any variables not accounted due to an all 0 row are called *free variables* and can take any real value.

Tips:

- Never divide / subtract in Gaussian elimination. Either multiply by $\frac{1}{x}$ or -1. Order is half of the work in maths.
- When switching rows to get pivots in the correct place, it is usually best to swap a line with zero pivot with the row that has the largest pivot in that place.

TODO: Consider Gaussian elimination for non square matrix and why the stagger skips some columns
TODO: Understand protocol matrix

In dem Ergebnis matrix, alle Zeilen sind lineare Kombiationen der Pivot Zeilen.