Mathematische Methoden

Contents

1 Fourier Analysis

The goal is to find a set of trigonmetric functions, such that their superposition matches the signal being analysed.

To be able to capture an arbitrary signal, we need to determine the amplitude, frequency and phase difference of each possible frequency. A possible basis could be a single sin function:

$$\int_{-\infty}^{\infty} \rho(\omega) \sin(\omega t + \varphi(\omega)) d\omega$$

Where $\rho(\omega)$ is the amplitude distribution and $\varphi(\omega)$ the phase shift distribution for all frequencies which we must determine. There are variants such as DCT and wavelet decomposition which work in this way. In fact, there are endless possible bases which can capture any signal, such bases are called **complete**.

However unless there is a compelling reason to use an alternative basis, Fourier transforms are done using sin and cos. They have many advantages making them a suitable:

- Infinitely differentiable
- When considering the space of continuous functions with inner product $< f, g >= \int_a^b f(t)\overline{g(t)} dt$, they are an orthogonal basis:

$$\begin{aligned} &\text{Orthogonal:} \\ &< \sin, \cos >= \int_0^{2\pi} \sin(x)\cos(x) dx = \sin^2(x) \mid_0^{2\pi} \\ &= 0 \\ &\text{Linearly Independent:} \\ &\alpha \sin(x) + \beta \cos(x) = 0 \forall x \in [0, 2\pi] \Leftrightarrow \alpha = \beta = 0 \end{aligned}$$

TODO: Show that they span the entire space.

Furthermore, we can represent them in a compact way using Euler's formula:

$$Re^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

1.1 Fourier Series

This is the process of converting a periodic signal against time to a series of trigonometric waves with certain amplitudes and phase differences for a specific frequency.

1.2 Fourier Transform

This is the more general form which can also convert **non-periodic** signals.

TODO:

- Express signal as fourier series
- How to sample and calculate the transform