Week 1 Bonusaufgaben

1. i.

$$N = \iiint_V \rho \ dV = \rho V$$

ii.

$$\begin{split} \rho &= 10^{28} m^{-3} \\ V &= 61.3 \times \left(10^{-2} m\right)^3 = 61.3 \times 10^{-6} m^3 \\ N &= 61.3 \times 10^{-6} \times 10^{28} \\ &= 61.3 \times 10^{22} \text{ Ladungstraeger} \end{split}$$

- 2. Antwort: c
- 3. i. Zylinderkoordinaten

$$r = \sqrt{3^2 + 5^2} = \sqrt{34}$$
$$\varphi = \arctan\left(\frac{5}{3}\right) = 1.03 \text{ rad}$$

ii. Kugelkoordinaten

$$2 = r \cos(\vartheta)$$

$$3 = r \sin(\vartheta) \cos(\varphi)$$

$$5 = r \sin(\vartheta) \sin(\varphi)$$

$$\frac{3}{\cos(\varphi)} = \frac{5}{\sin(\varphi)}$$

$$\tan(\varphi) = \frac{5}{3}$$

$$\varphi = \arctan\left(\frac{5}{3}\right) = 1.03 \text{ rad}$$

$$\frac{2}{\cos(\vartheta)} = \frac{3}{\sin(\vartheta)\cos(\varphi)}$$

$$\tan(\vartheta) = \frac{3}{2\cos(\varphi)}$$

$$\vartheta = \arctan\left(\frac{3}{2\cos(\varphi)}\right) = 1.24 \text{ rad}$$

4. r = constant

$$\begin{split} F &\coloneqq \frac{Q}{4\pi r^2} \vec{e_r} \\ \Psi &\coloneqq \int_0^{2\pi} \int_0^{\pi} r^2 \sin(\vartheta) \overline{F(r,\vartheta,\varphi)} \cdot d\vartheta d\varphi \\ &= \frac{Q r^2}{4\pi r^2} \int_0^{2\pi} \int_0^{\pi} \sin(\vartheta) \vec{e_r} \cdot d\vartheta d\varphi \end{split}$$

 $\vec{e_r} \cdot d\vartheta d\varphi$ is 1, as they are always parallel.

$$\begin{split} &=\frac{Q}{4\pi}\int_{0}^{2\pi}(-\cos(\pi))-(-\cos(0))d\varphi=\frac{Qr^{2}}{4\pi r^{2}}\int_{0}^{2\pi}2d\varphi\\ &=\frac{Qr^{2}}{4\pi r^{2}}((4\pi)-(0))=\frac{4\pi Q}{4\pi}=Q \end{split}$$