Integration Derivations

Integration by parts

$$\frac{d(f(x)g(x))}{dx} = f'(x)g(x) + f(x)g'(x)$$

$$f(x)g(x) = \int f'(x)g(x) + f(x)g'(x)$$

$$= \int f'(x)g(x) + \int f(x)g'(x)$$

$$\int f'(x)g(x) = f(x)g(x) - \int f(x)g'(x)$$

Choose $f'(x)$ as the function which gets nicer once integrated. This usually results in the following order:

- L ogarithms
- I nverse trig functions
- A lgebraic function (polynomial)
- T rigonometric function
- E xponential

Example:

$$\int x\cos(x)dx = x\sin(x) - \int \sin(x)dx$$

$$= x\sin(x) + \cos(x) + c$$

Integration by substitution

$$\frac{d(F(g(x)))}{dx} = F'(g(x)) \cdot g'(x) = \int_a^b f(g(x)) \cdot g'(x)dx$$

$$= F(g(b)) - F(g(a)) = \int_{g(a)}^{g(b)} f(y)dy$$

$$\int_a^b f(g(x)) \cdot g'(x)dx = \int_{g(a)}^{g(b)} f(y)dy$$

Example 1:

$$\int_0^1 e^{\sin(x)} \cdot \cos(x)dx$$

$$\text{Substitute } y = \sin(x)$$

$$\int_0^{\sin(1)} e^y dy = e^{\sin(1)} - e^0$$

$$e^{\sin(1)} - 1$$

Example 2:

$$\begin{aligned}\int \tan(x)dx &= -\int -\sin(x)\cdot\frac{1}{\cos(x)}dx\\ \text{Substitute } y &= \cos(x)\\ &= \int \frac{1}{y}dy\\ &= -\ln(|\cos(x)|)+c\\ &= \ln\left(\frac{1}{|\cos(x)|}\right)+c\end{aligned}$$