

# Technische Mechanik

*Kinematik* - how a model is currently at motion

*Statics* - Which conditions (forces & moments) are needed to keep a system at rest

*Dynamics* - Which conditions are needed to create movement in a system in a certain way

Time derivative -  $\dot{x} = \frac{dy}{dt}$

## Rigid bodies

A body in which deformation is negligible. There are no ideal rigid bodies in real life.

Let  $P, Q$  be points in a rigid body

$$\forall P, Q \in \mathbb{R}^3, |r_Q - r_P| = \text{Constant}$$

*Satz der Projizierten Geschwindigkeiten*: The velocities of any two points in a rigid body projected on the vector between them is always the same. This means the body is not getting shorter or longer (deforming).

Useful for determining the velocities of points on rigid bodies with relation to each other.

$$|r_Q - r_P| = \text{Konst} \quad \forall P, Q \in \mathbb{R}^3 \rightarrow v_Q \cdot e = v_P \cdot e$$
$$\text{wo } e = \frac{r_Q - r_P}{|r_Q - r_P|}$$

$v'_A$  = Velocity of A projected onto the vector alongside a rigid body.

## Coordinate Systems

Orthogonale Koordinatensystemen:

$$\text{Cartesian: } e_x \times e_y = e_z$$

$$\text{Cylindrical: } e_\rho \times e_\varphi = e_z$$

Position Vectors:

$$\text{Cartesian: } \mathbf{r} = xe_x + ye_y + ze_z$$

$$\text{Cylindrical: } \mathbf{r} = \rho e_\rho + ze_z$$

There is no separate  $e_\varphi$  component in a cylindrical position vector, as it's already accounted for by the  $e_\rho$  unit vector. The following derivatives are useful for calculations in the cylindrical co-ordinate system:

$$\text{Wegen des Einheitskreises: } e_\rho = \cos(\varphi)e_x + \sin(\varphi)e_y$$

$$\text{Intuitiv: } e_\varphi = \frac{d(e_\rho)}{d\varphi} = -\sin(\varphi)e_x + \cos(\varphi)e_y$$

$$\frac{d(e_\varphi)}{d\varphi} = -\cos(\varphi)e_x - \sin(\varphi)e_y = -e_\rho$$

$$\frac{d(e_\rho)}{d\varphi} = e_\varphi$$

The time derivatives can be found by deriving the cartesian formulae with respect to time and doing some substitution:

$$\frac{d(e_\varphi)}{dt} = -\dot{\varphi}e_\rho$$

$$\frac{d(e_\rho)}{dt} = \dot{\varphi}e_\varphi$$

Thus the velocity formula in the cylindrical co-ordinate system:

$$\vec{v} = \dot{\rho}e_\rho + \rho\dot{\varphi}e_\varphi + \dot{z}e_z$$

## Degrees of freedom

The minimum number of position vectors to clearly determine the state of a system.

Considering a system with several bodies. For a sum of degrees of freedom of  $n$ , and  $b$  linearly dependent connection equations, the resulting degrees of freedom of the whole system is:

$$f = n - b$$