BLG 202E - HW2

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Question 1:

$$\begin{aligned} x_1 - x_2 + 3x_3 &= 2 \\ x_1 + x_2 \quad\quad &= 4 \\ 3x_1 - 2x_2 + x_3 &= 1 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 1 & 1 & 0 & 4 \\ 3 & -2 & 1 & 1 \end{array}\right]$$

$$\begin{aligned} R_2 &\to R_2 - R_1 \\ R_3 &\to R_3 - 3R_1 \end{aligned} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 0 & 1 & -\frac{3}{2} & 1 \\ 0 & 1 & -8 & -5 \end{array}\right] R_3 \to R_3 - R_2 \left[\begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 0 & 1 & -\frac{3}{2} & 1 \\ 0 & 0 & -\frac{13}{2} & -6 \end{array}\right]$$

So,

$$-\frac{13}{2}x_3 = -6 \qquad x_3 = \frac{12}{13}$$

$$x_2 + \left(\frac{-3}{2}\right)\frac{12}{13} = 1$$

$$x_2 - \frac{18}{13} = 1 \qquad x_2 = \frac{31}{13}$$

$$x_1 - \frac{31}{13} + 3\,\frac{12}{13} = 2$$

$$x_1 - \frac{31}{13} + \frac{36}{13} = 2$$

$$x_1 + \frac{5}{13} = 2 \qquad x_1 = \frac{21}{13}$$

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Question 2 :

a.

$$I \cdot A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$, interchanging R_3 and R_4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} , \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Question 2 -

b.

$PA = LU$

$A = P^{-1}LU$

$Ax = b$

$P^{-1}LUx = b$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 26 \\ 9 \\ 1 \\ -3 \end{bmatrix}}_{b}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}}_{LUx} = \begin{bmatrix} 26 \\ 9 \\ 1 \\ -3 \end{bmatrix}$$

$a_1 = 26$
$a_2 = 9$
$a_3 = -3$
$a_4 = 1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{Ux} = \begin{bmatrix} 26 \\ 9 \\ -3 \\ 1 \end{bmatrix}$$

$b_1 = 26$
$b_2 = 9$
$b_3 = -3$
$b_4 = 1$

$$\underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{U} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 26 \\ 9 \\ -3 \\ 1 \end{bmatrix}$$

$$\underbrace{\begin{matrix} x_1 = 1 \\ x_2 = 1 \\ x_3 = 1 \\ x_4 = 1 \end{matrix}}_{x}$$

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q3. $A = \begin{bmatrix} -2 & 1 & 4 \\ 1 & 1 & 1 \\ 4 & 1 & -2 \end{bmatrix}$

```
Iteration 1 :
Eigenvalue 1 : 8
Eigenvector 1 : [-0.5   0.25  1.  ]
Iteration 2 :
Eigenvalue 1 : 5.25
Eigenvector 1 : [ 1.          0.14285714 -0.71428571]
Iteration 3 :
Eigenvalue 1 : 5.571428571428572
Eigenvector 1 : [-0.84615385  0.07692308  1.        ]
Iteration 4 :
Eigenvalue 1 : 5.769230769230769
Eigenvector 1 : [ 1.    0.04 -0.92]
Iteration 5 :
Eigenvalue 1 : 5.88
Eigenvector 1 : [-0.95918367  0.02040816  1.        ]


Iteration 1 :
Eigenvalue 2 : 4
Eigenvector 2 : [1. 1. 1.]
Iteration 2 :
Eigenvalue 2 : 3.0
Eigenvector 2 : [1. 1. 1.]
Iteration 3 :
Eigenvalue 2 : 3.0
Eigenvector 2 : [1. 1. 1.]
Iteration 4 :
Eigenvalue 2 : 3.0
Eigenvector 2 : [1. 1. 1.]
Iteration 5 :
Eigenvalue 2 : 3.0
Eigenvector 2 : [1. 1. 1.]
```

When $v_0 = [1,2,-1]^T$ eigenvalues converges to 6 according to iterations, eigenvectors converges to $v = [1,0,1]^T$.

```
Iteration 53 :
Eigenvalue 1 : 5.999999999999999
Eigenvector 1 : [-1.00000000e+00  9.25185854e-17  1.00000000e+00]
Iteration 54 :
Eigenvalue 1 : 6.0
Eigenvector 1 : [ 1.00000000e+00  3.70074342e-17 -1.00000000e+00]
Iteration 55 :
Eigenvalue 1 : 6.0
Eigenvector 1 : [-1.  0.  1.]
Iteration 56 :
Eigenvalue 1 : 6.0
Eigenvector 1 : [ 1.  0. -1.]
```

When $v_0 = [1,2,1]^T$ eigenvalues converges to 3 according to iterations, eigenvectors converges to $v = [1,1,1]^T$.

```
Iteration 3 :
Eigenvalue 2 : 3.0
Eigenvector 2 : [1. 1. 1.]
Iteration 4 :
Eigenvalue 2 : 3.0
Eigenvector 2 : [1. 1. 1.]
Iteration 5 :
Eigenvalue 2 : 3.0
Eigenvector 2 : [1. 1. 1.]
```

Limits did not seem to be same because of every array has unique eigenvalues and we reach them by using different initial vectors.