Analysis of Algorithms 1
(Fall 2013)
Istanbul Technical University
Computer Eng. Dept.

Chapter 9

Medians and Order Statistics

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Order Statistics

- The **ith order statistic** of a set of n elements is the **ith smallest element**.
- Minimum: first order statistic (i = 1),
- Maximum: nth order statistic (i = n).
- **Median,:** "halfway point" of the set.
  - Odd n: median occurs at i = (n + 1)/2
  - Even n:
    - lower median at floor((n + 1)/2)
    - Upper median at ceil((n + 1)/2)

Selection Problem

- **Input:** A set A of n (distinct) numbers and a number i, with $1 \le i \le n$.
- **Output:** The element $x \in A$ that is larger than exactly i - 1 other elements of A.
- can be solved in O(n lg n) time, since we can sort the numbers using heapsort or merge sort and then simply index the ith element in the output array
- But there are faster methods.

Minimum and Maximum

The best algorithm has complexity of θ(n)

Easiest algorithm:

- MINIMUM(*A)*
- 1 min ← A[1]
- 2 for i ← 2 to length[A]
- 3 do if min > A[i]
- 4 then min ← A[i]
- 5 return min

Randomized Select

- RANDOMIZED-SELECT(*A, p, r, i)*
- 1 **if *p = r***
- 2 **then return *A[p]***
- 3 *q* ← *RANDOMIZED-PARTITION(A, p, r)*
- 4 *k* ← *q - p + 1*
- 5 **if *i = k*** ▷ ***the pivot value is the answer***
- 6 **then return *A[q]***
- 7 **elseif *i < k***
- 8 **then return RANDOMIZED-SELECT(*A, p, q - 1, i)***
- 9 **else return RANDOMIZED-SELECT(*A, q + 1, r, i - k)***

expected time of RANDOMIZED-SELECT is Θ(*n).*

Randomized Select

- RANDOMIZED-SELECT(*A, p, r, i)*
- 1 **if *p = r***
- 2 **then return *A[p]***
- 3 *q* ← *RANDOMIZED-PARTITION(A, p, r)* $\Theta(n)$
- 4 *k* ← *q - p + 1*
- 5 **if *i = k*** ▷ ***the pivot value is the answer***
- 6 **then return *A[q]***
- 7 **elseif *i < k***
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T(n)=T(n/b)+ Θ(*n)*

T(n)=Θ(*n) //Master Thm, Case 3*

$$3.\ f(n) = \Omega\left(n^{\log_b a+\varepsilon}\right) \quad and \quad af(n/b) \le cf(n),$$

$$for \quad \exists c \quad c<1 \quad and \quad n > n_0$$

$$\Rightarrow T(n) = \Theta\left(f(n)\right)$$