EHB 211E: Basics of Electrical Circuits

Operational Amplifiers

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OP AMP (Operational Amplifier)

An op amp is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.

- The op amp is an electronic unit that behaves like a voltage-controlled voltage source
- The op amp can sum, amplify, integrate, differentiate a signal.
- The ability of an op amp to perform these operations is the reason it is called "operational amplifier"
- Op amps are among most widely used chips around the world

An op amp is designed so that it performs some mathematical operations when external components, such as resistors and capacitors, are connected to its terminals

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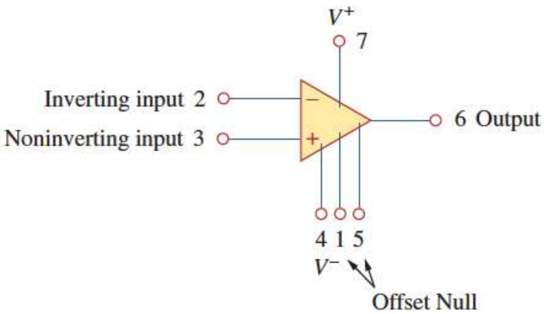
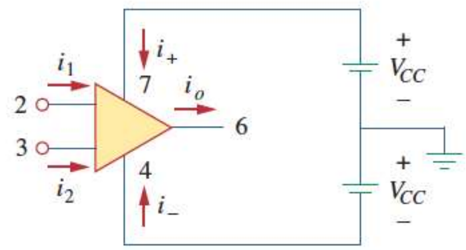
OP AMP (Operational Amplifier)

Circuit symbol



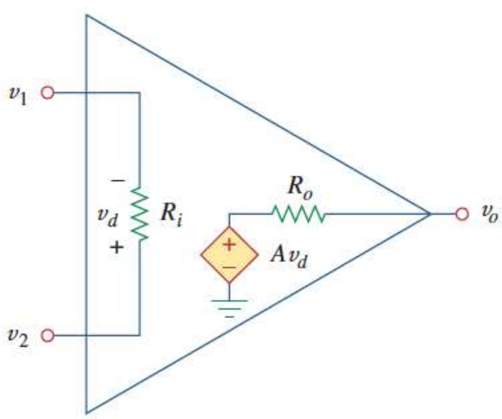
Powering the op amp



$$i_o = i_1 + i_2 + i_+ + i_-$$ (KCL)

OP AMP – equivalent circuit

Equivalent circuit



- R_i is the Thevenin equivalent resistance seen at the input terminal
- R_o is the Thevenin equivalent resistance seen at the output terminal

Differential input:

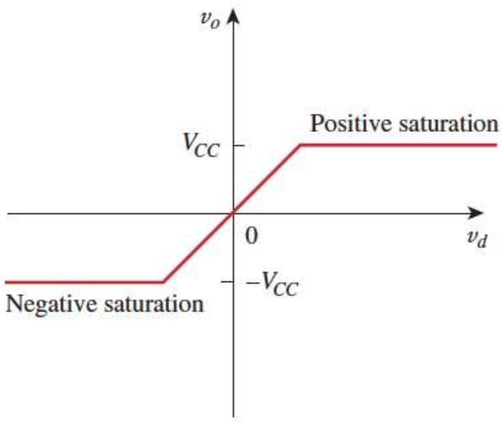
$$v_d = v_2 - v_1$$

Op amp output:
op amp introduces gain on the differential input:

$$\boxed{v_o = Av_d = A(v_2 - v_1)}$$

A: open-loop voltage gain
(without feedback from the input)

OP AMP – output limitation



- Magnitude of the output voltage can not exceed V_{CC}, output voltage is limited by the supply voltage!

$$-V_{CC} \leq v_o \leq V_{CC}$$

- Op amp operates in 3 modes:

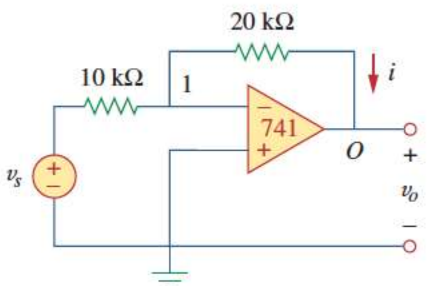
1. Positive saturation, $v_o = V_{CC}$.
2. Linear region, $-V_{CC} \leq v_o = Av_d \leq V_{CC}$.
3. Negative saturation, $v_o = -V_{CC}$.

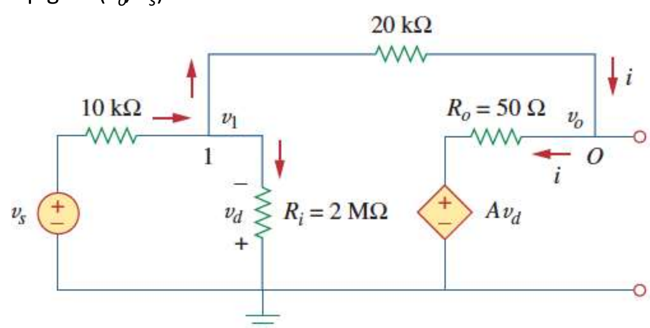
OP AMP – typical ranges for op amp parameters

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to 10^{13} Ω	$\infty\Omega$
Output resistance, R_o	10 to 100 Ω	0Ω
Supply voltage, V_{CC}	5 to 24 V	

Exercise

The below op amp (type 741) has an open-loop voltage gain of 2 x 10^5, input resistance of 2 MΩ, and output resistance of 50 Ω. Find the closed loop gain (v_o/v_s).





$$\frac{v_s - v_1}{10 \times 10^3} = \frac{v_1}{2000 \times 10^3} + \frac{v_1 - v_o}{20 \times 10^3} \quad \text{(Node 1)}$$

$$v_d = -v_1 \text{ and } A = 200{,}000$$

$$\frac{v_1 - v_o}{20 \times 10^3} = \frac{v_o - Av_d}{50} \quad \text{(Node 0)}$$

$$\Rightarrow \quad \frac{v_o}{v_s} = -1.9999699$$

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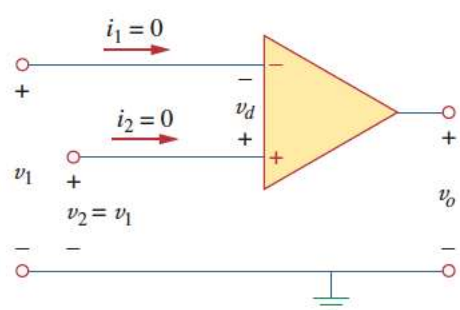
Ideal Op Amp

An ideal op amp is an amplifier with infinite open-loop gain, infinite input resistance and zero output resistance

1. Infinite open-loop gain, $A \simeq \infty$.
2. Infinite input resistance, $R_i \simeq \infty$.
3. Zero output resistance, $R_o \simeq 0$.



Current into both input terminals are zero

$$\boxed{i_1 = 0, \qquad i_2 = 0}$$

Due to infinite input resistance

Voltage across the input terminals is zero

$$v_d = v_2 - v_1 = 0 \qquad \boxed{v_1 = v_2}$$

Due to infinite open-loop gain

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Exercise

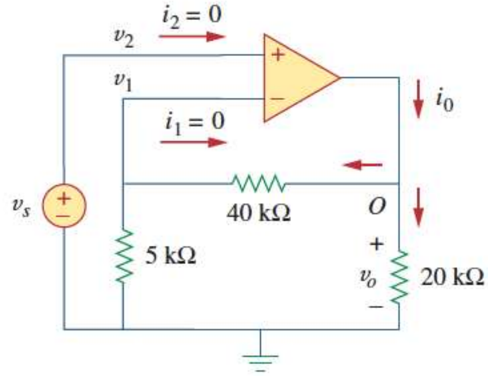
Calculate the closed-loop gain for the below circuit (assuming ideal op amp)

$$v_2 = v_s$$

$$v_1 = \frac{5}{5+40} v_o = \frac{v_o}{9} \quad \text{(voltage division)}$$

$$v_2 = v_1$$

$$v_s = \frac{v_o}{9} \quad \Rightarrow \quad \frac{v_o}{v_s} = 9$$



Inverting Amplifier

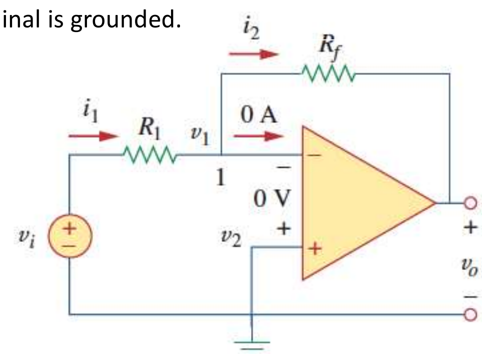
Input is applied to the inverting terminal. Non-inverting terminal is grounded.

$$i_1 = i_2 \quad \Rightarrow \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

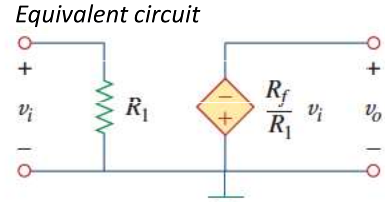
$$\frac{v_i}{R_1} = -\frac{v_o}{R_f}$$

$$\boxed{v_o = -\frac{R_f}{R_1} v_i}$$

Gain only depends on the external circuit elements

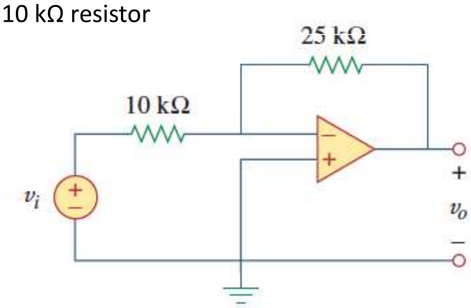


Equivalent circuit



Exercise

If v_i = 0.5 V, calculate output voltage (v_o) and the current in the 10 kΩ resistor



$$\frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

$$v_o = -2.5v_i = -2.5(0.5) = -1.25 \text{ V}$$

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50\ \mu\text{A}$$

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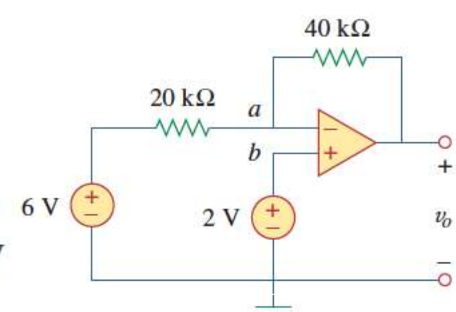
Exercise

Determine v_o



$$\frac{v_a - v_o}{40\text{ k}\Omega} = \frac{6 - v_a}{20\text{ k}\Omega}$$

$$v_a - v_o = 12 - 2v_a \quad \Rightarrow \quad v_o = 3v_a - 12$$

$$v_a = v_b = 2 \text{ V} \qquad v_o = 6 - 12 = -6 \text{ V}$$

If v_b would have been 0 V , v_o = -12 V (6V*-40/20)

If v_b is non-zero, one can not directly apply the non-inverting amplifier formula!

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Non-inverting Amplifier

Input is applied to the non-inverting terminal.
R1 is connected between the inverting terminal and the ground.

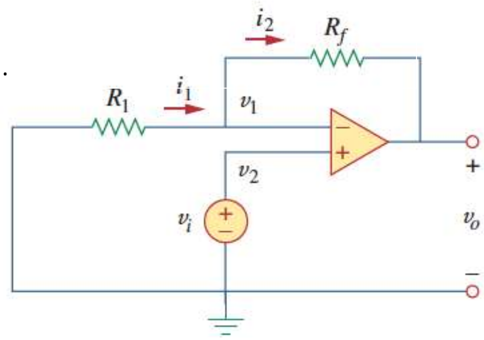
$$i_1 = i_2 \quad \Rightarrow \quad \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

$$v_1 = v_2 = v_i$$

$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f}$$

$$\boxed{v_o = \left(1 + \frac{R_f}{R_1}\right) v_i}$$

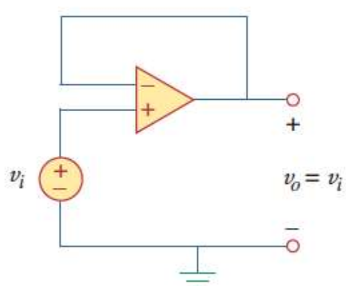
Positive gain!



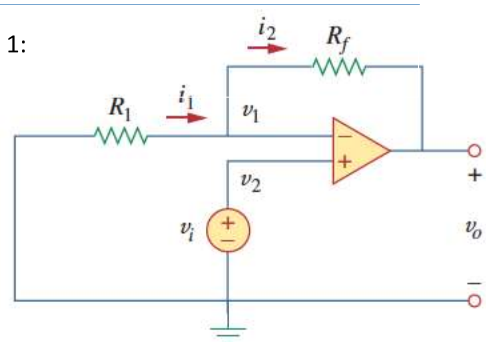
Once again, gain depends on external circuit elements

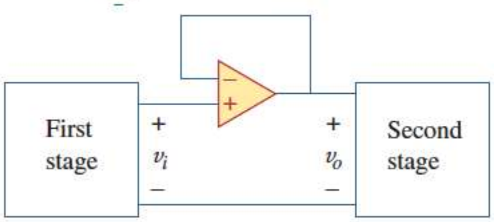
Non-inverting Amplifier – special cases

If $R_f = 0$, or $R_1 = \infty$, non-inverting amplifier circuit gain becomes 1:
This special circuit is called "voltage-follower"



$$\boxed{v_o = v_i}$$

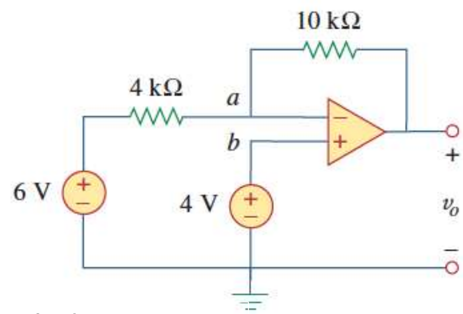




The voltage follower is used in minimizing the interaction between two stages of an amplifier. It is used as an intermediate stage (or-buffer) to isolate one circuit from the other

Exercise

Calculate v_o



Method 1: Superposition

$$v_o = v_{o1} + v_{o2}$$

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$ Due 6V source (inverting amplifier)

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14 \text{ V}$$ Due 4V source (non-inverting ampifier)

$$v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$$

Method 2: KCL

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

$$v_a = v_b = 4$$

$$v_o = -1 \text{ V}$$

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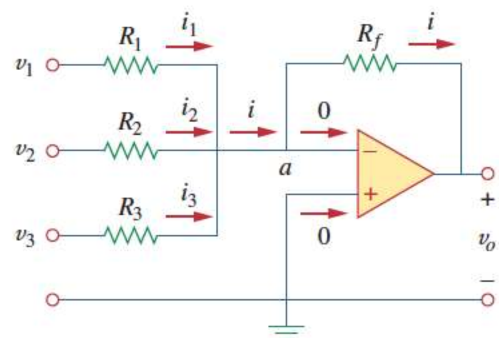
Summing Amplifier

A summing amplifier is an op amp circuit, which combines several inputs and produces an output that is the weighted sum of the inputs:



$$i = i_1 + i_2 + i_3$$

$$i_1 = \frac{v_1 - v_a}{R_1}, \quad i_2 = \frac{v_2 - v_a}{R_2}$$

$$i_3 = \frac{v_3 - v_a}{R_3}, \quad i = \frac{v_a - v_o}{R_f}$$

$$\boxed{v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)}$$

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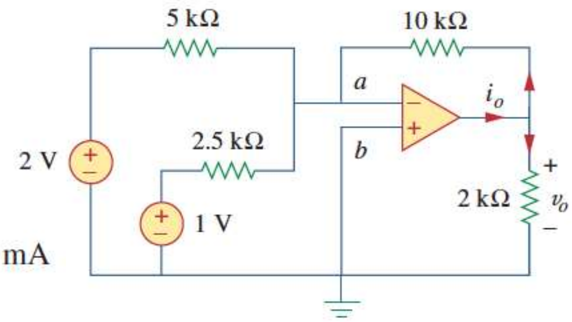
Exercise

Calculate v_o and i_o

Summer with 2 inputs:

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4 + 4) = -8 \text{ V}$$

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} \text{mA} = -0.8 - 4 = -4.8 \text{ mA}$$



Difference Amplifier

A difference amplifier is an op amp circuit, which amplifies the difference between two inputs, but rejects any signal that is common to the two inputs.

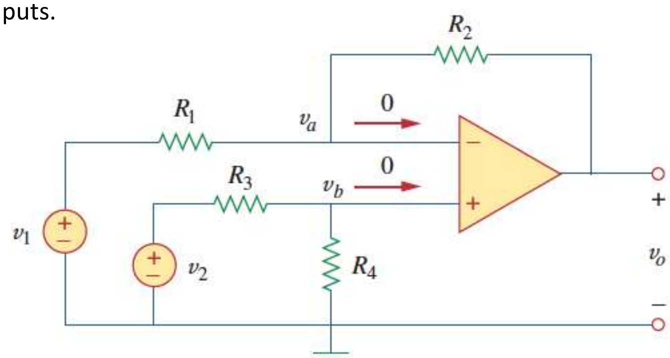
$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

$$v_o = \left(\frac{R_2}{R_1} + 1\right)v_a - \frac{R_2}{R_1}v_1$$

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

$$v_b = \frac{R_4}{R_3 + R_4}v_2$$ Voltage divider

$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$



$$\boxed{v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1}$$

Difference Amplifier

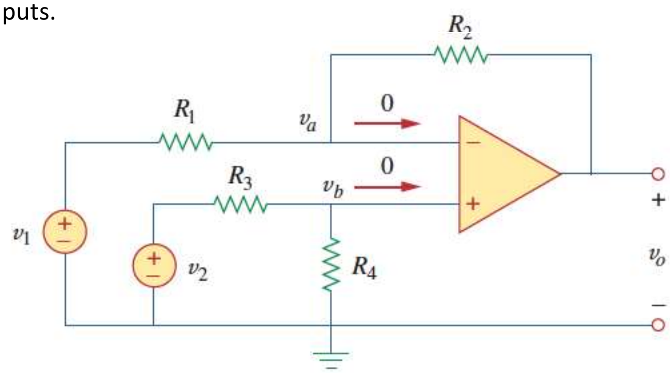
A difference amplifier is an op amp circuit, which amplifies the difference between two inputs, but rejects any signal that is common to the two inputs.

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

when

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$



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Exercise

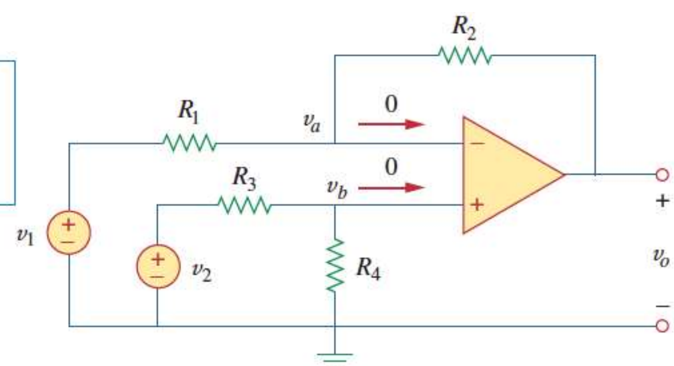
Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Design 1: use only one op amp

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

$$\frac{R_2}{R_1} = 5 \qquad 5\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3$$

$$R_3 = R_4$$



We may choose:

$R_1 = 10\ \text{k}\Omega$ and $R_3 = 20\ \text{k}\Omega$ $\qquad R_2 = 50\ \text{k}\Omega$ and $R_4 = 20\ \text{k}\Omega$

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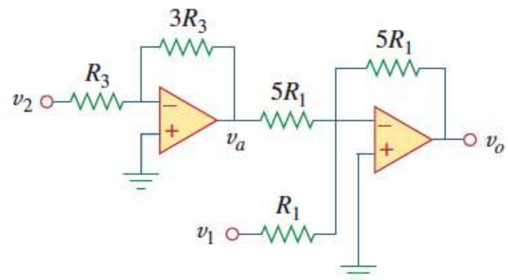
Exercise (continued…)

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Design 2: use 2 one op amps

Superposition:
-> v2 goes into 2 inverting amplifiers:
Overall gain: -3/1*-5/5 = 3
-> v1 goes into 1 inverting amplifier
Overall gain: -5/1 = -5



$$v_o = 3v_2 - 5v_1$$

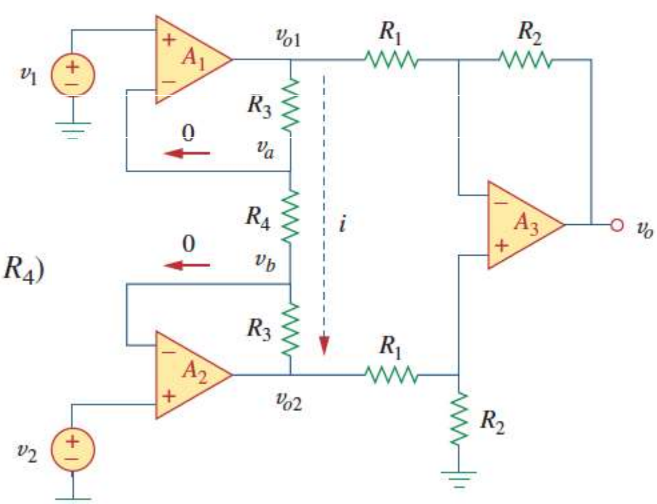
We may choose:

$$R_1 = R_3 = 10\ \text{k}\Omega$$

Exercise

The instrumentation amplifier, shown below is an amplifier of low-level signals used in process control or measurement applications and commercially available in a single package. Show that:

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$



A_3 is a difference amplifiler:

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1})$$

$$v_{o1} - v_{o2} = i(R_3 + R_4 + R_3) = i(2R_3 + R_4)$$

$$i = \frac{v_a - v_b}{R_4} \qquad i = \frac{v_1 - v_2}{R_4}$$

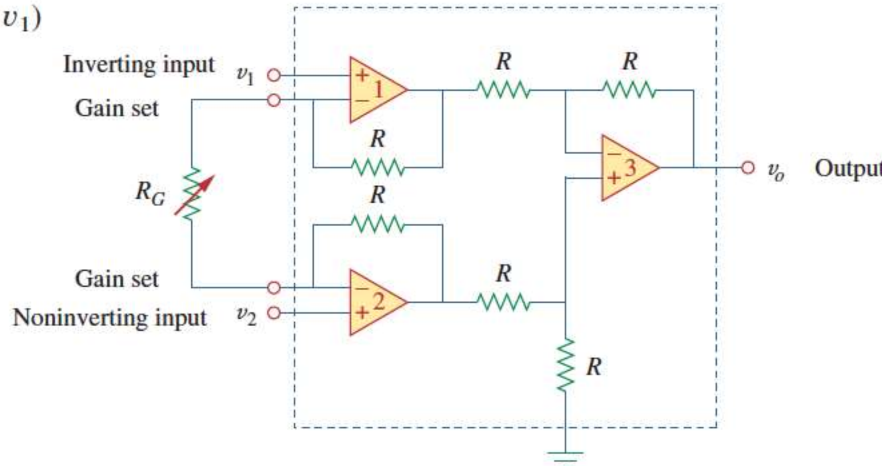
$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

Instrumentation Amplifier

Instrumentation amplifier amplifies small differential signal voltages superimposed on larger common-mode voltages

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

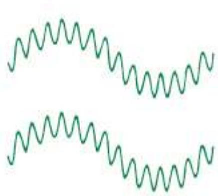
$$A_v = 1 + \frac{2R}{R_G}$$



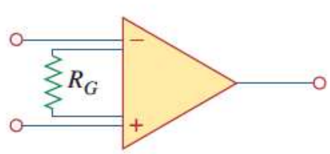
Instrumentation Amplifier (IA)

Instrumentation amplifier has 3 major characteristics

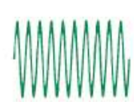
1) The voltage gain is adjusted by one external resistor: R_G
2) The input impedance of both inputs is very high and does not vary as the gain is adjusted
3) The output v_o depends on the difference between the inputs v_1 and v_2, not on the voltage common to them (common-mode voltage)



Small differential signals riding on larger common-mode signals



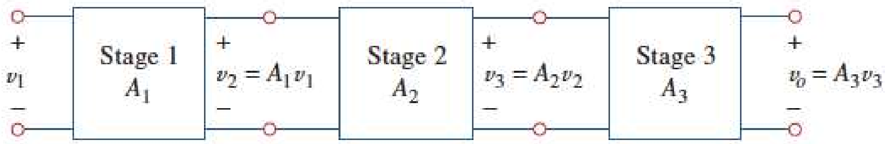
Instrumentation amplifier



Amplified differential signal, no common-mode signal

Cascaded Op Amp Circuits

A cascade connection is a head-to-tail arrangement of two or more op amp circuits such that the output of one is the input of the next:

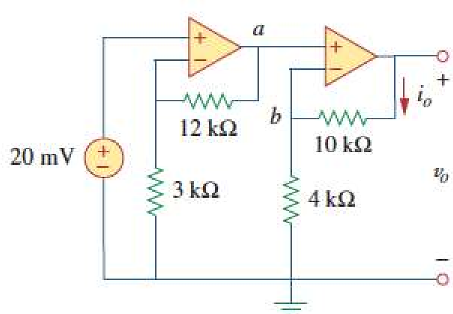


$$A = A_1 A_2 A_3$$

Exercise

Find v_o and i_o

Two non-inverting amplifiers are cascaded:



$$v_a = \left(1 + \frac{12}{3}\right)(20) = 100 \text{ mV}$$

$$v_o = \left(1 + \frac{10}{4}\right)v_a = (1 + 2.5)100 = 350 \text{ mV}$$

$$i_o = \frac{v_o - v_b}{10} \text{mA} \qquad v_b = v_a = 100 \text{ mV}$$

$$i_o = \frac{(350 - 100) \times 10^{-3}}{10 \times 10^3} = 25 \,\mu\text{A}$$

Exercise

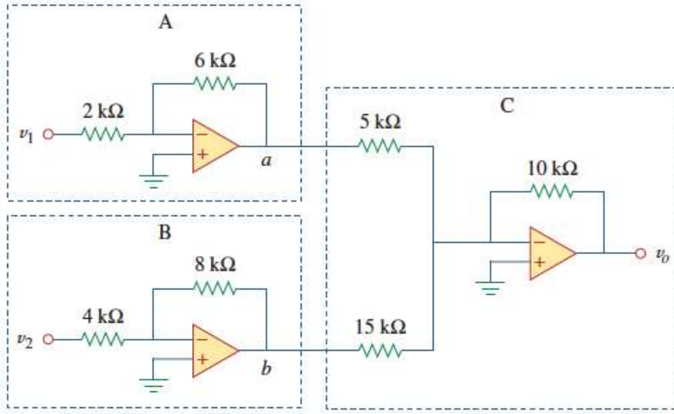
If v_1 = 1V and v_2 = 2V, find v_o.

Two inverting amplifiers are fed into a summing amplifier:

Gain of stage A: -3 -> $v_a = -3v_1$
Gain of stage B: -2 -> $v_b = -2v_2$
Stage C:

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

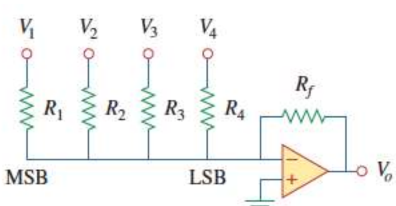
$v_o = -2v_a - 2/3v_b = 6v_1 + 4/3v_2$
$= 6 + 8/3 =$ **8.666 V**.



Digital to Analog Converter

The digital to analog converter (DAC) transforms digital signals into analog form.

A 4-bit DAC can be realized in many ways. A Simple realization is shown below (summing amplifier):



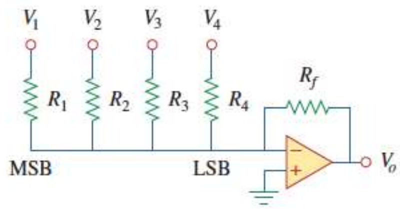
$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

Most Significant Bit (MSB)

Least Significant Bit (LSB)

Assume two voltage levels (binary) for the inputs V_1, V_2, V_3, V_4

Digital to Analog Converter



let $R_f = 10\ \text{k}\Omega$, $R_1 = 10\ \text{k}\Omega$
$R_2 = 20\ \text{k}\Omega$, $R_3 = 40\ \text{k}\Omega$, and $R_4 = 80\ \text{k}\Omega$

$$-V_o = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$
$$= V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

Binary input $[V_1V_2V_3V_4]$	Decimal value	Output $-V_o$
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
1010	10	1.25
1011	11	1.375
1100	12	1.5
1101	13	1.625
1110	14	1.75
1111	15	1.875

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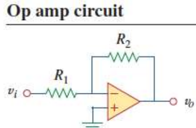
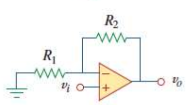
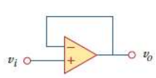
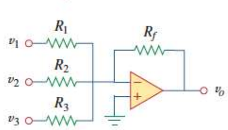
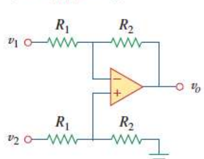
Summary

Op amp circuit	Name/output-input relationship
	Inverting amplifier $v_o = -\frac{R_2}{R_1}v_i$
	Noninverting amplifier $v_o = \left(1 + \frac{R_2}{R_1}\right)v_i$
	Voltage follower $v_o = v_i$
	Summer $v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$
	Difference amplifier $v_o = \frac{R_2}{R_1}(v_2 - v_1)$

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