EHB 211E: Basics of Electrical Circuits

Circuit Theorems

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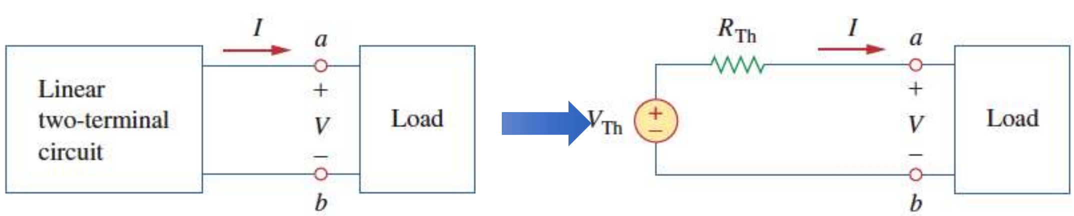
1

Thevenin's Theorem

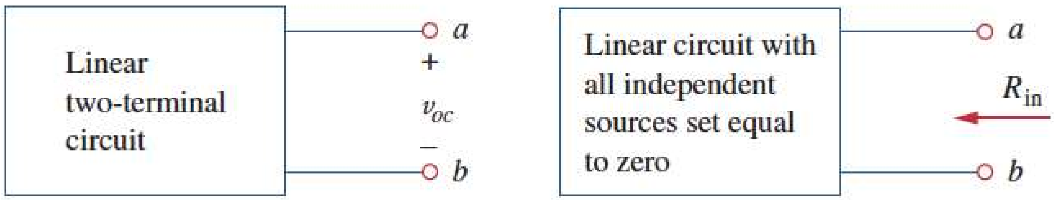
- A particular element in a circuit is variable (load) while other elements are fixed. *Example*: A household outlet terminal may be connected to different appliances constituting a variable load.
- Each time the variable element is changed, the entire circuit has to be analyzed all over again.
- To avoid this problem, Thevenin's Theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent, simplified circuit.



Thevenin's theorem states that a linear time-invariant resistive two-terminal one-port circuit can be replaced by an equivalent circuit consisting of a voltage source (V_{Th}) in series with a resistor (R_{Th}), where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input / equivalent resistance at the terminals when the independent sources are turned off.

2

Thevenin's Theorem - *summary*



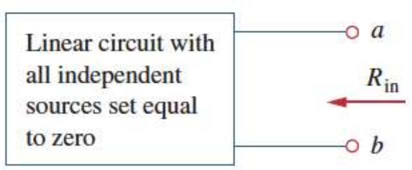
$V_{Th} = v_{oc}$

$R_{Th} = R_{in}$

$i_0 = (v_o - v_{Th}) / R_{Th}$
-> 0 intercept of the below graph gives v_{Th}
-> slope gives R_{Th}

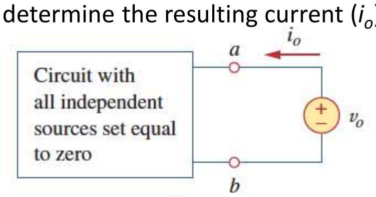
Thevenin's Theorem – *two cases*

- CASE 1: If the circuit has no dependent sources, we turn off all independent sources, R_{Th} is the input resistance of the network looking between terminals a-b.

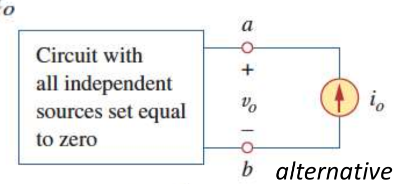


$R_{Th} = R_{in}$

- CASE 2: If the network has dependent sources, we turn of all independent sources (similar to superposition principle the dependent sources stay). We apply a voltage source (v_o) at terminals a-b, and determine the resulting current (i_o). Then: $R_{Th} = v_o / i_o$



$R_{Th} = \frac{v_o}{i_o}$



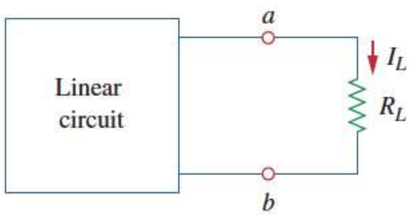
alternative

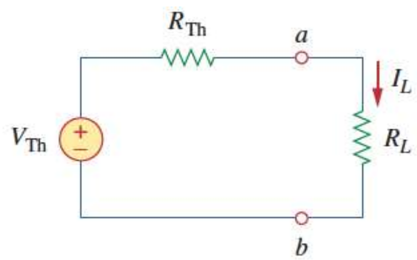
$R_{Th} = \frac{v_o}{i_o}$

Thevenin's Theorem – *benefits*

- Greatly simplifies the circuit! A large circuit is replaced by an independent source and a resistor.
- The equivalent circuit behaves the same way as the original circuit.
- Consider a linear circuit terminated by a load resistance, current through the node, and the voltage across the terminal can be calculated by:

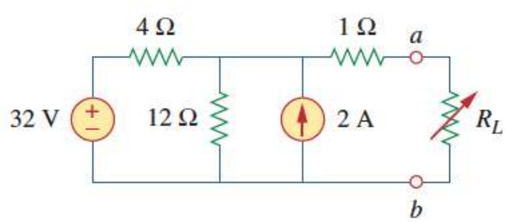
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \qquad V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$





Exercise

Find the Thevenin equivalent for the circuit below, to the left of terminals a-b. Then, find the current for R_L = 6, 16 Ω



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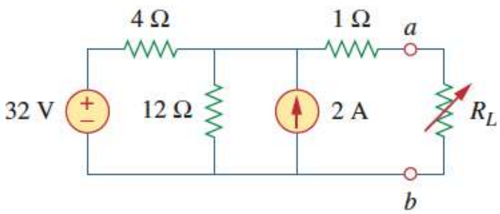
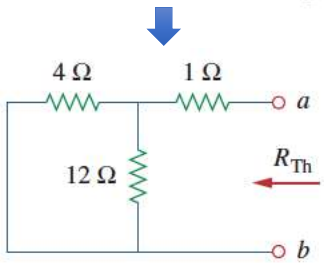
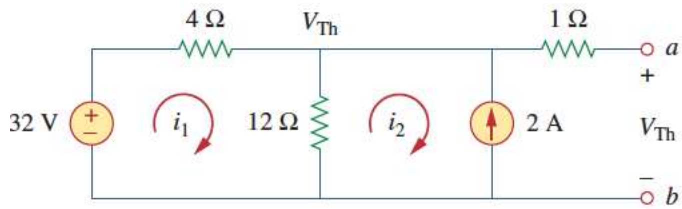


$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$



To find V_{Th}, apply mesh analysis



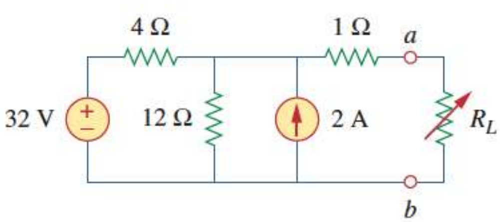
$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\ \text{A}$$

$$i_1 = 0.5\ \text{A}$$

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\ \text{V}$$

Exercise (continued…)

Find the Thevenin equivalent for the circuit below, to the left of terminals a-b. Then, find the current for R_L = 6, 16 Ω



Alternatively use nodal analysis,

Exercise (continued…)

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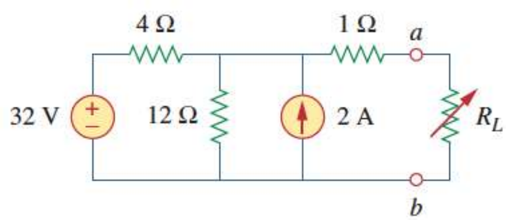
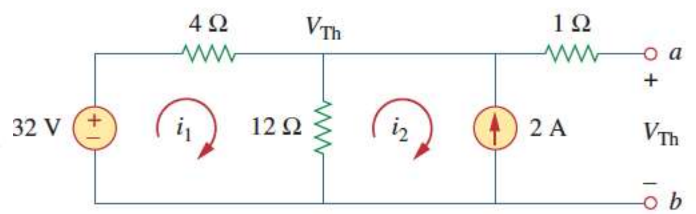




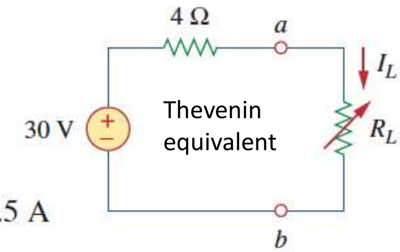
Alternatively use nodal analysis,

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12} \Rightarrow \quad V_{Th} = 30\text{ V}$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

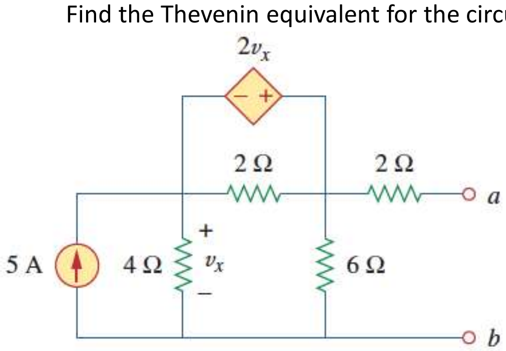
$R_L = 6$ $\qquad$ $I_L = \frac{30}{10} = 3\text{ A}$

$R_L = 16$ $\qquad$ $I_L = \frac{30}{20} = 1.5\text{ A}$

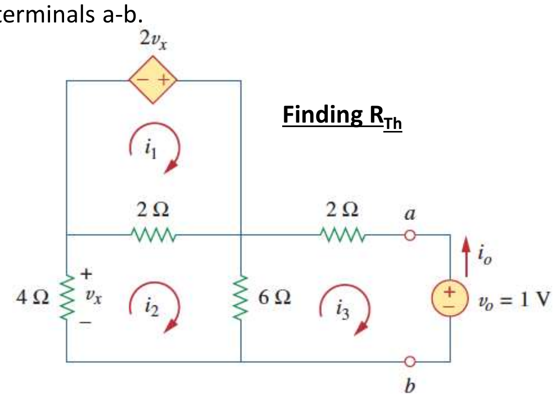


Exercise

Find the Thevenin equivalent for the circuit below, at terminals a-b.



Finding R_{Th}



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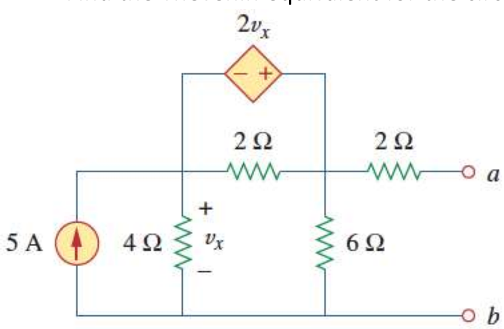
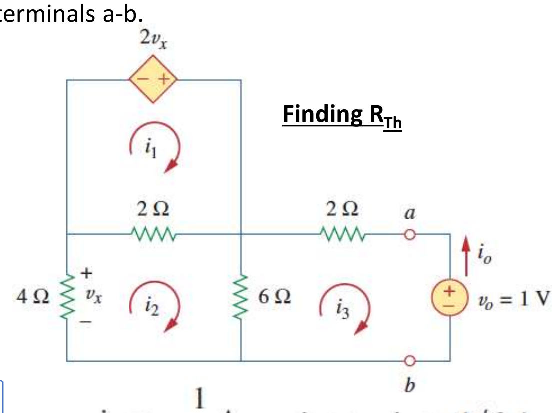




$$-2v_x + 2(i_1 - i_2) = 0$$ Loop 1

$$-4i_2 = v_x$$

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$ Loop 2

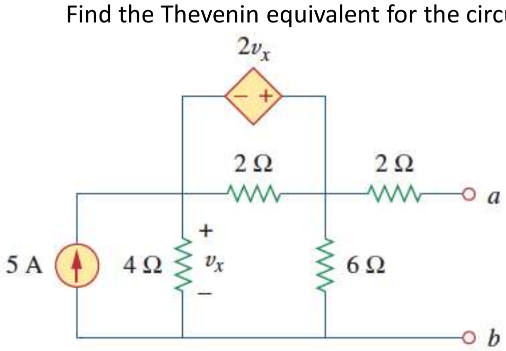
$$6(i_3 - i_2) + 2i_3 + 1 = 0$$ Loop 3

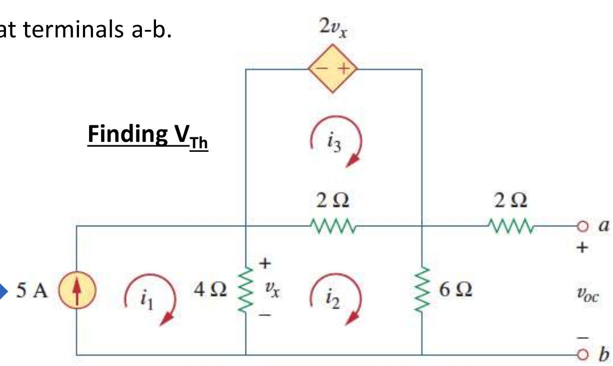
$$i_3 = -\frac{1}{6}\text{ A} \qquad i_o = -i_3 = 1/6\text{ A}$$

$$R_{\text{Th}} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

Exercise (continued)

Find the Thevenin equivalent for the circuit below, at terminals a-b.





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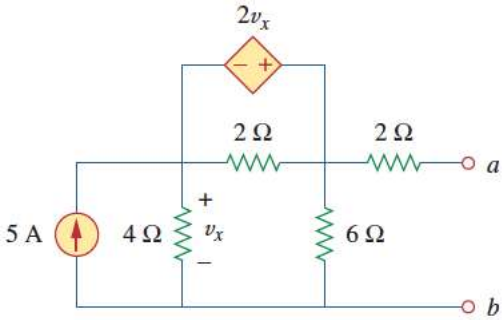
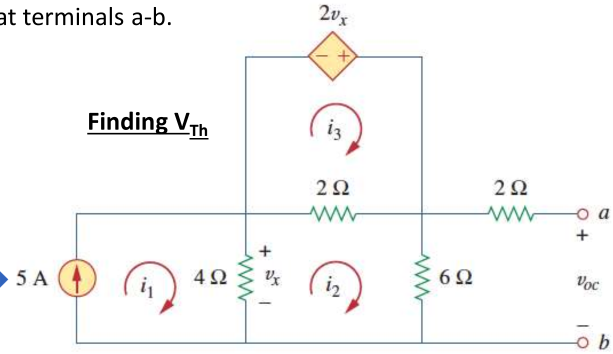


Finding V_{Th}



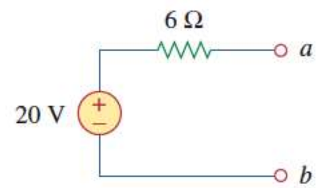
$i_1 = 5$ Loop 1

$-2v_x + 2(i_3 - i_2) = 0$ Loop 3

$4(i_1 - i_2) = v_x$

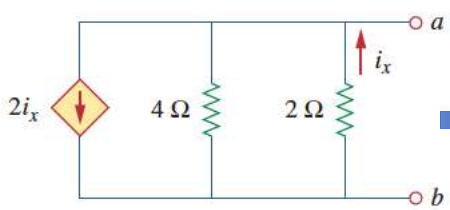
$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$ Loop 2

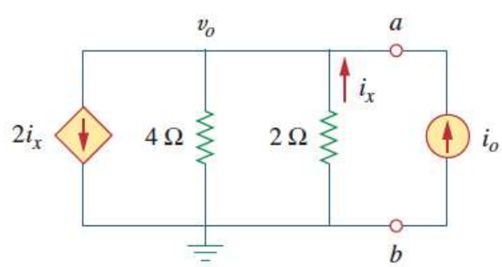
$i_2 = 10/3$ $V_{\text{Th}} = v_{oc} = 6i_2 = 20$ V



Exercise

Find the Thevenin equivalent for the circuit below, at terminals a-b.





$i_o = 1$ A

$2i_x + (v_o - 0)/4 + (v_o - 0)/2 + (-1) = 0$ KCL at node a

$i_x = (0 - v_o)/2 = -v_o/2$ Ohm's law

$v_o = -4$ V

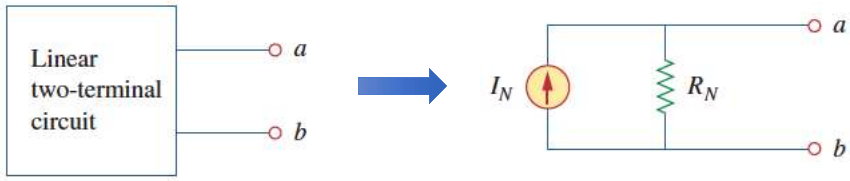
$R_{\text{Th}} = v_o/1 = \mathbf{-4\ \Omega}$

V_{Th} = 0V, no independent sources

Negative resistance implies that the circuit is supplying power due to dependent source

Norton's Theorem

Norton's theorem states that a linear time-invariant resistive two-terminal one-port circuit can be replaced by an equivalent circuit consisting of a current source (I_N) in parallel with a resistor (R_N), where I_N is the short circuit current at the terminals and R_N is the input / equivalent resistance at the terminals when the independent sources are turned off.

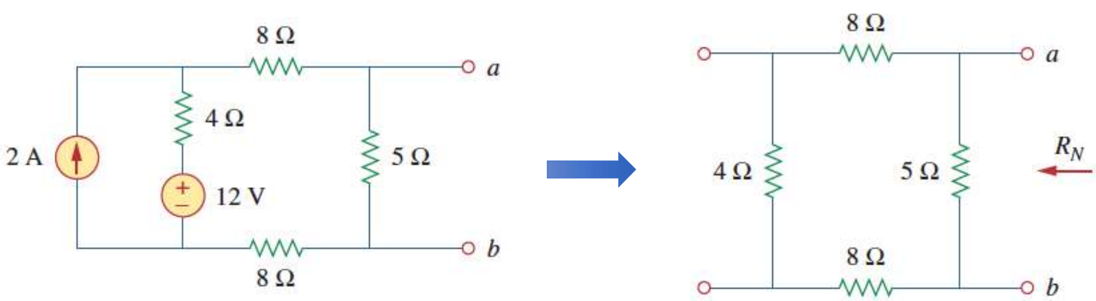


From source transformation:

$$\boxed{R_N = R_{Th}} \qquad \boxed{I_N = \frac{V_{Th}}{R_{Th}}} \qquad I_N = i_{sc}$$

Exercise

Find the Norton equivalent for the circuit below, at terminals a-b.

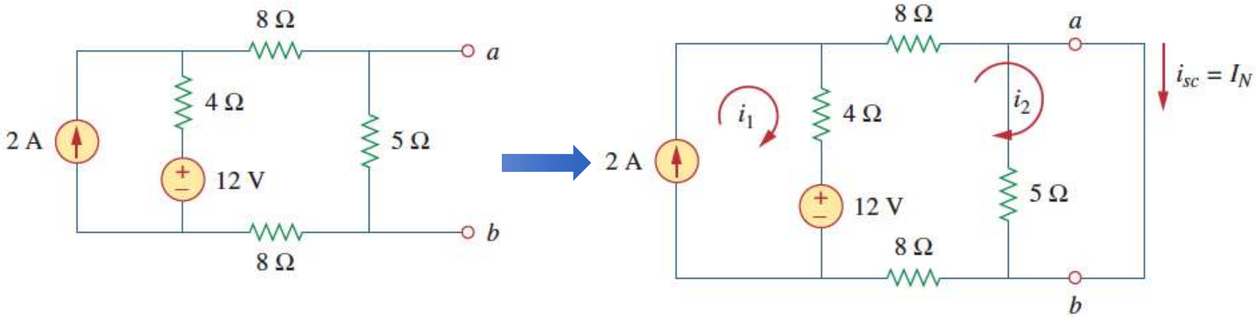


Finding R_N

$$R_N = 5 \,\|\, (8 + 4 + 8) = 5 \,\|\, 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

Exercise (continued…)

Find the Norton equivalent for the circuit below, at terminals a-b.

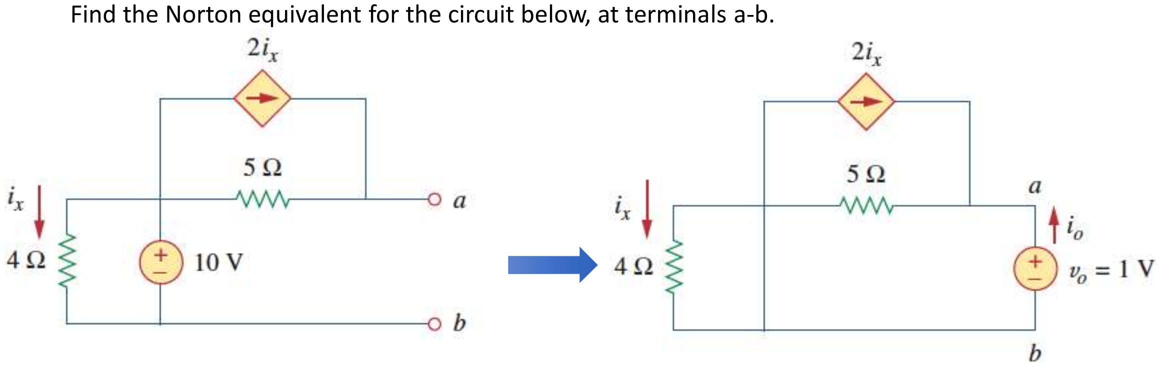


Finding I_N

$$i_1 = 2 \text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$
$$i_2 = 1 \text{ A} = i_{sc} = I_N$$

Exercise

Find the Norton equivalent for the circuit below, at terminals a-b.



Finding R_N
No current passes on 4 Ω:
-> $i_x = 0$
-> Dependent source is open circuited

$$i_o = \frac{1v}{5\Omega} = 0.2 \text{ A} \qquad v_o = 1 \text{ V}$$
$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\ \Omega$$

Exercise

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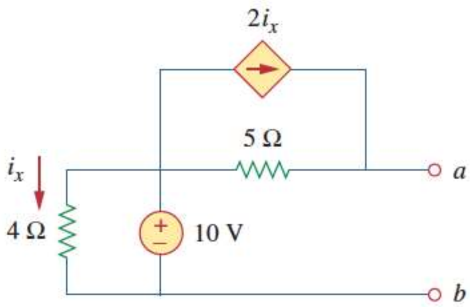
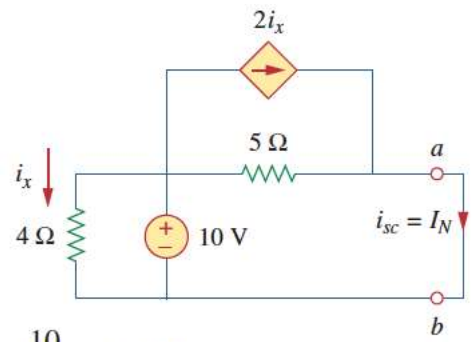




Finding I_N

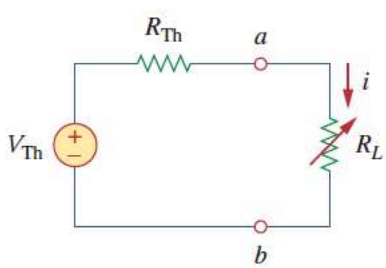
$$i_x = \frac{10}{4} = 2.5\ \text{A}$$

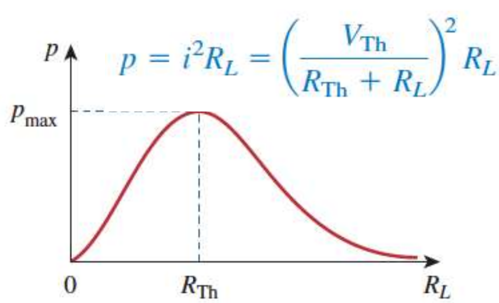
$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7\ \text{A} \quad \text{KCL at node a}$$

$$I_N = 7\ \text{A}$$

Maximum Power Transfer

For many applications, circuits are designed to deliver power to the load.
It is desirable to maximize the power, delivered to the load:

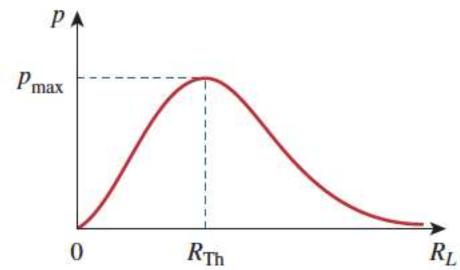




Maximum power is transferred to the load, when the load resistance equals the Thevenin resistance, seen from the load

Maximum Power Transfer - proof

$$p = i^2 R_L = \left(\frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L}\right)^2 R_L$$



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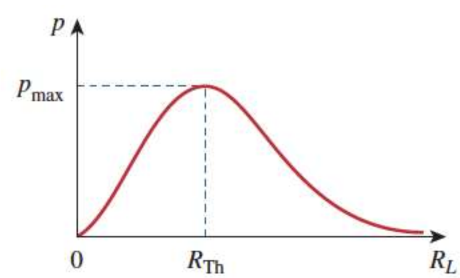


$$\frac{dp}{dR_L} = V_{\mathrm{Th}}^2\left[\frac{(R_{\mathrm{Th}} + R_L)^2 - 2R_L(R_{\mathrm{Th}} + R_L)}{(R_{\mathrm{Th}} + R_L)^4}\right]$$

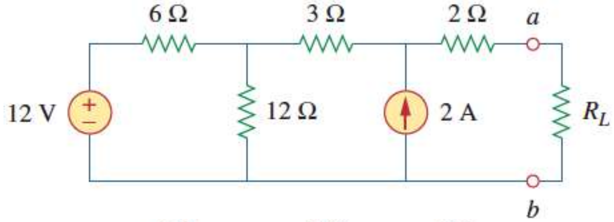
$$= V_{\mathrm{Th}}^2\left[\frac{(R_{\mathrm{Th}} + R_L - 2R_L)}{(R_{\mathrm{Th}} + R_L)^3}\right] = 0$$

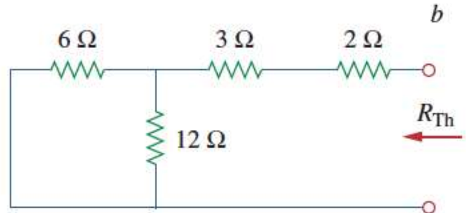
$$0 = (R_{\mathrm{Th}} + R_L - 2R_L) = (R_{\mathrm{Th}} - R_L)$$

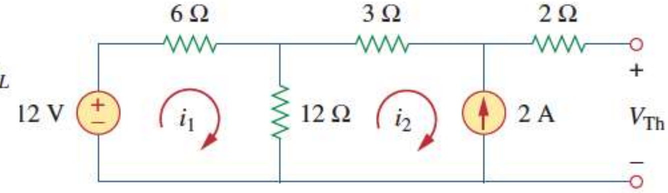
$$\boxed{R_L = R_{\mathrm{Th}}} \qquad \boxed{p_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}}}$$

Exercise

Find the value of R_L for maximum power transfer







$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

$$i_1 = -2/3$$

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$$

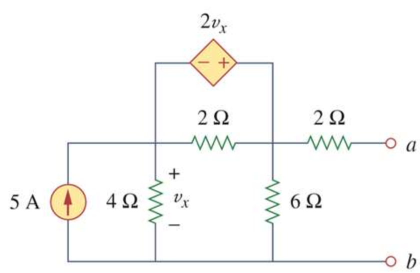
$$\Rightarrow \qquad V_{Th} = 22\text{ V}$$

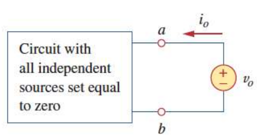
$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

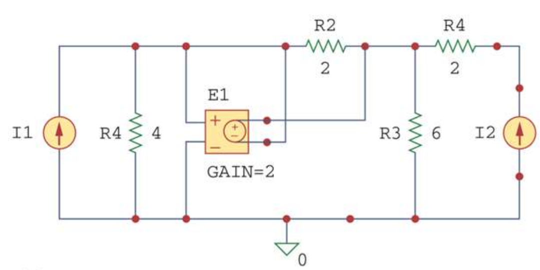
$$R_L = R_{Th} = 9\ \Omega$$

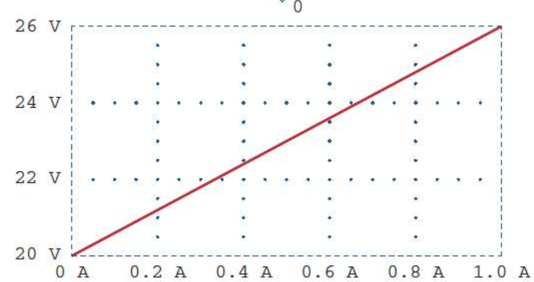
$$\boxed{p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\text{ W}}$$

Finding Thevenin equivalent using PSPICE









Analysis/Setup DC Sweep
Sweep Type Linear
Sweep Var Current Source
Name I2
Start Value 0
End Value 1
Increment 0.1

$i_0 = (v_o - v_{Th}) / R_{Th}$
-> 0 intercept of the below graph gives v_{Th}
-> slope gives R_{Th}