EHB 211E: Basics of Electrical Circuits
First Order Circuits

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Motivation

- We have considered three passive elements: resistors, capacitors, and inductors.
- We will consider circuits that contains various combinations of two or three passive elements:
- In this lecture, we will consider RC & RL circuits (1^{st} order circuits)
- RC & RL circuits produce differential equations of the first order
  A first order circuit is characterized by a first-order differential equation
- There are two ways to excite the circuits:
  - Initial conditions of the storage elements (*source-free circuits*)
    no independent sources, there may be dependent sources
  - Excitation by independent sources.

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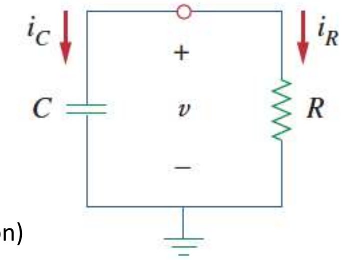
The Source-Free RC Circuit

Consider a series combination of R & C elements.
The capacitor is initially charged.



$$v(0) = V_0 \qquad w(0) = \frac{1}{2}CV_0^2$$

$$i_C + i_R = 0$$

$$C\frac{dv}{dt} + \frac{v}{R} = 0 \qquad \frac{dv}{dt} + \frac{v}{RC} = 0$$ (First-order differential equation)

$$\frac{dv}{v} = -\frac{1}{RC}dt$$

$$\ln v = -\frac{t}{RC} + \ln A$$ A is the integration constant $v(0) = A = V_0.$

$$\ln\frac{v}{A} = -\frac{t}{RC} \qquad v(t) = Ae^{-t/RC} \qquad v(t) = V_0e^{-t/RC}$$

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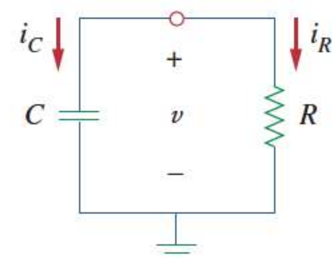
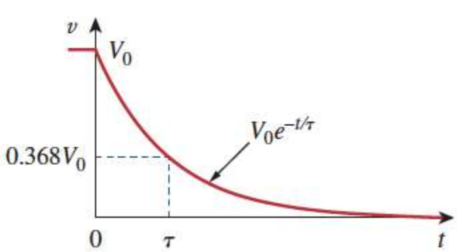
The Source-Free RC Circuit

$$v(t) = V_0e^{-t/RC}$$

Voltage response of the RC circuit is an exponential decay of the initial voltage. The response is due to initial energy stored -> ***Natural response***





At t = 0, v = v(0) = V_0

The time constant (τ) is the time required for the response to decay to a factor of 1/e (36.8%) of its initial value.

$$V_0e^{-\tau/RC} = V_0e^{-1} = 0.368V_0$$

$$\boxed{\tau = RC} \qquad \boxed{v(t) = V_0e^{-t/\tau}}$$

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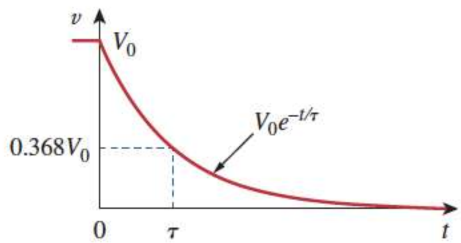
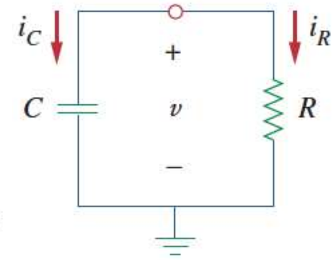
The Source-Free RC Circuit

$v(t) = V_0 e^{-t/RC}$

$$\boxed{v(t) = V_0 e^{-t/\tau}}$$





- Capacitor is fully discharged after 5 time constants (less than 1% remaining)
- Smaller the time constant, more rapidly the voltage decreases (faster the response)
- A circuit with small time constant reaches steady (final) state quickly.

Values of $v(t)/V_0 = e^{-t/\tau}$.

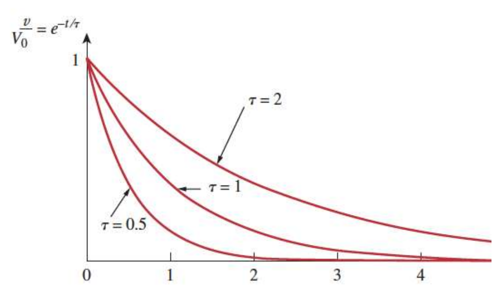
t	$v(t)/V_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

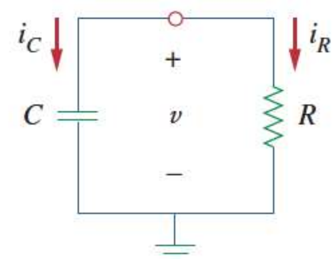
The Source-Free RC Circuit

$$\boxed{v(t) = V_0 e^{-t/\tau}}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}$$

$$p(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}$$





$$w_R(t) = \int_0^t p(\lambda)d\lambda = \int_0^t \frac{V_0^2}{R} e^{-2\lambda/\tau} d\lambda = -\frac{\tau V_0^2}{2R} e^{-2\lambda/\tau}\Big|_0^t = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau})$$

$$t \to \infty,\ w_R(\infty) \to \tfrac{1}{2} C V_0^2$$

The energy that was initially stored in the capacitor is eventually dissipated in the resistor.

Exercise

If $v_C(0) = 15V$, find v_C, v_x, i_x for $t > 0$.

$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$

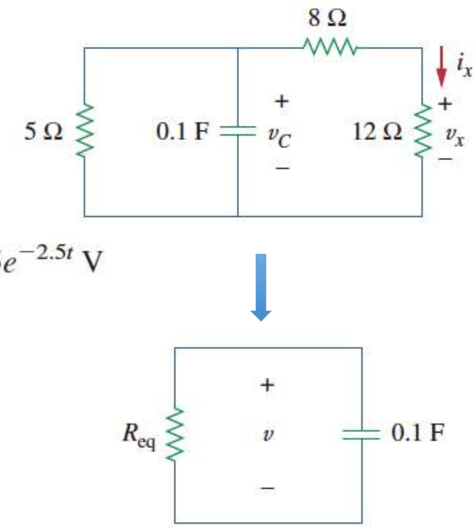
$$\tau = R_{eq}C = 4(0.1) = 0.4\ \text{s}$$

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

Voltage division

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\ \text{A}$$



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Exercise

The switch has been closed for a long time
The switch is opened at $t = 0$. Find $v(t)$ for $t > 0$. Calculate the initial energy stored in the capacitor.

Capacitor is open circuit for $t < 0$

$$v_C(t) = \frac{9}{9 + 3}(20) = 15\ \text{V}, \qquad t < 0$$

Voltage across capacitor can not change instantaneously:

$$v_C(0) = V_0 = 15\ \text{V}$$

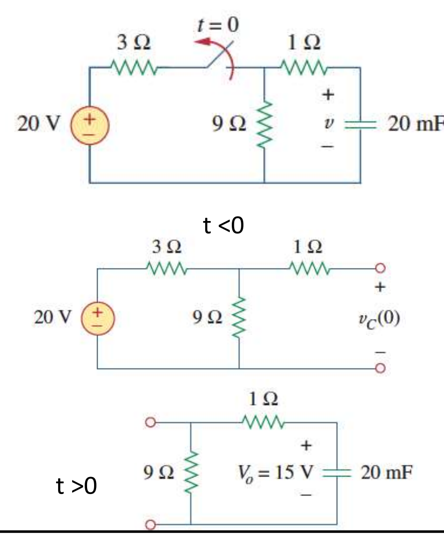
$t > 0$

$$R_{eq} = 1 + 9 = 10\ \Omega$$

$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2\ \text{s}$$

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2}\ \text{V} \qquad v(t) = 15e^{-5t}\ \text{V}$$

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25\ \text{J}$$



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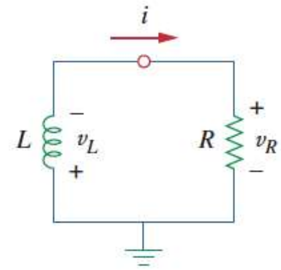
The Source-Free RL Circuit

Consider a series combination of R & L elements.
Now, initial inductor current is the response to the circuit



$$i(0) = I_0$$

$$w(0) = \frac{1}{2} L I_0^2$$

$$v_L + v_R = 0$$

$$L\frac{di}{dt} + Ri = 0 \Rightarrow \frac{di}{dt} + \frac{R}{L} i = 0 \Rightarrow \int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L} dt$$

$$\ln i \Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Big|_0^t \Rightarrow \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0 \Rightarrow \ln \frac{i(t)}{I_0} = -\frac{Rt}{L}$$

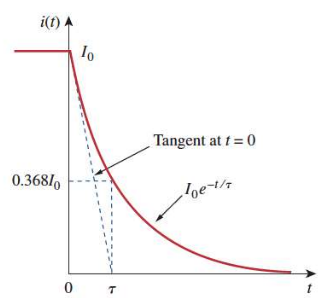
$$i(t) = I_0 e^{-Rt/L}$$

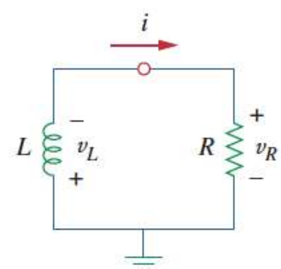
The Source-Free RL Circuit

$$i(t) = I_0 e^{-Rt/L}$$

$$\boxed{\tau = \frac{L}{R}}$$

$$\boxed{i(t) = I_0 e^{-t/\tau}}$$





$$v_R(t) = iR = I_0 R e^{-t/\tau}$$

$$p = v_R i = I_0^2 R e^{-2t/\tau} \qquad \tau = \frac{L}{R}$$

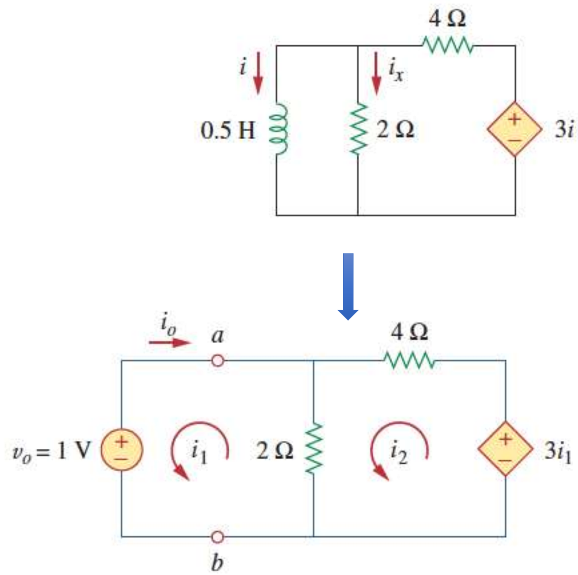
The energy that was initially stored in the inductor is eventually dissipated in the resistor. $t \to \infty, w_R(\infty) \to \frac{1}{2} L I_0^2$

$$w_R(t) = \int_0^t p(\lambda) d\lambda = \int_0^t I_0^2 e^{-2\lambda/\tau} d\lambda = -\frac{\tau}{2} I_0^2 R e^{-2\lambda/\tau}\Big|_0^t, \quad w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

Exercise

Assuming that i(0) = 10A, calculate i(t) and i_x(t).

Method 1: Find Equivalent (Thevenin) resistance



$$2(i_1 - i_2) + 1 = 0 \quad \Rightarrow \quad i_1 - i_2 = -\frac{1}{2}$$

$$6i_2 - 2i_1 - 3i_1 = 0 \quad \Rightarrow \quad i_2 = \frac{5}{6} i_1$$

$$i_1 = -3 \text{ A}, \qquad i_o = -i_1 = 3 \text{ A}$$

$$R_{eq} = R_{Th} = \frac{v_o}{i_o} = \frac{1}{3}\,\Omega$$

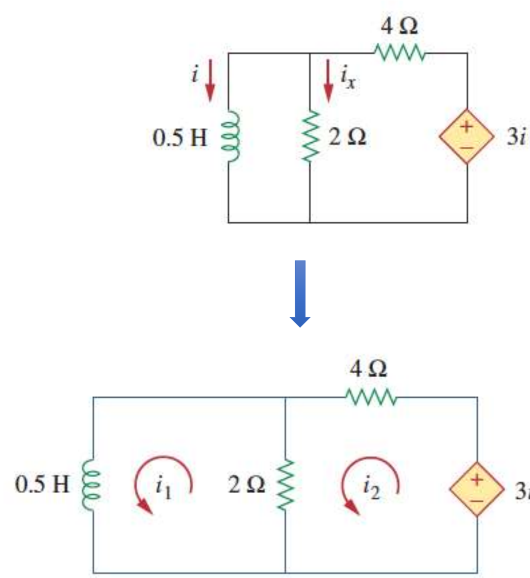
$$\tau = \frac{L}{R_{eq}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}\,\text{s}$$

$$i(t) = i(0)e^{-t/\tau} = 10e^{-(2/3)t}\text{ A}, \qquad t > 0$$

Exercise

Assuming that i(0) = 10A, calculate i(t) and i_x(t).

Method 2: Directly apply KVL



$$\frac{1}{2}\frac{di_1}{dt} + 2(i_1 - i_2) = 0 \qquad 6i_2 - 2i_1 - 3i_1 = 0$$

$$\frac{di_1}{i_1} = -\frac{2}{3}dt$$

$$\ln i\Big|_{i(0)}^{i(t)} = -\frac{2}{3}t\Big|_0^t \qquad \ln\frac{i(t)}{i(0)} = -\frac{2}{3}t$$

$$i(t) = i(0)e^{-(2/3)t} = 10e^{-(2/3)t}\text{ A}, \qquad t > 0$$

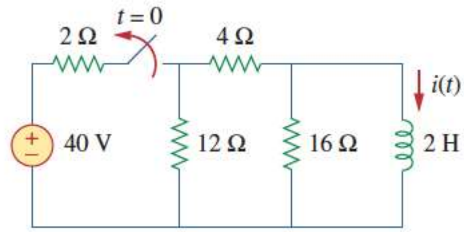
$$v = L\frac{di}{dt} = 0.5(10)\left(-\frac{2}{3}\right)e^{-(2/3)t} = -\frac{10}{3}e^{-(2/3)t}\text{ V}$$

$$i_x(t) = \frac{v}{2} = -1.6667e^{-(2/3)t}\text{ A}, \qquad t > 0$$

Exercise

The switch has been closed for a long time. At t= 0, the switch opens. Calculate i(t).

For t < 0, the inductor acts as a short circuit



$$\frac{4 \times 12}{4 + 12} = 3\ \Omega \qquad i_1 = \frac{40}{2 + 3} = 8\ \text{A}$$

$$i(t) = \frac{12}{12 + 4} i_1 = 6\ \text{A}, \qquad t < 0$$

Current through an inductor cannot change instantaneously

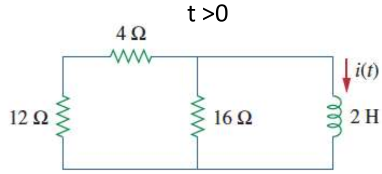
$$i(0) = i(0^-) = 6\ \text{A}$$

$$R_{\text{eq}} = (12 + 4) \parallel 16 = 8\ \Omega$$

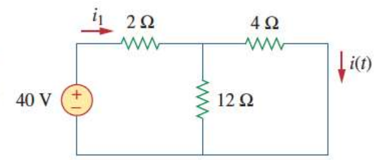
$$\tau = \frac{L}{R_{\text{eq}}} = \frac{2}{8} = \frac{1}{4}\ \text{s}$$

$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t}\ \text{A}$$

t >0

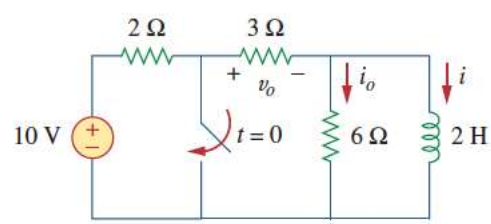


t <0



Exercise

The switch has been open for a long time. At t= 0, the switch closes. Calculate i_0(t), v_0(t), and i(t)



$$i(t) = \frac{10}{2 + 3} = 2\ \text{A}, \qquad t < 0 \qquad i(0) = 2.$$

$$v_o(t) = 3i(t) = 6\ \text{V}, \qquad t < 0$$

t > 0

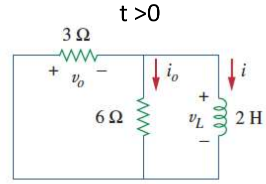
$$R_{\text{Th}} = 3 \parallel 6 = 2\ \Omega \qquad \tau = \frac{L}{R_{\text{Th}}} = 1\ \text{s}$$

$$i(t) = i(0)e^{-t/\tau} = 2e^{-t}\ \text{A}, \qquad t > 0$$

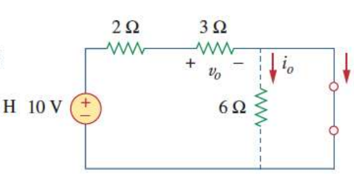
$$v_o(t) = -v_L = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t}\ \text{V}$$

$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t}\ \text{A}, \qquad t > 0$$

t >0



t <0



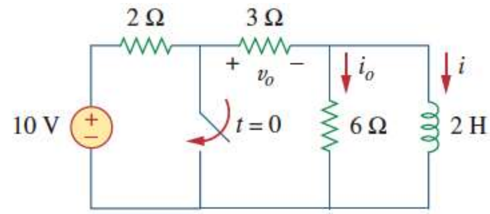
Exercise (continued…)

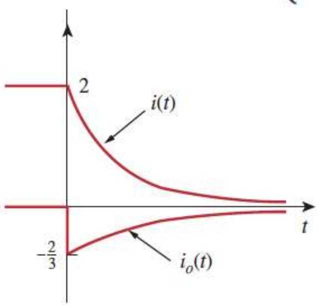
The switch has been open for a long time. At t= 0, the switch closes. Calculate $i_0(t)$, $v_0(t)$, and $i(t)$

$$i_o(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\frac{2}{3}e^{-t} \text{ A}, & t > 0 \end{cases}, \qquad v_o(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4e^{-t} \text{ V}, & t > 0 \end{cases}$$

All current flows through the inductor, not the resistor

$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2e^{-t} \text{ A}, & t \geq 0 \end{cases}$$





Inductor current is continuous while the resistor current makes a jump !

Time constant is the same regardless of what output is taken!

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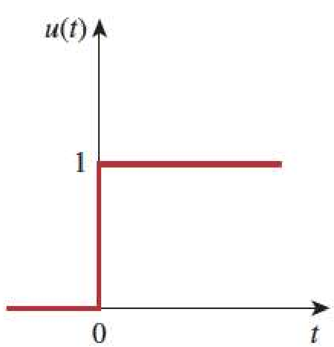
Singularity (switching) Functions

- Singularity functions serve as good approximations to the switching signals that arise in circuits.
- They are helpful in the compact description of step responses of RC and RL circuits.
- Singularity functions are either discontinuous or have discontinuous derivatives.

Unit step function



$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

u(t) is undefined for t = 0

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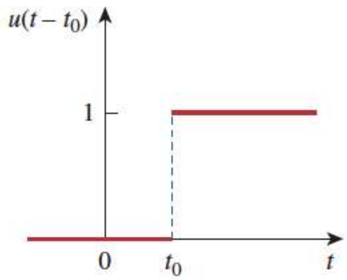
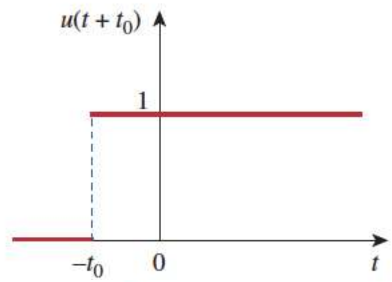
Unit Step Function



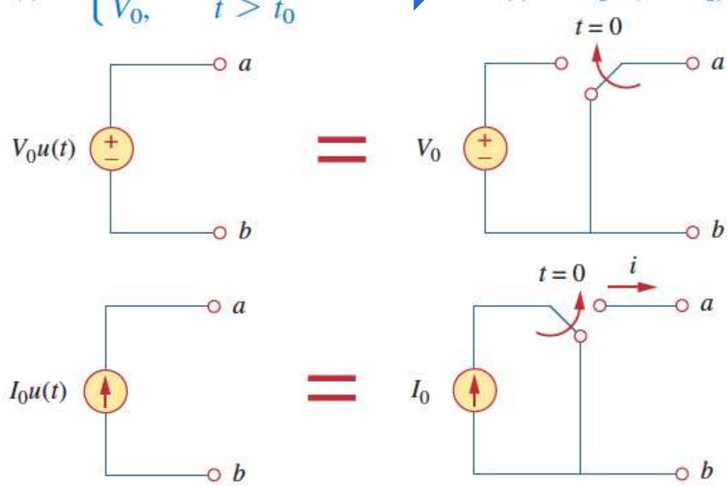


$$u(t-t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases} \qquad u(t+t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$

$$v(t) = \begin{cases} 0, & t < t_0 \\ V_0, & t > t_0 \end{cases} \Rightarrow v(t) = V_0 u(t - t_0)$$

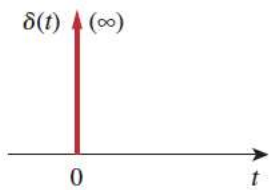
Unit Step Function

$$v(t) = \begin{cases} 0, & t < t_0 \\ V_0, & t > t_0 \end{cases} \Rightarrow v(t) = V_0 u(t - t_0)$$



Unit Impulse Function

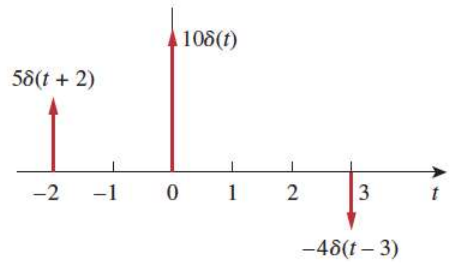
$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t < 0 \\ \text{Undefined}, & t = 0 \\ 0, & t > 0 \end{cases}$$



Unit impulse function may be regarded as an applied shock signal.
It may be visualized as a very short duration of pulse.

$$\int_{0^-}^{0^+} \delta(t)\, dt = 1$$

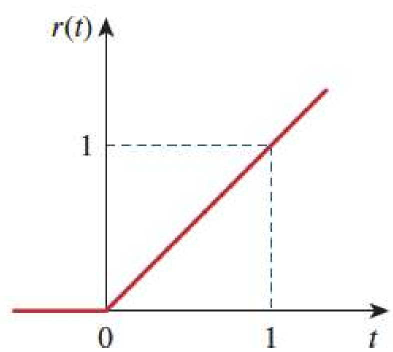
Strength of the impulse function



Unit Ramp Function

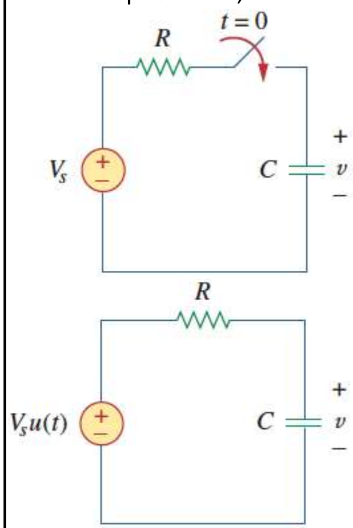
$$r(t) = \int_{-\infty}^{t} u(\lambda)d\lambda = tu(t)$$

$$r(t) = \begin{cases} 0, & t \le 0 \\ t, & t \ge 0 \end{cases}$$



Step Response of an RC Circuit

When the DC source of an RC circuit is suddenly applied, the voltage or current can be modeled as a step function, known as the step response.



$$v(0^-) = v(0^+) = V_0$$ Initial voltage on capacitor

$$C\frac{dv}{dt} + \frac{v - V_s u(t)}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}u(t) \;\Rightarrow\; \frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC} \quad t > 0$$

$$\frac{dv}{dt} = -\frac{v - V_s}{RC} \;\Rightarrow\; \frac{dv}{v - V_s} = -\frac{dt}{RC}$$

$$\ln(v - V_s)\Big|_{V_0}^{v(t)} = -\frac{t}{RC}\Big|_0^t \qquad \ln\frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

$$v - V_s = (V_0 - V_s)e^{-t/\tau} \qquad v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \quad t > 0$$

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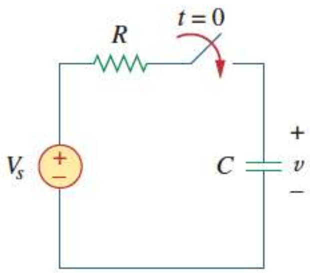
Step Response of an RC Circuit



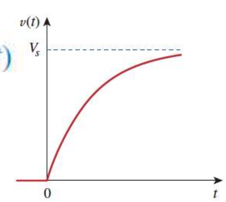
"Complete (total) response"

$$\boxed{v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}}$$

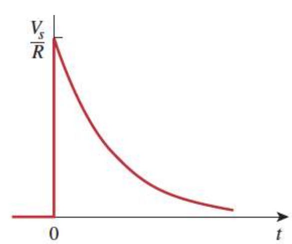
If capacitor is initially uncharged:

$$v(t) = \begin{cases} 0, & t < 0 \\ V_s(1 - e^{-t/\tau}), & t > 0 \end{cases} \qquad i(t) = C\frac{dv}{dt} = \frac{C}{\tau}V_s e^{-t/\tau}, \quad \tau = RC, \quad t > 0$$

$$v(t) = V_s(1 - e^{-t/\tau})u(t)$$



$$i(t) = \frac{V_s}{R}e^{-t/\tau}u(t)$$



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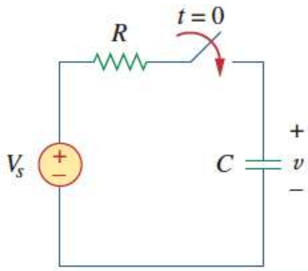
Step Response of an RC Circuit



Complete response = natural response + forced response
stored energy — independent source

$$v = v_n + v_f$$

$$v_n = V_o e^{-t/\tau} \qquad v_f = V_s(1 - e^{-t/\tau})$$

Alternative explanation:

Complete response = transient response + steady-state response
temporary part — permanent part

$$v = v_t + v_{ss}$$

$$v_t = (V_o - V_s)e^{-t/\tau} \qquad v_{ss} = V_s$$

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}}$$

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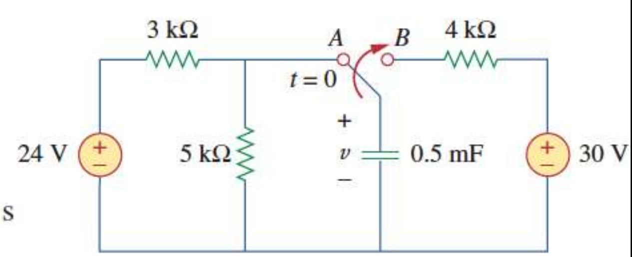
Exercise

At t = 0, the switch moves from position A to B. Determine v(t) for t > 0.



$$v(0^-) = \frac{5}{5+3}(24) = 15 \text{ V}$$

$$v(0) = v(0^-) = v(0^+) = 15 \text{ V}$$

$$\tau = R_{\text{Th}}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2 \text{ s}$$

$$v(\infty) = 30 \text{ V}$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$
$$= 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t}) \text{ V}$$

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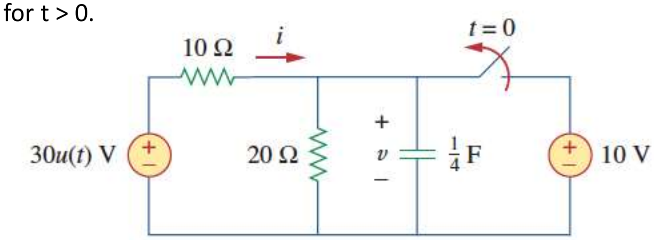
Exercise

At t = 0, the switch opens. Determine v(t) and i(t) for t > 0.



$$30u(t) = \begin{cases} 0, & t < 0 \\ 30, & t > 0 \end{cases}$$

$$v = 10 \text{ V}, \qquad i = -\frac{v}{10} = -1 \text{ A}$$

$$v(0) = v(0^-) = 10 \text{ V}$$

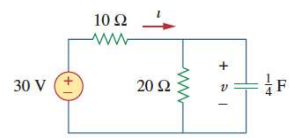
$$v(\infty) = \frac{20}{20 + 10}(30) = 20 \text{ V}$$

$$R_{\text{Th}} = 10 \parallel 20 = \frac{10 \times 20}{30} = \frac{20}{3}\,\Omega$$

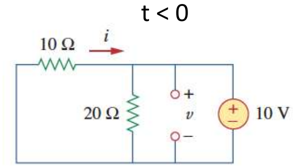
$$\tau = R_{\text{Th}} C = \frac{20}{3} \cdot \frac{1}{4} = \frac{5}{3}\text{ s}$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

t > 0



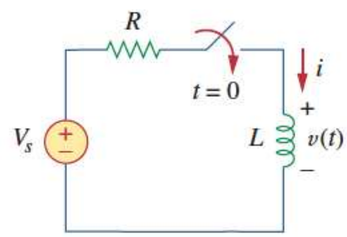
t < 0



$$i = \frac{v}{20} + C\frac{dv}{dt} = (1 + e^{-0.6t}) \text{ A}$$

Step Response of an RL Circuit

The current response is the sum of transient and steady state responses:



$$i = i_t + i_{ss}$$

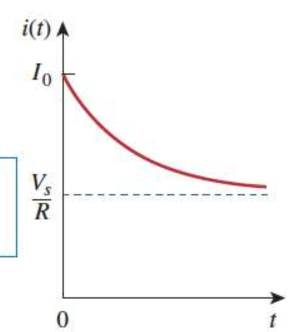
$$i_t = Ae^{-t/\tau}, \qquad \tau = \frac{L}{R}$$

$$i_{ss} = \frac{V_s}{R}$$

$$i = Ae^{-t/\tau} + \frac{V_s}{R} \qquad i(0^+) = i(0^-) = I_0$$

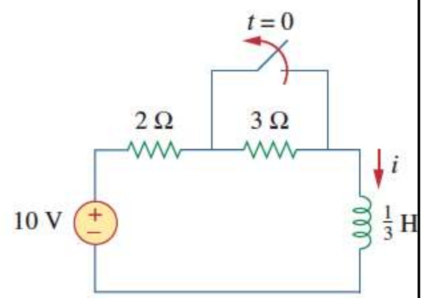
$$I_0 = A + \frac{V_s}{R} \Rightarrow A = I_0 - \frac{V_s}{R}$$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau} \Rightarrow \boxed{i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}}$$



Exercise

Find i(t) for t > 0.



$$i(0^-) = \frac{10}{2} = 5 \text{ A}$$

$$i(0) = i(0^+) = i(0^-) = 5 \text{ A}$$

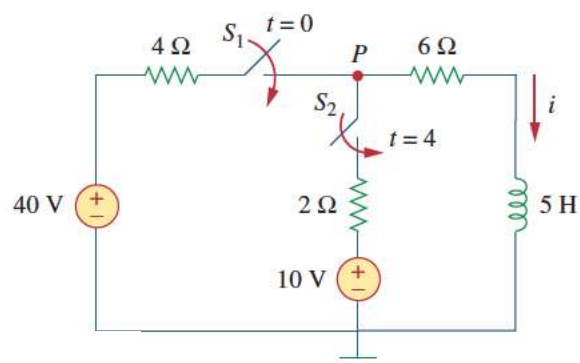
$$i(\infty) = \frac{10}{2+3} = 2 \text{ A}$$

$$R_{\text{Th}} = 2 + 3 = 5\ \Omega \qquad \tau = \frac{L}{R_{\text{Th}}} = \frac{\frac{1}{3}}{5} = \frac{1}{15} \text{ s}$$

$$\begin{aligned} i(t) &= i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \\ &= 2 + (5-2)e^{-15t} = 2 + 3e^{-15t} \text{ A}, \qquad t > 0 \end{aligned}$$

Exercise

At t = 0, switch 1 is closed.
At t = 4s, switch 2 is closed. Find i(t) for t > 0.



$$i(0^-) = i(0) = i(0^+) = 0$$

$$\boxed{0 \leq t \leq 4}$$

$$i(\infty) = \frac{40}{4+6} = 4 \text{ A}$$

$$R_{\text{Th}} = 4 + 6 = 10\ \Omega \qquad \tau = \frac{L}{R_{\text{Th}}} = \frac{5}{10} = \frac{1}{2}$$

$$\begin{aligned} i(t) &= i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \\ &= 4 + (0-4)e^{-2t} = 4(1 - e^{-2t}) \text{ A}, \qquad 0 \leq t \leq 4 \end{aligned}$$

$$\boxed{t \geq 4}$$

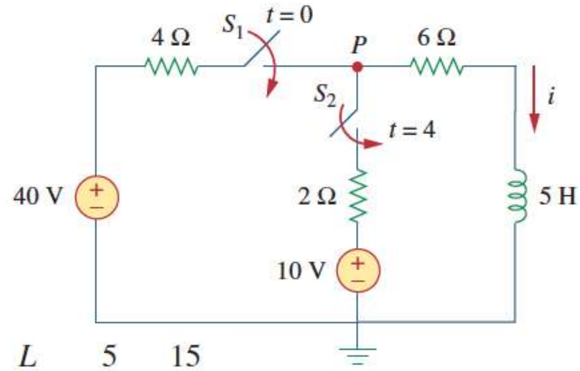
$$i(4) = i(4^-) = 4(1 - e^{-8}) \simeq 4 \text{ A}$$

Find the voltage(v) at node P

$$\frac{40 - v}{4} + \frac{10 - v}{2} = \frac{v}{6} \quad \Rightarrow \quad v = \frac{180}{11} \text{ V} \qquad i(\infty) = \frac{v}{6} = \frac{30}{11} = 2.727 \text{ A}$$

Exercise (continued)

At t = 0, switch 1 is closed.
At t = 4s, switch 2 is closed. Find i(t) for t > 0.



$\boxed{t \geq 4}$

$$i(4) = i(4^-) = 4(1 - e^{-8}) \simeq 4\text{ A}$$

Find the voltage(v) at node P

$$\frac{40 - v}{4} + \frac{10 - v}{2} = \frac{v}{6} \quad \Rightarrow \quad v = \frac{180}{11}\text{V}$$

$$i(\infty) = \frac{v}{6} = \frac{30}{11} = 2.727\text{ A}$$

$$R_{\text{Th}} = 4 \parallel 2 + 6 = \frac{4 \times 2}{6} + 6 = \frac{22}{3}\,\Omega \qquad \tau = \frac{L}{R_{\text{Th}}} = \frac{5}{\frac{22}{3}} = \frac{15}{22}\text{ s}$$

$$i(t) = i(\infty) + [i(4) - i(\infty)]e^{-(t-4)/\tau}, \qquad t \geq 4$$

$$i(t) = 2.727 + (4 - 2.727)e^{-(t-4)/\tau}, \qquad \tau = \frac{15}{22}$$

$$= 2.727 + 1.273e^{-1.4667(t-4)}, \qquad t \geq 4$$

OVERALL

$$\boxed{i(t) = \begin{cases} 0, & t \leq 0 \\ 4(1 - e^{-2t}), & 0 \leq t \leq 4 \\ 2.727 + 1.273e^{-1.4667(t-4)}, & t \geq 4 \end{cases}}$$

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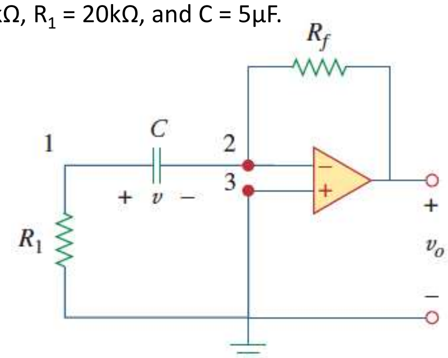
First-Order Op Amp Circuits - Exercise

For the op amp circuit, find v_o for t > 0, given that v(0) = 3V. Rf = 80kΩ, R_1 = 20kΩ, and C = 5µF.



Method 1

$$\frac{0 - v_1}{R_1} = C\frac{dv}{dt} \qquad \text{KCL at node 1}$$

$$v_1 = v$$

$$\frac{dv}{dt} + \frac{v}{CR_1} = 0 \qquad \text{(same equation as source free RC circuit)}$$

$$v(t) = V_0 e^{-t/\tau}, \qquad \tau = R_1 C$$

$$v(0) = 3 = V_0 \qquad \tau = 20 \times 10^3 \times 5 \times 10^{-6} = 0.1$$

$$v(t) = 3e^{-10t}$$

$$C\frac{dv}{dt} = \frac{0 - v_o}{R_f} \qquad \text{KCL at node 2} \qquad v_o = -R_f C\frac{dv}{dt}$$

$$v_o = -80 \times 10^3 \times 5 \times 10^{-6}(-30e^{-10t}) = 12e^{-10t}\text{ V}$$

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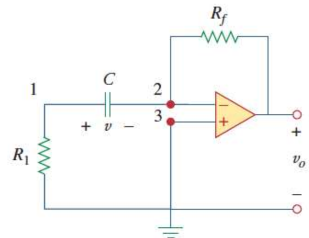
First-Order Op Amp Circuits - Exercise

For the op amp circuit, find v_o for $t > 0$, given that $v(0) = 3V$. $R_f = 80k\Omega$, $R_1 = 20k\Omega$, and $C = 5\mu F$.



Method 2

$$v(0^+) = v(0^-) = 3 \text{ V}$$

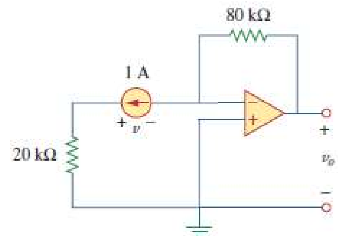
KCL

$$\frac{3}{20{,}000} + \frac{0 - v_o(0^+)}{80{,}000} = 0 \qquad v_o(0^+) = 12 \text{ V}$$

Since the circuit is source free: $v(\infty) = 0 \text{ V}$

KVL at input loop to find R_{eq} observed by the capacitor.

Remove capacitor and place a 1A current source



$$20{,}000(1) - v = 0 \qquad \Rightarrow \qquad v = 20 \text{ kV}$$

$$R_{\text{eq}} = \frac{v}{1} = 20 \text{ k}\Omega \qquad \tau = R_{\text{eq}}C = 0.1$$

$$\begin{aligned} v_o(t) &= v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/\tau} \\ &= 0 + (12 - 0)e^{-10t} = 12e^{-10t} \text{ V}, \qquad t > 0 \end{aligned}$$