# EHB 211E: Basics of Electrical Circuits

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## Recommended Texts

- Charles K. Alexander and Matthew Sadiku, “Fundamentals of Electric Circuits”, McGraw-Hill, 2012.
- Mauro Parodi and Marco Storace, “Linear and Nonlinear Circuits: Basic & Advanced Concepts Volume 1”, Springer, 2018.
- Leon O. Chua, Charles A. Desoer, Ernest S. Kuh, "Linear and Nonlinear Circuits”, McGraw-Hill, 1987.
- Müştak E. Yalçın, “Elektrik Devre Temelleri Ders Notlari”, 2011.
- Cevdet Acar, “Elektrik Devrelerinin Analizi”, ITU Yayınları, 1995.



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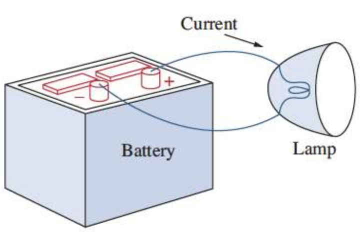
# Grading and Tips

- HomeWorks + 4-6 Short Exams :70%
- Final (FE) :30%
- VF – Short Exam Total < (Short Exam Class Average Total X 0.4)
- Study the course regularly.
- This course starts out easy and gets challenging pretty quickly.
- The best way to learn is to solve a lot of problems.
- Homework problems give you both a chance to prepare for the exams and to get credit for your efforts.
- Make an effort to learn PSpice and MATLAB.

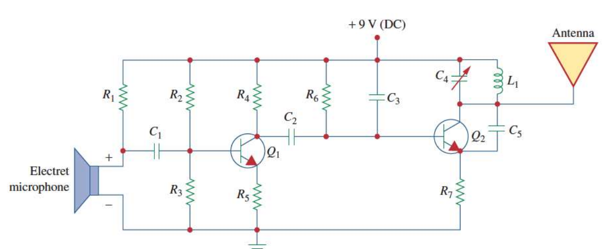


# Physical Circuits and Circuit Theory

- The goal of circuit theory is to predict the electrical behavior of physical circuits.




A simple electric circuit:
Elements:
1) Battery
2) Lamp
3) Connecting wires




A complicated circuit
(*radio receiver*)

## Distributed vs. Lumped Circuits

- Distributed Circuit:

A distributed circuit is one in which all dependent variables are functions of time and one or more spatial variables.

- Lumped Circuit:

A lumped circuit is one in which the dependent variables of interest are function of time alone.





## Distributed vs. Lumped Circuits

- For a circuit to be considered lumped, its physical dimension must be small enough so that, electromagnetic waves propagate across the circuit virtually instantaneously.

- Consider an audio circuit for which the highest frequency of interest is 25kHz.

$$\lambda = c/f = (3x10^8 \, m/s) / (25x10^3 \, s^{-1}) = 12 \times 10^3 \, m = 12km$$

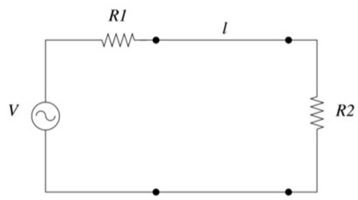
Even if the circuit is spread across a football stadium, the size of the circuit is very small compared to the shortest wavelength λ of interest.

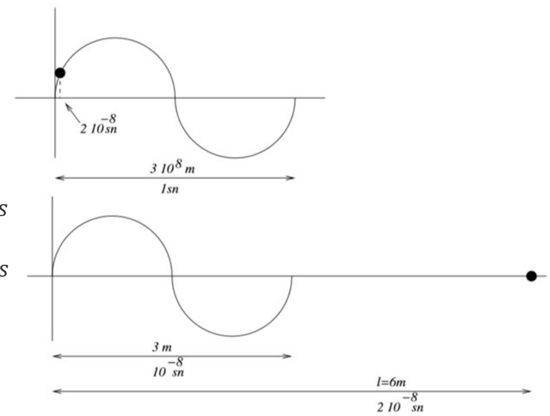
$$\lambda >> l$$

When lumped circuit theory holds, the current through any device terminal and voltage difference across any pair of terminals are well defined. The respective location of the element doesn't affect the circuit behavior.

# Distributed vs. Lumped Circuits

How long does it take to travel the connection between R1 and R2 if l=6m if the voltage source has a frequency of 1Hz?




Electromagnetic waves travel at the velocity of light c=3x10[8] m/s

f=1 Hz -> Wavelength $\lambda = \frac{3 \cdot 10^8}{1} = 3.10^8 m$ $t = \frac{6}{3 \cdot 10^8} = 20ns$

f=100MHz -> Wavelength $\lambda = \frac{3 \cdot 10^8}{100 \cdot 10^6} = 3m$ $t = \frac{6}{3 \cdot 10^8} = 20ns$





# Distributed vs. Lumped Circuits

Let l be the largest dimension of the circuit, λ the shortest wavelength of interest. If

λ>> l

then the circuit may be considered to be lumped.

While Lumped circuit is analyzed by solving a set of differential equations, Distributed circuit is analyzed by solving partial differential equations.

In distributed circuits the effects of physical dimensions are no longer negligible. Typical examples of distributed circuits are circuits made of waveguides and transmission lines.

In lumped circuits, the effects of physical dimensions are negligible. Any lumped circuit is made up of lumped components and the length of any connection is negligible. Therefore, any electrical perturbation immediately affects every part of the circuit so that the spatial position of a component with respect to the others doesn't affect the electrical behavior of the circuit.

In this course we will only deal with lumped circuits.

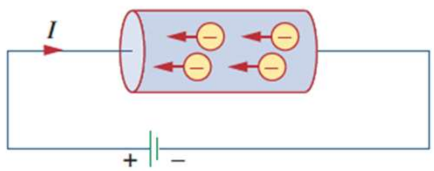
# The Fundamental Variables: Charge and Current

Charge is an electrical property of the atomic particles of which material consists. It is measured in Coulombs (C). The charge of an electron is $1.602 \times 10^{-19}$ C.

- In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons.
- Charges occur as integral multiples of electronic charge.
- Conservation of charge: charges can neither be created nor destroyed. Algebraic sum of charges in a system does not change
- Current is simply a measure of the net amount of positive charge that passes a plane in space per second in a reference direction

Conventionally, the current flow is taken as the movement of positive charges (opposite to the flow of negative charges).

$$i \triangleq \frac{dq}{dt}$$





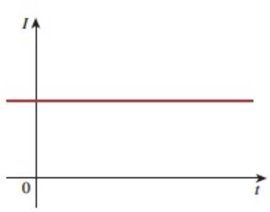



# Charge and Current

Electric current is the time rate of change of charge, measured in amperes (1A = 1 Coulomb/second).

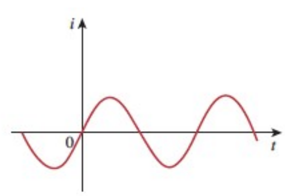
$$i \triangleq \frac{dq}{dt} \longleftrightarrow Q \triangleq \int_{t_0}^{t} i\,dt$$

Direct current (DC)
*(current remains constant with time)*




example: battery

Alternative current (AC)
*(current varies sinusoidally with time)*




example: current used in households to run refrigerator, washing machine, electrical appliances

# Andre-Marie Ampere
(20 January 1775-10 June 1836)

- Andre-Marie Ampere was a French physicist and mathematician who is generally regarded as one of the main founders of the science of classical electromagnetism, which he referred to as "electrodynamics". The SI unit of measurement of electric current, the ampere, is named after him.

- https://www.youtube.com/watch?v=RJb0r8dHzAo





# Exercises

**1)** How much charge is represented by 4,600 electrons?

**Solution:**
Each electron has $-1.602 \times 10^{-19}$ C. Hence 4,600 electrons will have

$$-1.602 \times 10^{-19} \text{ C/electron} \times 4{,}600 \text{ electrons} = -7.369 \times 10^{-16} \text{ C}$$

**2)** The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

**Solution:**

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

**3)** Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

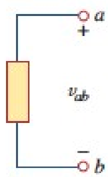
**Solution:**

$$Q = \int_{t=1}^{2} i\,dt = \int_{1}^{2} (3t^2 - t)dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Big|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

# The Fundamental Variables: Voltage

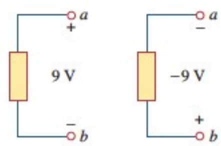
Moving electrons requires force (electro-motive), called *voltage* or *potential difference, measured in volts.*




The voltage $v_{ab}$ between points *a* and *b*, is the energy needed to move a unit charge from *a* to *b*.

$$v_{ab} \triangleq \frac{dw}{dq} \qquad 1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton-meter/coulomb}$$

Where, *w* is the energy (J), and q is the charge (C)




$$v_{ab} = -v_{ba}$$





## Alessandro Volta
## (18 February 1745-5 March 1827)

- Alessandro Giuseppe Antonio Anastasio Volta was an Italian physicist, chemist and pioneer of electricity and power who is credited as the inventor of electric battery and the discoverer of methane. The SI unit of measurement of voltage, the volt, is named after him.

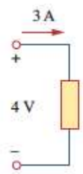
- https://www.youtube.com/watch?v=Is8wAeoTqHQ

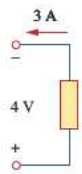
# Power and Energy

Power is the time rate of absorbed energy, measured in Watts (W).

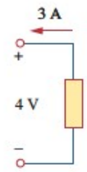
$$\boxed{p \triangleq \frac{dw}{dt}} \qquad p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \qquad \boxed{p = vi} \quad \longleftrightarrow \quad w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$$

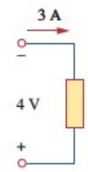
If power has a "+" sign, power is delivered to (absorbed by) the element.




If power has a "-" sign, power is supplied (generated) by the element.




$$\boxed{\sum p = 0}$$

<u>Conservation of energy /power (Tellegen's theorem)</u>:
Algebraic sum of power in a circuit is zero





# Exercises

**1)** An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

**Solution:**
The total charge is

$$\Delta q = i\,\Delta t = 2 \times 10 = 20\text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115\text{ V}$$

**2)** Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

**Solution:**
(a) The voltage is $v = 3i = 15 \cos 60\pi t$; hence, the power is

$$p = vi = 75 \cos^2 60\pi t \text{ W}$$

At $t = 3$ ms,

$$p = 75 \cos^2 (60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi = 53.48 \text{ W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3$ ms,

$$\begin{aligned} p &= -4500\pi \sin 0.18\pi \cos 0.18\pi \text{ W} \\ &= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW} \end{aligned}$$

**3)** How much energy does a 100-W electric bulb consume in two hours?

**Solution:**

$$\begin{aligned} w = pt &= 100\text{ (W)} \times 2\text{ (h)} \times 60\text{ (min/h)} \times 60\text{ (s/min)} \\ &= 720{,}000\text{ J} = 720\text{ kJ} \end{aligned}$$

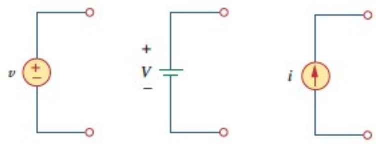
This is the same as

$$w = pt = 100\text{ W} \times 2\text{ h} = 200\text{ Wh}$$

# Circuit Elements

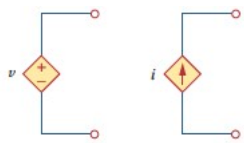
- ACTIVE elements: Capable of generating energy (*generators, batteries, operational amplifiers*)
- PASSIVE elements: absorbs / consumes energy (*resistors, capacitors, inductors*)

An ideal independent source, is an active element that provides a specified voltage or current that is completely independent of other circuit elements.




-> Independent voltage source delivers whatever current that is necessary to maintain its voltage.
-> Independent current source delivers whatever voltage that is necessary to maintain the delivered current.

An ideal dependent source, is an active element, whose source quantity is controlled by another voltage or current (*useful in modelling transistors, operational amplifiers, integrated circuits*).




TYPES:
- Voltage controlled voltage source (VCVS)
- Current controlled voltage source (CCVS)
- Voltage controlled current source (VCCS)
- Current controlled current source (CCCS)



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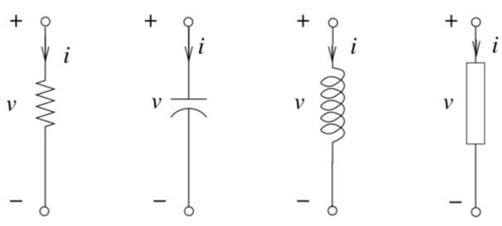
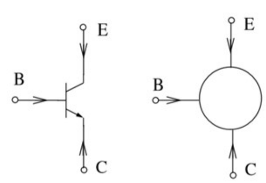
# Two Terminal and n-terminal Elements

- Typical examples of two terminal elements are resistor, capacitor, inductor, voltage and current source and diode.
- Typical examples of three terminal elements are transistors.

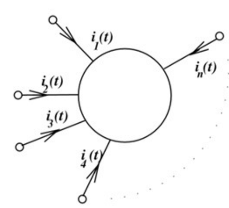



(a) (b) (c) (d)

3 terminal element




n-terminal element



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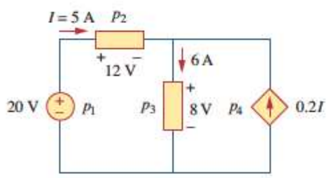
# Exercise

Calculate the power supplied or absorbed by each element




$p_1 = 20(-5) = -100$ W Supplied power

$p_2 = 12(5) = 60$ W Absorbed power

$p_3 = 8(6) = 48$ W Absorbed power

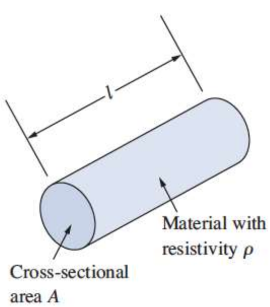
$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8$ W Supplied power

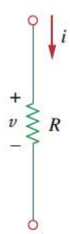
$$\boxed{p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0}$$





# Resistance and Ohm's Law




Materials have characteristic behavior of resisting flow of electrical charge: "*resistance*" (R).

$$R = \rho \frac{\ell}{A}$$

Resistivities of common materials.

| Material | Resistivity (Ω·m) | Usage |
|---|---|---|
| Silver | $1.64 \times 10^{-8}$ | Conductor |
| Copper | $1.72 \times 10^{-8}$ | Conductor |
| Aluminum | $2.8 \times 10^{-8}$ | Conductor |
| Gold | $2.45 \times 10^{-8}$ | Conductor |
| Carbon | $4 \times 10^{-5}$ | Semiconductor |
| Germanium | $47 \times 10^{-2}$ | Semiconductor |
| Silicon | $6.4 \times 10^{2}$ | Semiconductor |
| Paper | $10^{10}$ | Insulator |
| Mica | $5 \times 10^{11}$ | Insulator |
| Glass | $10^{12}$ | Insulator |
| Teflon | $3 \times 10^{12}$ | Insulator |

Ohm's Law states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through a resistor:

$$\boxed{v = iR}$$

$$R = \frac{v}{i}$$

$$1\,\Omega = 1\ \text{V/A}$$

# Conductance

Conductance is the ability of an element to conduct electrical current.

$$\boxed{G = \frac{1}{R} = \frac{i}{v}}$$

$$1\ \text{S} = 1\ \mho = 1\ \text{A/V}$$

$$i = Gv$$

Power dissipated by a resistor:

$$p = vi = i^2R = \frac{v^2}{R}$$

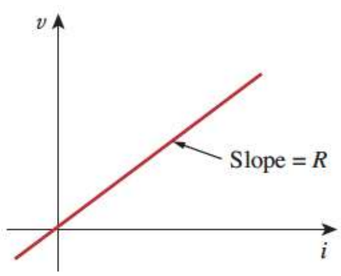
$$p = vi = v^2G = \frac{i^2}{G}$$

- The power dissipated in a resistor is a nonlinear function of either current or voltage.
- Since *R* and *G* are both positive quantities, power dissipated in a resistance is positive (resistors is a passive element and absorb power from the circuit).



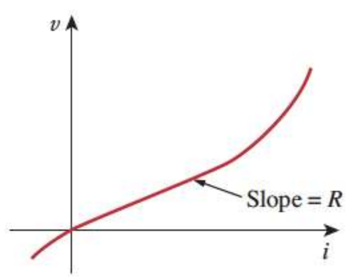
# Linear vs. Nonlinear Resistors

LINEAR



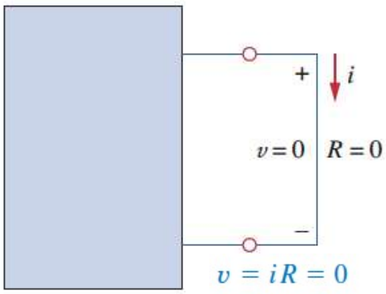

Obeys Ohm’s Law
(fixed slope for all values)

NONLINEAR

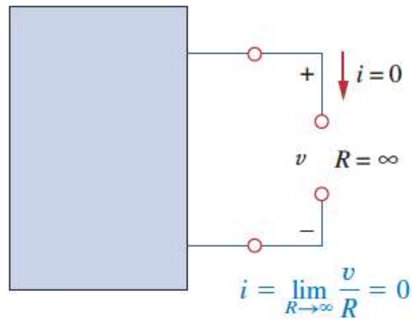



Does not obey Ohm’s Law
(slope changes, example: diodes)

# Short Circuit and Open Circuit




A “short circuit” is a circuit element with resistance approaching zero




An “open circuit” is a circuit element with resistance approaching infinity