MAT271E - HOMEWORK 3

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1. The hypothesis that mean length of battery charge time of iPhone 13 is under same circumstances is 50 minutes ($\boldsymbol{\mu_x = 50}$) will be tested with alternate hypothesis that is length of battery charge time different from 50 minutes.

The length of battery charge of iPhone 13 is assumed to be normally distributed, with a standart deviation of 6 minutes ($\boldsymbol{\hat{s}_x = 6}$).

Suppose the mean length of time estimated from measurements over $\boldsymbol{N = 11}$ times is: $\boldsymbol{\overline{x} = 48}$ minutes, at the level of significance of 0.05 ($\boldsymbol{\alpha = 0.05}$).

For small samples the sampling distribution of the mean is t distribution with degrees of freedom $\boldsymbol{N - 1 = 10}$.

For two-sided hypothesis test, $\boldsymbol{0.05/2 = 0.025}$ and $\boldsymbol{t_{10/0.025} = 2.228}$ (according to t-distribution table). The boundaries of the acceptence region:

$$b_1 = \mu_x - \frac{t \cdot \hat{s}_x}{\sqrt{N}} \qquad \text{and} \qquad b_2 = \mu_x + \frac{t \cdot \hat{s}_x}{\sqrt{N}}$$

$\boldsymbol{b_1 = 50 - \frac{(2.228) \cdot 6}{\sqrt{11}}}$ **= 45.96939635**

$\boldsymbol{b_2 = 50 + \frac{(2.228) \cdot 6}{\sqrt{11}}}$ **= 54.03060365**

As a result, $\boldsymbol{\overline{x} = 48}$ is in the interval (45.96939635-54.03060365). So, hypothesis is **ACCEPTED**.

2. Changing alternate hypothesis to $\boldsymbol{\mu_x < 50}$, for the same significance $\boldsymbol{\alpha = 0.05}$, same sample size $\boldsymbol{N = 11,}$ same standard deviation $\boldsymbol{\hat{s}_x = 6}$ and same mean of the sample $\boldsymbol{\overline{x} = 48}$.

For the one-sided hypothesis test, **0.05** and $\boldsymbol{t_{10/0.05} = 1.812}$. The lower boundary of the acceptance region:

$\boldsymbol{50 - \frac{(1.812) \cdot 6}{\sqrt{11}}}$ **= 46.72196866**

As a result, $\boldsymbol{\overline{x} =}$ **48 > 46.72196866**. So, hypothesis is **ACCEPTED**.

3. A large group of a students took a final exam in Statistics and its claimed to be standard deviation of the grades different from $\boldsymbol{\sigma_x = 18}$, at the level of significance of $\boldsymbol{\alpha = 0.10}$, for a sample size $\boldsymbol{N = 15}$. Samples are shown below:

7, 13, 27, 31, 35, 42, 50, 52, 59, 62, 70, 74, 75, 78, 90

$H_0: \sigma_x = 18$

$H_1: \sigma_x \neq 18$

For left-side, $\chi^2_{\left(\frac{N-1}{2}, \frac{1-\alpha}{2}\right)} = \chi^2_{(7,0.45)} \cong 6.346$

For right-side, $\chi^2_{\left(\frac{N-1}{2}, \frac{\alpha}{2}\right)} = \chi^2_{(7,0.05)} = 14.067$

$\chi^2(test) = \frac{(N-1)\cdot s^2}{\sigma^2}$

$\mu_x = 51$

$S^2 = \sum \frac{(x_i-\mu_x)^2}{N-1} = 614$

$\chi^2(test) = \frac{(N-1)\cdot s^2}{\sigma^2} = \frac{14\cdot 614}{324} = 26.5308642$

As a result, **26.530842** is not in the interval (6.346-14.067). So, hypothesis H_0 is rejected. Hypothesis H_1, Standard deviation of the grades are different from $\sigma_x = 18$ is **ACCEPTED** at the level of significance of $\alpha = 0.10$.

4. A large group of a students took a final exam in Statistics and its claimed to be standard deviation of the grades higher than $\boldsymbol{\sigma_x > 18}$, at the level of significance of $\boldsymbol{\alpha = 0.10}$, for a sample size $\boldsymbol{N = 15}$. Samples are shown below:

7, 13, 27, 31, 35, 42, 50, 52, 59, 62, 70, 74, 75, 78, 90

$H_0: \sigma_x = 18$

$H_1: \sigma_x > 18$

$\chi^2_{(N-1,\alpha)} = \chi^2_{(14,0.10)} = 21.064$

$\chi^2(test) = \frac{(N-1)\cdot s^2}{\sigma^2}$

$\mu_x = 51$

$S^2 = \sum \frac{(x_i-\mu_x)^2}{N-1} = 614$

$\chi^2(test) = \frac{(N-1)\cdot s^2}{\sigma^2} = \frac{14\cdot 614}{324} = 26.5308642$

As a result, **26.530842 > 21.064** and $\boldsymbol{\chi^2(test) > \chi^2(critical\ value)}$. So, hypothesis H_0 is rejected. Hypothesis H_1, Standard deviation of the grades are higher than $\sigma_x > 18$ is **ACCEPTED** at the level of significance of $\alpha = 0.10$.

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Chi-Square Table

Table 5-2

Critical Values of the χ^2 Distribution

df \ p	0.995	0.975	0.9	0.5	0.1	0.05	0.025	0.01	0.005	df
1	.000	.000	0.016	0.455	2.706	3.841	5.024	6.635	7.879	1
2	0.010	0.051	0.211	1.386	4.605	5.991	7.378	9.210	10.597	2
3	0.072	0.216	0.584	2.366	6.251	7.815	9.348	11.345	12.838	3
4	0.207	0.484	1.064	3.357	7.779	9.488	11.143	13.277	14.860	4
5	0.412	0.831	1.610	4.351	9.236	11.070	12.832	15.086	16.750	5
6	0.676	1.237	2.204	5.348	10.645	12.592	14.449	16.812	18.548	6
7	0.989	1.690	2.833	6.346	12.017	14.067	16.013	18.475	20.278	7
8	1.344	2.180	3.490	7.344	13.362	15.507	17.535	20.090	21.955	8
9	1.735	2.700	4.168	8.343	14.684	16.919	19.023	21.666	23.589	9
10	2.156	3.247	4.865	9.342	15.987	18.307	20.483	23.209	25.188	10
11	2.603	3.816	5.578	10.341	17.275	19.675	21.920	24.725	26.757	11
12	3.074	4.404	6.304	11.340	18.549	21.026	23.337	26.217	28.300	12
13	3.565	5.009	7.042	12.340	19.812	22.362	24.736	27.688	29.819	13
14	4.075	5.629	7.790	13.339	21.064	23.685	26.119	29.141	31.319	14
15	4.601	6.262	8.547	14.339	22.307	24.996	27.488	30.578	32.801	15