

The test phase result of the air defense system HİSAR, produced by ASELSAN, are as follows.

X is the number of missiles fired.

Y is the number of missiles blocked by the HİSAR.

	$y=0$	$y=1$	$y=2$	$y=3$	$y=4$	$y=5$
$X=0$	0,1	0	0	0	0	0
$X=1$	0,02	0,14	0	0	0	0
$X=2$	0	0,01	0,13	0	0	0
$X=3$	0,01	0,02	0	0,22	0	0
$X=4$	0	0,04	0,01	0,01	0,12	0
$X=5$	0	0	0,01	0,02	0,05	0,09

↖ (Joint probability mass function of the variables x and y)

Using the given table,

Marginal probability mass function of X :

$$P(X) = P(X=x) = \sum_{y_j} P(x, y_j)$$

x_i	0	1	2	3	4	5
$P(x_i)$	0,1	0,16	0,14	0,25	0,18	0,17

$$P(x_i) = \sum_{y_j=0}^5 P(x_i, y_j)$$

Marginal Probability mass function of y :

$$p(y) = P(Y=y) = \sum_{x_i} p(x_i, y)$$

y_j	0	1	2	3	4	5
$p(y_j)$	0,13	0,21	0,15	0,25	0,17	0,09

$$p(y_j) = \sum_{x_i=0}^5 p(x_i, y_j)$$

Conditional Probability mass function of x when $y=0$:

$$p(x_i | y_j) = P(X=x_i | Y=y_j) = \frac{P((X=x_i) \cap (Y=y_j))}{P(Y=y_j)} = \frac{p(x_i, y_j)}{p(y_j)}$$

x_i	0	1	2	3	4	5
$p(x_i y=0)$	$\frac{10}{13}$	$\frac{2}{13}$	0	$\frac{1}{13}$	0	0
	↑	↑	↑	↑	↑	↑
	$\frac{0,1}{0,13}$	$\frac{0,02}{0,13}$	$\frac{0}{0,13}$	$\frac{0,01}{0,13}$	$\frac{0}{0,13}$	$\frac{0}{0,13}$

$$p(x_i | y=0) = \frac{p(x_i, 0)}{0,13}$$

Conditional Probability mass function of x when $y=1$:

x_i	0	1	2	3	4	5
$p(x_i y=1)$	0	$\frac{2}{3}$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{4}{21}$	0
	↑	↑	↑	↑	↑	↑
	$\frac{0}{0,21}$	$\frac{0,14}{0,21}$	$\frac{0,01}{0,21}$	$\frac{0,02}{0,21}$	$\frac{0,04}{0,21}$	$\frac{0}{0,21}$

$$p(x_i | y=1) = \frac{p(x_i, 1)}{0,21}$$

Conditional probability mass function of X when $y = 2$:

x_i	0	1	2	3	4	5
$p(x_i y=2)$	0	0	$\frac{13}{15}$	0	$\frac{1}{15}$	$\frac{1}{15}$
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0}{0,15}$	$\frac{0}{0,15}$	$\frac{0,13}{0,15}$	$\frac{0}{0,15}$	$\frac{0,01}{0,15}$	$\frac{0,01}{0,15}$

$$p(x_i|y=2) = \frac{p(x_i, 2)}{0,15}$$

Conditional probability mass function of X when $y = 3$:

x_i	0	1	2	3	4	5
$p(x_i y=3)$	0	0	0	$\frac{22}{25}$	$\frac{1}{25}$	$\frac{2}{25}$
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0}{0,25}$	$\frac{0}{0,25}$	$\frac{0}{0,25}$	$\frac{0,22}{0,25}$	$\frac{0,01}{0,25}$	$\frac{0,02}{0,25}$

$$p(x_i|y=3) = \frac{p(x_i, 3)}{0,25}$$

Conditional probability mass function of X when $y = 4$:

x_i	0	1	2	3	4	5
$p(x_i y=4)$	0	0	0	0	$\frac{12}{17}$	$\frac{5}{17}$
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0}{0,17}$	$\frac{0}{0,17}$	$\frac{0}{0,17}$	$\frac{0}{0,17}$	$\frac{0,12}{0,17}$	$\frac{0,05}{0,17}$

$$p(x_i|y=4) = \frac{p(x_i, 4)}{0,17}$$

Conditional probability mass function of x when $y=5$:

x_i	0	1	2	3	4	5
$p(x_i y=5)$	0	0	0	0	0	1
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0}{0,09}$	$\frac{0}{0,09}$	$\frac{0}{0,09}$	$\frac{0}{0,09}$	$\frac{0}{0,09}$	$\frac{0,09}{0,09}$

$$p(x_i|y=5) = \frac{p(x_i, 5)}{0,09}$$

Conditional probability mass function of y when $x=0$:

y_j	0	1	2	3	4	5
$p(x=0 y_j)$	1	0	0	0	0	0
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0,1}{0,1}$	$\frac{0}{0,1}$	$\frac{0}{0,1}$	$\frac{0}{0,1}$	$\frac{0}{0,1}$	$\frac{0}{0,1}$

$$p(x=0|y_j) = \frac{p(0, y_j)}{0,1}$$

Conditional probability mass function of y when $x=1$:

y_j	0	1	2	3	4	5
$p(x=1 y_j)$	$\frac{1}{8}$	$\frac{7}{8}$	0	0	0	0
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0,02}{0,16}$	$\frac{0,14}{0,16}$	$\frac{0}{0,16}$	$\frac{0}{0,16}$	$\frac{0}{0,16}$	$\frac{0}{0,16}$

$$p(x=1|y_j) = \frac{p(1, y_j)}{0,16}$$

Conditional Probability mass function of y when $x=2$:

y_j	0	1	2	3	4	5
$p(x=2 y_j)$	0	$\frac{1}{14}$	$\frac{12}{14}$	0	0	0
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0}{0,14}$	$\frac{0,01}{0,14}$	$\frac{0,12}{0,14}$	$\frac{0}{0,14}$	$\frac{0}{0,14}$	$\frac{0}{0,14}$

$$p(x=2|y_j) = \frac{p(2, y_j)}{0,14}$$

Conditional probability mass function of y when $x=3$:

y_j	0	1	2	3	4	5
$p(x=3 y_j)$	$\frac{1}{25}$	$\frac{2}{25}$	0	$\frac{22}{25}$	0	0
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0,01}{0,25}$	$\frac{0,02}{0,25}$	$\frac{0}{0,25}$	$\frac{0,22}{0,25}$	$\frac{0}{0,25}$	$\frac{0}{0,25}$

$$p(x=3|y_j) = \frac{p(3, y_j)}{0,25}$$

Conditional probability mass function of y when $x=4$:

y_j	0	1	2	3	4	5
$p(x=4 y_j)$	0	$\frac{2}{9}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{2}{3}$	0
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0}{0,18}$	$\frac{0,04}{0,18}$	$\frac{0,01}{0,18}$	$\frac{0,01}{0,18}$	$\frac{0,12}{0,18}$	$\frac{0}{0,18}$

$$p(x=4|y_j) = \frac{p(4, y_j)}{0,18}$$

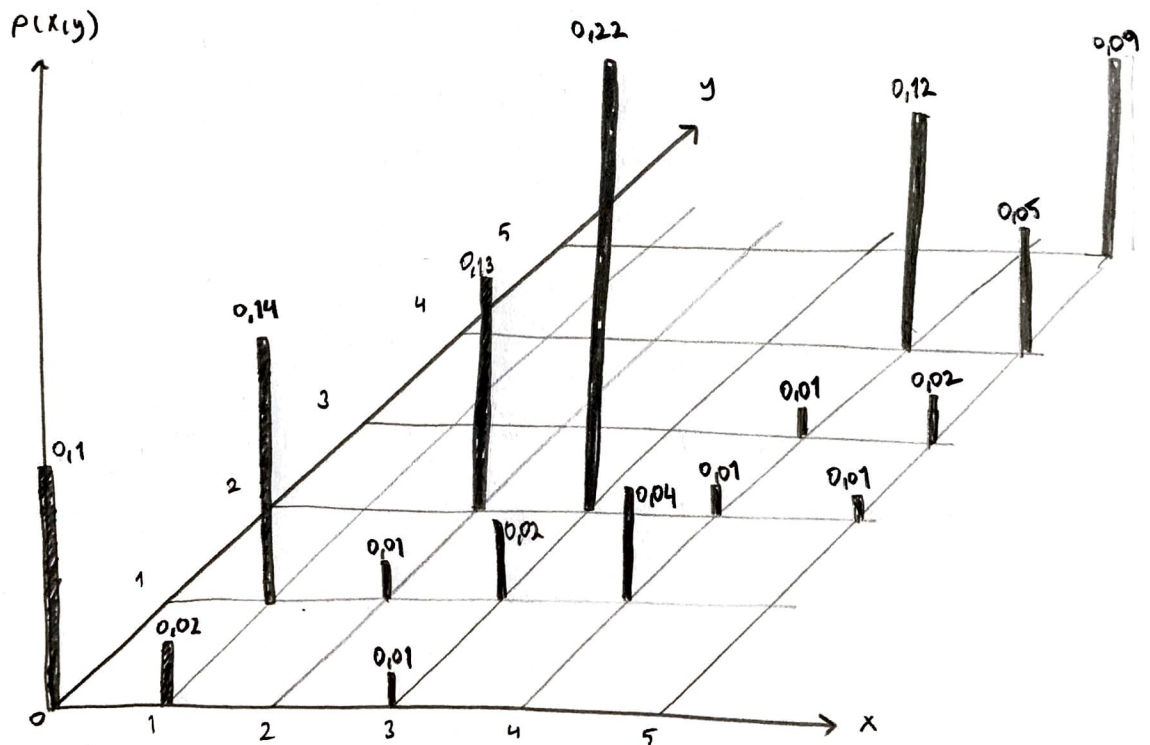
Conditional Probability mass function of y when $x=5$:

y_j	0	1	2	3	4	5
$P(x=5 y_j)$	0	0	$\frac{1}{17}$	$\frac{2}{17}$	$\frac{5}{17}$	$\frac{9}{17}$
	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow	\uparrow
	$\frac{0}{0,17}$	$\frac{0}{0,17}$	$\frac{0,01}{0,17}$	$\frac{0,02}{0,17}$	$\frac{0,05}{0,17}$	$\frac{0,09}{0,17}$

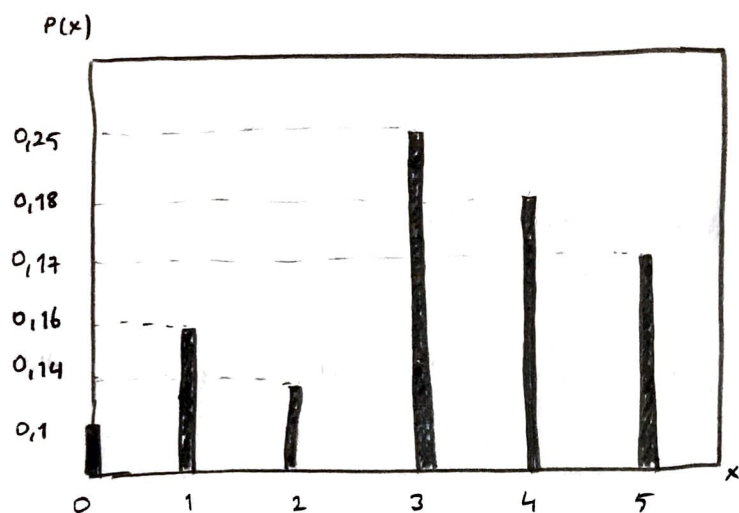
$$p(x=5|y_j) = \frac{r(5, y_j)}{0,17}$$

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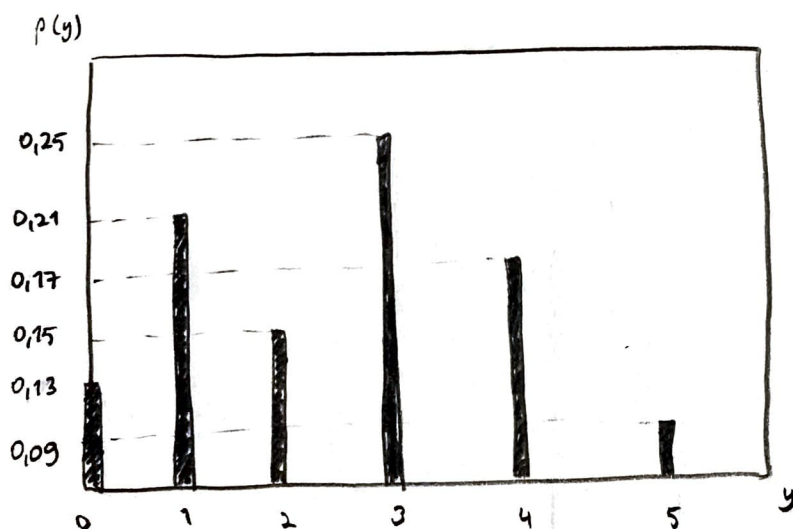
$p(x_i, y_j)$ joint probability mass function



$P(x_i)$ marginal probability mass function

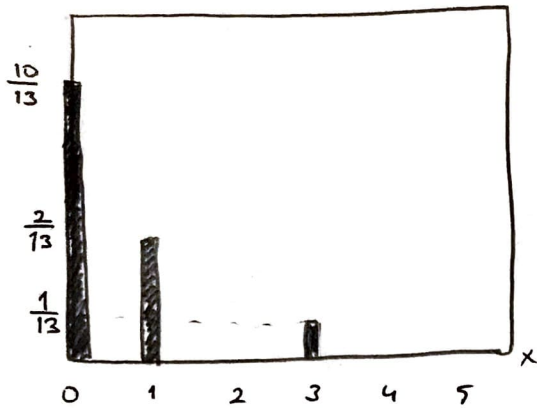


$P(y_j)$ marginal probability mass function



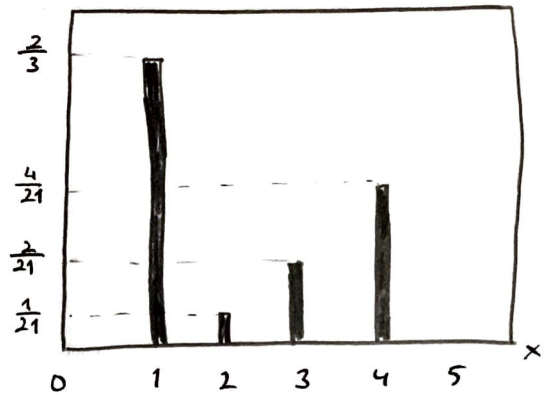
$P(x|y=0)$ conditional p.m.f.

$p(x|y=0)$



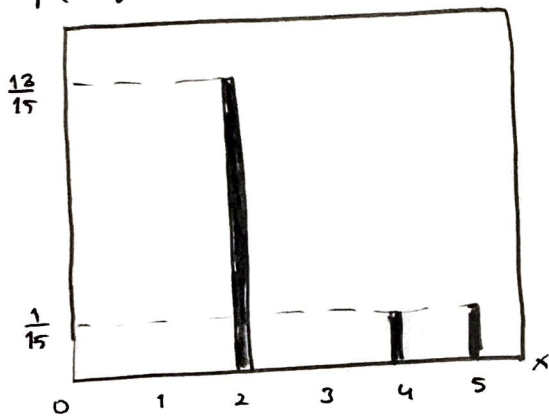
$p(x|y=1)$ conditional p.m.f.

$p(x|y=1)$



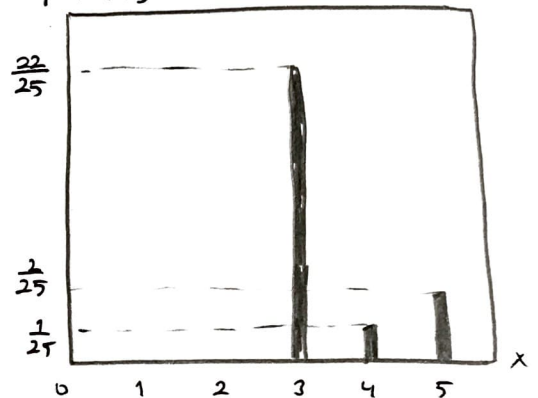
$p(x|y=2)$ conditional p.m.f.

$p(x|y=2)$



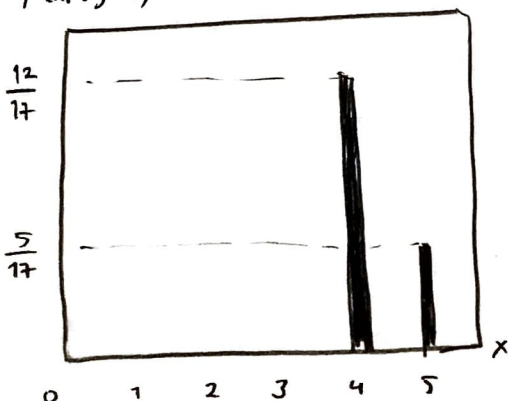
$p(x|y=3)$ conditional p.m.f.

$p(x|y=3)$



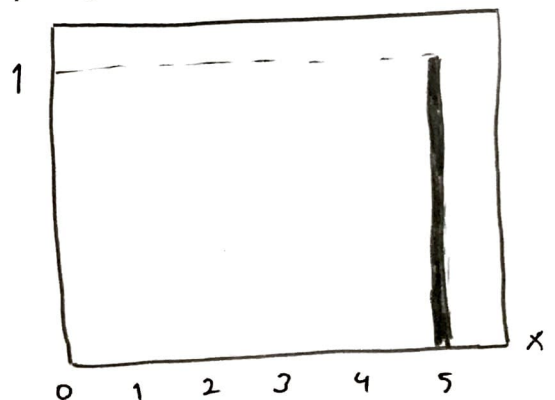
$p(x|y=4)$ conditional p.m.f.

$p(x|y=4)$



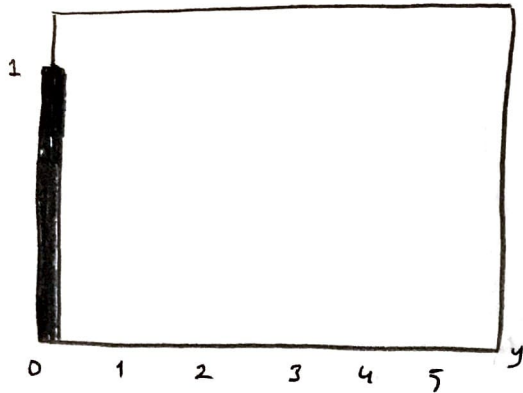
$p(x|y=5)$ conditional p.m.f.

$p(x|y=5)$



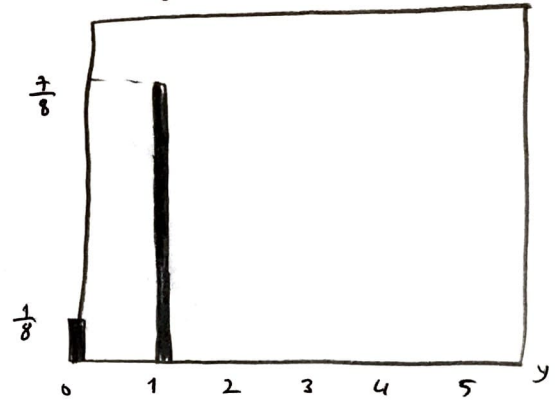
$p(x=0|y_j)$ conditional p.m.f

$p(x=0|y_j)$



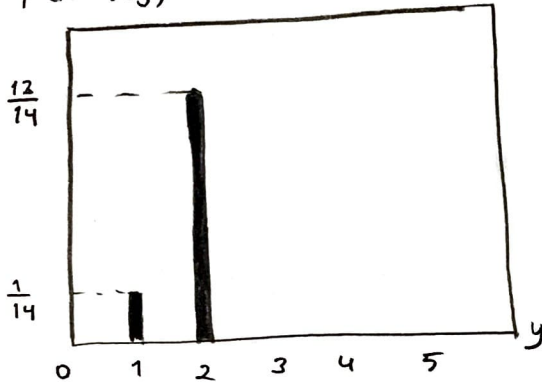
$p(x=1|y_j)$ conditional p.m.f

$p(x=1|y_j)$



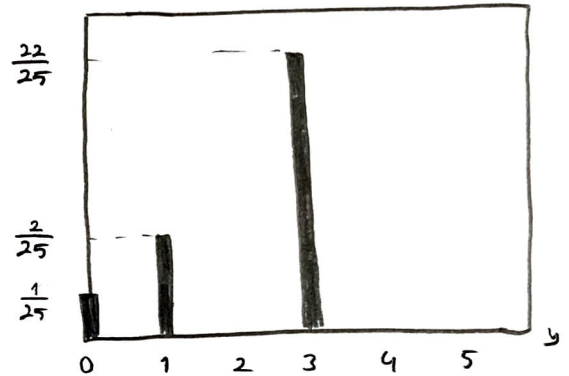
$p(x=2|y_j)$ conditional p.m.f.

$p(x=2|y_j)$



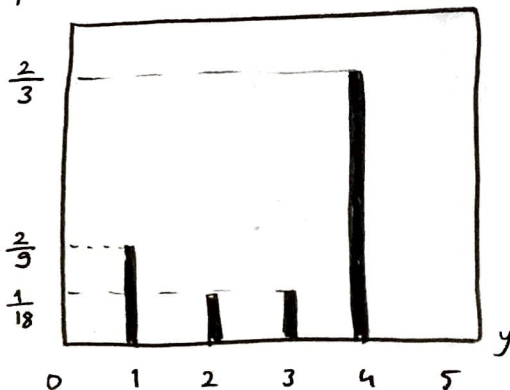
$p(x=3|y_j)$ conditional p.m.f

$p(x=3|y_j)$



$p(x=4|y_j)$ conditional p.m.f

$p(x=4|y_j)$



$p(x=5|y_j)$ conditional p.m.f

$p(x=5|y_j)$

