

Q1

(a)

$$\det(A) = (-1) \cdot (-1)^2 \cdot \begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} + 0 \cdot (-1)^3 \cdot \begin{vmatrix} -2 & -3 \\ 4 & 2 \end{vmatrix} + 0 \cdot (-1)^4 \cdot \begin{vmatrix} -2 & -3 \\ 3 & -1 \end{vmatrix}$$

$$\det(A) = -10$$

$$\det(B) = (-5) \cdot (-1)^2 \cdot \begin{vmatrix} 4 & 8 \\ -5 & -6 \end{vmatrix} + (-2) \cdot (-1)^3 \cdot \begin{vmatrix} 1 & 2 \\ -5 & -6 \end{vmatrix} + (-4) \cdot (-1)^4 \cdot \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix}$$

$$\det(B) = (-5)(-24 - (-40)) + (-2)(-1)(-6 - (-10)) + (-4) \cdot (8 - 8)$$

$$\det(B) = -72$$

(b)

$$A = \begin{bmatrix} -1 & -2 & -3 \\ 0 & 3 & -1 \\ 0 & 4 & 2 \end{bmatrix} \xrightarrow{\frac{1}{3} \cdot R_2 \rightarrow R_2} \begin{bmatrix} -1 & -2 & -3 \\ 0 & 1 & -1/3 \\ 0 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 - 4R_2 \rightarrow R_3} \begin{bmatrix} -1 & -2 & -3 \\ 0 & 1 & -1/3 \\ 0 & 0 & 10/3 \end{bmatrix}$$

$$\det(A) = (-1)(1)(10/3)(3)$$

$$\det(A) = -10$$

$$B = \begin{bmatrix} -5 & 1 & 2 \\ -2 & 4 & 8 \\ -4 & -5 & -6 \end{bmatrix} \xrightarrow{-\frac{R_1}{5} \rightarrow R_1} \begin{bmatrix} 1 & -1/5 & -2/5 \\ -2 & 4 & 8 \\ -4 & -5 & -6 \end{bmatrix} \xrightarrow{R_2 + 2R_1 \rightarrow R_2}$$

$$\begin{bmatrix} 1 & -1/5 & -2/5 \\ 0 & 18/5 & 36/5 \\ -4 & -5 & -6 \end{bmatrix} \xrightarrow{R_3 + 4R_1 \rightarrow R_3} \begin{bmatrix} 1 & -1/5 & -2/5 \\ 0 & 18/5 & 36/5 \\ 0 & -29/5 & -38/5 \end{bmatrix} \xrightarrow{\frac{5}{18} R_2 \rightarrow R_2}$$

$$\begin{bmatrix} 1 & -1/5 & -2/5 \\ 0 & 1 & 2 \\ 0 & -29/5 & -38/5 \end{bmatrix} \xrightarrow{R_3 + \frac{29}{5} R_2 \rightarrow R_3} \begin{bmatrix} 1 & -1/5 & -2/5 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\det(B) = (1)(1)(4)(-5)(18/5)$$

$$= -72$$

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -1 & 1 \\ 2 & 3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 7 \\ -2 \\ 10 \end{bmatrix}$$

$$\det(A_x) = \begin{vmatrix} 7 & -2 & 1 \\ -2 & -1 & 1 \\ -10 & 3 & -2 \end{vmatrix}, \det(A_y) = \begin{vmatrix} 1 & 7 & 1 \\ 3 & -2 & 1 \\ 2 & -10 & -2 \end{vmatrix}, \det(A_z) = \begin{vmatrix} 1 & -2 & 7 \\ 3 & -1 & -2 \\ 2 & 3 & -10 \end{vmatrix}$$

$$\det(A) = (1)(-1)^2 \begin{vmatrix} -1 & 1 \\ 3 & -2 \end{vmatrix} + (-2)(-1)^3 \begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} + (1)(-1)^4 \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$$

$$= 2 - (3) + 2(-6 - (-2)) + 9 - (-2)$$

$$= -6$$

$$\det(A_x) = (7)(-1)^2 \begin{vmatrix} -1 & 1 \\ 3 & -2 \end{vmatrix} + (-2)(-1)^3 \begin{vmatrix} -2 & 1 \\ -10 & -2 \end{vmatrix} + (1)(-1)^4 \begin{vmatrix} -2 & -1 \\ -10 & 3 \end{vmatrix}$$

$$= 7(2 - (3)) + 2(4 - (-10)) + (-6) - (10)$$

$$= 5$$

$$\det(A_y) = (1)(-1)^2 \begin{vmatrix} -2 & 1 \\ 10 & -2 \end{vmatrix} + (7)(-1)^3 \begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} + (1)(-1)^4 \begin{vmatrix} 3 & -2 \\ 2 & -10 \end{vmatrix}$$

$$= (4 - (-10)) + (-7)(-6 - (-2)) + (-30) - (-4)$$

$$= 44$$

$$\det(A_z) = (1)(-1)^2 \begin{vmatrix} -1 & -2 \\ 3 & -10 \end{vmatrix} + (-2)(-1)^3 \begin{vmatrix} 3 & -2 \\ 2 & -10 \end{vmatrix} + (7)(-1)^4 \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$$

$$= 10 - (-6) + (2)(-30 - (-4)) + (7)(9 - (-2))$$

$$= 41$$

$$x = \det(A_x) / \det(A)$$

$$x = -5/6$$

$$y = \det(A_y) / \det(A)$$

$$y = -22/3$$

$$z = \det(A_z) / \det(A)$$

$$z = -41/6$$

Q3

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$$\textcircled{a} \quad u + 2v = (3, -1, 2) + (4, -6, 10)$$

$$u + 2v = (7, -7, 12)$$

$$\|u + 2v\| = \sqrt{7^2 + (-7)^2 + 12^2} = \sqrt{242}$$

$$\textcircled{b} \quad 3v = (6, -9, 15) \quad 2w = (2, 12, -4)$$

$$\|3v\| = \sqrt{6^2 + (-9)^2 + 15^2} = \sqrt{342} = 3\sqrt{38}$$

$$\|2w\| = \sqrt{2^2 + 12^2 + (-4)^2} = \sqrt{164} = 2\sqrt{41}$$

$$\|3v\| - \|2w\| = 3\sqrt{38} - 2\sqrt{41}$$

$$\textcircled{c} \quad 2u = (6, -2, 4), \quad -v = (-2, 3, -5) \quad 3w = (3, 18, -6)$$

$$2u - v + 3w = (7, 19, -7)$$

$$\|2u - v + 3w\| = \sqrt{7^2 + 19^2 + (-7)^2} = \sqrt{459} = 3\sqrt{51}$$

Qu

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$$\textcircled{a} \quad u \cdot v = 2 \cdot 3 + 3 \cdot 3 + 1 \cdot 0 = 15$$

$$u \cdot u = 2 \cdot 2 + 3 \cdot 3 + 1 \cdot 1 = 14$$

$$v \cdot v = 3 \cdot 3 + 3 \cdot 3 + 0 \cdot 0 = 18$$

$$\textcircled{b} \quad u \cdot v = (-2) \cdot 2 + (-1) \cdot 1 + 0 \cdot 3 + (-3) \cdot 2 + 1 \cdot 4 = -7$$

$$u \cdot u = (-2)(-2) + (-1)(-1) + 0 \cdot 0 + (-3)(-3) + 1 \cdot 1 = 15$$

$$v \cdot v = 2 \cdot 2 + 1 \cdot 1 + 3 \cdot 3 + 2 \cdot 2 + 4 \cdot 4 = 34$$