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i)

a.
$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ -1 & 2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \xrightarrow[-3R_1 + R_3 \to R_3]{R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 0 & 3 & 6 & 9 \\ 0 & -10 & -5 & -14 \end{array}\right] \xrightarrow{\frac{10}{3}R_2 + R_3 \to R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 0 & 3 & 6 & 9 \\ 0 & 0 & 15 & 16 \end{array}\right]$$

$15z = 16$

$z = \frac{16}{15}$

$3y + 6z = 3y + 6 \cdot \frac{16}{15} = 9$

$3y + \frac{32}{5} = 9$, $y = \frac{13}{15}$

$x + y + 3z = 8$

$x + \frac{13}{15} + 3 \cdot \frac{16}{15} = 8$

$x + \frac{61}{15} = 8$, $x = \frac{59}{15}$

$x = \frac{59}{15}$

$y = \frac{13}{15}$

$z = \frac{16}{15}$

b.
$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ -1 & 2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \xrightarrow[-3R_1 + R_3 \to R_3]{R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 0 & 3 & 6 & 9 \\ 0 & -10 & -5 & -14 \end{array}\right] \xrightarrow{\frac{10}{3}R_2 + R_3 \to R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 0 & 3 & 6 & 9 \\ 0 & 0 & 15 & 16 \end{array}\right] \xrightarrow[\frac{1}{15}R_3 \to R_3]{\frac{1}{3}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & \frac{16}{15} \end{array}\right] \xrightarrow[-2R_3 + R_2 \to R_2]{-3R_3 + R_1 \to R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & \frac{72}{15} \\ 0 & 1 & 0 & \frac{13}{15} \\ 0 & 0 & 1 & \frac{16}{15} \end{array}\right] \xrightarrow{-R_2 + R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{59}{15} \\ 0 & 1 & 0 & \frac{13}{15} \\ 0 & 0 & 1 & \frac{16}{15} \end{array}\right]$$

$x = \frac{59}{15}$

$y = \frac{13}{15}$

$z = \frac{16}{15}$

ii)
$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right] \xrightarrow[-4R_1 + R_3 \to R_3]{R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & 0 \end{array}\right] \xrightarrow{R_2 + R_3 \to R_3}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \to$$ infinitely many solution

$z = a$

$7y + 4a = 1$

$y = \frac{1-4a}{7}$

$2x + 2\left(\frac{1-4a}{7}\right) + 2a = 0$

$7x + 1 - 4a + 7a = 0$

$7x + 3a = -1$

$x = \frac{-1-3a}{7}$

$x = \frac{-1-3a}{7}$

$y = \frac{1-4a}{7}$

$z = a$

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Q2

a)

$$M\cdot N = \begin{bmatrix} 2 & -1 \\ 3 & 6 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 7 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + (-1)\cdot 3 & 2\cdot 7 + (-1)(-6) \\ 3\cdot 1 + 6\cdot 3 & 3\cdot 7 + 6\cdot(-6) \\ -2\cdot 1 + 5\cdot 3 & -2\cdot 7 + 5\cdot(-6) \end{bmatrix}$$

$$M\cdot N = \begin{bmatrix} -1 & 20 \\ 21 & -15 \\ 13 & -44 \end{bmatrix} \qquad 3\cdot(M\cdot N) = \begin{bmatrix} -3 & 60 \\ 63 & -45 \\ 39 & -132 \end{bmatrix} \checkmark ,$$

$$3\cdot M = \begin{bmatrix} 6 & -3 \\ 9 & 18 \\ -6 & 15 \end{bmatrix} \qquad (3M)\cdot N = \begin{bmatrix} 6 & -3 \\ 9 & 18 \\ -6 & 15 \end{bmatrix} \cdot \begin{bmatrix} 1 & 7 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 6\cdot 1 + (-3)\cdot 3 & 6\cdot 7 + (-3)(-6) \\ 9\cdot 1 + 18\cdot 3 & 9\cdot 7 + 18\cdot(-6) \\ (-6)\cdot 1 + 15\cdot 3 & (-6)\cdot 7 + 15\cdot(-6) \end{bmatrix}$$

$$(3M)\cdot N = \begin{bmatrix} -3 & 60 \\ 63 & -45 \\ 39 & -132 \end{bmatrix} \checkmark , \qquad 3\cdot N = \begin{bmatrix} 3 & 21 \\ 9 & -18 \end{bmatrix} \qquad M\cdot(3N) = \begin{bmatrix} 2 & -1 \\ 3 & 6 \\ -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 21 \\ 9 & -18 \end{bmatrix}$$

$$= \begin{bmatrix} 2\cdot 3 + (-1)\cdot 9 & 2\cdot 21 + (-1)\cdot(-18) \\ 3\cdot 3 + 6\cdot 9 & 3\cdot 21 + 6\cdot(-18) \\ (-2)\cdot 3 + 5\cdot 9 & (-2)\cdot 21 + 5\cdot(-18) \end{bmatrix} = \begin{bmatrix} -3 & 60 \\ 63 & -45 \\ 39 & -132 \end{bmatrix} \checkmark \Rightarrow 3(M\cdot N) = (3M)N = M(3N)$$

b)

$$(M\cdot N)^T = \underbrace{\begin{bmatrix} -1 & 20 \\ 21 & -15 \\ 13 & -44 \end{bmatrix}}_{M\cdot N}{}^T = \begin{bmatrix} -1 & 21 & 13 \\ 20 & -15 & -44 \end{bmatrix}$$

$$N^T\cdot M^T = \underbrace{\begin{bmatrix} 1 & 3 \\ 7 & -6 \end{bmatrix}}_{N^T} \cdot \underbrace{\begin{bmatrix} 2 & 3 & -2 \\ -1 & 6 & 5 \end{bmatrix}}_{M^T} = \begin{bmatrix} 1\cdot 2 + 3\cdot(-1) & 1\cdot 3 + 3\cdot 6 & 1\cdot(-2) + 3\cdot 5 \\ 7\cdot 2 + (-6)\cdot(-1) & 7\cdot 3 + (-6)\cdot 6 & 7\cdot(-2) + (-6)\cdot 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 21 & 13 \\ 20 & -15 & -44 \end{bmatrix}$$

$$\Rightarrow (MN)^T = N^T\cdot M^T$$

Q 3

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$A^{-1}/$ $A^3 - 5A^2 + 7A = 0$

$A^2 \cdot (A \cdot A^{-1}) - 5A(A \cdot A^{-1}) + 7(A \cdot A^{-1}) = 0$

$\boxed{A \cdot A^{-1} = I}$

$A^{-1}/$ $= A^2 \cdot I - 5A \cdot I + 7I = 0$

$A(A \cdot A^{-1}) \cdot I - 5(A \cdot A^{-1}) \cdot I + 7A^{-1} \cdot I = 0$

$A \cdot I \cdot I - 5I \cdot I + 7A^{-1} \cdot I = 0$

$= I \underbrace{(A \cdot I - 5 \cdot I + 7 \cdot A^{-1})}_{0} = 0$

$A \cdot I - 5 \cdot I = -7 \cdot A^{-1}$

$\dfrac{5I - A \cdot I}{7} = A^{-1}$

$\dfrac{I(5 - A)}{7} = A^{-1}$