

1. Consider the function $f(x) = x^3$.
 - (a) (4 points) In what range is $f(x)$ monotonically increasing?
 - (b) (4 points) In what range is $f(x)$ monotonically decreasing?
2. Let $f(n)$ and $g(n)$ be monotonically increasing functions. Show that each of the following functions are also monotonically increasing.
 - (a) (4 points) $f(n) + g(n)$
 - (b) (4 points) $f(g(n))$
 - (c) (4 points) $f(n) \cdot g(n)$ (Assume that $f(n)$ and $g(n)$ are both nonnegative.)
3. (10 points) Prove equation 3.19 from the book. Stirling's approximation and the various properties of logarithms will be helpful.
4. (8 points) Let

$$p(n) = \sum_{i=0}^d a_i n^i$$
 where $a_d > 0$, be a degree- d polynomial in n , and let k be a constant such that $k \geq d$. Show that $p(n) = O(n^k)$.
5. Consider $\left(\frac{3}{2}\right)^n = \Omega(2^{\lg n})$.
 - (a) (5 points) Show that the equality is true.
 - (b) (5 points) What are all valid values of c when $n_0 = 1$?
6. Consider $n! = O((n+1)!)$.
 - (a) (5 points) Show that the equality is true.
 - (b) (5 points) What are all valid values of n_0 when $c = 1$?
7. Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.
 - (a) (5 points) $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.
 - (b) (5 points) $f(n) = O((f(n))^2)$.
 - (c) (5 points) $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.
 - (d) (5 points) $f(n) = \Theta(f(n/2))$.
8. Let $f(x) = \frac{x^2}{2}$. Give the following values. Show your work.
 - (a) (3 points) $f^3(1)$
 - (b) (3 points) $f^3(2)$
 - (c) (3 points) $f^3(3)$
9. (13 points) Rank the following functions by order of growth, slowest to fastest; that is, find an arrangement g_1, g_2, \dots of the functions satisfying $g_1 = O(g_2), g_2 = O(g_3), \dots$. In your ordering, circle any functions that have the same asymptotic rate of growth.

$n2^n$ n $\lg \lg n$ n^3 $4^{\lg n}$

$n \lg n$ \sqrt{n} $(3/2)^n$ $2^{\lg n}$

2^n n^2 $\lg n$ $n!$