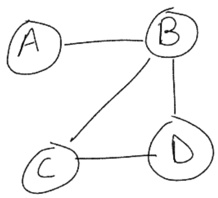
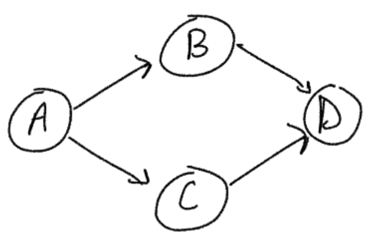
Homework 14
CS 3385
Due April 17 at the end of class

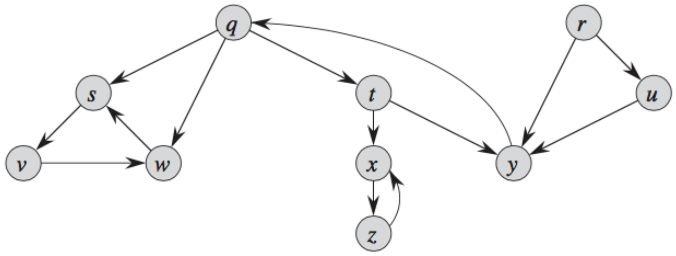
1. Given an adjacency-list representation of a directed graph, how long does it take to compute
   (a) the out-degree of a vertex?
   (b) the in-degree of a vertex?
2. Give the adjacency list and adjacency matrix representations for the following graph:



3. The **square** of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$ such that $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v. Describe an efficient algorithm for computing G^2 from G for the adjacency-list representation of G. Assume you have functions `addEdge(G,e)` and `neighbors(G,v)`.
4. Show the result from running breadth-first search on the directed graph of Figure 22.2(b), using vertex 3 as the source. Show the result by drawing the BFS tree with the d and π values in the vertices.
5. Show the result from running breadth-first search on the undirected graph of Figure 22.3, using vertex u as the source. Assume adjacency lists are in alphabetical order. Show the result by drawing the BFS tree with the d and π values in the vertices.
6. Argue that in a breadth-first search, the value $u.d$ assigned to a vertex u is independent of the order in which the vertices appear in each adjacency list. You can cite theorem 22.5 for your answer.
7. Show that the breadth-first tree computed by BFS can depend on the ordering within adjacency lists using the following tree. Using A as the starting vertex, show both possible BFS trees along with d and π values.



8. Give an example of a directed graph $G = (V, E)$, a source vertex $s \in V$, and a set of tree edges $E_\pi \subseteq E$ such that for each vertex $v \in V$, the unique simple path in the graph (V, E_π) from s to v is a shortest path in G, yet the set of edges E_π cannot be produced by running BFS on G, no matter how the vertices are ordered in each adjacency list.
9. Run a depth-first search on the following graph. Assume that the `for` loop of lines 5-7 of the DFS procedure considers the vertices in alphabetical order, and assume that each adjacency list is ordered alphabetically. Show the discovery and finishing times for each vertex, and show the classification (T, B, F, C) of each edge. You do not need to show the values for π.



10. Show the parenthesis structure of the depth-first search of Figure 22.4. Your answer should look similar to Figure 22.5(b).

11. Give a counterexample to the conjecture that if a directed graph G contains a path from u to v, and if $u.d < v.d$ in a depth-first search of G, then v is a descendant of u in the depth-first forest produced.

12. Give a counterexample to the conjecture that if a directed graph G contains a path from u to v, then any depth-first search must result in $v.d \leq u.f$.