Q1.

A1. mean $= \theta_1$, variance $= \theta_2$

$$f(x_i)(x_i, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\left(\frac{x_i-\theta_1}{2\theta_2}\right)^2}$$

likelihood fxn:

$$L = f(x_1,\theta) \cdot f(x_2,\theta) \cdots f(x_n,\theta)$$

$$= \frac{1}{\left(\sqrt{2\pi\theta_2}\right)^n} e^{-\frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}}$$

$$\log L = -\frac{n}{2}\log 2\pi - \frac{n}{2}\log\theta_2 - \frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}$$

$$\frac{\partial \log L}{\partial\theta_1} \Rightarrow \frac{-1}{2\theta_2} \times 2\Sigma(x_i-\theta_1) = 0$$

$$\Rightarrow \Sigma(x_i-\theta_1) = 0$$

$$\boxed{\theta_1 = \frac{\Sigma x_i}{n}}$$

$$\frac{\partial \log L}{\partial\theta_2} \Rightarrow \frac{-n}{2\theta_2} + \frac{\Sigma}{2\theta_2^2}(x_i-\theta_1)^2 = 0.$$

$$\Rightarrow -n\theta_2 + \Sigma(x_i-\theta_1)^2 = 0$$

$$\boxed{\theta_2 = \frac{\Sigma(x_i-\theta_1)^2}{n}}$$

Q2.

A2. $B(m, \theta) = {}^{n}C_{m}\, \theta^{m} (1-\theta)^{m-n}$; $x = 0, 1, 2, \dots, n$

Likelihood fn:

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{m}\, \theta^{m} (1-\theta)^{n-mi}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{m}\, \theta^{\sum mi} (1-\theta)^{\sum n - \sum mi}$$

$$L(\theta) = k\, \theta^{\sum mi} (1-p)^{n^2 - \sum mi}$$

taking log:

$$\ln L(\theta) = \ln k + \sum mi \ln \theta + (n^2 - \sum mi) \ln(1-\theta)$$

Differentiate w.r.t θ:

$$\frac{\partial}{\partial \theta} \ln L(\theta) = 0 + \sum mi \frac{\partial}{\partial \theta} \ln(\theta) + (n^2 - \sum mi) \frac{\partial}{\partial \theta} \ln(1-\theta)$$

$$\frac{\partial}{\partial \theta} \ln L(\theta) \Rightarrow \frac{\sum mi}{\theta} - \frac{(n^2 - \sum mi)}{(1-\theta)} = 0$$

$$\therefore \frac{\sum mi}{\theta} = \frac{n^2 - \sum mi}{1-\theta}$$

$$\therefore (1-\theta) \sum mi = \theta (n^2 - \sum mi)$$

$$\sum mi = \theta n^2$$

$$\boxed{\therefore \theta = \frac{\sum mi}{n^2}}$$