In [2]:

```
import pandas as pd
import numpy as np
df = pd.read_excel (r'C:\Users\dyu15\Downloads\finaldata.xlsx')

data = df.to_numpy()
data
```

Out[2]:

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In [3]:

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list = []
for d in data:
    list.append(d[1])
print(list)
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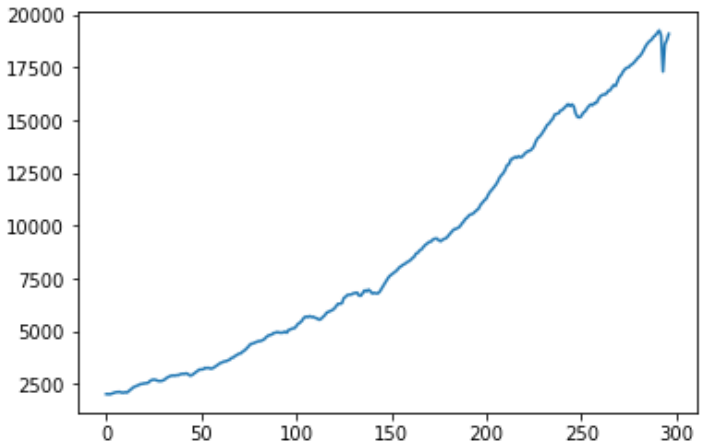
In [4]:

```
import matplotlib.pyplot as plt

xList = [i for i in range(len(list))]
```

```
yList = list

plt.plot(xList,yList)
```

Out[4]:

```
[<matplotlib.lines.Line2D at 0x2e0722458b0>]
```



In [5]:

```
import math

def gausselim(A,b):
    for i in range(0,len(A)):
        A[i].append(b[i])
    for i in range(0,len(A)):
        if len(A[i])!= len(A)+1:
            #check to make sure all the matricies are of the same length
            return "Matrix rows have non-uniform length"
    for k in range(len(A)):
        #find all the pivots, and the max one as well to check for singulatiry
        pivots = [abs(A[i][k]) for i in range(k, len(A))]
        i_max = pivots.index(max(pivots)) + k
        #check to see if the matrix is singular
        assert A[i_max][k] != 0, "Matrix is singular!"
        A[k], A[i_max] = A[i_max], A[k] #this code swaps rows
        for i in range(k + 1, len(A)):
            f = A[i][k] / A[k][k]
            for j in range(k + 1, len(A)+1):
                A[i][j] -= A[k][j] * f
            A[i][k] = 0
    x = []
    for i in range(len(A) - 1, -1, -1):
        x.insert(0, A[i][len(A)] / A[i][i]) #here we solve using elimination
        for k in range(i - 1, -1, -1):
            A[k][len(A)] -= A[k][i] * x[0]

    #for i in range(len(x)):
        #print("a", i, "is",x[i])
    return x

def plotData(x,y,n):
    sumx=[]
    sumy =[]
    xs=[]
    ys=[]
    i=0
    k=0
    while k <=2*n:
        while i <len(x):
            xs.append(x[i]**k)
            ys.append(((x[i])**k)*y[i])
            i=i+1
        sumx.append(sum(xs))
        sumy.append(sum(ys))
        i=0
```

```
        xs=[]
        ys=[]
        k=k+1
    g=[]
    sumy.pop(-1)
    j=0
    i=0
    matrix=[[]]
    arr=[]
    while j < n:
        for i in range(i,n+i):
            arr.append(sumx[i])
            i=i+1
        matrix.append(arr)
        arr=[]
        j=j+1
        i=j
    matrix.pop(0)
    gauss=gausselim(matrix,sumy)
    xj=np.linspace(0,297,1000)
    yj=[]
    for k in range(len(xj)):
        yjsum=0
        for i in range(n):
            yjsum=yjsum+gauss[i]*(xj[k]**i)
        yj.append(yjsum)
    getError(n-1)
    plt.title("polynomial of " + str(n - 1) + " degree")
    plt.plot(xj,yj)
    plt.plot(xList,yList)
    return

def plotDatas(x,y,n, start, end):
    sumx=[]
    sumy =[]
    xs=[]
    ys=[]
    i=0
    k=0
    while k <=2*n:
        while i <len(x):
            xs.append(x[i]**k)
            ys.append(((x[i])**k)*y[i])
            i=i+1
        sumx.append(sum(xs))
        sumy.append(sum(ys))
        i=0
        xs=[]
        ys=[]
        k=k+1
    g=[]
    sumy.pop(-1)
    j=0
    i=0
    matrix=[[]]
    arr=[]
    while j < n:
        for i in range(i,n+i):
            arr.append(sumx[i])
            i=i+1
        matrix.append(arr)
        arr=[]
        j=j+1
        i=j
    matrix.pop(0)
    gauss=gausselim(matrix,sumy)
    xj=np.linspace(start,end,1000)
    yj=[]
    for k in range(len(xj)):
        yjsum=0
        for i in range(n):
            yjsum=yjsum+gauss[i]*(xj[k]**i)
```

```
        yj.append(yjsum)
    getError(n-1)
    plt.title("polynomial of " + str(n - 1) + " degree")
    plt.plot(xj,yj)
    plt.plot(xList,yList)
    return

def get_dates(matrix): #Create a function that gets rid of dates, and turns it into time
starting at 0
    i=0
    rGDP=[]
    time=[]
    dates=[]
    while i<len(matrix):
        rGDP.append(matrix[i][1])
        time.append(i)
        dates.append(matrix[i][0])
        i=i+1
    return rGDP, time, dates
realGDP=get_dates(data)[0] #make results from functions into vectors
time=get_dates(data)[1]

def power(List,n):
    nList = []
    for x in List:
        nList.append(x**n)
    return nList

def xy(n):
    nList = []
    for i in range(len(xList)):
        nList.append((xList[i] ** n) * yList[i])
    return nList

def lnxy():
    nList = []
    for i in range(1, len(xList)):
        nList.append(math.log(xList[i]) * math.log(yList[i]))
    return nList
```

In [6]:

```
def A(n):
    start = 0
    end = n
    matrix = []
    while start != n+1:
        row = []
        for i in range(start, end + 1):
            row.append(sum(power(xList,i)))
        matrix.append(row)
        start += 1
        end += 1

    return matrix
A(6)
```

Out[6]:

```
[[297,
  43956,
  8688636,
  1932129936,
  458299483092,
  113237627245776,
  28778370446553156],
 [43956,
  8688636,
  1932129936,
  458299483092,
  113237627245776,
  28778370446553156,
```

```
 7466138941547482416],
[8688636,
 1932129936,
 458299483092,
 113237627245776,
 28778370446553156,
 7466138941547482416,
 196772747459532267157 2],
[1932129936,
 458299483092,
 113237627245776,
 28778370446553156,
 7466138941547482416,
 196772747459532267157 2,
 5250835969599563434380 96],
[458299483092,
 113237627245776,
 28778370446553156,
 7466138941547482416,
 196772747459532267157 2,
 5250835969599563434380 96,
 1415325552028779450014 00676],
[113237627245776,
 28778370446553156,
 7466138941547482416,
 196772747459532267157 2,
 5250835969599563434380 96,
 1415325552028779450014 00676,
 3846696796750060284244 4672496],
[28778370446553156,
 7466138941547482416,
 196772747459532267157 2,
 5250835969599563434380 96,
 1415325552028779450014 00676,
 3846696796750060284244 4672496,
 1052799347713282949771 2155944052]]
```

In [7]:

```
def B(n):
    matrix = []
    for i in range(0,n+1):
        matrix.append(sum(xy(i)))
    return matrix
B(6)
```

Out[7]:

```
[2629169.8690000004,
 520790388.6420001,
 117659711098.24602,
 28262910728412.695,
 7047318705684167.0,
 1.8028335872784353e+18,
 4.699712292631704e+20]
```

In [8]:

```
def getError(n):
    poly = 0
    mean = 0
    meanDiff = []
    diff = []
    r = gausselim(A(n), B(n))
    for i in range(len(xList)):
        for j in range(0, n+1):
            poly += r[j] * power(xList, j)[i]
        meanDiff.append(abs(yList[i] - poly))
        diff.append(yList[i] - poly)
        poly = 0
    mean = sum(meanDiff)/len(meanDiff)
    print("mean error: " + str(mean))
```

```
    print("total error: " + str(sum(power(diff, 2))))
    return

getError(46)
```

```
mean error: 108.511520735054
total error: 7615739.369965735
```

In [9]:

```
def ln(List):
    nList = []
    for i in range(1, len(List)):
        nList.append(math.log(List[i]))
    return nList

def a():
    num = (len(xList) * sum(lnxy())) - (sum(ln(xList)) * sum(ln(yList)))
    denum = (len(xList) * (sum(power(ln(xList), 2)))) - (sum(ln(xList)) ** 2)
    return num / denum

def b():
    return (sum(ln(yList)) - a()*sum(ln(xList)))/len(xList)

def expPoly():
    polyList = []
    for x in xList:
        polyList.append(math.exp(b()) * (x**a()))
    return polyList


import math

def expError():
    diff = 0
    sum = 0
    for i in range(len(xList)):
        diff += yList[i] - expPoly()[i]
        sum += diff ** 2
    return sum

expError()
```

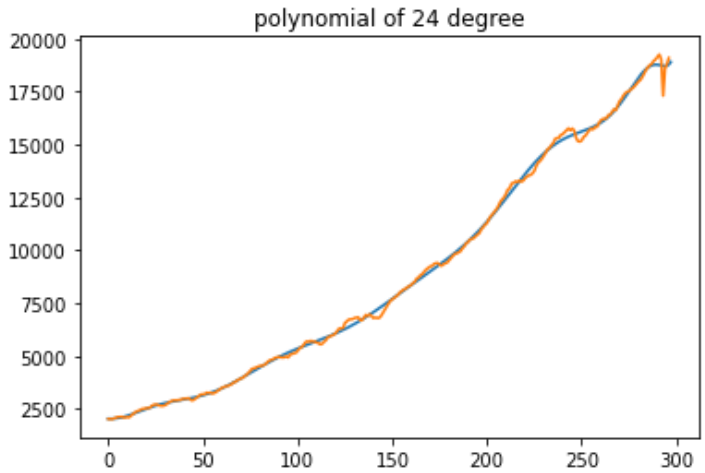
Out[9]:

```
1943223430676.449
```

In [10]:

```
print(plotData(xList, yList, 25))
```

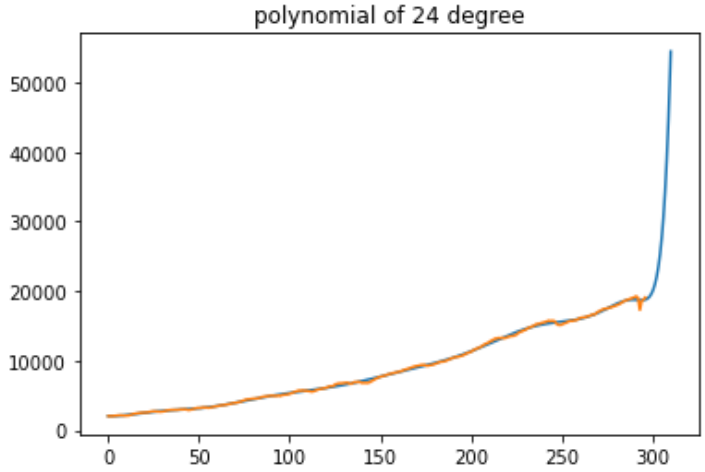
```
mean error: 118.40322385757311
total error: 9140292.902556535
None
```



In [11]:

```
print(plotDatas(xList, yList, 25, 0, 310))
```

```
mean error: 118.40322385757311
total error: 9140292.902556535
None
```

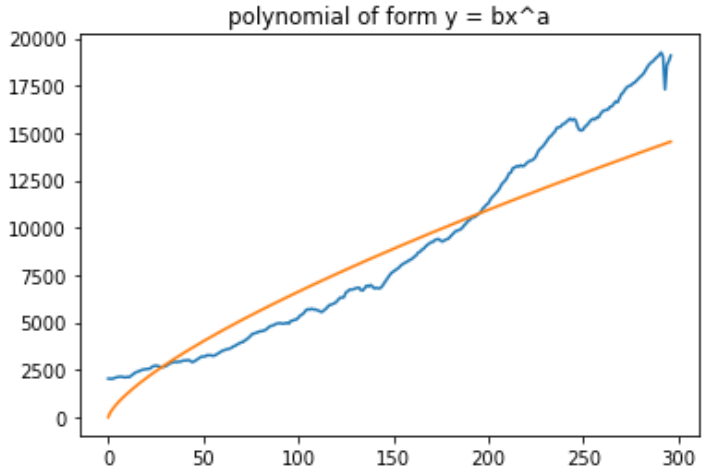


In [12]:

```
yPoly = expPoly()
plt.title("polynomial of form y = bx^a")
plt.plot(xList,yList)
plt.plot(xList,yPoly)
```

Out[12]:

```
[<matplotlib.lines.Line2D at 0x2e0723cefd0>]
```



In [13]:

```
def findLeastError(n):
    min = getError(0)
    index = 0
    for i in range(0,n+1):
        if(min > getError(i)):
            min = getError(i)
            index = i
    return index

print(findLeastError(50))
```

```
mean error: 4592.124969855684
total error: 8225196834.077089
mean error: 4592.124969855684
total error: 8225196834.077089

---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
<ipython-input-13-836ab869fcef> in <module>
      8     return index
```

```
      9
---> 10 print(findLeastError(50))

<ipython-input-13-836ab869fcef> in findLeastError(n)
      3     index = 0
      4     for i in range(0,n+1):
----> 5         if(min > getError(i)):
      6             min = getError(i)
      7             index = i

TypeError: '>' not supported between instances of 'NoneType' and 'NoneType'
```

In [ ]:

```
def logPoly(x,y):
    n=len(x)
    xi=sum(x)
    lnyi=sum([np.log(yj) for yj in y])
    xi2=sum([xj**2 for xj in x])
    lnyixi=[]
    for i in range(len(x)):
        lnyixi.append(x[i]*np.log(y[i]))
    lnyixi=sum(lnyixi)
    a=(n*lnyixi-xi*lnyi)/(n*xi2-xi**2)
    lnb=(xi2*lnyi-lnyixi*xi)/(n*xi2-xi**2)
    b=np.exp(lnb)
    y2=[b*np.exp(a*xj) for xj in x]
    plt.title("polynomial of form y = be^xa")
    plt.plot(x,y2)
    plt.plot(x,y)
logPoly(xList,yList)
```

In [ ]:

```
def logError(x, y):
    logList = []
    n=len(x)
    xi=sum(x)
    lnyi=sum([np.log(yj) for yj in y])
    xi2=sum([xj**2 for xj in x])
    lnyixi=[]
    for i in range(len(x)):
        lnyixi.append(x[i]*np.log(y[i]))
    lnyixi=sum(lnyixi)
    a=(n*lnyixi-xi*lnyi)/(n*xi2-xi**2)
    lnb=(xi2*lnyi-lnyixi*xi)/(n*xi2-xi**2)
    b=np.exp(lnb)
    for i in range(len(x)):
        logList.append((y[i] - b*np.exp(a*x[i])) ** 2)
    return sum(logList)

logError(xList, yList)
```

In [ ]:

```

```

In [ ]:

```

```