MA 114 Worksheet #11: Comparison and Limit Comparison Tests

1. (a) Explain the test for divergence. Why should you never use this test to prove that a series converges?

   (b) State the comparison test for series. Explain the idea behind this test.

   (c) Suppose that the sequences $\{x_n\}$ and $\{y_n\}$ satisfy $0 \leq x_n \leq y_n$ for all n and that $\sum_{n=1}^{\infty} y_n$ is convergent. What can you conclude? What can you conclude if instead $\sum_{n=1}^{\infty} y_n$ diverges?

   (d) State the limit comparison test. Explain how you apply this test.

2. Use the appropriate test — Divergence Test, Comparison Test or Limit Comparison Test — to determine whether the infinite series is convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt{n^2+2}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2+5^n}$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n+2}{3^n+1}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{10}{n}\right)^{10}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n+1}{3n^2+14n+7}$

   (h) $\displaystyle\sum_{n=0}^{\infty} \frac{1+2^n}{2+5^n}$

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+5n+2}$

   (j) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

   (k) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^2-\cos^2 n}$

   (l) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^4}$

   (m) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{(n+1)!}$

MathExcel Worksheet #11: Comparison and Limit Comparison Tests

1. For each of the following, determine which of the two is greater (for large n).

   (a) $\dfrac{1}{n}$, $\dfrac{1}{n^2}$

   (b) $\dfrac{1}{e^n}$, $\dfrac{1}{10^n}$

   (c) $\dfrac{1000}{n!}$, $\dfrac{2}{7^n}$

   (d) $\dfrac{1}{\ln(7n)}$, $\dfrac{1}{(12n)^2}$

   (e) $\dfrac{4}{\sqrt{n^5+8}}$, $\dfrac{1}{200n^2}$

   (f) $\dfrac{1}{10^n}$, $\dfrac{1}{n^{10}}$

2. (Review) For each of the following geometric series, determine the exact value of the sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} 5^{-n}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{3}{(-2)^n}$

   (c) $\displaystyle\sum_{n=-1}^{\infty} 0.75^n$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{2+4^n}{5^n}$