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Given an array of integers, find the highest product you can get from three of the integers.

The input arrayOfInts will always have at least three integers.

Gotchas

Does your function work with negative numbers? If array0fInts is [-10, -10, 1, 3, 2] we should return 300 (which we get by taking -10 * -10 * 3).

We can do this in O(n) time and O(1) space.

Breakdown

To brute force an answer we could iterate through array0fInts and multiply each integer by each other integer, and then multiply that product by each other other integer. This would probably involve nesting 3 loops. But that would be an $O(n^3)$ runtime! We can definitely do better than that.

Because any integer in the array could potentially be part of the greatest product of three integers, we must at least look at each integer. So we're doomed to spend at least O(n) time.

Sorting the array would let us grab the highest numbers quickly, so it might be a good first step. Sorting takes $O(n \lg n)$ time. That's better than the $O(n^3)$ time our brute force approach required, but we can still do better.

Since we know we must spend at least O(n) time, let's see if we can solve it in exactly O(n) time.

A great way to get O(n) runtime is to use a greedy approach. How can we keep track of the highestProduct0f3 "so far" as we do one walk through the array?

Put differently, for each new current number during our iteration, how do we know if it gives us a new highestProductOf3?

We have a new highestProductOf3 if the current number times two other numbers gives a product that's higher than our current highestProductOf3. What must we keep track of at each step so that we know if the current number times two other numbers gives us a new highestProductOf3?

Our first guess might be:

- 1. our current highestProductOf3
- 2. the threeNumbersWhichGiveHighestProduct

But consider this example:

```
int arrayOfInts[5] = {1, 10, -5, 1, -100};
```

Right before we hit -100 (so, in our second-to-last iteration), our highestProductOf3 was 10, and the threeNumbersWhichGiveHighestProduct were [10,1,1]. But once we hit -100, suddenly we can take -100*-5*10 to get 5000. So we should have "held on to" that -5, even though it wasn't one of the threeNumbersWhichGiveHighestProduct.

We need something a little smarter than threeNumbersWhichGiveHighestProduct. What should we keep track of to make sure we can handle a case like this?

There are at least two great answers:

- 1. **Keep track of the highest2 and lowest2 (most negative) numbers**. If the current number times some combination of those is higher than the current highestProductOf3, we have a new highestProductOf3!
- 2. **Keep track of the highestProductOf2 and lowestProductOf2** (could be a low negative number). If the current number times one of those is higher than the current highestProductOf3, we have a new highestProductOf3!

We'll go with (2). It ends up being slightly cleaner than (1), though they both work just fine.

How do we keep track of the highestProduct0f2 and lowestProduct0f2 at each iteration? (Hint: we may need to also keep track of something else.)

We also keep track of the lowest number and highest number. If the current number times the current highest—or the current lowest, if current is negative—is greater than the current highestProductOf2, we have a new highestProductOf2. Same for lowestProductOf2.

So at each iteration we're keeping track of and updating:

- highestProductOf3
- highestProductOf2
- highest
- lowestProductOf2
- lowest

Can you implement this in code? **Careful—make sure you update each of these variables in the right order**, otherwise you might end up e.g. multiplying the current number by itself to get a new highestProductOf2.

Solution

We use a greedy approach to solve the problem in one pass. At each iteration we keep track of:

- highestProductOf3
- highestProductOf2
- highest
- lowestProductOf2
- lowest

When we reach the end, the highestProductOf3 is our answer. We maintain the others because they're necessary for keeping the highestProductOf3 up to date as we walk through the array. At each iteration, the highestProductOf3 is the highest of:

- 1. the current highestProductOf3
- 2. current * highestProductOf2
- 3. current * lowestProduct0f2 (if current and lowestProduct0f2 are both low negative numbers, this product is a high positive number).

```
#define MAX(a, b) (((a) > (b)) ? (a) : (b))
#define MIN(a, b) (((a) < (b)) ? (a) : (b))
int highestProductOf3(const int *arrayOfInts, size_t length)
{
    size_t i;
    int highest, lowest;
    int highestProductOf2, lowestProductOf2, highestProductOf3;
    assert(length >= 3);
    // we're going to start at the 3rd item (at index 2)
    // so pre-populate highests and lowests based on the first 2 items.
    highest = MAX(arrayOfInts[0], arrayOfInts[1]);
    lowest = MIN(arrayOfInts[0], arrayOfInts[1]);
    highestProductOf2 = arrayOfInts[0] * arrayOfInts[1];
    lowestProductOf2 = arrayOfInts[0] * arrayOfInts[1];
    // except this one--we pre-populate it for the first *3* items.
    // this means in our first pass it'll check against itself, which is fine.
    highestProductOf3 = arrayOfInts[0] * arrayOfInts[1] * arrayOfInts[2];
    // walk through items, starting at index 2
    for (i = 2; i < length; i++) {
        int temp;
        int current = arrayOfInts[i];
        // do we have a new highest product of 3?
        // it's either the current highest,
        // or the current times the highest product of two
        // or the current times the lowest product of two
        temp = MAX(highestProductOf3, current * highestProductOf2);
        highestProductOf3 = MAX(temp, current * lowestProductOf2);
        // do we have a new highest product of two?
        temp = MAX(highestProductOf2, current * highest);
        highestProductOf2 = MAX(temp, current * lowest);
        // do we have a new lowest product of two?
```

```
temp = MIN(lowestProductOf2, current * highest);
lowestProductOf2 = MIN(temp, current * lowest);

// do we have a new highest?
highest = MAX(highest, current);

// do we have a new lowest?
lowest = MIN(lowest, current);
}
return highestProductOf3;
```

Complexity

O(n) time and O(1) additional space.

Bonus

- 1. What if we wanted the highest product of 4 items?
- 2. What if we wanted the highest product of k items?
- 3. If our highest product is really big, it could overflow. How should we protect against this?

What We Learned

Greedy algorithms in action again!

That's not a coincidence—to illustrate how one pattern can be used to break down several different questions, we're showing this one pattern in action on several different questions.

Usually it takes seeing an algorithmic idea from a few different angles for it to really make intuitive sense.

Our goal here is to teach you the right way of thinking to be able to break down problems you haven't seen before. Greedy algorithm design is a big part of that way of thinking.

For this one, we built up our greedy algorithm exactly the same way we did for the Apple stocks (/question/stock-price) question. By asking ourselves:

"Suppose we *could* come up with the answer in one pass through the input, by simply updating the 'best answer so far' as we went. What *additional values* would we need to keep updated as we looked at each item in our set, in order to be able to update the 'best answer so far' in constant time?"

For the Apple stocks question, the only "additional value" we needed was the min price so far.

For this one, we needed four things in order to calculate the new highestProductOf3 at each step:

- highestProductOf2
- highest
- lowestProductOf2
- lowest
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Next up: Product of All Other Numbers → (/question/product-of-other-numbers?course=fc1§ion=greedy)

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