

I'm making a search engine called MillionGazillion™.

I wrote a crawler that visits web pages, stores a few keywords in a database, and follows links to other web pages. I noticed that my crawler was wasting a lot of time visiting the same pages over and over, so I made a hash set, `visited`, where I'm storing URLs I've already visited. Now the crawler only visits a URL if it hasn't already been visited.

Thing is, the crawler is running on my old desktop computer in my parents' basement (where I totally don't live anymore), and it keeps running out of memory because `visited` is getting so huge.

How can I trim down the amount of space taken up by `visited`?

Gotchas

Your strategy shouldn't take a hit on runtime.

Replacing common substrings like `".com"` and `"www"` with characters that aren't allowed in URLs definitely wins us something, but we can do even better. How can we even further exploit overlaps or shared prefixes between URLs?

Breakdown

Notice that a boatload of URLs start with `"www."`

We could make `visited` a nested **dictionary** where the outer key is the subdomain and the inner key is the rest of the URL, so for example `visited["www."]["google.com"] = true` and `visited["www."]["interviewcake.com"] = true`. Now instead of storing the `"www."` for each of

these URLs, we've just stored it once in memory. If we have 1,000 URLs and half of them start with "www." then we've replaced $500 * 4$ characters with just 4 characters in memory.

But we can do even better.

What if we used this same approach of separating out shared prefixes recursively? How long should we make the prefixes?

What if we made the prefixes just one character?

Solution


We can use a trie. If you've never heard of a trie, think of it this way:

Let's make `visited` a nested dictionary where each map has keys of just one character. So we would store 'google.com' as `visited['g']['o']['o']['g']['l']['e']['.']['c']['o']['m']['*'] = true`.

The '*' at the end means 'this is the end of an entry'. Otherwise we wouldn't know what parts of `visited` are real URLs and which parts are just prefixes. In the example above, 'google.co' is a prefix that we might think is a visited URL if we didn't have some way to mark 'this is the end of an entry.'

Now when we go to add 'google.com/maps' to `visited`, we only have to add the characters '/maps', because the 'google.com' prefix is already there. Same with 'google.com/about/jobs'.

We can visualize this as a tree, where each character in a string corresponds to a node.



A trie is a type of tree.

To check if a string is in the trie, we just descend from the root of the tree to a leaf, checking for a node in the tree corresponding to each character in the string.



In our implementation, we chose to use nested dictionaries. To determine if a given site has been visited, we just call `AddWord()`, which adds a word to the trie if it's not already there.

```
using System.Collections.Generic;

public class TrieNode
{
    private Dictionary<char, TrieNode> _nodeChildren = new Dictionary<char, TrieNode>();

    public bool HasChildNode(char character)
    {
        return _nodeChildren.ContainsKey(character);
    }

    public void MakeChildNode(char character)
    {
        _nodeChildren[character] = new TrieNode();
    }

    public TrieNode GetChildNode(char character)
    {
        return _nodeChildren[character];
    }
}

public class Trie
{
    private const char EndOfWordMarker = '\0';

    private TrieNode _rootNode = new TrieNode();

    public bool AddWord(string word)
    {
        var currentNode = _rootNode;
        bool isNewWord = false;

        // Work downwards through the trie, adding nodes
        // as needed, and keeping track of whether we add
        // any nodes.
        foreach (var character in word)
        {
            if (!currentNode.HasChildNode(character))
            {
```

```

        isNewWord = true;
        currentNode.MakeChildNode(character);
    }

    currentNode = currentNode.GetChildNode(character);
}

// Explicitly mark the end of a word.
// Otherwise, we might say a word is
// present if it is a prefix of a different,
// longer word that was added earlier.
if (!currentNode.HasChildNode(EndOfWordMarker))
{
    isNewWord = true;
    currentNode.MakeChildNode(EndOfWordMarker);
}

return isNewWord;
}
}

```

If you used a **ternary search tree**, that's a great answer too. A **bloom filter** also works—specially if you use run-length encoding.

Complexity

How much space does this save? This is about to get MATHEMATICAL.

How many characters were we storing in our flat dictionary approach? Suppose visited includes *all possible URLs of length 5 or fewer characters*. Let's ignore non-alphabetical characters to simplify, sticking to the standard 26 English letters in lowercase. There are 26^5 different possible 5-character URLs (26 options for the first character, times 26 options for the 2nd character, etc), and of course 26^4 different possible 4-character URLs, etc. If we store each 5-character URL as a normal string in memory, we are storing 5 characters per string, for a total of $5 * 26^5$ characters for all possible 5-character strings (and $4 * 26^4$ total characters for all 4-character strings, etc). **So for all 1, 2, 3, 4, or 5 character URLs, our total number of characters stored is:**

$$5 * 26^5 + 4 * 26^4 + 3 * 26^3 + 2 * 26^2 + 1 * 26^1$$

So for all possible URLs of length n or fewer, our total storage space is:

$$n26^n + (n - 1)26^{(n-1)} + \dots + 1 * 26^1$$

This is $O(n26^n)$.

How many characters are stored in our trie? The first layer has 26 nodes (and thus 26 characters), one for each possible starting character. On the second layer, each of those 26 nodes has 26 children, for a total of 26^2 nodes. The fifth layer has 26^5 nodes. **To store all 1, 2, 3, 4, or 5 character URLs our trie will have 5 layers. So the total number of nodes is:**

$$26^5 + 26^4 + 26^3 + 26^2 + 26^1$$

So for all URLs of length n or fewer, we have:

$$26^n + 26^{(n-1)} + \dots + 26^1$$

This is $O(26^n)$. We've shaved off a factor of n .

Bonus trivia: although the HTTP spec allows for unlimited URL length, in practice many web browsers won't support URLs over 2,000 characters.

What We Learned

We ended up using a trie. Even if you've never heard of a trie before, you can reason your way to deriving one for this question. That's what we did: we started with a strategy for compressing a common prefix ("www") and then we asked ourselves, "How can we take this *idea* even further?" That gave us the idea to treat *each character* as a common prefix.

That strategy—starting with a small optimization and asking, "How can we take this same *idea* even further?"—is hugely powerful. It's one of the keys to unlocking complex algorithms and data structures for problems you've never seen before.

Ready for more?