Over the next two classes we will be covering probabilities, permutations and combinations.

Probabilities

Probability is just a number that expresses the likelihood that an event will occur.

A general question that we may want to ask is given a set of items what is the chance that a particular item will come up. More concretely, given a six-sided die what is the chance that you will roll a four? How about a two?

The answer is one out of six. The reason is that only one side can come up at a time that there are a total of six sides.

A formula for the probability of event A occurring would look something like:

P(A) = Number of ways event A can occur/The total number of possible outcomes

For our die example

Probability of getting a four would be

P(4) = Number of ways to roll a 4/Total number of sides = 1/6

At this point you can continue this for all the sides. Or compute them as a class exercise.

You can show the class the probability of a certain number of a range of numbers in Excel using the PROB function. In one column have the class insert the six values on the die and in another column place all the probabilities of each die. Using the PROB as follows

PROB(die range, probability range, value or range of values)

Another example to solidify this point is to use an object that has a different number of sides and ask the class to tell you the probabilities.

Let's take this the next step and ask ourselves what is the probability of rolling an even(odd) number.

Probability of rolling an even number would be

P(even) = Number of ways to roll an even number/Total number of sides = 3/6 = 1/2

At this point you can show the class that it is the same for rolling an odd number. Or as an exercise have the class compute them

You can see that the probability of any of the six sides coming up is the same 1/6. This can also be seen for the even(odd) probability.

It is said that these outcomes are equally likely to occur.

What happens when the outcomes are not equally likely to occur?

Pose the question to the class of if you have a jar with colored marbles in it and there are not an equal amount of each color then what is the probability of choosing one particular colored marble

Suppose that there are three different color marbles in the jar with a population defined as follows: 8 red, 6 white and 4 blue

The outcomes are as follows
P(red) = # of ways to choose red/total # of marbles = 8/18 = 4/9
P(white) = # of ways to choose white/total # of marbles = 6/18 = 1/3
P(blue) = # of ways to choose blue/total # of marbles = 4/18 = 2/9

As a class exercise choose a different set of numbers and have the class tell you what the probabilities for each one are.

Independent Probabilities

Now let's look at how we can compute the probabilities of independent events that occur in a sequence. Two events are said to be independent when the occurrence of the first event does not affect the probability of the second event to occur.

An example of this is rolling two die. Rolling a 4 on the first die has nothing to do with rolling a 5 on the second. These two events are independent. As a side note, rolling a 4 on the first die and rolling a value on the second die where the sum of the two is 9 are dependent or conditional which we'll discuss later.

To compute the probability of two independent events we first must compute the probability of each event occurring separately and then multiple the probabilities.

A formula for computing the probability of two independent events occurring would look something like this:

P(A and B) = P(A) * P(B)

NOTE: Remind the class that when you multiple two numbers that are less then 1 the results is smaller. Since all probabilities are equal to or less than 1 multiplying them together will result in a smaller number.

Going back to the die example above.
Now suppose that we have two dice and we want to know the probability of rolling a 4 on the first one and a 3 on the second.
We know from earlier that the probability of rolling a 4 is 1/6.
We also know that the probability of rolling a 3 is 1/6.

So following our formula from above the probability of rolling a 4 and a 3 is:
P(4 and 3) = P(4) * P(3) = 1/6 * 1/6 = 1/36

Select other examples and work as a class exercise.

We just saw what the probability of rolling a 4 on the first die and a 3 on the second is 1/36. The total of these two dice are 7. Maybe we want to now ask our self what is the probability of rolling a 7. We have found part of the answer but not all because there are more ways to roll a 7 then just rolling a 4 on the first die and a 3 on the second. Can we list other ways to roll a 7?

Die 1	Die 2
1	6
2	5
3	4
4	3
5	2
6	1

There are 6 ways to roll a 7 with two 6-sided die, each with a probability of 1/36. Therefore the probability of rolling a 7 following our formula from above is:

P(Rolling two die to equal 7) = P(1 and 6) * P(2 and 5) * P(3 and 4) * P(4 and 3) * P(5 and 2) * P(6 and 1) = 1/36 * 1/36 * 1/36 * 1/36 * 1/36 * 1/36 = 6/36 = 1/6

As a class exercise you can select another number and have the class compute the probability.

In Excel you can have the class list all the combination of two dice sum them up in a third column and determine the most common frequent outcome using the MODE function. You can also have them graph the sums, which will show the class the most common outcomes.

Now let's look at the jar of marbles example.
Suppose we want to know what the probability of selecting a red marble and white marble from our jar.
We know from earlier that the probability of selecting a red marble is 4/9.
We also know that the probability of selecting a white marble is 1/3.

So again following our formula from above the probability of selecting a red marble and a white marble is:
P(red and white) = P(red) * P(white) = 4/9 * 1/3 = 4/27

At this point you can have the class compute the probabilities of selecting a red and a blue, a white and a blue or a red, white and a blue as a class exercise.

Conditional or dependent probabilities

To continue this thread let's look at what happens when our next event are dependent or conditional on the previous event. Stated another way, two events are said to be dependent if the occurrence of the first affects the probability of the second event to occur.

Give the following concrete example to the class to help solidify the idea.
Start with a deck of playing cards. Select a card from the deck and without putting the card back in the deck select another card. Our question then is what is the probability that the first card was a 10 and the second was 5.

We know that there are 52 cards in the deck and since there are 4 of each card the probability of selecting a 10 is 4/52. Now since we didn't replace our first card there are now only 51 cards in the deck. Therefore the probability that our second card was a 5 is now 4/51. As you can see our first choice has affected the outcome of selecting our second card.

Our total probability is P(10 and 5) = 4/52 * 4/51 = 16/2652 = 4/633

Mathematically we would state this as:
P(A and B) = P(A) * P(A|B)
where P(A|B) is our conditional probability.
Note: Conditional probability of B in relationship to A is the probability that B occurs given that A has already occurred. We'll discuss this later.

Another example would be that we select three cards from our full deck and want to know that is the probability that we choose 3 7's. Following the same logic as above our formula would look like:

P(7 and 7 and 7) = 4/52 * 3/51 * 2/50 = 24/132600 = 1/5525

Continue this theme with the class by choosing other sets of cards and have them compute the probability.

Looking at the above equation we can solve for the conditional probability.
P(A|B) = P(A and B)/P(A)

As an example let's look at back at our marble example. Earlier we discussed the probability of selecting a red and white marble. Now let's ask what is the probability of selecting and white marble on the second draw given that we selected a red marble on our first draw.

P(white|red) = P(red and white)/P(red) = (4/27)/(4/9) = 0.33 = 33%

As an extended class exercise or homework compute the conditional probability of selecting a 7 from a deck of cards after you have selected a 5. You can assign the class several other combinations for homework.